TH-NRDCSK: A Non-Coherent Time Hopping Chaotic System for Anti-Jamming Communications

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The authors gratefully acknowledge the support from the Electronic Warfare Research Center at the Gwangju Institute of Science and Technology (GIST), originally funded by Defense Acquisition Program Administration (DAPA) and Agency for Defense Development (ADD).

ABSTRACT Time hopping spread spectrum (THSS) system and chaotic system have been studied as candidates of the anti-jamming (AJ) systems with substantial advantages. However, THSS systems suffer from the strict synchronization prerequisite. In order to overcome the shortcomings of the THSS system and utilize the profits of the non-coherent chaotic system, we first propose a novel TH-NRDCSK system to enhance the bit error rate (BER) performance of the conventional non-coherent THSS system and relieve the strict synchronization issue of coherent THSS systems. Then, we analyze the performance of the proposed system under additive white Gaussian noise (AWGN) channel and jamming environment. Finally, we implement the simulation to demonstrate the consistency of the analysis and the simulation results and to disclose the effects of the system parameters on system’s AJ performance. Moreover, the performance of the proposed system is compared to that of conventional counterparts to look at the benefits of integrating chaotic and THSS systems. The simulation results show that the proposed system significantly outperforms conventional counterparts in a practical environment.

INDEX TERMS Time hopping - noise reduction differential chaos shift keying (TH-NRDCSK) system, time hopping (TH) system, noise reduction - differential chaos shift keying (NR-DCSK) system, anti-jamming (AJ) system, non-coherent system

I. INTRODUCTION

ELECTRONIC warfare (EW) includes attacking an enemy (electronic attack, EA), protecting against enemy attacks (electronic protection, EP), and detecting/intercepting electromagnetic energy (electronic support, ES) [1]. In an EW environment, military operations are executed along with increasingly complex demands for electromagnetic spectrum. Among many military operations, EP has been a crucially important issue due to the development of attacking technologies. EP basically focuses on eliminating, reducing, or mitigating the implication of the desired or undesired signals. Anti-jamming (AJ) is one of EP technologies in the physical layer of wireless communication system and has been investigated. In other words, AJ technology is of utmost concern to realize reliable and covert communication [2].

Time hopping spread spectrum (THSS) technology is considered to be one of the most efficient AJ technologies with its powerful characteristics, such as ultra-wide band (UWB) and low transmit power density and robustness to jamming signals [3]-[6]. In addition, this system inherently offers resilience to multipath fading and multiple access capabilities. These advantages are due the fact that the THSS system obtains UWB by transmitting information bits with extremely narrow pulses (called monocycle). Despite these advantages, THSS systems require strict synchronization, which is why their applications are limited in reality. To address this issue, non-coherent THSS systems have been proposed in [7], yet its BER performance are much worse than that of the coherent ones.

On the other hand, chaotic systems and their related applications for covert communication have widely been studied over the past several decades [8]-[17]. Chaotic systems also have significant advantages in AJ and LPI applications by utilizing non-linear signals [18]. Chaotic systems can be
classified as coherent or non-coherent ones [19]. Chaotic shift keying (CSK) is a fundamental coherent system. CSK system is complicated in practice due to the prerequisite of chaotic synchronization at the receiver. Another system is the differential chaotic shift keying (DCSK) system, which is a basic and simple non-coherent chaotic system, that does not require chaotic synchronization as well as channel state information for decoding. Even though the DCSK system has inherently worse performance than the CSK counterpart, it has been preferred in practical wireless communication due to its simplicity. In order to improve the performance of the DCSK system, several advanced alternatives have recently been proposed, i.e. high efficiency-DCSK [20], improved-DCSK [21], short reference-DCSK [22], reference modulated-DCSK [23], noise reduction-DCSK [24], among which the NR-DCSK scheme seems to be the most efficient. The NR-DCSK system utilizes repetition of chaotic samples so that the repeated samples are averaged at the receiver to reduce the effect of noise.

