Improved adaptive backstepping approach on STATCOM controller of nonlinear power systems

WANZHEN QUAN1,2, QINGYU SU1

1The School of Automation Engineering, Northeast Electric Power University, Jilin, 132012, P.R.China
2Rocket Force University of Engineering, Xi’an, 710025, P.R. China

Corresponding author: Qingyu Su (e-mail: suqingyu@neepu.edu.cn).

This work was supported by the National Natural Science Foundation of China (61503071, 61873057), Natural Science Foundation of Jilin Province (20180520211JH) and Education Department of Jilin Province (201693, JJKH20170106KJ).

ABSTRACT This paper proposes an improved backstepping control approach which uses error-compensation and the sliding mode variable structure control to achieve stability for a single machine infinite bus system (SMIBS) with a static synchronous compensator (STATCOM). An adaptive sliding mode control was also utilised to obtain strong response performance in order to avoid excitation and the unknown parameters generated by the recursive steps of traditional backstepping control. Increasing the error-compensation term was used to generate a new virtual control to guarantee that the system states are more rapid stable and eliminate system chattering effectively generated by traditional sliding mode control. The boundedness and convergence of the closed-loop system were then obtained based on Lyapunov stability theory. Finally, a simulation is demonstrated to illustrate the effectiveness and the practicality of the proposed control method.

INDEX TERMS Error compensation, adaptive sliding mode variable structure control, single machine infinite bus system, static synchronous compensator.

I. INTRODUCTION

It is well known that the stability of a single machine infinite bus system is becoming more significant and has been explored by many researchers, as demands for power generation continuously increases [1]. The abundant demand-side interactions in the smart grid would increase the stochastic fluctuation of end-users load demand in the power grid, which further increases the voltage variations in the system, as compared with the traditional power grid [2]. Moreover, since the customers may have different behavioral patterns, the ambiguity in their actions could make the current measurements of the network vary continuously with time and end-use consumption [3]. As the scale of the grid continues to expand, the control of the power system is becoming more and more difficult, the systems are being operated under high stress conditions such as unplanned power outages [4]. Furthermore, more effective and efficient control means are required for stability control. Nowadays flexible alternate current transmission system (FACTS) devices are highly-developed, including static synchronous compensators (STATCOMs), thyristor controlled series compensations (TCSCs), static var compensators (SVCs), etc. Such FACTS equipment combines powerful electronic technology and modern control technology to adjust the phase of power system voltage and power angle, greatly improves the transmission capacity of the transmission line, and reduces the level of transmission loss. The STATCOM is one of the most hopeful members of the FACTS family, which is capable of absorbing reactive power and enhancing the speed of transient response [5].

In the past several years, the algorithms of backstepping have received much attention in various fields for nonlinear systems with a lower triangular structure, and it has been extensively used in electric power systems [6]-[9]. The first attempt is that the authors in [10] apply the adaptive control algorithms to the problems for a single machine infinite bus system. In order to overcome the uncertainty of the power systems, an adaptive fuzzy logic control approach in [11] is proposed in generator excitation and thyristor controlled series compensation. In [12] and [13], the improved robust adaptive backstepping control is the main techniques, which is applied to ensure the global asymptotic stability for power
systems, and uniformly ultimate boundedness of tracking error. In [14], the control law was designed via an improved backstepping method for single-machine infinite-bus power systems with SMES. Furthermore, the control law is derived using variable structure control in [15] so as to the stability of the system, while the terminal voltage of the generator is within the safety boundary. Now, in order to influence the parameter uncertainty such as damping coefficient, people pay more and more attention to the extended adaptive backstepping methods. A controller based on the improved backstepping method of a single machine infinite bus power systems with uncertain parameters is investigated in [16].

Intelligent controllers such as fuzzy logic controllers as nonlinear control methods have been used in [17] in the control of STATCOM. For the multi-engine power system with boiler turbo generator set, the nonlinear adaptive controller of the power system is designed in [18], and the global control technology is used to improve the transient performance of the power system. Both parameter updating laws and nonlinear adaptive controllers for a single machine infinite bus system with damping coefficient uncertainties have been studied in [19] and [20]. In practice, [21]-[23] put forward a model-based adaptive control approach for satellites and hypersonic vehicles. This paper continues to study the development of adaptive back-stepping method [24]-[29], providing a new solution for the design of adaptive controller for a single machine infinite bus system with parametric uncertainties. Although algorithm proposed by [30] does not follow the deterministic equivalence principle for the design of adaptive control law, the chattering of the system is effectively eliminated. Inspired by the above viewpoint, this paper extends this approach to improve STATCOM controllers.

Due to [27] just considers the single-machine infinite system on generator steam valve of nonlinear power system, in this paper an improved backstepping method with error-compensation is proposed to solve the stabilisation problem of STATCOM. On the basis of [30] for chaotic system, the improved backstepping method with error-compensation is derived for the power system, which combines the advantages of error compensation and proposed sliding mode control. The contributions of this paper are summarised as follows:

- The STATCOM controller can improve the voltage stability and power supply quality by generating or absorbing reactive power to improve the power factor.

- Compared with the previous work in [9], a virtual controller is designed to increase error compensation, improve the speed of parameter identification, compensate the dynamic characteristics of unknown error, at the same time effectively eliminate the chattering brought by the traditional sliding mode variable structure control. Furthermore, the proposed control approach allows accurate asymptotic tracking and eventually reaches a stable value. Therefore, the improved method of this paper is also applicable to engineering applications.

