ABSTRACT
In this paper, we investigate the effects of residual hardware impairments (RHIs), channel estimation errors (CEEs) and imperfect successive interference cancellation (ipSIC) on the cooperative non-orthogonal multiple access (NOMA) system over Nakagami-m channels, where the amplify-and-forward (AF) relay can harvest energy from the source. The exact expressions for outage probability and ergodic sum rate are derived in closed-form. In addition, the asymptotic outage analyses in the high signal-to-noise (SNR) regime are carried out. The results show that the outage probability exists an error floor due to the existence of CEEs and compared with RHIs, CEEs have a more serious impact on the system outage performance. The close simulation results of Monte Carlo verify the accuracy of our theoretical derivation. Finally, the performance of energy efficiency is examined with RHIs, CEEs and ipSIC.

INDEX TERMS
NOMA, Residual hardware impairments, SWIPT, Imperfect successive interference cancellation.

I. INTRODUCTION
With the development of communication technology, the fifth generation (5G) mobile communication network has gradually entered the field of vision. Mobile Internet and Internet of things (IoTs) services will become the main driving forces of mobile communication development in the future 5G era [1]. Currently, communication networks primarily utilized orthogonal multiple access (OMA) technique [2]. The conventional OMA scheme needs to guarantee orthogonality of the resource to avoid interferences among users, such as time/frequency/code resources and has the disadvantage of low spectral efficiency and connectivity [3]. Although the orthogonal frequency division multiple access (OFDMA) technique can improve spectral efficiency by using overlapping subcarriers, it is still difficult to achieve the maximum Shannon capacity bound. In contract, power domain multiplexing, non-orthogonal multiple access (NOMA) can achieve higher spectral efficiency by serving multiple user in the same resource [4]. At transmitter, the signals are encoded by superposition coding and sent to destinations with different power levels. At the receiver, the signal can be separately detected by successive interference cancellation (SIC) [5, 6]. Moreover, NOMA can ensure the fairness of users by allocating more power to the users under weaker channel conditions and less power to the user under stronger channel conditions [7].

To enhance the reception reliability of far users, cooperative NOMA is proposed by introducing cooperative communication into NOMA [8]. In order to improve the performance of the cell-edge user, the authors proposed two cooperative NOMA relay schemes in [9], the results show that the proposed scheme can improve the outage performance of cell-edge users. To exploit the prior information of NOMA systems, the authors proposed a cooperative NOMA
scheme in [10], the simulation results indicated that the performance of cooperative NOMA systems is superior to the non-cooperative NOMA. Most of the above articles are based on the perfect SIC (pSIC), while the execution of pSIC requires extremely high precision receivers, which is obviously impossible in the real communication networks [11]. In [12], the authors studied the bit error rate performance of NOMA systems under the conditions of ideal and non-ideal SIC, and presented a more realistic method than the pSIC. The expressions of outage probability and throughput of code-domain NOMA (CD-NOMA)/power-domain NOMA (PD-NOMA) with pSIC/imperfect SIC (ipSIC) over Rayleigh fading channels in delay-limited transmission mode were derived in [13]. All of the works showed that the study of ipSIC is more relevant to the actual situation.

Simultaneous wireless information and power transfer (SWIPT) is an effective way to prolong the lifetime of energy-constrained wireless networks, such as IoTs and vehicle-to-everything (V2X) [3, 14]. In the applications, the devices are deployed in remote areas or mobility, where the wireless devices are infeasible to support by power line [15]. In general, SWIPT are classified into two type of protocols, power splitting (PS) [16] and time switch (TS) [17]. In [18], authors studied the effects of SWIPT under both PS and TS schemes in MIMO wireless networks and presented the outer bound for the achievable rate-energy region. In [19], the system performances of both single-input single-output (SISO) and multiple-input single-output (MISO) SWIPT NOMA systems were studied and the results showed that the performance of SWIPT NOMA is better than that of NOMA, which indicated that SWIPT can provide higher gain for the system. Considering TS protocol, authors in [20] derived exact analytical expressions for the outage probability and network throughput of cooperative relaying systems.

Unfortunately, the main drawback of the above research works is that they assume all radio frequency (RF) components are perfect, which is over-realistic. In practice, by deploying low-cost and low power components, all RF front-ends are prone hardware imperfections, and they suffer from a variety of hardware impairments, such as phase noise, in phase/quadrature phase (I/Q) imbalance, high power amplifier nonlinearity and quantization error [21, 22]. To this end, a great deal of works have proposed many algorithms to compensate the loss caused by hardware impairments [23–25]. However, the residual hardware impairments have significant effects on the system performance, authors in [26] studied the system effects of multiple-relay AF network in the presence of RHIs and derived the closed-form expression of the outage probability. In [27], authors investigated the effects of RHIs on the NOMA system networks and proved that the outage performance of the system will be reduced slightly by RHIs at the low signal-to-noise ratios (SNRs).

In practical communication systems, due to the continuous movement of users, the path loss is uncertain, especially for the access of 5G massive users, it is a great challenge to obtain perfect channel state information (CSI) [28]. The common way to do this is to estimate channel at relay by the training sequence. In these cases, the channel estimation errors (CEEUs) are thus inevitable due to the imperfection of estimation algorithms and some types of noise [29]. The effects of CEEs on multiple-input multiple-output (MIMO)-OFDM systems over Rayleigh fading channels were discussed in [30]. The influence of CEEs on the performance of wireless-powered with a DF relay over the Rayleigh channels was studied in [31]. The authors demonstrated that CEEs can reduce the performance of demodulator seriously and improve the bit error rate (BER) of the considered system [32]. Specifically, the authors in [33] considered a more practical system, where the joint effects of RHIs and CEEs on the AF cooperative system were investigated and it is proved that RHIs has a great negative influence on the system performance at high transmission rate, while CEEs has the opposite effect. The outage and the expected spectral efficiency performances of the cooperative DF systems in the presence of RHIs and CEEs were studied in [34]. Recently, in [35], the authors considered the performance of the NOMA system in the presence of both RHIs and CEEs, while the presence of direct link communication between the base station and the far users was not considered.

A. MOTIVATION AND CONTRIBUTIONS

The previous literature has provided solid foundation of cooperative NOMA, however the study on NOMA SWIPT AF systems in the presence of RHIs, ICSI and ipSIC is still invisible yet. In order to fill this gap, this paper makes an in-depth study of the joint effects of the three practical factors on the system performance. In particular, the Nakagami-m channel is adopted as our channel model. The reason is that Nakagami-m is a general fading channel, which can be used to capture some common fading channels. In this paper, the source node can transmit the supposed signals to the near users and the far users with the aid of an AF relay or through the direct connection. To intuitively reflect the state of communication, the exact analytical expressions of the outage probability and the approximate expressions at high SNRs are derived. In addition, the ergodic sum rate and energy efficiency in ideal and non-ideal conditions are calculated. The main contributions of this paper are summarized as follows:

- We consider a practical wireless communication system, where all transceivers suffer from hardware impairments and CSIs from all links are imperfect due to CEEUs. Owing to the above ideal factors, the ipSIC at the receivers is taken into account.
- By considering the three deleterious factors, we investigate the outage performance of the considered system by deriving the closed-form analytical expressions of the outage probability for the far users and the near users.
- In order to obtain deeper insights, we derive the approximate outage probability in the high SNR region. The simulation results show that an error floor exists in
the non-ideal conditions. This happens because that the asymptotic outage performance is limited by the CEEs.

- We analyze the ergodic sum rate and energy efficiency of the considered system by deriving the closed-form expressions. The results indicate that, as the SNR increases, the ergodic sum rate and energy efficiency will increase all the time in the ideal conditions, while for the non-ideal conditions, due to the RHIs, ICSI and ipSIC, the ergodic rate and energy efficiency will reach the upper bound.

The remainder of this paper is organized as follows: Section II describes the system model by considering RHIs, ICSI and ipSIC. Section III presents the exact and asymptotic outage probability expressions of the far users and near users. In Section IV, the ergodic capacity of the far and near users are investigated. In Section V, we investigate the energy efficiency in both ideal and non-ideal conditions. Our analyses are proved by the numeric results in Section VI before the conclusion in Section VII.

B. NOTATION
Throughout this paper, Pr (·) denotes probability for a random variable; \( \Delta \) represents definition operation; \( \mathbb{E} (\cdot) \) symbolizes the expectation operator; the cumulative distribution function (CDF) and the probability density function (PDF) of a random variable \( X \) are represented by \( F_X (\cdot) \) and \( f_X (\cdot) \), respectively.