To enjoy advantages of both the THSS system and the NR-DCSK system with less complexity for AJ communication, we propose a novel TH-NRDCSK system, which is a combination of the THSS system and the NR-DCSK system. The integration of these two systems will allow us to utilize the advantages of both systems. In order to verify the AJ performance of the proposed system, we analyze the system performance under an additive white Gaussian noise (AWGN) channel and a broad band noise jamming (BBNJ) environment. We derive a closed form expression of the system bit error rate (BER). Moreover we simulate the proposed TH-NRDCSK system performance under several simulation setups to illustrate the effect of system parameters, i.e. the number of repeated bit for one bit \( N_s \), the number of time hopping slot for one repeated bit \( N_h \), the number of repeating one chaotic sample \( P \), and the length of subslot \( \beta \). The AJ performances of the TH-NRDCSK system and other conventional counterparts are also compared. Eventually, we observe several insights about the effects of system parameters through the analytic and simulation results and show that the TH-NRDCSK system is more robust than some of the other aforementioned systems. We also show that the proposed system can outperform coherent THSS and chaotic systems if synchronization errors occur.

The remainder of this paper is organized as follows. The conventional THSS system and NR-DCSK system are briefly introduced and the proposed TH-NRDCSK system is proposed in Section II. The BBNJ, partial band noise jamming (PBNJ), tone jamming (TJ), sweep jamming (SJ) models are depicted in section III. The BER expression for the proposed system under AWGN and BBNJ environment is derived in section IV. The simulation results of the proposed TH-NRDCSK system under several conditions, and the conclusions are presented in Section V and VI, respectively.

## II. SYSTEM MODELS

In this section, we briefly introduce the conventional THSS system and NR-DCSK system, and thoroughly elaborate on the TH-NRDCSK system. Table I shown at the top of this page describes the system parameters and whether they are page describes the system parameters and whether they are

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Meaning</th>
<th>THSS</th>
<th>NR-DCSK</th>
<th>TH-NRDCSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l )</td>
<td>The index of bits</td>
<td>o</td>
<td>x</td>
<td>o</td>
</tr>
<tr>
<td>( i )</td>
<td>The index of repeated bits</td>
<td>o</td>
<td>x</td>
<td>o</td>
</tr>
<tr>
<td>( c_i )</td>
<td>( i )-th time hopping code</td>
<td>o</td>
<td>x</td>
<td>o</td>
</tr>
<tr>
<td>( k )</td>
<td>The index of chaotic samples</td>
<td>x</td>
<td>o</td>
<td>o</td>
</tr>
<tr>
<td>( \beta )</td>
<td>The length of sequence in one subslot</td>
<td>x</td>
<td>o</td>
<td>o</td>
</tr>
<tr>
<td>( P )</td>
<td>The number of repeating one chaotic sample</td>
<td>x</td>
<td>o</td>
<td>o</td>
</tr>
<tr>
<td>( f_s )</td>
<td>Sampling frequency</td>
<td>o</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T_c )</td>
<td>Chip time, one time hopping slot interval</td>
<td>o</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N_{th} )</td>
<td>The number of time hopping slot in one repeated bit</td>
<td>o</td>
<td>x</td>
<td>o</td>
</tr>
<tr>
<td>( N_s )</td>
<td>The number of repeating one bit</td>
<td>o</td>
<td>x</td>
<td>o</td>
</tr>
<tr>
<td>( N_t )</td>
<td>The period of time hopping code</td>
<td>o</td>
<td>x</td>
<td>o</td>
</tr>
<tr>
<td>( E_b/N_0 )</td>
<td>Energy per bit to noise power spectral density ratio</td>
<td>o</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SJR</td>
<td>Signal to jamming power ratio</td>
<td>o</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
by multiplying information bit with the mask signal. The transmitted signal \( s_{tr}^{(j)}(u, t^{(j)}) \) with antipodal modulation can be expressed as \([4]\)

\[
s_{tr}^{(j)}(u, t^{(j)}) = \sum_{i=-\infty}^{\infty} d_i^{(j)}(u)w_{tr}(t - iT_j - \beta c^{(j)}_i(u)T_e),
\]

(1)

where \( t^{(j)} \) is the \( j \)-th transmitter clock time, \( w_{tr}(t) \) is the transmitted monocyte waveform. \( T_j, c_i, T_e, \) and \( d_i \) represent the pulse train spacing (frame) time, TH sequence, chip time, and modulation repeated bits, respectively. Then, the receiver receives the signal, which has been distorted by the noise and interference, and correlates the received signal with mask signal coherently. The original bits are restored by summing the correlated results and comparing the results with a threshold.