- Motivated by the previous works for a uncertain chaotic system [30], the improved additional sliding mode variable structure control with error-compensation in STATCOM is proposed is proposed for external disturbances with unknown parameters and unknown mechanical input power.

The paper is organized as follows. In Section II an error-compensation problem under consideration and preliminaries is briefly introduced, and the traditional backstepping methods of the SMIBS with STATCOM in detail is given by section III. Section IV proposes an improved adaptive sliding mode variable structure control method according to error-compensation. In section V, the SMIBS with STATCOM is simulated, and the effectiveness of the improved method is verified. Finally, Several conclusions in the paper can be drawn in section VI.

II. PRELIMINARIES

A. THE DYNAMIC MODEL

The SMIB power system with STATCOM is shown in following figure 1. The model is introduced by [31], where the generator equation of state model can be written as follows (1).

\[
\begin{alignat}{2}
\delta &= \omega - \omega_0, \\
\dot{\omega} &= \frac{w_0}{H} [P_m - \frac{E'_q V_s \sin \delta}{X_{1} + X_{2}} + \frac{X_{1} X_{2} I_q}{\sqrt{(X_{2} E'_q)^2 + (X_{1} V_s)^2 + 2 X_{1} X_{2} V_s \cos \delta}}] - \frac{D}{\omega_0} (\omega - \omega_0)], \\
\dot{I}_q &= \frac{1}{T_q} (-I_q + I_{q0} + u),
\end{alignat}
\]

(1)

The geometric parameters and the testing data of the no load and the full load characteristics are also offered. The parameters of the generator are also offered as follows. In which \(P_m\) is the mechanical power, \(\omega\) is the angular velocity, \(\delta\) is the power angle, \(V_s\) is the infinite bus voltage for generator, about 1.0, \(D\) is the damping coefficient, \(H\) is the moment of inertia for the generator rotor, \(E'_q\) is the generator transient potential, \(I_q\) is the power systems reactive current, \(I_{q0}\) is the initial stable value \(I_q\), \(T_q\) is the STATCOM control system balance time constant, about 0.02, \(X_1 = X_{q} + X_{T} + X_{L1}\), about 0.84, \(X_2 = X_{L2}\), about 0.52, \(B_c\) and \(B_L\) are the...
susceptance of the capacitor and the inductor, \( u \) is the balance input of STATCOM, \((\delta_0, \omega_0, I_{q0})\) is the steady state operating point of the system.

Because the damping coefficient is difficult to measure accurately, we define the unknown parameter \( \theta = \frac{\partial f}{\partial \theta} \) to facilitate the design of the system, and define the state variables of the system (1) \( x_1 = \delta - \delta_0, x_2 = \omega - \omega_0, x_3 = I_q - I_{q0} \), then the dynamics can be chosen in the following equations:

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \theta x_2 + a_0 - k \sin(x_1 + \delta_0) [1 + f(x_1)(x_3 + I_{q0})], \\
\dot{x}_3 &= -\frac{1}{\tau_e} x_3 + \frac{1}{\tau_q} u,
\end{align*}
\]

where \( a_0 = \frac{\omega_0}{\tau_p} P_m, k = \frac{\omega_0 E'_V}{H} \), \( \theta = -D/H \).

**B. THE STATEMENT OF PROBLEM AND CONTROL OBJECTIVE**

In this subsection, we put forward the problem to be studied at first. At the same time, the nonlinear uncertain systems with strict parameter feedback are considered as follow:

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 + \Phi_1^T(x_1) \theta, \\
\dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)x_3 + \Phi_2^T(x_1, x_2) \theta, \\
&\vdots \\
\dot{x}_i &= f_i(x_1, \cdots, x_i) + g_i(x_1, \cdots, x_i)x_{i+1} \\
&\quad + \Phi_i^T(x_1, \cdots, x_i) \theta, \\
&\vdots \\
\dot{x}_n &= f_n(x_1, \cdots, x_n) + g_n(x_1, \cdots, x_n)x_{n+1} \\
&\quad + \Phi_n^T(x_1, \cdots, x_n) \theta
\end{align*}
\]

(3)

among them \( x(\in R^n) \) and \( u(\in R) \) are state variables and control input, respectively; smooth function \( f_i, g_i, i = 1, 2, \cdots \) is to satisfy the conditions \( f_i(0) = 0, g_i(x_1, \cdots, x_i) \neq 0; \theta(\in R^n, 1 < p \leq n) \) are the unknown parameter vectors, smooth vector \( \Phi_i(x_1, \cdots, x_n) \) meet the condition \( \Phi_i(0) = 0 \).

Our control goal is to construct a continuous STATCOM controller \( u \) to make the lower triangular structure (3) closed-loop error system globally asymptotically stable. In other words, we are looking at the design of a controller for a system (2) satisfying the lower triangular structure.

**C. THE BASIC DEFINITION AND LEMMA**

According to system (2), \( x_2^* \) and \( x_3^* \) are virtual control variables, and then the error variables showing in the following \( e_i, i = 1, 2, 3 \) can be derived.

\[
\begin{align*}
e_1 &= x_1, \\
e_2 &= x_2 - x_2^*, \\
e_3 &= x_3 - x_3^*
\end{align*}
\]

**Lemma 1:** (Barbalat’s lemma [32]) Considering a differentiable function \( h(t) \). If \( \lim_{t \to \infty} h(t) \) is finite and \( \dot{h}(t) \) is uniformly continuous, then \( \lim_{t \to \infty} h(t) = 0 \).

