Leading-following consensus for multi-agent systems with event-triggered delayed impulsive control

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ABSTRACT In this paper, the exponential leader-following consensus problem is investigated for networked multi-agent systems. In order to further reduce the usage of the communication resources, event-triggered delayed impulsive control strategy is put forward, which successfully combines the delayed impulsive control and event-triggered mechanism, and the delayed impulsive controller is implemented only at event-triggered instants. By constructing the novel Lyapunov based event-triggered strategy and impulsive control theory, some easy-to-check conditions are derived such that the error system is exponentially stable. Furthermore, it is shown that the Zeno behavior can be eliminated from the event-triggering rules. Numerical simulation is proposed to illustrate the effectiveness of the theoretical results.

INDEX TERMS Leader-following, consensus, multi-agent, event-triggered, delayed impulsive control.

I. INTRODUCTION

O

VER the past decades, consensus of multi-agent has draw great attention owing to its important applications, such as cooperative control of robots [1]–[3], flocking phenomenon of biological and social system [5], [6], and so forth [7]–[9]. Therefore, designing control law for each agent, using local information from its neighbors such that the prescribed collective behaviors or consensus can be achieved, has became the central problem of the multi-agent systems. The control of multi-agent systems has been a very active area of research. Up to now, Various efforts have been paid for the realization of consensus of multi-agent in more recent decades [10]–[14], and multifarious control schemes have been imposed to design effective controllers [15]–[19], [31].

Although the multi-agent systems provide us with unprecedented opportunities, the limitations of devices and source consumptions do introduce new challenges to the applications. For instance, an agent is only equipped with microprocessor for some practical reasons. Generally, those equipments have limited handling capacity for information, especially in battery-powered systems. That is to say the constraints do not allow frequent communication. Therefore, how to design a resource-efficient transmission strategy has become a typical issue for multi-agent systems. So far, in spite of a number of resource-efficient schemes had been presented in the literature [15], [16], [18], [19], in most of control strategy, the packets are transmitted periodically. As a result, some "unnecessary" data is sent frequently and the resources are excessively used to some extent. Therefore, an alternative control scheme, namely, event-triggered method was proposed urgently which stands out in reducing unnecessary waste of networks resources. A guiding ideology behind such a strategy is to reduce the communication frequency though introducing an event generator to decide whether the...
signals shall be sent to the neighbor nodes or not. To be specific, in event-triggered control, the sampling is happened only if the state-dependent control error exceeds a tolerable bound. The event-triggered law decides whether the sampling should be acted with a fast or slow rate, and it also decides whether the sampled information deserve to be transmitted for control updates. For example, in [21], an event-triggered control strategy which depends on the error of signal agents’ state between current and last sampling instants was proposed. If the norm of measure error surpasses the tolerable bound, the control information will be updated with the sampling date. Recently, a rich body of research results also have been reported on the event-triggered control schemes [22]–[26]. Specifically, in [25], an event-triggered protocol was proposed to reach the consensus of multi-agent systems subject to state-dependent nonlinear coupling, in which the neighboring agents are coupled via nonlinear function with local passivity. In [26], the event-based distributed filtering problem for continuous-time stochastic systems over sensor networks has been considered. By introducing a triggering condition to regulate the communication rates for each component of the system state, the dynamics of estimation error is exponentially mean-square bounded. On the other hand, it is interesting to find that another control scheme may give us a opportunity to further improve the performance of involving systems based on event triggered strategy. Impulsive control, as a powerful discontinuous control method, has been favored by most researchers owing to it changes the state of the system instantaneously only at certain discrete instants [15], [26]–[28], [31]. Compared with continuous control methods, this instantaneous impulsive jumps can reduce the amount of transmitted information and the control cost to some extent. In past decades, impulsive control has been successfully applied in many disciplines [29], [30]. However, in many practical application, the impulsive control still has certain aspects to be improved for networks. For instance, the impulse instants usually be set in advance, and the impulsive frequency must be designed high enough in order to guarantee the fast convergence rate. For the aspect of optimizing the control, we have find that the event-triggered control is a suitable candidate to synthesize the new strategy, named event-triggered impulsive control. The main idea of the novel scheme is that impulsive control is imposed only at the event-triggering instants, but there is no longer any control any control effect within the event-triggering interval. Obviously, compared with general impulsive control, where impulse instants need to be preset, event-triggered impulsive strategy is more flexible as the impulse instants are determined by state-dependent condition. Meanwhile, the new scheme inherits the property of discontinuous, which makes up the shortcoming of event-triggered control, such that the control cost and number of packets sent can be greatly reduced. The two approaches complement each other. In addition, pocket loss sometimes occurs due to packet collision, in practical, caused by frequent date sampling, thus it is urgent and necessary to consider impulsive control with delay such that the instantaneous impulsive jumps depend not only on the current state of systems but also on the past state. Nevertheless, a thorough literature search has revealed the fact that the results on the problem of event-triggered impulsive control strategy to realize the consensus of multi-agent systems have been very few [31]. However, in [31], the consensus is addressed under distributed event-triggering impulsive rules, in which the impulsive instants is generated by each individual agents. It may lead to frequent control, and using the single agents’ instants to facilitate control is unreasonable. Beside, the existing of delays in impulsive control is ignored which may be also an important factor during the strategy.