The coherent THSS system utilizes the nanosecond pulses at the transmitter, and correlates the received signal with a mask signal at the receiver. This causes time synchronization issues on the coherent THSS system. Consequently, non-coherent THSS systems were proposed to address the time synchronization problem \([7]\). However, the non-coherent THSS systems are shown to have poor performance.

**B. NR-DCSK SYSTEM**

Chaotic communication has also been considered to be a powerful method in AJ communication applications. The NR-DCSK system repeats each chaotic sample \( P \) times at the transmitter side so that the received signal can be averaged at the receiver side to reduce the noise effect. The specific process of the NR-DCSK system is as follows. First, a chaotic sequence of length \( \beta / P \) is generated by the logistic map which provides simplicity and good performance. The logistic map is expressed as \([25]\)

\[
x_{k+1} = 1 - 2x_k^2,
\]

(2)

where the expectation of \( x_k, E[x_k] = 0 \), and the variance of \( x_k, V[x_k] = 1 \) for normalized \( x_k \). Secondly, \( \beta / P \) chaotic samples are repeated \( P \) times to make the reference sequence of \( \beta \) length. Then, the information sequence is created by multiplying the \( l \)-th bit and the reference sequence. The sequence of the transmitted signal is made by concatenating the reference sequence and the information sequence. As a result, the transmitted signal of the \( l \)-th bit, \( b_l \), can be represented by \([24]\)

\[
s_{l,k} = \begin{cases} 
    x_{l,k}, & 0 < k \leq \beta, \\
    b_l x_{l,k}, & \beta < k \leq 2\beta,
\end{cases}
\]

(3)

The bit generation process of the NR-DCSK system is illustrated in Fig. 1. At the receiver, the received signal is averaged using a block average filter of size \( P \) and correlated with its time delayed version by a factor of \( \beta / P \). The transmitted bit is then estimated by summing the correlation results and passing the outcome through a threshold detector.

**C. TH-NRDCSK SYSTEM**

The TH-NRDSCK system is a combination of THSS system and the NR-DCSK system. The system block diagram is illustrated in Fig. 2. Instead of using conventional THSS pulses, i.e. rectangle monocyte, Gaussian pulse, Gaussian monocyte, and Gaussian doublet, NR-DCSK signal is generated at the chaotic signal generator. In addition, the received NR-DCSK signal is averaged by moving average filter at the receiver side. Fig. 3 shown at the next page presents how the TH-NRDSCK transmitted signal \( s_{l,k} \) is generated. Each transmitted bit, \( b_l \) is repeated \( N_e \) times in its respective subframe. These repeated bits, \( c_i \), can be written as \( b_l x_{l,k} \). Then each repeated bit, \( c_i \) in the subframe is expanded into \( N_h \) time hopping slots. Only one of these slots contains useful information for each \( c_i \). This specific slot is selected through a predetermined TH code, \( c_i \), which corresponds to a specific \( i \) value for each \( c_i \). This slot is decomposed into two subslots of length \( \beta \), where the first subslot has a reference sequence of \( x_{l,k} \) repeated \( P \) times for each \( k \) until \( k = \beta / P \), and the second subslot has an information.

![FIGURE 1. Bit generation process of the NR-DCSK system](image1)

![FIGURE 2. System architecture of the TH-NRDCSK system](image2)
sequence equal to the reference sequence multiplied by \( e_i \). 
\( x_{i,k} \) is a chaotic sequence generated by a logistic chaotic map. In addition, \( x_{i,k} \left[ \frac{P}{\beta} \right] \) represents a \( x_{i,k} \) element repeated \( P \) times in a NR-DCSK subslot. Note that \( P \) is the number of times that one chaotic sample is repeated so that \( P \) is divisor of \( \beta \), and the length of chaotic sequences required to complete a single subframe is \( \beta / P \). A graphical illustration of the bit structure of the TH-NRDCSK system is given in the Fig. 4. Mathematically, \( s_{i,k} \) can be expressed as

\[
s_{i,k} = \begin{cases} 
  x_{i,k} \left[ \frac{P}{\beta} \right], & c_i T_c f_s < k \leq c_i T_c f_s + \beta, \\
  e_i x_{i,k} \left[ \frac{P}{\beta} \right] - \beta, & c_i T_c f_s + \beta < k \leq c_i T_c f_s + 2\beta, 
\end{cases}
\]