**D. THE TRADITIONAL CONTROLLER DESIGN**

The basic principle of backstepping scheme is divided into the following steps. Firstly, a suitable virtual error variable is defined. And then in each step, we should construct a virtual controls \( x_i^* \) and the control input \( u \), making the subsystem stable by using the appropriate Lyapunov function \( V_i \). Adaptive control law and update law are obtained, with respect to an over all Lyapunov function \( V \) is analyzed.

**Step 1:** Firstly, we select a virtual virtual control variables function:

\[
x_2^* = -k_1 e_1,
\]

where \( k_1 > 0 \). Then we derived further according to the definition, \( \dot{e}_1 = \dot{x}_1 = x_2 \).

Choosing Lyapunov function of system (2)

\[
V_1 = \frac{1}{2} e_1^2,
\]

furthermore, the derivative curve of \( V_1 \) is

\[
\dot{V}_1 = e_1 \dot{e}_1 = e_1 x_2 = -k_1 e_1^2 + e_1 e_2.
\]

It is clearly that when \( e_2 = 0 \), there is \( \dot{V}_1 < 0 \) which is satisfied the stability criteria.

**Step 2:** Choosing the following Lyapunov function:

\[
V_2 = V_1 + \frac{1}{2} e_2^2,
\]

it is possible to derive the following expression for \( \dot{e}_2 \)

\[
\dot{e}_2 = \dot{x}_2 - \dot{x}_2^* = \theta x_2 + a_0 - k \sin(x_1 + \delta_0) [1 + f(x_1)(x_3 + I_{q0})] + k_1 x_2.
\]

The derivative of \( V_2 \) is

\[
\begin{align*}
\dot{V}_2 &= \dot{V}_1 + e_2 \dot{e}_2 \\
&= -k_1 e_1^2 + e_2 \{e_1 + \theta x_2 + a_0 - k \sin(x_1 + \delta_0) [1 + f(x_1)(x_3 + I_{q0})] + k_1 x_2 \}. \\
&= -k_1 e_1^2 + e_2 \{k_2 e_2 + \theta x_2 + a_0 - k \sin(x_1 + \delta_0) [1 + f(x_1)(x_3 + I_{q0})] + k_1 x_2 \}.
\end{align*}
\]

Meanwhile, according to the definition \( e_3 = x_3 - x_3^* \), choosing virtual stabilization function:

\[
x_3^* = \frac{e_1 + \hat{\theta} x_2 + a_0 + k_1 x_2 + k_2 e_2 k \sin(x_1 + \delta_0) f(x_1)}{1 - \frac{1}{f(x_1)} - I_{q0}},
\]

(8)

among them \( k_2 \) is a constant, \( \hat{\theta} \) is an estimate of \( \theta \), and \( \tilde{\theta} \) is an estimated error which meets \( \theta = \tilde{\theta} + \theta \), hence

\[
\dot{V}_2 = -k_1 e_1^2 + k_2 e_2 e_3 \sin(x_1 + \delta_0) f(x_1) - k_2 e_2^2 + \hat{\theta} e_2 x_2.
\]

**Step 3 :** According to system (2), we make the overall Lyapunov function

\[
V = V_2 + \frac{1}{2} e_1^2 + \frac{1}{2} \hat{\theta}^2,
\]

(9)
where the parameter \( \rho > 0 \) is a given adaptive gain, and
\[
\dot{e}_3 = \dot{x}_3 - \hat{x}_3^2
\]
\[
= -\frac{1}{T_q} x_3 + \frac{1}{T_q} u - \frac{(1 + \hat{\theta} + k_1 k_2) x_2}{k \sin(x_1 + \delta_0) f(x_1)}
\]
\[
+ \frac{(e_1 + \hat{\theta} x_2 + a_0 + k_1 x_2 + k_2 e_2) \hat{f}(x_1) \sin(x_1 + \delta_0)}{k \hat{f}(x_1) \sin(x_1 + \delta_0)}
\]
\[
+ \frac{f(x_1) \cos(x_1 + \delta_0) x_2}{k \hat{f}(x_1) \sin(x_1 + \delta_0)} - \frac{\hat{f}(x_1) x_2}{\hat{f}^2(x_1)}.
\]

The derivative of \( V \) is:
\[
\dot{V} = \dot{V}_2 + e_3 \dot{e}_3 + \frac{1}{\rho} \dot{\theta} \hat{\theta}
\]
\[
= -k_1 \dot{e}_1^2 - k_2 \dot{e}_2^2 + \hat{\theta} x_2 e_3 - e_3 \dot{\theta} e_2 - e_3 \dot{e}_3 - \frac{1}{\rho} \dot{\theta} \hat{\theta},
\]

since \( \theta = \hat{\theta} + \hat{\theta} \).

\[
\dot{V} = -k_1 \dot{e}_1^2 - k_2 \dot{e}_2^2 + \hat{\theta} x_2 e_3 - e_3 \dot{\theta} e_2 - k_1 \dot{e}_1^2 - k_2 \dot{e}_2^2 + \hat{\theta} x_2 e_3 - e_3 \dot{\theta} e_2 - e_3 \dot{e}_3 - \frac{1}{\rho} \dot{\theta} \hat{\theta},
\]