II. SYSTEM MODEL
As can be seen in FIGURE 1, we consider a NOMA SWIPT AF relaying network, which consists one source \( S \), one relay \( R \) and \( M \) destinations \( (D_1, D_2, \ldots, D_M) \). Note that it is unrealistic to perform NOMA for all users. The practical way is to separate all users into multiple clusters, where users perform NOMA in the same cluster and OMA in the different clusters. To facilitate the analysis, users in the same cluster are divided into two groups, one is near user and another is far user.\(^1\) In one cluster, \( S \) wants to send the signals \( s_1 \) and \( s_2 \) to the far users \( D_f \) and the near users \( D_n \) directly or with the aid of the AF relay where the signals satisfy with \( \mathbb{E} |s_1|^2 = \mathbb{E} |s_2|^2 = 1 \). Moreover, it is almost impossible to obtain the exact CSI. The practical way to obtain the channel knowledge is training by using pilots. By using linear minimum mean square error (LMMSE) [36], the channel coefficient can be defined as \( h_i = \bar{h}_i + e_i, i = \{SR, SD_f, SD_n, RD_f, RD_n\} \) where \( h_i \) and \( \bar{h}_i \) denote the real channel coefficient and estimation channel coefficient, respectively. \( e_i \sim CN (0, \sigma_i^2) \) denotes the CEEs [37]\(^2\). Without loss of generality, the estimated channel gains between the source and destinations are sorted as \( |\hat{h}_{SD_1}|^2 \leq |\hat{h}_{SD_2}|^2 \leq \ldots \leq |\hat{h}_{SD_M}|^2 \).

The whole process is divided into two phases: EH phase and signals transmission phase.

A. EH PHASE
In this paper, the power splitting protocol is adopted as shown in FIGURE 2. The transmit signal was split into two streams: one part of the power \((1 - \varsigma)\) is used for EH and another part \((\varsigma)\) is used for information transmission, where \( \varsigma \) is power allocation factor for information transmission. It is assumed that the total power transmitted from the source node is \( P_S \), then the harvested energy at \( R \) can be expressed as

\[
Q_{EH} = \varsigma (1 - \varsigma) |h_{SR}|^2 P_S, \tag{1}
\]

where \( \varsigma \in [0, 1] \) denotes the energy conversion efficiency. In this phase, the RHIs and CEEs are not considered, as all non-ideal factors can be encompassed in \( \varsigma \).

According to the PS protocol, the received power at the relay can be expressed as

\[
P_R = \eta |h_{SR}|^2 P_S. \tag{2}
\]

B. SIGNALS TRANSMISSION PHASE
The signals transmission is divided into two time slots. In the first time slot, the superposed signals are simultaneously transmitted to destinations and relay. In the second time slot, the relay amplifies and forwards the signals to the destinations.

The first time slot: The source node \( S \) sends \( \sqrt{a_1 P_S} s_1 + \sqrt{a_2 P_S} s_2 \) to the relay \( R \) and the destinations \( D_f \) and \( D_n \), where \( a_1 \) and \( a_2 \) represent the power coefficients of the transmission power for \( D_f \) and \( D_n \) with \( a_1 + a_2 = 1 \) and \( a_1 a_2 = 0 \).

\(^1\)In this paper, we only study signals of far user and near user in one cluster.

\(^2\)Note that, the estimation channel and estimation are orthogonal [38] and the CEEs can be approximated as a Gaussian random variable [39].
\[ a_1 > a_2. \] Thus, the received signal at the relay and the users can be expressed as
\[
y_t = (h_{i_1} + e_{i_1}) \left( \sqrt{a_1 P_S s_1} + \sqrt{a_2 P_S s_2 + \eta_{r,i_1}} \right) + \eta_{r,i_1} + v_{i_1},
\]
where \( \eta_{r,i_1} \sim \mathcal{CN}(0, \xi_{t,i_1}) \) denotes the additive white Gaussian noise (AWGN) at \( R; D_f \) and \( D_n \), \( \eta_{r,i_1} \sim \mathcal{CN}(0, \xi_{r,i_1}) \) and \( \eta_{r,i_1} \sim \mathcal{CN}(0, \xi_{t,i_1} P_S | h_{i_1} |^2) \) denote the distortion noises from the transmitter and receiver, respectively [40]. Combining the distribution of \( \eta_{r,i_1} \) and \( \eta_{t,i_1} \), after some mathematical calculations, the received signal can be rewritten as
\[
y_t = (h_{i_1} + e_{i_1}) \left( \sqrt{a_1 P_S s_1} + \sqrt{a_2 P_S s_2 + \eta_{i_1}} \right) + v_{i_1},
\]
where \( \eta_{t,i_1} \sim \mathcal{CN}(0, \xi_{t,i_1} P_S) \) and \( \xi_{t,i_1} = \sqrt{\xi_{t,i_1}^2 + \xi_{i_1}^2} \).

The second time slot: The relay amplifies and forwards the received signal to the destinations as
\[
y_2 = (G y_{SR} + \eta_{i_2}) (h_{i_2} + e_{i_2}) + \eta_{r,i_2} + v_{i_2},
\]
where \( G = P_{r,f} / \left( (|P_S|^2 + \sigma_{SR}^2)(1 + \chi_{r,i_2}) \right) \) represents the amplifying gain factor, \( \eta_{r,i_2} \sim \mathcal{CN}(0, \xi_{r,i_2} P_{Rr}) \) and \( \eta_{r,i_2} \sim \mathcal{CN}(0, \xi_{r,i_2} P_{Rr} | h_{i_2} |^2) \) denote the distortion noises, while \( \eta_{r,i_2} \sim \mathcal{CN}(0, \xi_{r,i_2} P_{Rr}) \) is AWGN. Similar to (4), we can rewrite (5) as
\[
y_2 = (h_{i_2} + e_{i_2}) (G y_{SR} + \eta_{i_2}) + v_{i_2},
\]
where \( \eta_{t,i_2} \sim \mathcal{CN}(0, \xi_{t,i_2} P_{Rr}) \) and \( \xi_{t,i_2} = \sqrt{\xi_{t,i_2}^2 + \xi_{i_2}^2} \).

C. FADING CHANNEL
In this study, we assume that the channel coefficient \( h_t \) follows Nakagami-\( m \)-distribution. The PDF and CDF of the estimated channel gain \( h_t \) can be expressed as
\[
f_{h_t}(x) = \frac{x^{\alpha_i - 1} e^{-\frac{x}{\beta_i}}}{\Gamma(\alpha_i) \beta_i^\alpha e^{-\frac{x}{\beta_i}}},
\]
\[
F_{h_t}(x) = 1 - \sum_{g_i=0}^{\alpha_i-1} \frac{(-x)^g_i}{g_i!} \left( \frac{x}{\beta_i} \right)^{g_i} \frac{1}{\Gamma(g_i)},
\]
where \( \Gamma(\cdot) \) denotes the Gamma function, \( \alpha_i \) and \( \beta_i \) are the multipath fading and the control spread parameters, respectively. Using the order statistics, the PDF and CDF of the \( m \)-th user estimated channel gain \( h_m \) can be expressed as [41]
\[
F_{h_m}(x) = b_m \sum_{z=0}^{M-m} \left( \begin{array}{c} M-m \\ z \end{array} \right) \left[ F_{h_t}(x)^{m+z} \right]^{m+z},
\]
where \( b_m = M!(m-1)!(M-m)! \).