The signal \( s_{i,k} \) is transmitted through an AWGN channel and corrupted by jamming signal. The received \( r_{i,k} \) at the destination can be expressed as

\[
r_{i,k} = s_{i,k} + n_{i,k} + j_{i,k},
\]

where \( n_{i,k} \) is the AWGN and \( j_{i,k} \) is the jamming signal. The AWGN channel is considered for analysis simplicity. The receiver gets \( r_{i,k'} \) by using the same TH code, \( c_i \), to select one time hopping slot which contains valid information signals.

After that, the receiver creates \( r'_{i,k'} \), by passing \( r_{i,k'} \) through the moving average filter of window size \( P \). \( r'_{i,k'} \) is given by

\[
r'_{i,k'} = \frac{\beta}{P} \sum_{m=1}^{mP} r_{i,k'},
\]

\[
= s_i + \frac{1}{P} \sum_{k'=(m-1)P+1}^{mP} n_{i,k'} + \frac{1}{P} \sum_{k'=(m-1)P+1}^{mP} j_{i,k'}.
\]

(6)

This operation makes the sequence of length \( \beta / P \) and reduces the noise effect. The signal \( r'_{i,k'} \) is correlated with its time delay version \( r'_{i,k'+\beta/P} \). Then, the reconstructed repeated bits \( e_i \) are recovered by comparing the correlation output with a threshold. By doing so, the original bit \( b_i \) is recovered by applying the hard decision method that uses \( N_r \) repeated bits. The decision variable of the repeated bits \( e_i \) at the correlation output, \( D_i \), can be written as

\[
D_i = \sum_{k=1}^{\beta/P} \left( x_i \left[ \frac{k}{P} \right] + \bar{n}_{i,k'} + \bar{j}_{i,k'} \right) \left( e_i x_i \left[ \frac{k}{P} \right] + \bar{n}_{i,k'+\beta} + \bar{j}_{i,k'+\beta} \right),
\]

(7)

where \( \bar{n}_{i,k'} \) and \( \bar{j}_{i,k'} \) are \( \frac{1}{P} \sum_{k'=(m-1)P+1}^{mP} n_{i,k'} \) and \( \frac{1}{P} \sum_{k'=(m-1)P+1}^{mP} j_{i,k'} \) which are the noise and jamming signal after passing through the moving average filter. In addition, \( \bar{n}_{i,k'+\beta} \) and \( \bar{j}_{i,k'+\beta} \) denote the \( \beta / P \) delayed version of \( \bar{n}_{i,k'} \) and \( \bar{j}_{i,k'} \). Then, the decision variable may be further expanded as

\[
D_i = \sum_{k=1}^{\beta/P} \left[ e_i \left( x_i \left[ \frac{k}{P} \right] \right)^2 + x_i \left[ \frac{k}{P} \right] n_{i,k'} + e_i x_i \left[ \frac{k}{P} \right] n_{i,k'} + n_{i,k'} - n_{i,k'+\beta} + j_{i,k'} + j_{i,k'+\beta} \right] \\
+ \bar{n}_{i,k'} \bar{j}_{i,k'+\beta} + \bar{j}_{i,k'} \bar{n}_{i,k'+\beta} + \bar{n}_{i,k'} \bar{n}_{i,k'+\beta} + \bar{j}_{i,k'} \bar{j}_{i,k'+\beta} \\
+ x_i \left[ \frac{k}{P} \right] \bar{j}_{i,k'} + e_i x_i \left[ \frac{k}{P} \right] \bar{j}_{i,k'} + \bar{j}_{i,k'} \bar{j}_{i,k'+\beta} \\
\right]
\]

(8)

where \( U \) and \( N_1, N_2, N_3, N_4 \) and \( N_5 \) represent the data and noise components, and \( I_1, I_2, \) and \( I_3 \) represent the interference components.