In view of the above discussions, in this paper, we aim at designing the event-triggered delayed impulsive control such that the leader-following consensus of multi-agent systems can be achieved. To the best of our knowledge, the development of the novel control schemes are still open and remain challenging. The main purpose of this paper is therefore to shorten such a gap. The main contributions of this paper can be summarized as follows.

1) By exploiting a Lyapunov energy function, the centralized event-triggered delayed impulsive strategy is proposed, in which the Zeno behavior is successfully circumvented and the bound of the times delays for impulsive control is also obtained.

2) With the delayed impulsive controller is adopted only at the event-triggering generated instants, the proposing of novel control machine not only fully exploits the advantages of event-triggered control and impulsive control, but also realize the complement of each other.

3) The leader-following consensus problem for multi-agent systems is investigated, for the first time, via event-triggered delayed impulsive control strategy. Not only some criteria guaranteeing the consensus are given, but also an useful proposition is provided in the form of lemma.
The structure of this paper is organized as follows. In Section II, the networked leader-following multi-agent systems is described, and a main lemma is introduced. Section III presents the main results of this paper. To illustrate the effectiveness of analytical results, a numerical example is given in Section IV. Section V concludes this paper.

Notations: The notation is quite standard. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote, respectively, the $n$-dimensional Euclidean space and the set of all $n \times m$ real matrices. $\lambda_{\max}(P)$ denotes the maximum eigenvalue of matrix $P$, and $\mathbb{N}, \mathbb{R}_+, \mathbb{Z}_+$ refer to the natural numbers, the set of positive real numbers and the set of positive integer numbers, respectively. $\text{diag}\{\cdots\}$ denote a block-diagonal matrix. $a \vee b$ means the maximum value of $a$ and $b$. $\psi$ is the symmetric block in one symmetric matrix. The function set $PC(R, R^n) = \{\psi : R \to R^n \}$ is continuous everywhere except at finite number of point $t$, at which $\psi(t^+), \psi(t^-)$ exist and $\psi(t^+) = \psi(t^-)$. $v_0$ represents a function class in which the function $V(t, x) \in PC([t_0 - \tau, +\infty), \mathbb{R}_+)$ is positive definite, locally Lipschitzian in $x$.

II. MODEL DESCRIPTION AND PRELIMINARIES

In this section, some preliminaries about the models and necessary definitions and lemma are given. Consider the following reference state or the leader state:

$$\dot{s}(t) = As(t) + Bf(s(t)), \tag{1}$$

where $s(t) \in \mathbb{R}^n$ is the state of the leader; $A, B$ are the constant matrices which are defined on $\mathbb{R}^{n \times n}$; $f(s(t)) = (f_1(s(t)), f_2(s(t)), \ldots, f_n(s(t)))^T$ is continuous nonlinear vector functions. Correspondingly, the following nonlinear networked multi-agent system is considered, which can be forced to the leader state $s(t)$:

$$\dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) - u_i(t) + d \sum_{j=1}^N g_{ij}x_j(t - \tau_0), i = 1, 2, \ldots, N, \tag{2}$$

where $x_i(t) \in \mathbb{R}^n$ is the agent’s state; $d$ stands for the control gain; $G = (g_{ij})_{N \times N}$ is the undirected coupling matrix representing the coupling topology, which is defined as follows: if there is a connection between nodes $i$ and $j$, $\forall (i \neq j)$, $g_{ij} = g_{ji} = 1 > 0$; otherwise $g_{ij} = g_{ji} = 0 (i \neq j)$. For diagonal elements of $G$, $g_{ii} = -\sum_{j=1,j \neq i}^N g_{ij}$, $i = 1, 2, \ldots, N$. $u_i(t) \in \mathbb{R}^n$ indicates the control input to be designed, and $\tau_0$ is a coupling constant delay. Obviously, the leader’s dynamics is independent of others. We take the system matrices for all the agents and the leader to be identical because this case has practical background such as group of birds, school of fished etc. Throughout this paper, we assume that the network topology is strongly connected.