During the first time slot, \( D_n \) first decodes the high intensity signal \( s_1 \) according to the NOMA protocol. Therefore, the received signal-to-interference-plus-noise ratio (SINR) for \( D_n \) to decode \( D_f \)’s signal is given by
\[
\gamma_{SDn} = \frac{a_1 \rho_{SDn} \gamma}{(a_2 + \rho_{SDn}^2 \gamma)^{\frac{1}{2}}} + (\gamma_{RDn} + 1)^{\frac{1}{2}},
\]
where \( \gamma = P_{SR}/N_0 \) denotes the transmit SNR at \( S \). Owing to the iSIC, the SINR of \( D_n \) to decode its own signal can be given as
\[
\gamma_{SDn} = \frac{a_2 \rho_{SDn} \gamma}{(a_2 + \rho_{SDn}^2 \gamma)^{\frac{1}{2}}} + (\gamma_{RDn} + 1)^{\frac{1}{2}} \sigma_{SDn}^2 \gamma + 1,
\]
where the parameter \( \varepsilon (0 \leq \varepsilon \leq 1) \) denotes the level of residual interference due to that \( D_n \) cannot remove \( D_f \)'s information.\(^4\) In addition, it is worth noting that \( \varepsilon = 0 \) represents the system performs perfect SIC and \( \varepsilon = 1 \) represents that SIC is not implemented in the system. The received SINR of the user \( D_f \) is given by
\[
\gamma_{RDf} = \frac{\rho_{SR} \rho_{RDf} \alpha_1 \gamma'}{(\rho_{SR} \rho_{RDf} + (a_2 + \rho_{SR}) \gamma')^\frac{1}{2}} + (\gamma_{RDn} + 1)^{\frac{1}{2}} \sigma_{RDf} \gamma + 1.
\]

During the second time slot, the received SINR of user \( D_f \) is given by
\[
\gamma_{RDf} = \frac{\rho_{SR} \rho_{RDf} \alpha_1 \gamma'}{(\rho_{SR} \rho_{RDf} + (a_2 + \rho_{SR}) \gamma')^\frac{1}{2}} + (\gamma_{RDn} + 1)^{\frac{1}{2}} \sigma_{RDf} \gamma + 1 + \rho_{SR} \rho_{RDf} \alpha_1 \gamma',
\]
where \( \gamma' = P_{SR}/N_0 \) denotes the transmit SNR at \( R; d_1 = \kappa_{SR}^2 + \kappa_{RDf}^2 \), \( \kappa_{SR}^2 = \sigma_{SR}^2 (d_1 + 1) \), \( \kappa_{RDf}^2 = \sigma_{RDf}^2 (d_1 + 1) \), \( \varepsilon_{SR} = \sigma_{SR}^2 \kappa_{SR}^2 \), \( \varepsilon_{RDf} = \sigma_{RDf}^2 \kappa_{RDf}^2 \), \( \varepsilon_{RD} = \sigma_{RD}^2 \kappa_{RD}^2 \), \( \varepsilon_{SR} = \sigma_{SR}^2 \kappa_{SR}^2 \), \( \varepsilon_{RDf} = \sigma_{RDf}^2 \kappa_{RDf}^2 \), and \( \varepsilon_{RD} = \sigma_{RD}^2 \kappa_{RD}^2 \) are established. (11)-(16) simplified to ideal conditions.

III. OUTAGE PROBABILITY ANALYSIS
In this section, the outage probability of \( D_f \) and \( D_n \) are derived to analyze the performance of the considered system. In order to obtain more insights, we also analyze the asymptotic behavior of outage probability in the high SNR region.

\(^4\)The parameter of iSIC \( \varepsilon \) follows Gaussian distribution [42] and we assume that \( \varepsilon \) is a fixed value for simplicity in this paper.
A. EXACT OUTAGE PROBABILITY

An outage event will occur at $D_f$ if the SINRs of the signal transmitted by the source node and the relay node cannot reach the target threshold $\gamma_{\text{thf}}$. Thus, the outage probability for $D_f$ can be expressed as

$$p_{\text{out}}^{D_f} = \Pr(\gamma_{SD_1} < \gamma_{\text{thf}}) \Pr(\gamma_{RD_1} < \gamma_{\text{thf}}).$$

(17)

The exact expression of $D_f$’s outage probability is given in the following theorem.

Theorem 1. The closed-form expression for the outage probability of the far user is expressed as

$$p_{\text{out}}^{D_f} = b_f \sum_{z=0}^{M-f} \left(\begin{array}{c}
M-f \\
z
\end{array}\right) \frac{(-1)^z}{f+z} \left[1 - \frac{\alpha_{SD}^{-1}}{\alpha_{SD}} e^{-\frac{\alpha_{SD}}{\beta_{SD}}} g_{\alpha}^{f+z}
\right]$$

$$\times \left[1 - \frac{2}{F(\alpha_{SR})} \frac{\alpha_{SR}}{\beta_{SR}^2} \sum_{g=0}^{\alpha_{SR}-1} \frac{1}{\alpha_{SR}} \sum_{u=0}^{\alpha_{SR}-1} \left(\begin{array}{c}
\alpha_{SR} \\
\alpha_{SR}
\end{array}\right) u\right]$$

$$\times \frac{\alpha_{SR}}{\beta_{SR}^2} g_{\alpha}^{f+z-g_{\alpha}} \left(\frac{1}{\beta_{RD_1} \gamma_{\text{thf}}} \right) \left(\alpha_{\gamma_{\text{thf}}} + \phi_{\alpha}\right)$$

$$\times (\gamma_{\alpha})^{g_{\alpha} - g_{\alpha} - g_{\alpha} + 1} \beta_{SR}^{-1} K_{\alpha-1} (2 \left(\frac{\alpha_{\gamma_{\text{thf}}} + \phi_{\alpha}}{\beta_{SR} \beta_{RD_1} \gamma_{\text{thf}}}\right)),$$

(18)

where $b_f = M! / ((f - 1)! (M - f)!)$ and $f$ denotes the $f$-th user (far user), $\theta_1 = \gamma_{\text{thf}} \left(\alpha_{\gamma_{\text{thf}}} - \alpha_{\gamma_{\text{thf}}}^+ \right),$ $\theta_2 = \left(\gamma_{\gamma_{\text{thf}}}^+ + 1\right) \alpha_{\gamma_{\text{thf}}}^+ \gamma_{\text{thf}} + \gamma_{\text{thf}}$ with $a_1 > \left(\alpha_{\gamma_{\text{thf}}}^+ \right) \gamma_{\text{thf}}$ and $a_1 > \left(\alpha_{\gamma_{\text{thf}}}^+ \right) \gamma_{\text{thf}}, K_v (\cdot)$ denotes the modified Bessel function of the second kind with order $v$.

Proof. See Appendix A.

The outage event will occur at $D_n$ for two scenarios: i) $D_n$ cannot decode $D_f$’s signal successfully; and ii) $D_n$ fails to decode its own signal. Hence, the outage probability of $D_n$ can be expressed as

$$p_{\text{out}}^{D_n} = \left[1 - \Pr(\gamma_{RD_n} > \gamma_{\text{thf}}, \gamma_{RD_n} > \gamma_{\text{thn}})\right]$$

$$\times \left[1 - \Pr(\gamma_{SD_n} > \gamma_{\text{thf}}, \gamma_{SD_n} > \gamma_{\text{thn}})\right],$$

(19)

where $\gamma_{\text{thn}}$ denotes the target threshold at $D_n$.

The exact expression of $D_n$’s outage probability is given in the following theorem.

Theorem 2. The closed-form expression for the outage probability of the near user is expressed as

$$p_{\text{out}}^{D_n} = b_n \sum_{z=0}^{M-n} \left(\begin{array}{c}
M-n \\
z
\end{array}\right) \frac{(-1)^z}{f+z} \left[1 - \frac{\alpha_{SDn}^{-1}}{\alpha_{SDn}} e^{-\frac{\alpha_{SDn}}{\beta_{SDn}}} g_{\alpha}^{f+z}
\right]$$

$$\times \left[1 - \frac{2}{F(\alpha_{SR})} \frac{\alpha_{SR}}{\beta_{SR}^2} \sum_{g=0}^{\alpha_{SR}-1} \frac{1}{\alpha_{SR}} \sum_{u=0}^{\alpha_{SR}-1} \left(\begin{array}{c}
\alpha_{SR} \\
\alpha_{SR}
\end{array}\right) u\right]$$

$$\times \frac{\alpha_{SR}}{\beta_{SR}^2} g_{\alpha}^{f+z-g_{\alpha}} \left(\frac{1}{\beta_{RD_n} \gamma_{\text{thf}}} \right) \left(\alpha_{\gamma_{\text{thf}}} + \phi_{\alpha}\right)$$

$$\times (\gamma_{\alpha})^{g_{\alpha} - g_{\alpha} - g_{\alpha} + 1} \beta_{SR}^{-1} K_{\alpha-1} (2 \left(\frac{\alpha_{\gamma_{\text{thf}}} + \phi_{\alpha}}{\beta_{SR} \beta_{RD_n} \gamma_{\text{thf}}}\right)),$$

(20)

where $b_n = M! / ((n - 1)! (M - n)!)$ and $n$ denotes the $n$-th user (near user), $\tau = \max(\gamma_{\text{thf}}, \gamma_{\text{thn}})$ and $\gamma_{\text{thf}} = \left(\alpha_{\gamma_{\text{thf}}}^+ + 1\right) \alpha_{\gamma_{\text{thf}}}^+ \gamma_{\text{thf}} + \gamma_{\text{thf}} / \left(\alpha_{\gamma_{\text{thf}}} + \alpha_{\gamma_{\text{thf}}}^+ \right),$ $\gamma_{\text{thn}} = \left(\alpha_{\gamma_{\text{thn}}}^+ + 1\right) \alpha_{\gamma_{\text{thn}}}^+ \gamma_{\text{thn}} + \gamma_{\text{thn}} / \left(\alpha_{\gamma_{\text{thn}}} + \alpha_{\gamma_{\text{thn}}}^+ \right); \gamma_{\text{thf}} > \gamma_{\text{thn}}.$ Note that all the conditions that make the equation establish are $a_1 > \left(\alpha_{\gamma_{\text{thf}}} + \alpha_{\gamma_{\text{thf}}}^+ \right) \gamma_{\text{thf}}, a_2 > \left(\alpha_{\gamma_{\text{thn}}}^+ + 1\right) \gamma_{\text{thn}}; a_1 > \left(\alpha_{\gamma_{\text{thf}}} + \alpha_{\gamma_{\text{thf}}}^+ \right) \gamma_{\text{thf}}, a_2 > \left(\alpha_{\gamma_{\text{thn}}}^+ + 1\right) \gamma_{\text{thn}}.$

Proof. See Appendix B.