## III. JAMMING MODELS

A. BROAD BAND NOISE JAMMING (BBNJ) [26]

Broad band noise jamming (BBNJ) is generated by allocating jamming energy over the whole frequency bandwidth with equal amounts of power. The frequency bandwidth is assumed to be the bandwidth used by the target signal. In other words, BBNJ increases the noise intensity at the receiver evenly, which makes legitimate communicate more difficult to operate properly. The BBNJ is commonly regarded as a
zero-mean Gaussian random variable with power $P_j$. Then, the power spectral density of BBNJ is $P_j/2W$, where $W$ is the bandwidth of transmitted signal.

**B. PARTIAL BAND NOISE JAMMING (PBNJ) [27]**

Partial band noise jamming (PBNJ) signal disrupts a portion of the bandwidth of the transmitted signal with equally concentrated power. Since the jammer uses less bandwidth and more power for the given bandwidth, PBNJ is considered to be more effective than BBNJ. Once the jammer concentrates its power to affect a valid fraction of the transmitted signal, the jammed signal will be incorrectly decoded with a high probability. The power of PBNJ signal will be concentrated to a bandwidth $\rho W$, where $\rho$ is the ratio of frequency domain occupancy ($0 < \rho \leq 1$). The PBNJ signal is a band-limited Gaussian noise with a flat power spectral density over the jammed bandwidth, i.e., the power spectral density in the jammed band is $P_j/2\rho W$.

**C. TONE JAMMING (TJ) [28]**

Tone Jamming (TJ) is also considered as an efficient jamming method due to the fact that the jamming signal attacks one or more frequencies of the target signals' spectrum strategically. TJ is categorized as single tone jamming (STJ) or multi tone jamming (MTJ). STJ includes only one sinusoidal signal having the concentrated power, while MTJ includes several sinusoidal signals which have equally divided power. Mathematically, the TJ can be expressed as

$$J_{TJ}(t) = \sum_{m=1}^{M} \sqrt{\frac{2P_j}{M}} \sin (2\pi f_m t + \theta_m),$$

where $M$, $P_j$, $f_m$, and $\theta_m$ are the number of jamming tones, total jamming power, the frequency, and the phase of the $m$-th jamming signal, respectively.

**D. SWEEP JAMMING (SJ) [29]**

Sweep Jamming (SJ) signal changes its frequency periodically over a certain frequency band and time. In this way, SJ is considered to be efficient by utilizing the advantages of both TJ and PBNJ. The frequency $f_{SJ}$ of the SJ signal can be expressed by the following equation

$$f_{SJ}(t) = f_l + (f_u - f_l) t/T_{SJ},$$

where $f_l$, $f_u$, and $T_{SJ}$ are lower bound of the jamming frequency, upper bound of the jamming frequency, and the period of sweep, respectively. Then, a sweep jamming signal can be expressed as

$$J_{SJ}(t) = \sqrt{2P_j} \cos \left( f_l t + \frac{f_u - f_l}{2T_{SJ}} t^2 + \theta_{SJ} \right),$$

where $P_j$ is the power of the jamming signal and $\theta_{SJ}$ is the phase delay of the jamming signal.

**IV. PERFORMANCE ANALYSIS**

In this section, we derive the theoretical performance of the proposed TH-NRDCSK system under the AWGN channel and BBNJ jamming environment. For $i$-th repeated bit $b_i$, the repeated bit energy $E_i$ is considered to be $E_0/N_s$ due to the fact that one bit is repeated $N_s$ times. Since the chaotic signal, AWGN, and BBNJ are independent of each other and the mean of the AWGN and BBNJ are zero, the mean of the decision variable $D_i$ is the mean of data component $U$ which can be readily expressed as

$$E[D_i] = E[U] = \frac{E_c}{2P_j},$$

where the repeated bit energy $E_c = 2P \sum_{k=1}^{\beta/P} E\left[\sqrt{x_{t,k}^2}\right]$ and $E_c = E_0/N_s$. In addition, the repeated bit energy $E_c$ is assumed to be constant for a high spreading factor $\beta/P$. Thus, the variance of the decision variable can be expressed as

$$V[D_i] = \sum_{u=1}^{5} V[N_u] + \sum_{v=1}^{3} V[I_v].$$

The variance of $\sum_{u=1}^{5} V[N_u]$, and $\sum_{v=1}^{3} V[I_v]$ can be obtained as

$$V[N_1] = \frac{E_c N_0}{2P_j} = V[N_2],$$

$$V[N_3] = \frac{\beta N_0^2}{4P_j^3},$$

$$V[I_1] = \frac{E_c P_j}{2P_j^2} = V[I_2],$$

$$V[I_3] = \frac{\beta P_j^2}{P_3^3},$$

where $N_0$ and $P_j$ is the noise power spectral density and jamming power spectral density, respectively. By substituting (13) for (14), (15), (16), (17), (18), $V[D_i]$ can be expressed as

$$V[D_i] = \frac{E_c (P_j + \frac{N_0}{2})}{P_j^2} + \beta \frac{(P_j + \frac{N_0}{2})^2}{P_3^3}$$