Let $e_i(t) = x_i(t) - s(t)$ be the tracking error of agent $i$ between the current state $x_i(t)$ and the reference state $s(t)$. In order to achieve the leader-following consensus of system (1) and (2), we introduce the delayed impulsive effect in accordance with the references [32], [33]:

$$u_i(t) = \sum_{k=1}^{\infty} \delta(t - t_k)[D_k e_i(t - \tau_k) - e_i(t)], k \in \mathbb{Z}_+, \tag{3}$$

where $D_k$ is the impulsive control gain matrix to be designed, $\delta(\cdot)$ refers to the Dirac delta function, and $\tau_k$ is the time delay in impulsive controller. The time sequence $\{t_k, k \in \mathbb{N}_+\}$ is the impulsive instants generated by event-triggered strategy, which shows the typical improvement and difference than previous works.

Remark 1: It should be noted that the event-triggered delayed impulsive control strategy is discontinuous control implement technique based on the multi-agent systems. Whenever the pre-designed event-triggered strategy is violated, the triggering instants are generated, meanwhile, the sampled date $e_i(t^-_k)$ and $e_i(t^+_k)$ is transmitted to the impulsive controller, and the sampled date $e_i(t^+_k)$ is updated to a new date $z_i(t_k)$ at $t_k$. Finally, the actuator adjusts the running state of the plant $i$ to achieve the desired control effect. According to traditional control strategy, the event-triggered control schemes have to continuous adopt control on the systems, and the instants of the impulsive control must be given in advance. Obviously, the novel method successfully combines the merits of the previous strategy, which not only will be more flexible then before, but also can effectively improve the resource usage.

From (1) and (2), we can obtain the following error system:

$$\begin{cases}
\dot{e}_i(t) = Ae_i(t) + B\tilde{f}(e_i(t)) \\
+ d \sum_{j=1}^N g_{ij}e_j(t - \tau_0) + u_i(t) \\
\end{cases} \tag{4}$$

where $\tilde{f}(e_i(t)) = f(x_i(t)) - f(s(t))$, and $\vartheta_i(s) \equiv \vartheta_i(s) \in PC([t_0 - \tau, t_0], \mathbb{R}^n)$ is an initial value vector function, $\tau = \max\{\tau_i, i \in \mathbb{N}\}$.

Substituting (3) into (4) and integrating (4) from $t_k - s$ to $t_k - s$ and letting $s \to 0^+$, we have from the properties of the Dirac delta function that

$$e_i(t_k^+) - e_i(t_k^-) = D_k e_i(t_k^- - \tau_k) - e_i(t_k^-). \tag{5}$$
where \( e_i(t_k^+) = \lim_{t \to t_k^+} e_i(t), e_i(t_k^-) = \lim_{t \to t_k^-} e_i(t) \).

Throughout this paper, we always assume that \( e_i(t) \) is right continuous as \( t_k \), i.e., \( e_i(t_k) = e_i(t_k^-) \). In this case, it follows from (5) that

\[
e_i(t_k) = D_k e_i(t_k^- - \tau_k).
\]

Therefore, under the event-triggered delayed impulsive control (3), the error system can be described as

\[
\begin{cases}
\dot{e}_i(t) = A e_i(t) + B \tilde{f}(e_i(t)) + d \sum_{j=1}^{N} g_{ij} e_j(t), & i = 1, 2, \ldots, N, \\
e_i(t_k) = D_k e_i(t_k^- - \tau_k), & k \in Z_+.
\end{cases}
\]  

(6)

**Remark 2:** In this paper, we aim to design the event-triggered delayed impulsive control for handling the leader-following consensus of the multi-agent system. To this end, some issues should be considered in process of the investigation. Firstly, different from the previous results of event-triggered control [21]-[26] and impulsive control [29], [30], how to establish a theoretical framework to combine the entirely different control approaches. Meanwhile, the merits of each method can be retained. Secondly, as the existing of delays in the impulsive control, the proposed impulsive control strategy in [29], [30] are no longer valid. It mainly lies in how to deal with the relationship of the impulsive delays and impulsive instants. Thirdly, in distributed event-triggered strategy proposed in [21], [25], [26], each agent updates according to the local information. The distributed event-triggered impulsive strategy [31] may lead to frequent and unbefitting impulse. Instead, consider the global information, namely centralized event-triggered strategy [23], [34], will be the better candidate to synthesis the novel control method. As such, how to design a suitable event-triggered function has been another significant problem, which not only decides the information transmitted times but also guarantees the excluding of the Zeno phenomenon. In this sense, the above difficulties exhibit essential challenges for our works.