B. ASYMPTOTIC OUTAGE BEHAVIOR

In order to obtain more insights, the asymptotic outage performances for $D_f$ and $D_n$ are investigated in this subsection. At high SNRs, the asymptotic PDF and CDF of the channel gain $\rho = |h|^2$ are given as [43]

$$f_{\rho}^{\infty} (x) \approx \frac{x^{\alpha_{\gamma_{\text{thf}}} + \phi_{\alpha}}}{\alpha_{\gamma_{\text{thf}}} + \phi_{\alpha}},$$

(21)

$$F_{\rho}^{\infty} (x) = \frac{M!}{m! (M - m)!} \left(\frac{1}{\alpha_{\gamma_{\text{thf}}} + \phi_{\alpha}}\right)^m \left(\frac{x}{\beta_{\gamma_{\text{thf}}} + \phi_{\alpha}}\right)^{\alpha_{\gamma_{\text{thf}}} + \phi_{\alpha}},$$

(22)

In the following corollaries, we describe the asymptotic expressions of outage probability of both near users and far users in ideal and non-ideal conditions.

Corollary 1. At high SNRs, the asymptotic expressions of outage probability for $D_f$ are expressed as

- Ideal conditions ($\kappa = \sigma_e = \varepsilon = 0$)

$$p_{\text{out}}^{D_f} \approx b_f \left(\frac{1}{\alpha_{SD}}\right)^f \left(\frac{\alpha_{SD}}{\beta_{SD}}\right)^{\alpha_{SD} f} \left(\frac{1}{\alpha_{SD}^{\alpha_{SR}} \beta_{SR}}\right)^{\alpha_{SR}} \left(\frac{\phi_{b_{1}}}{\gamma_{\text{thf}}}\right)^{\alpha_{RD}},$$

(23)

- Non-ideal conditions ($\kappa, \sigma_e, \varepsilon \neq 0$)
Proof. See Appendix D.

**Remark 1.** Corollary 1 and Corollary 2 provide some insights on the derived analytical results. The results show the effects of channel fading parameters and non-ideal factors on asymptotic outage performance intuitively. For the ideal conditions, the parameters of RHHs, CEEs and ipSIC are all reduced to 0, and the asymptotic outage probability decreases with the increase of SNR, especially in the high SNR regime, the outage probability increases almost linearly. While for non-ideal conditions, RHHs, CEEs and ipSIC have detrimental effects on the outage performance of the considered system. Moreover, the outage probability tends to be a fixed constant value with the increase of SNR which is due to the existence of CEEs. We can conclude that for the non-ideal conditions, the outage probability of the considered system depends on the channel fading parameters, distortion noise, CEEs and the performance of SIC, while the outage probability only depends on the channel fading parameters in the ideal conditions.

**IV. ERGODIC SUM RATE**

The ergodic capacity is the time average of the maximum information rate of the random channel in all fading states which can well reflect the fading performance of the system. Thus, in this section, we study the ergodic sum rate of the considered system.

The achievable rate of $D_f$ and $D_n$ can be expressed as

$$R_f = \frac{1}{2} \log_2 \left(1 + \max \left[\gamma_{SD_f}, \gamma_{RD_f}\right]\right),$$

$$R_n = \begin{cases} \frac{1}{2} \log_2 \left(1 + \max \left[\gamma_{SD_n}, \gamma_{RD_n}\right]\right), & \text{if } \rho_{RD_n} > \rho_{RD_f} \\ \frac{1}{2} \log_2 \left(1 + \gamma_{SD_n}\right), & \text{if } \rho_{RD_n} < \rho_{RD_f} \end{cases}$$

where $1/2$ indicates that the communication process is divided into two time slots. It is difficult to derive the exact expression of the ergodic sum rate, thus, in this paper, we only study the approximate expression of the ergodic sum rate in high SNR regime.

The ergodic rate of the $D_f$ can be expressed as

- **Ideal conditions ($\kappa, \sigma_e, \varepsilon = 0$)**

$$R_f^{\text{ave}} = \mathbb{E} \left[\frac{1}{2} \log_2 \left(1 + \max \left[\gamma_{SD_f}, \gamma_{RD_f}\right]\right)\right],$$

where $\gamma, \gamma' \to \infty$, substituting (13) and (14) into (29), after some mathematical calculations the ergodic rate of $D_f$ can be rewritten as

$$R_f^{\text{ave}} \approx \frac{1}{2} \log_2 \left(1 + \frac{a_1}{a_2}\right).$$

- **Non-ideal conditions ($\kappa, \sigma_e, \varepsilon \neq 0$)**
where $\chi_5 = (\kappa^2_{SD} + 1) \sigma^2_{eSD} \lambda$, $\chi_6 = \sigma^2_{eSR} (d_1 + 1)$, $\chi_7 = \sigma^2_{eRD} (d_1 + 1)$, $\chi_8 = \sigma^2_{eSR} \sigma^2_{eRD} (d_1 + 1)$, $\Lambda_1 = \left( \alpha_{SD} + 1 \right) \beta_{SR}$, $\Lambda_5 = \left( \alpha_{SD} + 1 \right) \beta_{SD}$, $\Lambda_2$ can be expressed as

$$
\Lambda_2 = \frac{b_3 \Gamma(c_2)}{\Gamma(\nu)} \beta^{\nu}_{RDj} \prod_{t=0}^{\nu-1} \left( \frac{1}{t!} \right) \sum_{p_0 + \cdots + p_{\nu-1} = c_1} \left( \frac{c_1}{p_0 \cdots p_{\nu-1}} \right) \prod_{g_2 = 0}^{\nu-1} \left( \frac{1}{g_2!} \right) g_2^{p_2} \left( \frac{c_1 + 1}{\nu} \right) \left( \frac{1}{\beta_{RDj}} \right),
$$

(32)

where $c_1 = M - f + t$, $c_2 = g_2 p_2 + \alpha_{RDj} - 1$.

**Proof.** See Appendix E.

The ergodic rate of the near users can be expressed as

- Ideal conditions ($\kappa = \sigma_e = \varepsilon = 0$)

$$
R_{ave}^{n,id} = \Pr(\rho_{RD,n} > \rho_{RD}) \left[ \frac{1}{2} \log_2 \left( 1 + \max \left( \gamma_{SD,n} \gamma_{RD,n} \right) \right) \right] + \Pr(\rho_{RD,n} < \rho_{RD}) \left[ \frac{1}{2} \log_2 \left( 1 + \gamma_{SD,n} \right) \right].
$$

(33)

As for $D_n$, $\rho_{RD,n}$ and $\rho_{RDj}$ are independent random variables, we thus assume $\Pr(\rho_{RD,n} > \rho_{RD}) = \Pr(\rho_{RD,n} < \rho_{RD}) = 1/2$. The ergodic rate of $D_n$ can be rewritten as

$$
R_{ave}^{n,id} = \frac{a_2 b_n}{4 n^2} \sum_{z=0}^{M-n} \left( M-n \right) \left( \frac{1}{n+z} \right) a_2 (\Phi_1 + \Phi_2 - \Phi_3 - \Phi_4).
$$

(34)