(i) The controller \( u \) of this system was deduced through backstepping adaptive approach as follows:
\[
\dot{u} = \frac{T_q}{k q} \left( k e_2 \sin(x_1 + \delta_0) f(x_1) + \frac{1}{T_q} x_3 + \frac{(1 + \hat{\theta} + k_1 k_2) x_2}{k \sin(x_1 + \delta_0) f(x_1)}
\]
\[
+ \frac{(e_1 + \hat{\theta} x_2 + a_0 + k_1 x_2 + k_2 e_2) \hat{f}(x_1) \sin(x_1 + \delta_0)}{k \hat{f}(x_1) \sin(x_1 + \delta_0)}
\]
\[
+ \frac{f(x_1) \cos(x_1 + \delta_0) x_2}{k \hat{f}(x_1) \sin(x_1 + \delta_0)} - \frac{\hat{f}(x_1) x_2}{\hat{f}^2(x_1)} \right)
\]

(ii) Because of the effect of the controller (13), the closed-loop system in the new coordinate system \((e_1, e_2, e_3)\) is
\[
\dot{e}_1 = -k_1 e_1 + e_2,
\]
\[
\dot{e}_2 = -e_1 - k_2 e_2 - k e_3 \sin(x_1 + \delta_0) f(x_1) + \hat{\theta} x_2,
\]
\[
\dot{e}_3 = k \sin(x_1 + \delta_0) f(x_1) e_2 - \frac{(\hat{\theta} + k_1 + k_2) \hat{f}(x_1) \sin(x_1 + \delta_0) \hat{f}(x_1)}{k \sin(x_1 + \delta_0) f(x_1)} \hat{\theta} x_2 - k_3 e_3.
\]

(iii) Under the effect of the controller (13), the closed-loop tracking error system (14) is regulated to zero asymptotically.

**Proof:** Firstly, we know \( V(t) < V(0) \) according to (12), that is to say \( e_1, e_2, x_1, x_2 \) are bounded, then we choose \( \Omega = -\dot{V}, \int_0^t \Omega(\tau) d\tau = V(0) - V(t) \), because \( V(0) \) is bounded, \( V(t) \) is also diminish and bounded, and that is \( \lim_{t \to \infty} \int_0^t \Omega(\tau) d\tau < \infty \), that is to say \( \Omega \) is bounded, so \( \lim_{t \to \infty} \Omega = 0 \) obtained by the Barbalat Lemma 1. If \( t \to \infty \), there are \( e_1 \to 0, e_2 \to 0, x_1 \to 0, x_2 \to 0 \). According to the relationships of \( x_1, x_2, x_3 \) and \( x_2^*, x_3^* \), it can be seen that \( e_3 \to 0, x_3 \) is also bounded.

**III. THE IMPROVED CONTROLLER DESIGN**

**A. THE IMPROVED ADAPTIVE BACKSTEPPING SLIDING MODE VARIABLE STRUCTURE CONTROLLER**

The first two steps are the same as the backstepping adaptive sliding mode approach in chapter III. Here we will introduce improved method from the third step.

**Step 3:** The sliding mode surface \( s = d_1 e_1 + d_2 e_2 + e_3 = 0 \) is selected, where \( d_1, d_2 \) are constants, respectively. The overall Lyapunov function consists of
\[
\dot{V} = V_2 + \frac{1}{2} s^2 + \frac{1}{2p} \dot{\theta}^2,
\]