**Assumption 1:** The nonlinear function \( f_i(\cdot) \) satisfies the Lipschitz condition, i.e., there exist positive constants \( l_i, i = 1, 2, \ldots, N \) such that for any \( \xi_1, \xi_2 \in \mathbb{R} \),

\[
|f_i(\xi_1) - f_i(\xi_2)| \leq l_i|\xi_1 - \xi_2|,
\]

and denotes \( L = \text{diag}\{l_1, l_2, \ldots, l_n\} \).

**Definition 1:** The leader-following consensus of the system (1) and (2) is said to be achieved, if there exist some positive constants \( \sigma, T, M > 1 \), such that

\[
\|e(t)\| \leq M \|\vartheta\| e^{-\sigma(t-t_0)}, \forall t > T,
\]

hold for the initial values, where \( \|\vartheta\| = \sup_{t_0 - \tau \leq t_0 \leq t_0} |\vartheta(s)| \).

Before proceeding, the following lemma should be introduced which will be useful in our subsequent analysis, in which the Lyapunov-based event-triggered strategy is constructed.

**Lemma 1:** The system (6) is said to be globally exponentially stable, if there exist a function \( V \in \mathcal{V} \) and some positive constants \( c_1, c_2, m, \gamma, \mu, M, \alpha, \beta, \tau_k < \tau, k \in Z_+ \) such that the following inequalities hold:

1) \( c_1\|z\|^m \leq V(t, z) \leq c_2\|z\|^m \)

\((t, z) \in [t_0 - \tau, +\infty) \times \mathbb{R}^N; \)

2) \( D^+ V(t, z(t)) \leq -\gamma V(t, z(t)) \) for all \( t \in [t_k^{-}, t_k] \);

3) \( V(t_k) \leq e^{-\beta_k + \mu \tau_k} V(t_k^- \tau_k) \), \( k \in Z_+ \),

where the event-triggering impulsive instants are determined by the following Lyapunov-based strategy:

\( t_k = \inf\{t > t_{k-1} : V(t) \geq e^{-\mu t} (e^{\mu t_{k-1}} V(t_{k-1})) \cap \mathcal{V}(V_{t_0})\}, \)

in which \( V_{t_0} = \sup_{t_0 - \tau \leq t \leq t_0} V(s), t_{k-1} = \sup\{t \in [t_{k-1}, t_k) : V(t) \leq e^{-\mu t} (e^{\mu t_{k-1}} V(t_{k-1}) \cap \mathcal{V}(V_{t_0}))\}, \)

\( \tau = \frac{\alpha}{\beta + \gamma}, \Pi_k = \max_{l \in S_k}\{\alpha + \sum_{i=l}^{k} (\alpha - \beta_i + \mu \tau_i)\}, \)

\( S_k = \{1, 2, \ldots, k-1\} \) and parameters \( \alpha, \beta, \mu, \tau_k \) satisfy

\[
\lim_{k \to +\infty} \Pi_k < M.
\]  

(7)

**Proof 1:** For the sake of simplification, we take \( V(t) := V(t, z(t)) \). Before describing the detailed proof, a vector-valued auxiliary function is introduced as follows

\[
U(t) = \begin{cases}
\begin{aligned}
e^{-\mu(t-t_0)} V(t), & t \in [t_0, +\infty), \\
V_{t_0}, & t \in [t_0 - \tau, t_0).
\end{aligned}
\end{cases}
\]

(8)

According to the definition of \( U(t) \), the event-triggered delayed impulsive control strategy in Lemma 1 can be transformed into the form that

\[
t_k = \inf\{t > t_{k-1} : U(t) \geq e^{\mu U(t_{k-1})}\}, k \in Z_+.
\]  

(9)

where \( U(t_{k-1}) = U(t_{k-1}) \cap \mathcal{V}(V_{t_0}), U_{t_0} = V_{t_0} \). In particular, when \( t = t_0 \), it is easily to verify that \( U(t_{0}) \leq U_{t_0} < e^{\mu} U_{t_0} \).

According to the secondly challenging in Remark 2, we will give the bound of delays with the assistance of demonstrating lower bound of two triggering instants.