- Non-ideal conditions ($\kappa, \sigma_e, \varepsilon \neq 0$)

$$
R_{ave}^{n,id} \approx \frac{1}{4} \log_2 \left( 1 + \frac{a_2 \Lambda_4}{(\kappa^2_{SD} + a_1 \varepsilon) \Lambda_4 + \chi_1} \right) + \frac{1}{4} \log_2 \left( 1 + \max \left[ \frac{a_2 \Lambda_4}{(\kappa^2_{SD} + a_1 \varepsilon) \Lambda_4 + \chi_1}, \frac{a_1 \Lambda_1 \Lambda_2}{(a_1 \varepsilon + d_2) \Lambda_3 + \chi_2 \Lambda_3 + \chi_3 \Lambda_1 + \chi_4} \right] \right),
$$

(35)

where $\chi_1 = (\kappa^2_{SD} + 1) \sigma^2_{eSD} \lambda$, $\chi_2 = \sigma^2_{eSR} (d_2 + 1)$, $\chi_3 = \sigma^2_{eRD} (d_2 + 1)$, $\chi_4 = \sigma^2_{eSR} \sigma^2_{eRD} (d_2 + 1)$, $\Lambda_4 = (\alpha_{SD} + 1) \beta_{SR}$, $\Lambda_3$ can be expressed as

$$
\Lambda_3 = \frac{b_3 \Gamma(c_4)}{\Gamma(\nu)} \beta^{\nu}_{RD,n} \prod_{z=0}^{n-1} \left( \frac{1}{z!} \right) \sum_{p_0 + \cdots + p_{\nu-1} = c_3} \left( \frac{c_3}{p_0 \cdots p_{\nu-1}} \right) \prod_{g_3 = 0}^{\nu-1} \left( \frac{1}{g_3!} \right) g_3^{p_3} \left( \frac{c_3 + 1}{\nu} \right) \left( \frac{1}{\beta_{RD,n}} \right)^{c_3},
$$

(36)

where $c_3 = M - n + q$, $c_4 = g_3 p_3 + \alpha_{RD,n} - 1$.

**Proof.** See Appendix F.

According to the above results, we can get the ergodic sum rate in two conditions as follows:

- Ideal conditions ($\kappa = \sigma_e = \varepsilon = 0$)

$$
R_{ave}^{sum,id} \approx \frac{1}{4} \log_2 \left( 1 + \frac{a_2 b_n}{4 n^2} \sum_{z=0}^{M-n} \left( M-n \right) \left( \frac{1}{n+z} \right) a_2 (\Phi_1 + \Phi_2 - \Phi_3 - \Phi_4) \right) \times \left( \frac{1}{n+z} \right) a_2 (\Phi_1 + \Phi_2 - \Phi_3 - \Phi_4).
$$

(37)

- Non-ideal conditions ($\kappa, \sigma_e, \varepsilon \neq 0$)

The ergodic sum rate for the users is given at the top of next page.

V. ENERGY EFFICIENCY

Energy efficiency is another important metric to evaluate the performance of wireless communication system. It refers to the useful information transmitted to the receivers by each unit of energy consumed by the transmitters, which is expressed as [44]

$$
\eta_{ee} = \frac{R_i}{Q_{total}},
$$

(39)

where $R_i$ denotes the achievable rate of the $i$-th user, and $Q_{total}$ denotes the total energy consumption. $Q_{total}$ can be expressed as

$$
Q_{total} = P_S + P_R + P_C,
$$

(40)

where $P_C$ is the fixed energy consumption which caused by transmitter and the receiver.

VI. NUMERICAL RESULTS

In this section, the correctness of our theoretical analysis is verified by the Monte-Carlo simulation. For the purpose of comparison, the performance of the system in the ideal conditions is also provided. The main parameters for the simulation are provided in Table 1.

FIGURE 3 plots the outage probability versus the transmit SNR for ideal conditions ($\kappa_i = \sigma_{e_i} = \varepsilon = 0$) and non-ideal conditions ($\kappa, \sigma_e, \varepsilon \neq 0$). From FIGURE 3, we can see that the simulated results are completely coincident with the theoretical analysis values, which proves the correctness of our theoretical analysis. In the ideal conditions, we can see that
\[ P_{\text{sum}, \text{nid}} = \frac{1}{4} \log_2 \left( 1 + \frac{a_2 \Lambda_4}{\kappa S D_n^2 \Lambda_4 + \chi_5 + a_1 \varepsilon \Lambda_4} \right) \]
\[ + \frac{1}{4} \log_2 \left( 1 + \max \left( \frac{a_2 \Lambda_4}{\kappa S D_n^2 \Lambda_4 + \chi_5 + a_1 \varepsilon \Lambda_4}, \frac{a_2 \Lambda_1 \Lambda_3}{a_2 \Lambda_1 \Lambda_3 + \chi_6 \Lambda_3 + \chi_8 \Lambda_3 + \chi_3 \Lambda_3 + \chi_4} \right) \right) \]
\[ + \frac{1}{2} \log_2 \left( 1 + \max \left( \frac{a_1 \Lambda_5}{a_2 \Lambda_5 + \kappa S D_n^2 \Lambda_5 + \chi_1}, \frac{a_2 \Lambda_1 \Lambda_2}{a_2 \Lambda_1 \Lambda_2 + d_1 \Lambda_1 \Lambda_2 + \chi_3 \Lambda_1 + \chi_4} \right) \right) \].

(38)

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<td>(\varepsilon (\text{nid}))</td>
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**FIGURE 3:** Outage probability of users versus transmit SNR in different conditions

The outage probability is always reducing without limited, while the outage probability gradually becomes stable in the non-ideal conditions. The reason is that in the ideal conditions, as the SNR increases, there are more desirable transmission signals, while in the non-ideal conditions, the outage probability is nearly a fixed value due to the CEEs (we will explain the reason in the next diagram). The results show that system performance cannot be always improved by increasing the SNR in non-ideal conditions.

**FIGURE 4:** Outage probability of users versus different non-ideal factors

The outage probability. The results show that the impact of CEEs on system performance is more serious than that of RHIs or ipSIC.

**FIGURE 5** illustrates the impact of the ipSIC parameter on the system performance when SNR = 20dB. For \(D_f\), we can see that the outage probability is always constant whether for the ideal or non-ideal conditions which is due to that SIC has no effect on the far users, which is determined by the definition of SIC. For \(D_n\), when the system is in the ideal conditions, the outage probability of the considered system is independent of the ipSIC parameter, thus the image of the outage probability is a parallel line. When the system is in the non-ideal conditions, the outage performance deteriorates with the ipSIC parameter increasing before \(\varepsilon < 0.2\). From **FIGURE 5**, we can see that when \(\varepsilon = 0.2\), the outage probability is close to 1, which is due to the conditions.
It is noticed from FIGURE 7 that in the non-ideal conditions, the transmit SNR both in ideal and non-ideal conditions.

On the other hand, the performance is more sensitive to CEEs than RHIs on the system. This phenomenon indicates that the system is very obvious. This phenomenon indicates that the system performance is more sensitive to CEEs than RHIs on the other hand.

When $\sigma$ increases from 0 to 0.2, the change of outage probability of $D_n$ and $D_f$ versus RHIs and CEEs with SNR = 20dB. From FIGURE 6, we can see that the outage probability image of $D_f$ is above $D_n$ due to the large power allocation. From FIGURE 6, we can see that whether $D_f$ or $D_n$, for a certain $\sigma$, when $\kappa$ linearly increases from 0 to 0.2, the change of the outage probability is less pronounced. When $\sigma$ increases from 0 to 0.2, for a certain $\kappa$, the change of outage probability of $D_f$ and $D_n$ is very obvious. This phenomenon indicates that the system performance is more sensitive to CEEs than RHIs on the other hand.

FIGURE 7 investigates ergodic sum rate of the users versus RHIs and CEEs on the large power allocation. From FIGURE 6, we can see that the outage probability image of $D_f$ is above $D_n$ due to the large power allocation. From FIGURE 6, we can see that whether $D_f$ or $D_n$, for a certain $\sigma$, when $\kappa$ linearly increases from 0 to 0.2, the change of the outage probability is less pronounced. When $\sigma$ increases from 0 to 0.2, for a certain $\kappa$, the change of outage probability of $D_f$ and $D_n$ is very obvious. This phenomenon indicates that the system performance is more sensitive to CEEs than RHIs on the other hand.

FIGURE 7 investigates ergodic sum rate of the users versus RHIs and CEEs on the near users versus RHIs and CEEs with SNR = 20dB. From FIGURE 6, we can see that the outage probability image of $D_f$ is above $D_n$ due to the large power allocation. From FIGURE 6, we can see that whether $D_f$ or $D_n$, for a certain $\sigma$, when $\kappa$ linearly increases from 0 to 0.2, the change of the outage probability is less pronounced. When $\sigma$ increases from 0 to 0.2, for a certain $\kappa$, the change of outage probability of $D_f$ and $D_n$ is very obvious. This phenomenon indicates that the system performance is more sensitive to CEEs than RHIs on the other hand.