since \( e_3 = s - d_1 e_1 - d_2 e_2, \theta = \hat{\theta} + \hat{\theta} \) the derivative of \( V \) is:
\[
\dot{V} = \dot{V}_2 + s \dot{s} + \frac{1}{\rho} \dot{\theta} \hat{\theta}
\]
\[
= -k_1 \dot{e}_1^2 - k_2 \dot{e}_2^2 + k_1 e_3 \sin(x_1 + \delta_0) f(x_1) - k_3 e_2^2 + \hat{\theta} e_2 x_2
\]
\[
+ s(d_1 \dot{e}_1 + d_2 \dot{e}_2 + \dot{e}_3) - \frac{1}{\rho} \dot{\theta} \hat{\theta}.
\]
While \( \dot{e}_1 = \dot{x}_1, \dot{e}_2 = \dot{x}_2 + k_1 x_2 \) and
\[
\dot{e}_3 = -\frac{1}{T_q} x_3 + \frac{1}{T_q} u - \frac{(1 + \theta + k_1 k_2) x_2}{k \sin(x_1 + \delta_0) f(x_1)}
\]
\[
\frac{(\theta + k_1 + k_2) [\theta x_2 + a_0 - k \sin(x_1 + \delta_0)] f(x_1)(x_3 + I_{q0})}{k \sin(x_1 + \delta_0) f(x_1)}
\]
\[
\frac{(\theta + k_1 + k_2) [\theta x_2 + a_0 - k \sin(x_1 + \delta_0)] f(x_1)(x_3 + I_{q0})}{k \sin(x_1 + \delta_0) f(x_1)}
\]
\[
+ \frac{\theta x_2 + a_0 + k_1 x_2 + k_2 e_2}{k \sin(x_1 + \delta_0) f(x_1)}
\]
\[
+ \frac{\dot{f}(x_1) x_2}{f^2(x_1)}.
\]
Therefore, it yields that
\[
\dot{V} = -[k_1 - \frac{k^2 d_1^2 \sin^2(x_1 + \delta_0) f^2(x_1)}{4}] e_1^2 - [k_2 - 1 - 2 k_1 - 1 - k \sin(x_1 + \delta_0)]
\]
\[
+ e_2^2 + e_2 \theta x_2 + s \delta_2 \theta x_2 - \frac{(1 + \theta + k_1 k_2) \theta x_2}{k \sin(x_1 + \delta_0)} + \frac{1}{\rho} \theta \theta
\]
\[
+ s \left(-k e_2 \sin(x_1 + \delta_0) f(x_1) + d_1 x_2 + d_2 k_1 x_2
\right)
\]
\[
- \frac{1}{T_q} x_3 + \frac{1}{T_q} u - \frac{(1 + \theta + k_1 k_2) x_2}{k \sin(x_1 + \delta_0) f(x_1)}
\]
\[
- \frac{(\theta + k_1 + k_2 - d_2 k \sin(x_1 + \delta_0) f(x_1))}{k \sin(x_1 + \delta_0)}
\]
\[
\cdot \theta x_2 + a_0 - k \sin(x_1 + \delta_0) [1 + f(x_1)(x_3 + I_{q0})]
\]
\[
+ (e_1 + \theta x_2 + a_0 + k_1 x_2 + k_2 e_2)
\]
\[
\cdot \left(\frac{\dot{f}(x_1) \sin(x_1 + \delta_0) + f(x_1) \cos(x_1 + \delta_0) \theta x_2}{k \sin(x_1 + \delta_0) f(x_1)} + \frac{\dot{f}(x_1) x_2}{f^2(x_1)} - \beta s\right).
\]
By choosing the parameter update law
\[
\dot{\theta} = \rho [e_2 + s \delta_2 - \frac{s \theta + k_1 + k_2}{k \sin(x_1 + \delta_0) f(x_1)}] x_2.
\]
Then
\[
\dot{V} = -\frac{1}{4} \left(k_1 - \frac{k^2 d_1^2 \sin^2(x_1 + \delta_0) f^2(x_1)}{4} \right) e_1^2
\]
\[
- \frac{k_2 - 1 - k \sin(x_1 + \delta_0) f(x_1) d_2}{2} e_2^2
\]
\[
- \frac{k d_1 \sin(x_1 + \delta_0) f(x_1)}{2} e_1 + e_2^2
\]
\[
(17)
\]
\[\text{Step 4: It is easy to see } \dot{V} < 0 \text{ by } (17). \text{ Moreover, in Section II basing on the problem statement and control objective, we can use the following theorem to get a backstepping adaptive controller } u, \text{ as shown below.}
\]
\[\text{Theorem 2: Considering the model of the static synchronous compensator system } (1) \text{ under equation } (2), \text{ we select good parameters } c_1, d_i (i = 1, 2), \text{ which satisfy the conclusion } k_1 - \frac{k^2 d_1^2 \sin^2(x_1 + \delta_0) f^2(x_1)}{4} > 0, k_2 - 1 - k \sin^2(x_1 + \delta_0) f(x_1) d_2 > 1, \text{ the parameter } \beta (> 0) \text{ is the sliding mode gain factor, and the other should be correctly selected according to actual control requirements.}
\]
\[\text{(i) The controller } u \text{ was deduced through backstepping adaptive sliding mode variable structure control of the non-linear system:}
\]
\[
u = T_q \left(k e_2 \sin(x_1 + \delta_0) f(x_1) - d_1 x_2 - k_1 d_2 x_2 + \frac{1}{T_q} x_3
\]
\[
+ \frac{(1 + \frac{\hat{\theta}}{\beta}) + k_1 k_2 x_2}{k \sin(x_1 + \delta_0) f(x_1)} - \frac{\hat{\theta} + k_1 + k_2 - d_2 k \sin(x_1 + \delta_0)}{k \sin(x_1 + \delta_0)} f(x_1)
\]
\[
\cdot \left[\hat{\theta} x_2 + a_0 - k \sin(x_1 + \delta_0) [1 + f(x_1)(x_3 + I_{q0})]ight]
\]
\[
+ \frac{\dot{f}(x_1) \sin(x_1 + \delta_0) + f(x_1) \cos(x_1 + \delta_0) x_2}{k \sin(x_1 + \delta_0)}
\]
\[
\cdot \left(\frac{\dot{f}(x_1) x_2}{f^2(x_1)} - \beta s\right).
\]
\[
(18)
\]
\[(\text{ii) Because of the effect of the controller } (18), \text{ the closed-loop system in the new coordinate system for } (e_1, e_2, e_3) \text{ are}
\]
\[
\dot{e}_1 = -k_1 e_1 + e_2,
\]
\[
\dot{e}_2 = -e_1 - k_2 e_2 - k e_3 \sin(x_1 + \delta_0) f(x_1) + \hat{\theta} x_2,
\]
\[
\dot{e}_3 = k \sin(x_1 + \delta_0) (e_2 + d_2 e_3) - d_1 x_2 + d_2 e_1
\]
\[
+k_2 d_2 e_2 - k \sin(x_1 + \delta_0) f(x_1) \hat{\theta} x_2 - \beta s.
\]
\[
(19)
\]
\[\text{(iii) Under the effects of the controller } (18), \text{ the closed-loop tracking error system } (19) \text{ is regulated to zero asymptotically.}
\]
\[\text{Proof: We choose } V(t) < V(0) \text{ according to } (17), \text{ that is to say } e_1, e_2, s, x_1, x_2 \text{ are bounded, then we choose } \Omega = -V,
\]
\[
\int_0^t \Omega(\tau) d\tau = V(0) - V(t). \text{ The Barbalat lemma 1 derived the result; the proof process is also analogous to the proof of Theorem 1. Due to the restriction of space, we leave out the proof of Theorem 2.}
\]
\[\text{Remark 1: Compared with the adaptive backstepping design, the sliding mode gain coefficient introduced by the adaptive backstep sliding mode variable structure controller effectively compensate the unknown error for the dynamic effect of the system.}
\]
\[\text{B. THE IMPROVED CONTROLLER DESIGN OF ADAPTIVE BACKSTEPPING SLIDING MODE VARIABLE STRUCTURE CONTROL BASED ON ERROR COMPENSATION}
\]
\[\text{Step 1: Firstly, an error-compensation and virtual stabilization function are introduced:}
\]
\[
x^*_1 = -k_1 e_1 - p_1 e_2,
\]
where \( k_1 \) is a positive constant, \( p_1 e_2 \) is an error compensation used to compensate for the dynamic effects of unknown errors on system stability. Furthermore, yielding an equation,
\[
\dot{e}_1 = -k_1 e_1 + (1 - p_1)e_2. \quad (20)
\]