Firstly, we eliminate the Zeno phenomenon, i.e., the time interval between any two consecutive triggering instants is always lower bounded by some finite positive quantity.
From the definition of event-triggering strategy, it implies that for $t \in [t_{k-1}, t_k)$,

$$U(t) \geq e^{\alpha U}(t_{k-1}) > U(t_{k-1}),$$

holds. Then, there exists $t^*_k = \sup \{t \in [t_{k-1}, t_k) : U(t) \leq U(t_{k-1}) \}$ such that $U(t^*_k) = U(t_{k-1})$. According to condition (2), it follows that

$$D^+U(t) = \mu e^{\mu(t-t_0)}V(t) + e^{\mu(t-t_0)}D^+V(t)$$

$$\leq (\mu + \gamma)e^{\mu(t-t_0)}V(t)$$

$$= (\mu + \gamma)U(t), \quad [t^*_k, t_k).$$

Integrating both sides of (10) with respect to $t$ over the time interval $[t^*_k, t_k)$ gives rise to

$$U(t_k^*) \leq e^{(\mu + \gamma)(t-t^*_k)}U(t^*_k) \leq e^{(\mu + \gamma)(t-t^*_k)}U(t^*_k).$$

Notice that, according to the rule of (9) of event-triggering, the next event will be triggered at $U(t_k) = U(t) = e^{\alpha U}(t_{k-1})$, for $t > t_{k-1}$. Therefore

$$e^{\alpha U(t_{k-1})} \leq e^{(\mu + \gamma)(t-t_{k-2})}U(t_{k-1}), \quad k \in \mathbb{Z}_+.$$

It can be further derived that

$$t_k - t_{k-1} \geq \frac{\alpha}{\mu + \gamma}, \quad k \in \mathbb{Z}_+.$$

Taking $\tau = T = \frac{\alpha}{\mu + \gamma}$, it implies that the Zeno behavior is excluded, and $T$ is found as the lower bound of triggering instants. Hence, according to the property of delays of impulsive, for any delay $\tau_k$, $k \in \mathbb{Z}_+$, we denote $t_k - \tau_k \in [t_{k-1}, t_k)$, i.e., $0 < \tau_k \leq T$. In what follows, the globally exponentially stable of the system will be investigated.

For $k = 1$, it can be derived from the event-triggered impulsive control strategy (9) that

$$U(t_1^*) = e^{\alpha U}(t_0),$$

and

$$U(t) \leq e^{\alpha U}(t_0) \leq e^{\alpha U}(t_0), \quad t \in [t_0, t_1).$$

When $t = t_1$, it follows from condition (3) that

$$U(t_1) \leq e^{-\beta_1 + \mu \tau_1}U(t_1^* - \tau_1) \leq e^{-\beta_1 + \mu \tau_1}U(t_1^*)$$

$$\leq e^{-\beta_1 + \mu \tau_1}U(t_0) = e^{-\beta_1 + \mu \tau_1}U(t_0)$$

Similarly, for $k = 2$, we have

$$U(t_2^*) = e^{\alpha U}(t_1),$$

and

$$U(t) \leq e^{\alpha U}(t_1) \leq (e^{\alpha} \sqrt{e^{2\alpha - \beta_1 + \mu \tau_1}})U(t_0), \quad t \in [t_1, t_2).$$

When $t = t_2$, it holds that

$$U(t_2) \leq e^{-\beta_2 + \mu \tau_2}U(t_2 - \tau_2) \leq e^{-\beta_2 + \mu \tau_2}U(t_2)$$

$$= (e^{\alpha - \beta_2 + \mu \tau_2} \vee e^{\alpha - \beta_2 + \mu \tau_2 + \alpha - \beta_1 + \mu \tau_1})U(t_0).$$

For $k = 3$, it follows

$$U(t_3^*) = e^{\alpha U}(t_2),$$

and for $t \in [t_2, t_3)$, we have

$$U(t_3) \leq e^{\alpha U}(t_3) \leq e^{\alpha \vee e^{\Pi_k}}U(t_0).$$

For simplicity, let $\Pi_k = \max_{i \in \mathbb{N}} \{\alpha + \sum_{i=1}^{k} (\alpha - \beta_1 + \mu \tau_i)\}$. Repeating the above derivation by using the recursive method for $k = 1, 2, \cdots$, it can be verified that for $t \in [t_{k-1}, t_k), k = 1, 2, \cdots$, one has

$$U(t) \leq e^{\alpha U}(t_{k-1}) \leq (e^{\alpha} \vee e^{\Pi_k})U(t_0).$$

Therefore, according to the convergence of $\lim_{k \to \infty} \Pi_k < +\infty$, we have

$$U(t) \leq e^{M}U(t_0), \quad t > t_0,$$

and implies that

$$V(t) \leq e^{M}V(t_0)e^{\mu(t-t_0)}, \quad t \geq t_0,$$

together with the condition (1), yields that

$$\|z(t, t_0, \varphi)\| \leq \sqrt{c_2 e^{M} \|\varphi\| e^{\mu(t-t_0)}}, \quad t \geq t_0,$$

from Definition 1, which means that the error system is globally exponentially stable. The proof of the lemma is completed.