FIGURE 7 shows that the energy efficiency of the considered system versus transmit SNR in the ideal and non-ideal conditions. As can be observed from FIGURE 8, the energy efficiency is almost completely coincident and very negligible in the both ideal and non-ideal conditions when the SNR is lower than 10dB, while with the SNR increasing, the gap of energy efficiency between ideal and non-ideal conditions is noticeable.

$2 \sigma > 2 \sigma \gamma_{thn}$ and $2 \sigma \gamma_{thn}$ that are mentioned in Section 3 are not satisfied.

$0.05 \, 0.1 \, 0.15 \, 0.2 \, 10^{-4} \, 10^{-3} \, 10^{-2} \, 10^{-1} \, 10^0 \, \kappa$
conditions becomes large. This situation is closely related to the outage probability. In the non-ideal conditions, when the system is in the high SNR region, there is an upper bound for the energy efficiency, which is the result of the joint action of RHIs, CEEs, and ipSIC.

VII. CONCLUSION

In this paper, we investigate the impact of NOMA AF relaying networks under the influence of RHIs, CEEs and ipSIC. The exact and asymptotic outage probability expressions are derived. Furthermore, the ergodic sum rate and the energy efficiency are also investigated. It is shown that RHIs, CEEs, and ipSIC all have a significant negative impact on system performance. In particular, we can observe that the effect of CEEs on the considered system outage performance is more obvious than that of RHIs. It can also be seen that the ergodic sum rate and energy efficiency have upper bounds, which means that the system performance will not be improved with the increase of SNR due to the existence of non-ideal factors.

APPENDIX A: PROOF OF THEOREM 1

Substituting (13) and (14) into (17), the outage probability of \( D_f \) can be rewritten as

\[
P_{out}^{D_f} = \Pr\left[ \frac{a_1 \rho_{SD_f} \gamma}{(a_2 + c_{SD_f}) \rho_{SD_f} \gamma + (k_{SD_f} + 1) c_{SD_f} \gamma + 1} \leq \gamma_{thf} \right] \\
\times \left[ 1 - \Pr\left( \frac{\rho_{SR} \rho_{RD_f} a_1 \gamma'}{(\rho_{SR} a_2 + d_1) \gamma' + \rho_{RD_f} \gamma' + \rho_{SR} \gamma' + \varphi_3} \geq \gamma_{thf} \right) \right].
\]

(A.1)

\( I_1 \) and \( I_2 \) are calculated as follows:

\[
I_1 = \Pr\left( \rho_{SD_f} < \frac{(k_{SD_f} + 1) \alpha_{e_s} \gamma_{thf} \gamma + \gamma_{thf}}{a_1 \gamma - (a_2 + k_2) \gamma_{thf}} \leq \theta_2 \right)
\]

\[
= b_f \sum_{z=0}^{M-f} \binom{M-f}{z} \left( \frac{-1}{f+z} \right) \left[ 1 - \sum_{g_1=0}^{\alpha_{e_1} d_1} e^{-\frac{g_1}{c_{SD_f}}} \left( \frac{\theta_2}{c_{SD_f}} \right)^{g_1+f+z} \right]^{g_1+f+z}.
\]

(A.2)

\[
I_2 = 1 - \Pr\left( \frac{\rho_{SR} \rho_{RD_f} a_1 \gamma'}{(\rho_{SR} a_2 + d_1) \gamma' + \rho_{RD_f} \gamma' + \rho_{SR} \gamma' + \varphi_3} \geq \gamma_{thf} \right)
\]

\[
= 1 - \Pr\left( \frac{\rho_{SR}}{\rho_{RD_f}} \geq \frac{\rho_{SR} \gamma' + \varphi_3}{\theta_1} \frac{\theta_1}{\rho_{SR} - \theta_1} \gamma' \frac{\varphi_2}{\gamma_2} \right)
\]

\[
= 1 - \int_{\theta_1}^{\infty} f_{\rho_{SR}}(y) dy \int_{\frac{\gamma_{thf} a_1 \gamma'}{(\gamma_{thf} + 1) \gamma' + \rho_{RD_f} \gamma' + \rho_{SR} \gamma' + \varphi_3}}^{\gamma_{thf}} f_{\gamma'}(x) dx
\]

\[
= 1 - \frac{2}{\Gamma(\alpha_{SR})} \frac{\theta_1}{\rho_{SR}} e^{-\frac{\theta_1}{\rho_{SR}}} \left[ \frac{\alpha_{SR} - 1}{\alpha_{SR}} \sum_{g_2=0}^{\alpha_{SR} - 1} \frac{1}{g_2!} \sum_{u=0}^{\alpha_{SR} - 1} \left( \frac{\theta_1}{\rho_{SR}} \right)^u \right]
\]

\[
\times \left( \sum_{q=0}^{g_2} \frac{g_2}{q} \frac{\theta_{1}^{g_2+1}}{\theta_{1}^{g_2+1}} + \frac{1}{\beta_{RD_f} \gamma_2} \right)^{q-\gamma}\gamma_{thf}^{\gamma_{thf}}.
\]

(A.3)

Combining (A.2) and (A.3), we can end the proof.

APPENDIX B: PROOF OF THEOREM 2

Substituting (11), (12), (15) and (16) into (19), the outage probability of \( D_n \) can be written as

\[
P_{out}^{D_n} = I_3 \times I_4,
\]

(B.1)

where \( I_3 \) and \( I_4 \) can be further expressed as

\[
I_3 = 1 - \Pr\left( \frac{\rho_{SR} \gamma' + \gamma_2}{\rho_{RD_n}} \geq \frac{(\rho_{SR} \gamma' + \gamma_2) \lambda_1}{(\rho_{SR} - \lambda_1) \gamma' \varphi_4} \right)
\]

\[
= 1 - \Pr\left( \frac{\rho_{SR} \gamma' + \gamma_2}{\rho_{RD_n}} \geq \frac{(\rho_{SR} \gamma' + \gamma_2) \lambda_2}{(\rho_{SR} - \lambda_2) \gamma' \varphi_4} \right)
\]

\[
= 1 - \int_{\lambda}^{\infty} f_{\rho_{SR}}(y) dy \int_{\frac{\lambda \gamma' + \gamma_2}{(\lambda - \gamma') \gamma' + \rho_{RD_n} \gamma' + \rho_{SR} \gamma'}^{\gamma_{thf}} f_{\gamma'}(x) dx
\]

\[
= 1 - \frac{2}{\Gamma(\alpha_{SR})} \frac{\theta_1}{\rho_{SR}} e^{-\frac{\theta_1}{\rho_{SR}}} \left[ \frac{\alpha_{SR} - 1}{\alpha_{SR}} \sum_{g_3=0}^{\alpha_{SR} - 1} \frac{1}{g_3!} \sum_{u=0}^{\alpha_{SR} - 1} \left( \frac{\theta_1}{\rho_{SR}} \right)^u \right]
\]

\[
\times \left( \sum_{t=0}^{g_3} \frac{g_3}{t} \frac{\lambda \gamma' + \gamma_2}{(\lambda - \gamma') \gamma' + \rho_{RD_n} \gamma' + \rho_{SR} \gamma'} \right)^{t-\gamma} \frac{1}{\beta_{RD_n} \gamma_4} \gamma_{thf}^{\gamma_{thf}}.
\]

(B.2)

\[
I_4 = 1 - \Pr\left( \rho_{SD_n} > \tau_1, \rho_{SD_n} > \tau_2 \right)
\]

\[
= 1 - \Pr\left( \rho_{SD_n} > \max(\tau_1, \tau_2) \right)
\]

\[
= b_n \sum_{z=0}^{M-n} \binom{M-n}{z} \left( \frac{-1}{n+z} \right) \left[ 1 - \sum_{g_4=0}^{\alpha_{e_4} d_4} e^{-\frac{g_4}{c_{SD_n}}} \left( \frac{\tau}{c_{SD_n}} \right)^{g_4+n+z} \right]^{g_4+n+z}.
\]

(B.3)
The reasoning processes of $I_3$ and $I_4$ are similar to that of $I_1$ and $I_2$. After some mathematical calculations, we can obtain (20).