Choosing Lyapunov function for the system (2)
\[
V_1 = \frac{1}{2} e_1^2, 
\]
therefore, the derivative curve of \( V_1 \) is
\[
\dot{V}_1 = e_1 \dot{e}_1 = -k_1 e_1^2 + (1 - p_1)e_1 e_2. \quad (21)
\]

It can be seen that when \( e_2 = 0 \), \( \dot{V}_1 < 0 \) satisfies the stability criterion.

**Step 2**: The Lyapunov function is defined below:
\[
V_2 = V_1 + \frac{1}{2} e_2^2, 
\]

meanwhile,
\[
\dot{e}_2 = \ddot{x}_2 - \dot{x}_2^* \\
= \theta x_2 + a_0 - k \sin(x_1 + \delta_0)\left(1 + f(x_1)(x_3 + I_{q_0})\right) \\
+ k_1 x_2 + p_1 \dot{e}_2, 
\]

and further result
\[
\dot{e}_2 = \frac{1}{1-p_1}\{\theta x_2 + a_0 - k \sin(x_1 + \delta_0)[1 + f(x_1)(x_3 + I_{q_0})] + k_1 x_2\}. 
\]

The derivative curve of \( V_2 \) is
\[
\dot{V}_2 = \dot{V}_1 + e_2 \dot{e}_2 \\
= -k_1 e_1^2 + (1 - p_1)e_1 e_2 + e_2 \left(\frac{1}{1-p_1}\{\theta x_2 + a_0 - k \sin(x_1 + \delta_0)[1 + f(x_1)(x_3 + I_{q_0})] + k_1 x_2\}\right) \\
= -k_1 e_1^2 - \frac{k \sin(x_1 + \delta_0)f(x_1)}{1-p_1} e_2 \dot{e}_3 + \frac{e_2}{1-p_1}\{(1 - p_1)^2 e_1 + \theta x_2 + a_0 - k \sin(x_1 + \delta_0)[1 + f(x_1)(x_3 + I_{q_0})] + k_1 x_2\}. 
\]

According to Definition \( e_3 = x_3 - x_3^* \), we can select virtual stabilization function:
\[
x_3^* = \frac{(1-p_1)^2 e_1 + \theta x_2 + a_0 + k_1 x_2 + k_2 e_2}{k \sin(x_1 + \delta_0)f(x_1)} \\
- \frac{1}{f(x_1)} - I_{q_0} - p_2 e_3, 
\]

where \( k_2, p_2 \) are constants, \( \hat{\theta} \) is the estimate of \( \theta \), \( \hat{\theta} \) is estimated error and \( \theta = \hat{\theta} - \hat{\theta} \), so
\[
\dot{V}_2 = -k_1 e_1^2 - \frac{k_2 e_2^2}{1-p_1} + \frac{e_2}{1-p_1}\{\theta x_2 \}
- k \sin(x_1 + \delta_0)f(x_1)(1-p_2)e_3. 
\]

**Step 3**: The sliding mode surface \( s = d_1 e_1 + d_2 e_2 + e_3 = 0 \) is selected, where \( d_1, d_2 \) are constants, respectively. The overall Lyapunov function consists of
\[
V = V_2 + \frac{1}{2} s^2 + \frac{1}{2\rho}\hat{\theta}^2, 
\]

since \( \dot{s} = d_1 \dot{e}_1 + d_2 \dot{e}_2 + \dot{e}_3 \) and \( \theta = \hat{\theta} + \hat{\theta} \), the derivative of \( V \) is:
\[
\dot{V} = \dot{V}_2 + s \dot{s} + \frac{1}{\rho}\hat{\theta} \ddot{\hat{\theta}} \\
= -k_1 e_1^2 - \frac{k_2 e_2^2}{1-p_1} + \frac{e_2}{1-p_1}\{\theta x_2 - k \sin(x_1 + \delta_0)(1 - p_2)e_3\} + s(d_1 \dot{e}_1 + d_2 \dot{e}_2 + \dot{e}_3) - \frac{1}{\rho}\hat{\theta} \ddot{\hat{\theta}}. 
\]