**Remark 3:** It should be noted that time delays are ubiquitous in real-world systems due to various factors such as signal transmission, network bandwidth, sampling, and controller calculations. The information in a long time ago may be unable to provide reference values in the following analysis. Therefore, in most references [12], [31]–[33], the delays in general are given a upper bound to guarantee the information possessing the authenticity. In this paper, by utilizing the event-triggered machines, the lower bound of any two consecutive triggering instants can be also regarded as the bound of the time delays.
III. MAIN RESULTS

In this section, the leader-following consensus problem for networked multi-agent and leader under a Lyapunov-based event-triggered control strategy is discussed, and a sufficient criterion will be established. First of all, we design the following quadratic Lyapunov-based event-triggered impulsive control strategy:

\[ t_k = \inf \{ t > t_{k-1} : \Phi(t, e(t)) \leq 0 \}, k \in \mathbb{Z}_+ , \]  

where \( \Phi(t, e(t)) = e^{\alpha - \mu t} [e^{\mu t_k} e^T(t_k - 1) Pe(t_{k-1})] \) or \( e^T(t_k) Pe(t_k) \), \( \alpha \in \mathbb{R}_+ \), \( P = I_N \otimes P \), \( e(t) = (e_1^T(t), e_2^T(t), \ldots, e_n^T(t)) \), and \( P \) is a candidate definite matrix to be chosen that satisfy some conditions. Based on impulsive control theory and above-mentioned event-triggered impulsive control strategy, the following theorem and corollary can be derived.

**Theorem 1:** Under the Assumption 1, definition of lemma 1, and condition (6), if there exist some scalars \( \gamma > 0, \beta_k > 0, k \in \mathbb{Z}_+ \), a positive definite matrix \( P \in \mathbb{R}^{n \times n} \), and a positive definite diagonal matrix \( S \in \mathbb{R}^{n \times n} \) such that the following LMIs hold

1. \( \begin{pmatrix} \Lambda & PB \\ * & -S \end{pmatrix} < 0, t \in [t_k, t_{k+1}) \),
2. \( D_k^T P D_k \leq e^{-\beta_k + \mu \tau_k} P \),

where \( \Lambda = (PA + A^T P + L^T SL) + (d - \gamma + d\lambda_{\max}(G^T G) e^{M - \mu \tau_0} P) \). Then, the leader-following consensus between system (1) and (2) will be achieved under the event-triggered impulsive control strategy (11), and the Zeno phenomenon is excluded.

**Proof 2:** Construct the following Lyapunov function for the error system (6):

\[ V(t) = \sum_{i=1}^{N} e_i^T(t) Pe_i(t) . \]

Taking the right-upper Dini derivative of \( V(t) \) over interval \( t \in [t_{k-1}, t_k) \), \( k \in \mathbb{Z}_+ \) along the trajectory of the error system (6), we have

\[
D^+ V(t) = \sum_{i=1}^{N} e_i^T(t) (PA + A^T P)e_i(t)
+ 2 \sum_{i=1}^{N} e_i^T(t) PB \tilde{f}(e_i(t), t)
+ 2d \sum_{i=1}^{N} \sum_{j=1}^{N} g_{ij} e_i^T(t) Pe_j(t - \tau_0) .
\]  

It follow from Assumption 1 that

\[
2e_i^T(t) PB \tilde{f}(e_i(t), t) \leq \sum_{i=1}^{N} e_i^T(t) PBS^{-1} P e_i(t) + \tilde{f}(e_i(t)) S \tilde{f}(e_i(t))
\leq \sum_{i=1}^{N} e_i^T(t)(PBS^{-1} P + L^T SL)e_i(t) .
\]  

Moreover, through some algebraic manipulations, it can be verified that

\[
2d \sum_{i=1}^{N} \sum_{j=1}^{N} g_{ij} e_i^T(t) Pe_j(t - \tau_0)
\leq 2d e^T(t)(G \otimes P)e(t - \tau_0)
\leq de^T(t)(I_N \otimes P)e(t) + de^T(t)(G^T G \otimes P)e(t - \tau_0) .
\]  

On the other hand, by recalling the definition of \( t_k = \sup \{ t \in [t_{k-1}, t_k) : V(t) \leq e^{\mu (t - t_{k-1})} V(t_{k-1}) \} \) and the auxiliary function \( U(t) \), we have

\[
U(t_k) \leq U(t) , t \in [t_k, t_{k+1}) , k \in \mathbb{Z}_+ .
\]