**APPENDIX C: PROOF OF COROLLARY 1**

Based on (17), the asymptotic expression of outage probability of $D_f$ in the ideal conditions can be expressed as

$$P_{D_f}^{\infty, id} = \Pr(RSD_n < \theta_2) \left[ 1 - \Pr\left( \frac{\rho_{SR} \rho_{RD_f} \gamma' \varphi_2 + \varphi_3}{\rho_{SR} \varphi_1 + \rho_{RD_f} \gamma' \varphi_2 + \varphi_3} > b_1 \right) \right].$$

(C.1)

Using the inequality $xy/(1 + x + y) < \min(x, y)$, $I_2^\infty$ can be rewritten as

$$I_2^\infty = 1 - \Pr\left( \frac{\rho_{SR} \gamma' \varphi_1, \rho_{RD_f} \gamma' \varphi_2}{\varphi_3} > \frac{b_1 \varphi_1 \varphi_2}{\varphi_3} \right) \approx F_{\rho_{SR}}^{\infty}\left( \frac{\varphi_2 b_1}{\gamma'} \right) + F_{\rho_{RD_f}}^{\infty}\left( \frac{\varphi_1 b_1}{\gamma'} \right).$$

(C.2)

Substituting (C.4) into (C.1), using (21) and (22), after some mathematical calculations, we can obtain (23).

In the non-ideal conditions, $I_1^\infty$ and $I_2^\infty$ can be expressed as

$$I_1^\infty = \Pr(RSD_n < \theta_2'),$$

(C.3)

$$I_2^\infty = 1 - \Pr(\rho_{SR} \geq \theta_1', \rho_{RD} \geq \frac{(\rho_{SR} + \sigma_{SR}^2) \theta_1' \sigma_{RD}^2}{\rho_{SR} - \theta_1' \sigma_{SR}^2}).$$

(C.4)

Substituting (10) into (C.3) and substituting (8), (10) into (C.4), after some mathematical calculations, we can obtain (24).

**APPENDIX D: PROOF OF COROLLARY 2**

Substituting (11), (12), (15) and (16) into (19), the asymptotic expression of outage probability of $D_n$ in the ideal conditions is given at the top of the next page.

Using the inequality $xy/(1 + x + y) < \min(x, y)$ [45], $I_3^\infty$ can be rewritten as

$$I_3^\infty = 1 - \Pr\left( \min\left( \frac{\rho_{SR} \gamma' \varphi_5, \rho_{RD_n} \gamma' \varphi_4}{\varphi_6} \right) > \frac{\varphi_4 \varphi_5 b_2}{\varphi_6}, \min\left( \frac{\rho_{SR} \gamma' \varphi_5, \rho_{RD_n} \gamma' \varphi_4}{\varphi_6} \right) > \frac{\varphi_4 \varphi_5 b_3}{\varphi_6} \right) \approx F_{\rho_{SR}}^{\infty}(\psi) + F_{\rho_{RD_n}}^{\infty}(\xi).$$

(D.2)

Substituting (D.2) into (D.1), using (21) and (22), after some mathematical calculations, we can obtain (25).

In the non-ideal conditions, $I_3^\infty$ and $I_4^\infty$ can be expressed as

$$I_3^\infty = 1 - \Pr(\rho_{SR} > \lambda'_1, \rho_{RD_n} > \frac{(\rho_{SR} + \sigma_{SR}^2) \sigma_{RD}^2 \lambda'_1}{(\rho_{SR} - \lambda'_1) \sigma_{SR}^2});$$

$$I_4^\infty = 1 - \Pr(\rho_{SD_n} > \tau'_1, \rho_{RD_n} > \frac{(\rho_{SR} + \sigma_{SR}^2) \sigma_{RD}^2 \lambda'_2}{(\rho_{SR} - \lambda'_2) \sigma_{SR}^2}).$$

(D.3)

Substituting (7), (8) into (D.3) and substituting (10) into (D.4), after some mathematical calculations, we can obtain (26).

**APPENDIX E**

Substituting (13) and (14) into (27), the ergodic rate of $D_f$ in the non-ideal conditions can be expressed as

$$R_{nave}^{\text{id}} = \frac{1}{2} \log_2 \left[ 1 + \max \left( \frac{a_1 \rho_{SD_f} \gamma^2}{\rho_{SR} \rho_{RD_f} \gamma^2 \sigma_{RD}^2}, \frac{a_2 + \kappa_{SD}^2}{(a_2 + d_1) \gamma^4 + \rho_{RD_f} \gamma^2 \sigma_{SD}^2 + \rho_{SR} \gamma^2 \phi_1 + \phi_3^2} \right) \right].$$

(E.1)

Following the inequality [46]

$$\mathbb{E}\left[ \log_2 \left( 1 + \frac{x}{y} \right) \right] \approx \log_2 \left( 1 + \frac{\mathbb{E}(x)}{\mathbb{E}(y)} \right),$$

(E.2)

(E.1) can be rewritten as

$$R_{nave}^{\text{id}} = \frac{1}{2} \log_2 \left( 1 + \max \left[ \frac{a_1 \mathbb{E}[\rho_{SD_f}] }{(a_2 + \kappa_{SD_f}^2) \mathbb{E}[\rho_{SD_f}] + \frac{\kappa_{SD_f}^2 + 1}{\sigma_{SD_f}^2} \gamma + 1}, \mathbb{E}[\rho_{SR}] \mathbb{E}[\rho_{RD_f}] \frac{a_1 \gamma^2}{(a_2 + d_1) \gamma^4 + \mathbb{E}[\rho_{RD_f}] \gamma^2 \sigma_{SD}^2 + \mathbb{E}[\rho_{SR}] \gamma^2 \phi_1 + \phi_3^2} \right] \right).$$

(E.3)

Defining $\Lambda_1 \triangleq \mathbb{E}[\rho_{SR}], \Lambda_2 = \mathbb{E}[\rho_{RD_n}]$ and using [47, Eq. 3.478.1], therefore $\Lambda_1$ and $\Lambda_2$ can be expressed as

$$\Lambda_1 = \int_0^\infty x f_{\rho_{SR}}(x) dx = \int_0^\infty x^\alpha \beta_{\rho_{SR}} e^{-\frac{x}{\beta_{\rho_{SR}}}} dx = (\alpha + 1) \beta_{\rho_{SR}},$$

(E.4)
\[
P_{D_n}^{\infty, id} = \Pr \left( \frac{\rho_{SD_n}}{\Gamma} < \tau \right) \times \left[ 1 - \Pr \left( \frac{\rho_{SR} \rho_{RD_n} \gamma' \phi_4 + \rho_{SR} \gamma_5 + \phi_6}{\rho_{RD_n}} > b_2, \frac{\rho_{SR} \rho_{RD_n} \gamma' \phi_4 + \rho_{SR} \gamma_5 + \phi_6}{\rho_{RD_n}} > b_3 \right) \right].
\]

\[
\Lambda_2 = \int_0^\infty x f_{\rho_{RD_f}}(x) dx
\]
\[
= \frac{1}{\Gamma(a_{RD_f})} \frac{\beta_{RD_f}^{-1}}{\beta_{RD_f}} \left[ 1 - \sum_{g_2=0}^{\alpha_{RD_f} - 1} \frac{e^{-\beta_{RD_f}} \left( \frac{x}{\beta_{RD_f}} \right)^{g_2}}{g_2!} \right] f^{-1} \times e^{-\beta_{RD_f}} \left[ \sum_{g_2=0}^{\alpha_{RD_f} - 1} \frac{e^{-\beta_{RD_f}} \left( \frac{x}{\beta_{RD_f}} \right)^{g_2}}{g_2!} \right] dx
\]
\[
= \frac{b_f \Gamma(2)}{\Gamma(a_{RD_f})} \frac{1}{\beta_{RD_f}} \sum_{t=0}^{f-1} \left( \frac{1}{t!} \right) (-1)^t \sum_{p_0 + \cdots + p_{\alpha_{RD_f}} - 1 = c_1} \left( c_1 \right) ! \prod_{g_2=0}^{c_1} \left( \frac{1}{g_2!} \right) (c_1 + 1) ! \beta_{RD_f} \left( \frac{1}{\beta_{RD_f}} \right) \left( \frac{1}{\rho_{RD_f}} \right) x^{c_1} \frac{d}{d^c_1}
\]

Similarly, we can obtain \( \Lambda_3, \Lambda_4, \) and \( \Lambda_5 \) in the same way, respectively. Substituting \( \Lambda_1, \Lambda_2 \) and \( \Lambda_5 \) into (E.3), we can obtain (31).