While \( \dot{e}_1 = \ddot{x}_1, \dot{e}_2 = \ddot{x}_2 + k_1 x_2 + p_2 e_3, \) and
\[
\dot{e}_3 = -\frac{1}{T_q} x_3 - \frac{1}{T_q} u - \frac{[(1-p_1)^2 + \theta + \frac{k_1 k_2}{1-p_1}]x_2}{k \sin(x_1 + \delta_0)f(x_1)} \\
\cdot \{\theta x_2 + a_0 - k \sin(x_1 + \delta_0)[1 + f(x_1)(x_3 + I_{q_0})]\} \\
+ \frac{[(1-p_1)^2 e_1 + \theta x_2 + a_0 + k_1 x_2 + k_2 e_2\]}{k \sin(x_1 + \delta_0)^2} \\
\cdot \{f(x_1) \sin(x_1 + \delta_0) + f(x_1) \cos(x_1 + \delta_0)]x_2 + p_2 e_3\}, 
\]

that is
\[
\dot{e}_3 = \frac{1}{1-p_1}\{\frac{1}{T_q} x_3 - \frac{1}{T_q} u - \frac{[(1-p_1)^2 + \theta + \frac{k_1 k_2}{1-p_1}]x_2}{k \sin(x_1 + \delta_0)f(x_1)} \\
- \frac{\{\theta x_2 + a_0 - k \sin(x_1 + \delta_0)[1 + f(x_1)(x_3 + I_{q_0})]\}}{k \sin(x_1 + \delta_0)^2} \\
+ \frac{[(1-p_1)^2 e_1 + \theta x_2 + a_0 + k_1 x_2 + k_2 e_2\]}{k \sin(x_1 + \delta_0)^2} \\
\cdot \{f(x_1) \sin(x_1 + \delta_0) + f(x_1) \cos(x_1 + \delta_0)]x_2 + p_2 e_3\}. 
\]

Therefore, it yields that
\[
\dot{V} = \left[-k_1 - \frac{k^2 \sin^2(x_1 + \delta_0) f^2(x_1) d_1^2}{4(1-p_1)^2}\right] e_1^2 \\
- \left[\frac{k \sin(x_1 + \delta_0)f(x_1)(1-p_2)d_1 e_1}{1-p_1}\right] + e_2^2 - \left[\frac{k_2}{1-p_1}\right] \\
\cdot \frac{k \sin(x_1 + \delta_0)f(x_1)(1-p_2)d_2}{1-p_1} - e_2^2 + \frac{e_2}{1-p_1}\theta x_2 \}
+ s \left[\frac{k \sin(x_1 + \delta_0)f(x_1)}{1-p_1}\right] e_2 + d_1 x_2 + \frac{d_2 k_1 x_2}{1-p_1} \\
+ \frac{1}{1-p_1}\{\frac{1}{T_q} x_3 - \frac{1}{T_q} u - \frac{[(1-p_1)^2 + \theta + \frac{k_1 k_2}{1-p_1}]x_2}{k \sin(x_1 + \delta_0)f(x_1)} \\
- \frac{\{\theta x_2 + a_0 - k \sin(x_1 + \delta_0)[1 + f(x_1)(x_3 + I_{q_0})]\}}{k \sin(x_1 + \delta_0)^2} \\
+ \frac{[(1-p_1)^2 e_1 + \theta x_2 + a_0 + k_1 x_2 + k_2 e_2\]}{k \sin(x_1 + \delta_0)^2} \\
\cdot \{f(x_1) \sin(x_1 + \delta_0) + f(x_1) \cos(x_1 + \delta_0)]x_2 + p_2 e_3\}. 
\]
While we can get the parameter update law is that
\[
\dot{\hat{\theta}} = \rho \frac{e_2}{1-p_1} + \frac{sd_2}{1-p_1} - \frac{s(\hat{\theta} + k_1 + \frac{k_2}{1-p_1})}{(1-p_2)k\sin(x_1 + \delta_0)f(x_1)}x_2. \tag{23}
\]

**Remark 2:** In (23), the adaptive law \(\dot{\hat{\theta}}\) different from that in (11) and (16). The advantages of the improved adaptive law are listed as follows. On the basis of (11), a sliding surface (11) and (16). The advantages of the improved adaptive law

\[
\dot{\hat{\theta}} = \rho \frac{e_2}{1-p_1} + \frac{sd_2}{1-p_1} - \frac{s(\hat{\theta} + k_1 + \frac{k_2}{1-p_1})}{(1-p_2)k\sin(x_1 + \delta_0)f(x_1)}x_2.
\]

**Step 4:** It is easy to see \(\dot{V} < 0\) by (24). Moreover, based on the problem statement and the controller objective in Section II, we can use the following theorem to get a backstepping adaptive controller \(u\), as shown below.

**Theorem 3:** Considering the model of the static synchronous compensator system (1) under equation (2), we select good parameters \(e_i, d_i, p_i, i = 1, 2\), which satisfy the conduction \(k_1 - \frac{k^2\sin^2(x_1 + \delta_0)f^2(x_1)d_1^2}{4(1-p_1)^2} > 0\), \(\frac{k_2}{1-p_1} - \frac{k\sin(x_1 + \delta_0)f(x_1)(1-p_2)d_2}{4(1-p_1)^2} > 0\), \(\beta > 0\) is a sliding mode gain coefficient and the parameter \(\beta > 0\) is the sliding mode gain factor, and the other should be correctly selected according to actual control requirements.