For a Gaussian noise and BBNJ environment, the $BER$ for repeated bits can be represented as

$$BER = \frac{1}{2} \Pr(D_l < 0 \mid b_l = +1) + \frac{1}{2} \Pr(D_l > 0 \mid b_l = -1),$$

which can be obtained as
TABLE 2. Simulation Value of System Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
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<tbody>
<tr>
<td>(\beta)</td>
<td>200</td>
<td>200</td>
<td>25</td>
<td>25 ~ 400</td>
<td>25</td>
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<tr>
<td>(P)</td>
<td>1 ~ 40</td>
<td>25</td>
<td>5</td>
<td>Divisor of (\beta)</td>
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<td>(f_s\ [Hz])</td>
<td>2(\beta/T_c)</td>
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<td>(T_c\ [s])</td>
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<td>(N_h)</td>
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<td>(N_t)</td>
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</tr>
<tr>
<td>Jamming type</td>
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<td>SJ</td>
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<td>SJ</td>
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<td>(f_l\ [Hz])</td>
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<td>(f_s\ [Hz])</td>
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<tr>
<td>(E_b/N_0\ [dB])</td>
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<td>15</td>
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<tr>
<td>(SJR\ [dB])</td>
<td>-30 ~ 10</td>
<td>-30 ~ 10</td>
<td>-30 ~ 10</td>
<td>-30 ~ 10</td>
<td>-30 ~ 10</td>
<td>-30 ~ 10</td>
</tr>
</tbody>
</table>

\[
\overline{BER} = \frac{1}{2} \text{erfc}\left(\left(\frac{2V[D_i]}{E[D_i]}\right)^{\frac{1}{2}}\right)
\]

\[
= \frac{1}{2} \text{erfc}\left(8\left(\frac{P_j + N_0}{E_c}\right) + \frac{8\beta (P_j + N_0)^2}{P E_c^2}\right)^{\frac{1}{2}}
\]

\[
= \frac{1}{2} \text{erfc}\left(\left[8\left(\frac{P_j / N_s N_h + N_0 / 2}{E_b / N_s}\right) + \frac{8 \beta (P_j / N_s N_h + N_0 / 2)^2}{P (E_b / N_s)^2}\right]^{\frac{1}{2}}\right),
\]

where erfc(x) is the complementary error function defined as \(\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-\mu^2) d\mu\) and \(P_{j0}\) is the jamming power having the bandwidth before time hopping spreading of the NR-DCSK signal. Under the only AWGN environment, the system BER can also be easily obtained from (21) by fixing \(P_j = 0\)

\[
\overline{BER}_{AWGN} = \frac{1}{2} \text{erfc}\left(\left(\frac{4N_0}{E_c} + \frac{2\beta N_0^2}{PE_c^2}\right)^{\frac{1}{2}}\right)
\]

\[
= \frac{1}{2} \text{erfc}\left(\left[4\left(\frac{N_0}{E_b / N_s} + \frac{\beta N_0^2}{P (E_b / N_s)^2}\right)\right]^{\frac{1}{2}}\right).
\]

Considering hard decision of \(N_s\) repeated bits, the ultimate BER performance of the TH-NRDCSK system can be calculated as

\[
BER = \sum_{z=1}^{\lfloor N_s/2 \rfloor} \left(\frac{N_s}{N_s - z}\right) \overline{BER}^{N_s - z} (1 - \overline{BER})^z.
\]

**Remark:** Through the equation (21) and (22), we notice that the jamming power \(P_{j}\) is affected by both \(N_h\) and \(N_s\) and the repeated bit energy \(E_c\) is altered by \(N_s\). Therefore, if the jamming power \(P_{j}\) is much smaller than the noise power spectral density \(N_0\), i.e. \(P_{j} \ll N_0\), we can approximate equation (21) to equation (22). In this case, the BER performance of the TH-NRDCSK system is affected by only \(N_s\). Otherwise, when the jamming power \(P_{j}\) is much bigger than the noise power spectral density \(N_0\), i.e. \(P_{j} \gg N_0\), the BER performance of the TH-NRDCSK system is affected by only \(N_h\).