**APPENDIX F**

Substituting (12) and (16) into (28), and using the inequality \( xy / (1 + x + y) < \min(x, y) \), we can obtain

\[
R_{\text{ave}}^{n, id} = E \left[ \frac{1}{2 \ln 2} \ln \left( 1 + \max \left[ \gamma_{SD_n}, \gamma_{RD_n} \right] \right) \right]
\]
\[
< \frac{1}{2 \ln 2} \left( \ln \left( 1 + a_2 \max \left[ \rho_{SD_n} \gamma, \min \left( \rho_{SR} \gamma, \rho_{RD_n} \gamma \right) \right] \right) \right) .
\]

Denote \( W = \max \left[ \rho_{SD_n} \gamma, \min \left( \rho_{SR} \gamma, \rho_{RD_n} \gamma \right) \right] \), according to the knowledge of probability theory, we can get

\[
F_W (w) = \Pr \left( \max \left[ \rho_{SD_n} \gamma, \min \left( \rho_{SR} \gamma, \rho_{RD_n} \gamma \right) \right] \leq w \right)
\]
\[
= \Pr \left( \rho_{SD_n} \gamma \leq w \right) \left[ 1 - \Pr \left( \rho_{SR} \gamma > \frac{w}{\gamma} \right) \right] \left( \rho_{RD_n} \gamma > \frac{w}{\gamma} \right)
\]
\[
= b_n \sum_{z=0}^{M-n} \left( \frac{M-n}{z} \right) (-1)^t \left[ \sum_{g_1=1}^{\alpha_{RD} - 1} \frac{e^{-\beta_{SD_n} \gamma} \left( \frac{w}{\beta_{SD_n} \gamma} \right)^{g_1}}{g_1!} \right] \sum_{g_2=0}^{\alpha_{SD} - 1} \frac{e^{-\beta_{SD_n} \gamma} \left( \frac{w}{\beta_{SD_n} \gamma} \right)^{g_2}}{g_2!}
\]
\[
\times e^{-\beta_{SD_n} \gamma} \left( \frac{w}{\beta_{SD_n} \gamma} \right) \left( \frac{1}{\beta_{RD_n} \gamma} \right) \left( \frac{1}{\beta_{RD_n} \gamma} \right) \left( \frac{1}{\beta_{RD_n} \gamma} \right) \left( \frac{1}{\beta_{RD_n} \gamma} \right) \left( \frac{1}{\beta_{RD_n} \gamma} \right) \left( \frac{1}{\beta_{RD_n} \gamma} \right) \left( \frac{1}{\beta_{RD_n} \gamma} \right) \left( \frac{1}{\beta_{RD_n} \gamma} \right) \left( \frac{1}{\beta_{RD_n} \gamma} \right) \left( \frac{1}{\beta_{RD_n} \gamma} \right) \left( \frac{1}{\beta_{RD_n} \gamma} \right) \left( \frac{1}{\beta_{RD_n} \gamma} \right)
\]

Base on (F.3) and (F.2), \( R_{\text{ave}}^{n, id} \) can be rewritten as

\[
\Phi_1 = \sum_{g_1=0}^{\alpha_{SD} - 1} \sum_{g_2=0}^{\alpha_{SD} - 1} \sum_{q=0}^{1} \sum_{z=0}^{M-n} \frac{1}{g_1} e^{-\beta_{SD_n} \gamma} \left( \frac{w}{SD_n \gamma} \right)^{q_1} \frac{1}{g_2} e^{-\beta_{SD_n} \gamma} \left( \frac{w}{SD_n \gamma} \right)^{q_2}
\]

\[
\times \int_0^\infty \frac{e^{-\beta_{SD_n} \gamma} \left( \frac{w}{SD_n \gamma} \right)^{q_3}}{1 + a_2 w} dw,
\]

\[
\Phi_2 = \sum_{g_1=0}^{\alpha_{RD} - 1} \sum_{g_2=0}^{\alpha_{RD} - 1} \sum_{q=0}^{1} \sum_{z=0}^{M-n} \frac{1}{g_1} e^{-\beta_{SD_n} \gamma} \left( \frac{w}{SD_n \gamma} \right)^{q_1} \frac{1}{g_2} e^{-\beta_{SD_n} \gamma} \left( \frac{w}{SD_n \gamma} \right)^{q_2}
\]

\[
\times \int_0^\infty \frac{e^{-\beta_{SD_n} \gamma} \left( \frac{w}{SD_n \gamma} \right)^{q_3}}{1 + a_2 w} dw,
\]

\[
\Phi_3 = \sum_{q=1}^{M-n} \left( \frac{M-n}{q} \right) (-1)^q \sum_{g_1=0}^{\alpha_{SD} - 1} \sum_{g_2=0}^{\alpha_{SD} - 1} \frac{1}{g_1} e^{-\beta_{SD_n} \gamma} \left( \frac{w}{SD_n \gamma} \right)^{q_1} \frac{1}{g_2} e^{-\beta_{SD_n} \gamma} \left( \frac{w}{SD_n \gamma} \right)^{q_2}
\]

\[
\times \int_0^\infty \frac{e^{-\beta_{SD_n} \gamma} \left( \frac{w}{SD_n \gamma} \right)^{q_3}}{1 + a_2 w} dw,
\]

where \( \vartheta = (g_1 g_3)^{-1} \left( \beta_{SR} \gamma \right)^{-g_1} \left( \beta_{RD_n} \gamma \right)^{-g_3} \). With the aid of [47, Eq. 3.352.4] and [47, Eq. 3.353.5], after some mathematical calculations, we can obtain the final representation

\[
\Phi_1 = \left\{ \begin{array}{l}
\frac{-1}{a_2} \sum_{g_1=0}^{\alpha_{SD} - 1} \sum_{g_3=0}^{\alpha_{RD} - 1} \vartheta e^{-\vartheta} E^{-1}(f_1 - f_3), l_1 = 0
\end{array} \right.
\]

\[
+ \sum_{L=1}^{L_1} \sum_{g_1=0}^{\alpha_{SD} - 1} \sum_{g_3=0}^{\alpha_{RD} - 1} \left[ (L-1) \vartheta e^{-\vartheta} E^{-1}(f_1 - f_3), l_1 > 0
\end{array} \right.
\]

\[
(7.7)
\]
where $f_1 = (a_2\beta SD \gamma)^{-1}$, $f_2 = (a_2\beta RD \gamma)^{-1}$, $f_3 = (a_2\beta SD n)^{-1}$, $l_1 = g_1 + g_3$, $l_2 = g_1 + g_3 + g_4 p g_4$, and $\text{Ei} (\cdot)$ denotes the exponential integral functions. $\Xi_1$ and $\Xi_2$ are expressed as

$$\begin{align*}
\Xi_1 &= \alpha_{in} R_D n^{-1} M - n \sum_{q=0}^{M-n} \binom{M-n}{q} (-1)^q \frac{1}{a_2} l_{q+1} \\
&\quad \times \sum_{p=0}^{\alpha_{in} R_D n^{-1} q} \binom{q}{p} \prod_{g=0}^{\alpha_{in} R_D n^{-1} - q} \frac{1}{g!} \, . \tag{F.10}
\end{align*}$$

Substituting (F.7) into (F.3), we can obtain $R_{ave}^{n2, id}$. Similarly, $R_{ave}^{n2, id}$ can be expressed as follows

$$R_{ave}^{n2, id} = E \left[ \frac{1}{2} \log_2 (1 + a_2 p SD \gamma) \right] = \frac{a_2 \gamma}{2 \ln 2} \int_0^\infty \frac{1 - F_{SD n}(x)}{1 + a_2 \gamma x} dx \\
= -\frac{b_n}{2 \ln 2} \sum_{z=0}^{M-n} \binom{M-n}{z} \frac{1}{z+2} \Phi_4 \, . \tag{F.12}
$$

Following [47, Eq. 3.352.4] and [47, Eq. 3.353.5], $\Phi_4$ can be further expressed as

$$\Phi_4 = \left\{ \begin{array}{ll}
\sum_{L=1}^{g_4 p g_4} (L-1)! (-1)^{g_4 p g_4 - L} f_2^L & \\
+(-1)^{g_4 p g_4 - 1} e^{f_3 f} (-f_3) g_4 p g_4 & > 0
\end{array} \right. \, . \tag{F.13}
$$

$\Xi_3$ is expressed as

$$\Xi_3 = \sum_{t=1}^{n+2} (-1)^t \sum_{p_0+\cdots+p_{\alpha_{in} R_D n^{-1}}=t} \binom{t}{p_0, \ldots, p_{\alpha_{in} R_D n^{-1}}} \times \prod_{g=0}^{\alpha_{in} R_D n^{-1}} \left( \frac{1}{g!} \right) g_4 p g_4 \, . \tag{F.14}
$$

Combining the above formulas, we can end the proof.

**REFERENCES**