Given any \( \theta \in [-\tau_0, 0] \), it has

\[
U(t + \theta) \leq \begin{cases} 
U(t_k) , t + \theta \in [t_k, t_{k+1}) \\
U(t_k) , t + \theta \in [t_{k-1}, t_k)
\end{cases}
\leq \begin{cases} 
\mu e(t - \tau_0) , t + \theta \in [t_k, t_{k+1}) \\
\mu e(t - \tau_0) , t + \theta \in [t_{k-1}, t_k)
\end{cases}
\leq \mu e^T(t - \tau_0) , t \in [t_k, t_{k+1}) , k = 2, 3, \ldots .
\]

Therefore, one can be concluded that

\[
U(t + \theta) \leq U(t_k) \leq \mu e^T(t - \tau_0) , t \in [t_k, t_{k+1}) , k \in \mathbb{Z}_+ ,
\]

which further implies that, for \( \theta \in [-\tau_0, 0] \),

\[
e^{\mu(t+\theta-\tau_0)} V(t + \theta) = \mu e^T(t + \theta - \tau_0) , t \in [t_k, t_{k+1}) , k \in \mathbb{Z}_+ .
\]

Then, the second term in (14), together with (15), we have

\[
de^T(t - \tau_0)(G^T G \otimes P)e(t - \tau_0)
\leq d\lambda_{\max}(G^T G) V(t - \tau_0)
\leq d\lambda_{\max}(G^T G) e^{M - \mu \tau_0} V(t) .
\]

Substituting the above-mentioned inequality into (14) and considering condition (1), we obtain

\[
D^+ V(t) \leq e^T(t)(PA + A^T P + PBS^{-1} P)
\leq de^T(t)(G^T G \otimes P)e(t - \tau_0)
\leq de^T(t)(G^T G \otimes P)e(t - \tau_0)
\leq de^T(t)(G^T G \otimes P)e(t - \tau_0) .
\]
Following LMIs hold

\[ \lambda_{\text{max}}(G^T G + M - \mu \tau_0)(I_N \otimes \mathcal{P}) \leq \gamma \mathcal{V}(t) \]

When \( t = t_k \), in view of condition (2), we have

\[ \mathcal{V}(t_k) = \sum_{i=1}^{N} e_i^T(t_k) P e_i(t_k) \]

\[ = \sum_{i=1}^{N} e_i^T(t_k - \tau_k) D P D e_i(t_k - \tau_k) \leq e^{-\beta_k + \mu \tau_k} \mathcal{V}(t_k - \tau_k). \]

Therefore, it is easy to verify that the conditions of Lemma 1 have been satisfied, thus the error system is reaching globally exponential stable, that is, the leader-following consensus between the multi-agent has been achieved and the Zeno phenomenon is also excluded according to lemma 1. The proof is completed.

Furthermore, in order to decrease the computational load, the identical parameters may be better to see. Therefore, we assume the \( \beta_k = \alpha, k \in \mathbb{Z}_+ \), and the positive-defined matrix \( \mathcal{P} \), in Lyapunov function, is replaced by identity matrix. Based on the following Lyapunov event-triggered impulsive control strategy

\[ t_k = \inf \{ t > t_{k-1} : e_i^T(t_k - 1)(e(t_k) - e(t_{k-1})) \leq 0, k \in \mathbb{Z}_+ \}, \]

one can has the following corollary.

**Corollary 1:** Under the Assumption 1, definition of lemma 1, and condition (6), if there exist some scalars \( \gamma > 0, M > 0, \alpha > 0, k \in \mathbb{Z}_+ \), a positive definite matrix \( \mathcal{P} \in \mathbb{R}^{n \times n} \), and a positive definite diagonal matrix \( S \in \mathbb{R}^{n \times n} \) such that the following LMIs hold

\( \lambda_{\text{max}}(A + A^T + BB^T + L^T L) < d - \gamma + \lambda_{\text{max}}(G^T G)e^{M - \mu \tau_0}, t \in [t_{k-1}, t_k], \)

\( \lambda_{\text{max}}(D^T D) \leq e^{-\alpha + \mu \tau_k}, \)

Then, the leader-following consensus between system (1) and (2) will be achieved under the event-triggered impulsive control strategy (11), and the Zeno phenomenon is excluded.

**Remark 4:** The nonidentical time delays in controller are growing great challenging to the investigation owing to the triggering impulsive sequence cannot be redetermined and it is completely determined by the event-triggering strategy. However, under the Lyapunov-based event-triggering strategy, it is interesting to find the lower of any two consecutive triggering instants can be cast into the bound of the time delays, which overcomes the essential problem in the process of researches.