V. SIMULATION RESULTS

In this section, we simulate the proposed TH-NRDCSK system under six simulation setups to validate the theoretical analysis and observe the effect of system parameters on AJ performance and compare it to conventional counterparts. The BER performance of the TH-NRDCSK system for the parameter \(P\) is illustrated in A under the AWGN channel. In addition, B, C, and D shows the AJ performance of the TH-NRDCSK system for several \(N_h\) and \(N_s\), under four types of jamming environments, and for several \(\beta\) and \(P\) combinations, respectively. Moreover, E and F depicts the AJ performance comparison to conventional counterparts under SJ environment and under time synchronization error condition. The value of simulation parameters for each cases are given in Table II.

A. BER PERFORMANCE OF THE TH-NRDCSK SYSTEM UNDER AWGN CHANNEL

The analytic and simulated results for the proposed TH-NRDCSK system under the AWGN channel for various \(P\) are presented in Fig. 5. This simulation demonstrates that the simulation results entirely validate the analytic BER expression. In addition, the performance is improved as \(P\) increases. Specifically, at \(BER = 10^{-2}\), the proposed TH-NRDCSK system can obtain about 1dB SJR gain as \(P\) is doubled. It is
also noteworthy that as P increases, the gap between analysis and simulation curves enlarges. This is because the number of chaotic samples, which can be calculated by $\beta/P$, decreases so that the approximation of analysis can not be guaranteed.

B. AJ PERFORMANCE OF THE TH-NRDCSK SYSTEM FOR DIFFERENT $N_h$ AND $N_s$

Fig. 6 and Fig. 7 present the BER of the proposed TH-NRDCSK system versus SJR with different $N_s$ and $N_h$ values. Since $\beta$ and P is 200 and 25, respectively in this simulation, the performance gap between analysis and simulation results is identified in Fig. 6 and Fig. 7. Specifically, Fig. 6 illustrates that the AJ performance of the TH-NRDCSK system is degraded as $N_s$ increases due to the fact that the repeated bit energy $E_c$ is decreased by $N_s$ times, i.e. $E_c = E_b/N_s$. Even though the jamming power is lower as $N_s$ increases, the graph in Fig. 6 shows that the AJ performance of the TH-NRDCSK system is degraded. It can be seen that the effect of noise is more effective than the effect of jamming due to the distributed power by $N_s$.

Nonetheless, the simulation result gets closer to theoretical analytic results as $N_s$ increases. This is because the weak law of large numbers theory applies as $N_s$ increases. In addition, Fig. 7 represents that the AJ performance of the TH-NRDCSK system is improved as $N_h$ increases. Unlike $N_s$, the larger $N_h$ value gives the more SJR power gain. Particularly, at BER $= 10^{-2}$, the proposed TH-NRDCSK system can obtain about 3dB SJR gain as $N_h$ is doubled. Eventually, the AJ performance of the TH-NRDCSK system is improved as $N_s$ decreases and $N_h$ increases.

C. AJ PERFORMANCE OF THE TH-NRDCSK SYSTEM UNDER DIFFERENT JAMMING TYPE

In Fig. 8, we illustrate the comparison of AJ performance of the TH-NRDCSK system under different jamming types. BBNJ has less impact on the TH-NRDCSK system than other jamming types. Moreover, the TH-NRDCSK system has similar AJ performance against PBNJ and SJ attacks. However, we notice that SJ has the most powerful effect to
FIGURE 9. AJ performance of the TH-NRDCSK system according to $\beta$, $P$

the TH-NRDCSK system in low SJR region. As a results, SJ is the most aggressive opponent to TH-NRDCSK system. SJ is also shown to be the most powerful jamming against NR-DCSK system [30]. Beyond the 5dB SJR, however, the effect of all jamming types becomes similar. In subsequent simulations, AJ performance against SJ will continue to be explored.