(i) The controller \(u\) was deduced through backstepping adaptive sliding mode variable structure control for the nonlinear systems:
\[
u = T_q \left( \frac{k\sin(x_1 + \delta_0)f(x_1)}{1-p_1}e_2 - d_1x_2 - \frac{d_2}{1-p_1}k_1x_2 \right) \tag{25}
\]

(ii) Because of the effect of the controller (25), the closed-loop system in the new coordinate system for \((e_1, e_2, e_3)\) are
\[
\begin{align*}
e_1 &= -k_1e_1 + (1-p_1)e_2, \\
e_2 &= -(1-p_1)e_1 - \frac{k_2}{1-p_1}k_1e_2 + (1-p_2)k\sin(x_1 + \delta_0)f(x_1)e_3 - \beta x_2, \\
e_3 &= \frac{1-p_2}{1-p_1}k\sin(x_1 + \delta_0)f(x_1)(e_2 + d_2 e_3) - d_1 x_2 + d_2(1-p_1)e_1 + \frac{k_2d_1}{1-p_1}e_2 \tag{26}
\end{align*}
\]

(iii) Because of the effect of the controller equation (25), the closed-loop tracking error system (26) is regulated to zero asymptotically.

**Proof:** We choose \(V(t) < V(0)\) according to (24), that is to say, \(e_1, e_2, s, x_1, x_2\) are bounded, then we choose \(\Omega = -V, \int_{0}^{t} \Omega(\tau) d\tau = V(0) - V(t)\). The Barbalat lemma 1 derived the result; the proof process is also analogous to the proof of Theorem 1. Due to the restriction of space, we leave out the proof of Theorem 3.

**Remark 3:** As can be seen from the previous discussion, the adaptive backstepping controller proposed in this section is better than others. The emphasis is on introducing error compensation in the first step. On this basis, the virtual control, error variables and error compensation are combined to extend the Lyapunov function, improving the stability and performance of the power system, and it is more likely to enter a steady state in a short period of time and quickly eliminate the error.

**IV. SIMULATION**

To validate the improved controller derived in the previous section, several simulations for a single machine infinite system have been achieved. Some parameters of systems are selected from [24]: \(\rho = 1, V = 1, T = 0.2, H = 10, E = 1.08; \Delta H = 0.3; X = 0.9; w_0 = 314; P M_0 = 0.8; p_1 = 0.471; p_2 = 0.778; k_1 = 30; k_2 = 10\); there are \(x_1 = \delta - \delta_0, x_2 = \omega - \omega_0, x_3 = I_q - I_q 0\) as well.

![FIGURE 2: The simulation system structure of ECABSMVSC](image)
1) Parameters $p_1$ and $p_2$ are error-compensations, which are one or near to one in a general sense.

2) The parameters $d_1$ and $d_2$, varying from zero to one, are obtained by constructing a sliding surface.

3) The parameters $k_1$ and $k_2$ guarantee the stability of the system (2) by using the virtual controls $x_2^*$ and $x_3^*$, it only needs $k_1 > 0$, $k_2 > 0$. However, in order to ensure the derivative of the Lyapunov function (24) $< 0$, parameter $k_1$ should satisfy $k_1 - \frac{k^2 \sin^2(x_1 + \delta_0) f_x(x_1) d_2^2}{(1 - p_1)^2} > 0$, and $k_2$ should be chosen to satisfy $k_2 > 0$.

4) $\rho > 0$ is chosen to adjust the adaptive parameter according to literature[19]. Parameter $\beta > 0$ is a constant sliding mode gain.

In Figure 2, the simulation system structure basing on error-compensation adaptive backstepping sliding mode variable structure controller (ECABSMVSC) is shown. As shown in Figure 3, $\delta$ gradually tends to $\delta_0(57.3)$, which is the power angle of stable working point, namely the state variable $x_1$ gradually return at the point 0. It’s worth noting that, the proposed method of basing on ECABSMVSC has better transient power angle performance than the traditional method of adaptive backstepping controller (ABC) or the improved adaptive backstepping sliding mode variable structure controller (IABSMVSC), such as small chattering, smaller overshoot and more quickly return at the balance point. Figure 3 also indicates that the proposed backstepping sliding mode variable structure controller performance is better as compared to the other traditional controllers.

It can be seen from the simulation curve shown in figure 4, $\omega$ gradually tends to stabilize the rotor speed of the generator at the working point of $\omega_0(314)$, hence the state variable $x_2$ gradually tends to 0. In other words, all of these responses can be achieved in their stable state. However, the improved ECABSMVSC method maintains transient stability at a more sensitive rotational speed and seeks a faster steady state in a shorter time. At the same time, transient response the overshoot is smaller, more smooth than traditional ABSMVSC and ABC methods. That is to say, the improved controller improves the state convergence speed and makes the transient performance better.

Figure 5 present that, in the improved ECIABSMVSC method, the states of $\delta_0$ reach stable at a new balance point. Namely, when $t = 5s$, the balance point changes ranging from 57.3 (deg) to 57.5 (deg), and the rotor speed of the generator varies with the change of the balance point. Furthermore, it is shown that the improved controller maintains good convergence performance in Figure 5.

V. CONCLUSION

In this paper, the nonlinear controller of the STATCOM for generators is designed by proposed adaptive backstepping method. There were three points in this method to be noted. Firstly, it is important to realize that the additional error compensation has the capability of eliminating the chattering phenomena compared to the traditional sliding mode controller. Secondly, the proposed controller enhances the convergence speed of the identification of unknown parameters. In addition, the controller has a fast response to various uncertain parameters. Thirdly, the approach of the backstepping
ping controller is feasible and constructive to engineering applications. In the end, the simulation results further prove that the control method can effectively improve the transient stability.

REFERENCES