### IV. NUMERICAL SIMULATION

In this section, we present a simulation example to illustrate the theoretical results obtained in this paper. Consider the Chua’s circuits model, of which the dynamic is described by

\[
\begin{align*}
\dot{s}_1(t) &= a(s_2(t) - G(s_1(t))), \\
\dot{s}_2(t) &= s_1(t) - s_2(t) + s_3(t), \\
\dot{s}_3(t) &= -bs_2(t),
\end{align*}
\]

where \( G(s_1(t)) = 0.32s_1 - 0.295(|s_1(t) + 1| - |s_1(t) - 1|) \), \( a = 10, b = 14.87 \). With loss of generality, we consider the system (16) as the leader of the networked multi-agent system. Therefore, we can obtain the system parameters in the following

\[
A = \begin{pmatrix} -0.32a & -a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -a & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\]

the nonlinear function \( f(s_1(t)) = -0.295(|s_1(t) + 1| - |s_1(t) - 1|) \). Thus the Lipschitz condition in Assumption can be calculated as \( L = 0.61I_3 \).

To verify the feasibility of Theorem 1, we consider a networked multi-agent system of six agents, where the dynamic of each agent is depicted by the Chua’s circuits (16), the coupling delay \( \tau_0 = 0.2 \), coupling strength \( d = 0.5 \), and the coupling matrix can be chosen as

\[
G = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}.
\]

obviously, according to the graph theory, the network topology of the networked system (2) is strongly connected. By simple computation, one has that \( \lambda_{\text{max}}(G^T G) = 4 \). Noting that, in condition (1) of Theorem 1, the positive matrix \( S \) and positive constant \( \gamma \) can be adjusted to guarantee the inequality satisfied. By using the MATLAB LMI toolbox, after some trail, the feasible solution of condition (1) can be obtained that

\[
P = \begin{pmatrix} 0.0673 & 0.0179 & 0.0528 \\ 0.0179 & 0.1630 & -0.0554 \\ 0.0528 & -0.0554 & 0.1006 \end{pmatrix},
\]

and \( S = 1.0446I_3, \gamma = 10.6 \). Therefore, the condition (1) can be satisfied. Next, we will compute the coupling matrix \( D \) such that the condition (2) hold. It should be noted that,
for the sake of simplicity, we can consider the $D_k$, $\beta_k$ and $\tau_k$ is fixed. We assume the coupling gain matrix is

$$D = \begin{pmatrix} -0.86 & 0 & 0 \\ 0 & -0.78 & 0 \\ 0 & 0 & -0.85 \end{pmatrix},$$

then the inequality $\text{diag}\{0.0092, 0.0849, 0.1275\} \leq e^{-\beta_{k}+\mu_{k}} I_3$ has to be hold. It is easy to find that the parameter can be chosen as $\beta = 0.8$, $\tau = 0.35$ according the condition (2). Consequently, the conditions in Theorem 1 are satisfied.

Therefore, the exponentially leader-following consensus of the multi-agent system (1) and (2) will be achieved under the event-triggered impulsive control, and the state trajectories of the leader and following agents are shown in Fig (a), the error system of (6) under event-triggered impulsive control is shown in Fig (b). In addition, the multi-agent system under traditional impulsive control is presented in Fig (c), in which the impulsive instants is given in advance. Meanwhile, the event-triggered impulsive instants and traditional impulsive instants is given in Fig (d). Obviously, according to Fig (c) and (d), the event-triggered impulsive scheme need much less impulsive instants during the process of achieving the consensus, and when $t = 0.6$, the error systems both approximate to zero. Noting the fact that impulsive instants are set before, some impulsive instants is unnecessary and the instants sequence have to be unrelenting owing to the unknown terminal time. Therefore, under the similar performance, the property of event-triggered impulsive control giving controller at the suitable time, can greatly reduce the usage of the control cost.

(a) Each agents’ and the leader’s evolution of the system.
(b) Trajectories of the error system of the following agent with the leader under the event-triggered delayed impulsive control.
(c) Trajectories of the error system of the following agent with the leader under traditional impulsive control.
(d) Impulsive instants for event-triggered impulsive control and single impulsive control.
V. CONCLUSION
In this paper, the exponential leader-following consensus problem is investigated for multi-agent systems. It should be noted that, different from traditional methods, a novel Lyapunov based event-triggered delayed impulsive control is proposed which fully merges the advantages of the two control strategy, and the delayed impulsive control is adopted only at the event-triggering instants. Based on the novel control strategy and recursive method, some less constraint conditions have been provided such that the error system is exponentially stable, and the Zeno behavior is excluded. In addition, a lemma is given to show the general criterion. On the end, a numerical example has been given to demonstrate the effectiveness of developments. In future, the research topic would be focused on extension of event-triggered delayed impulsive strategy to more complex situations such as switching topologies or exogenous disturbances.

REFERENCES