D. AJ PERFORMANCE OF THE TH-NRDCSK SYSTEM FOR DIFFERENT $P$ AND $\beta$

We then simulate the effect of $P$ and $\beta$ on the AJ performance of the TH-NRDCSK system under the SJ environment. In the simulation, we set $\beta$ equal to 25, 50, 100, 200, 300, and 400. Correspondingly, $P$ is set to the divisors of $\beta$ because the number of generated chaotic samples required to present one bit should be $\beta/P$.

Fig. 9 shows the BER performance of the proposed system versus $P/\beta$. It is shown that the best BER performance is obtained when $P/\beta$ is 0.5 for any fixed $\beta$. When $P$ is equal to $\beta$, the AJ performance of the TH-NRDCSK system becomes worse even though $P$ increases. Moreover, Fig. 9 shows that the AJ performance of the TH-NRDCSK system has a similar trend versus $P/\beta$ for all $\beta$, i.e. BER performance is improved until $P/\beta = 0.5$ and degraded at $P/\beta = 1$. One more thing to note is that at $P/\beta = 0.5$, AJ performance is improved as $\beta$ increases.

E. AJ PERFORMANCE OF THE TH-NRDCSK SYSTEM COMPARED TO CONVENTIONAL SYSTEMS

In Fig. 10, the graph shows a comparison of BER provided by the TH-BPSK, non-coherent TH-BPSK, CSK, NR-DCSK, and proposed TH-NRDCSK systems. It is shown that the coherent TH-BPSK and CSK systems provide a better performance than the non-coherent counterparts. Compared to non-coherent TH-BPSK system at $BER = 10^{-2}$, the proposed TH-NRDCSK system acquires 8.5dB SJR gain. This improvement is definitely achieved by applying the NR-DCSK structure to the non-coherent TH-BPSK system. Moreover, the TH-NRDCSK system exhibits a similar performance to the coherent CSK system below -10dB SJR. Furthermore, the TH-NRDCSK system exhibits a more robust AJ performance than the NR-DCSK system under 8dB SJR. This enhancement is due to the application of THSS system. Eventually, Fig. 10 shows the advantages of integration of two systems definitely.

F. AJ PERFORMANCE OF THE TH-NRDCSK SYSTEM AND COHERENT COUNTERPARTS UNDER THE SYNCHRONIZATION ERROR CONDITION

The AJ performance of the TH-NRDCSK system and coherent counterparts under the time synchronization error condition is shown in Fig. 11. We compare the performance of the TH-NRDCSK system with that of the TH-BPSK
and CSK system by simulating under no, 1, and 2 samples synchronization error conditions. In the Fig. 11, the performance of the non-coherent TH-NRDCSK system is inherently worse than that of the coherent TH-BPSK and CSK systems under no synchronization error. However, when the synchronization error occurs, the performance of the coherent system is drastically degraded. When 2 samples error occurs, both coherent TH-BPSK and CSK systems would be ineffective. On the other hand, the BER performance of the TH-NRDCSK system is barely affected by synchronization error. This simulation illustrates the reason why the non-coherent system is more preferable in reality.

VI. CONCLUSION

In this paper, we first proposed a hybrid TH-NRDCSK system to enhance the BER performance of the conventional non-coherent THSS system against jamming attacks. The TH-NRDCSK system is a THSS system that utilizes NRDCSK signal instead of conventional THSS pulses. Secondly, we analytically derived the BER expression of the proposed system under a AWGN channel and under BBNJ environment and verified the derived BER performance by simulation results. The BER performance versus \( E_b/N_0 \) is improved by increasing \( P \). In addition, the AJ performance of the TH-NRDCSK system is improved as \( N_s \) decreases and \( N_h \) increases independently.

Other simulations are provided to observe the AJ performance of the TH-NRDCSK system under several simulation setups. We present that SJ is shown to be the most powerful among the four jamming types to the TH-NRDCSK system which is the reason why SJ is used in subsequent simulations. Moreover, the case when \( P \) is half of \( \beta \) has the best performance for any fixed \( \beta \). We also demonstrate that the time synchronization problem has a serious impact on coherent systems. Finally, we showed that the TH-NRDCSK outperforms other non-coherent counterparts.

REFERENCES