Novel Analytical Method for Overhang Effects in Surface-Mounted Permanent-Magnet Machines

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ABSTRACT

The torque and power densities of surface-mounted permanent-magnet (SPM) machines can be improved significantly using an overhang structure. However, it is difficult to analyze and design SPM machines with overhang because a 3D finite-element method (FEM) that uses high computation time is required to consider the overhang effects accurately. To reduce the computation time significantly, we propose an analytical method in which a magnetic equivalent circuit (MEC) model and an analytical solution (AS) of the Laplace and Poisson equations are combined. This method can accurately estimate the performances of SPM machines with overhang as well as significantly reduce the computation time compared with that required for the 3D FEM. The accuracy and generality of the proposed method were verified by comparison with the 3D FEM in different cases.

INDEX TERMS

Analytical method, magnetic equivalent circuit (MEC) model, overhang effect, overhang structure, surface-mounted permanent-magnet (SPM) machine.

I. INTRODUCTION

In a radial-flux machine, when the rotor stack length is longer than the stator stack length, as shown in Fig. 1, it is referred to as an overhang. By adopting an overhang structure that provides additional sources and paths of magnetic flux, machine performance parameters such as torque and power densities can be enhanced effectively without expanding the size of the machine [1], [2]. This is so because the space used up by windings is utilized by adopting a rotor overhang structure. Because of these advantages, overhang structures have been used in various types of radial-flux machines such as claw pole machines [3], spoke-type permanent-magnet (PM) machines [4]–[6], surface-mounted PM (SPM) machines [7]–[9], and interior PM (IPM) machines [2].

A 3D finite-element method (FEM) is essential for the accurate assessment of overhang effects, which cause non-uniform magnetic flux distributions in the axial direction. However, the FEM is computationally expensive. Recently, some studies on analysis methods for SPM machines with overhang have been conducted with the goal of reducing computational time. Firstly, a computational method which was a 2-D FEM combined with the modified permeance...
coefficient of a PM to consider overhang effects was proposed in [6], [7]. Although this method is simple and effective, a 3D FEM is necessary for calculating the modified coefficient. Moreover, the 2-D FEM is still time-consuming. Another method is the magnetic equivalent circuit (MEC) model, which involves reluctances and magnetic flux sources based on the flux paths and material properties of machines. The magnetic flux of an MEC can be calculated easily using Kirchhoff’s voltage and current laws. The performance of a machine can be predicted using the calculated air-gap magnetic flux. An MEC with simplified components has been widely used for various types of machines such as transverse flux machines [10], SPM machines [11]–[16], IPM machines [17]–[20], axial-flux PM (AFPM) machines [21], [22], flux-switching PM machines [23], reluctance machines [24], and induction machines [25], [26] with the aim of reducing computational costs significantly. There have also been some attempts to use MECs to consider the overhang effects of SPM machines [27], [28]. In [27], an effective air-gap length between the rotor overhang and stator core was defined for such an MEC. Thus, the average flux densities within the air gap and PM were calculated accurately. However, its applicability was limited to linear SPM machines. In [28], an MEC considering an effective overhang length was proposed. Although the overhang effects of an SPM machine can simply be predicted by the MEC, the accuracy of the MEC might be low in cases where the effective overhang length cannot be appropriately defined. Moreover, the minute magnetic flux distributions are rarely calculated by the MEC because it is composed of simplified components. Therefore, accurate and more generally applicable analysis methods are still required for the analysis and design of SPM machines with overhang.

To calculate minute magnetic flux distributions and predict machine performances accurately, an analytical solution of the Laplace and Poisson equations (AS) has been popularly utilized for various machines such as flux switching machines [29], transverse flux machines [30], SPM machines [31]–[37], IPM machines [38], AFPM machines [39], [40], reluctance machines [41], [42], induction machines [43]. As shown in the aforementioned studies, an AS can be applied to practical machines with complex configurations. However, there have been few studies using an AS because it is difficult to directly calculate complex magnetic flux distributions resulting from overhang through it.

To overcome the aforementioned limitations of using an MEC or an AS when considering overhang effects, we proposed an analytical method (AM) which combined the two methods. The proposed AM can calculate the minute magnetic flux distributions of an SPM machine with overhang precisely as well as reduce computational costs significantly. To verify the accuracy and wide applicability of the proposed AM, the results were compared with the 3D FEM results for various cases.

II. ANALYTICAL METHOD FOR SPM MACHINE WITH OVERHANG STRUCTURE

We proposed an analytical method (AM) effectively considering the overhang effects of SPM machines. The proposed AM combines an analytical solution of the Laplace and Poisson equations (AS) and a magnetic equivalent circuit (MEC) model. Utilizing both the AS and the MEC model, the proposed AM can accurately calculate the magnetic flux distributions of SPM machines with overhang as well as effectively reduce computational costs.

To simplify the proposed AM, it is assumed that PMs are radially magnetized and there is no magnetic saturation in the rotor and stator cores. The procedure to assess the performance of SPM machines with overhang consists three steps: 1) estimating the performance of SPM machines without overhang using the AS, 2) calculating the performance enhancement effected by adopting an overhang structure using the MEC, 3) predicting the performances of SPM machines with overhang by combining the results of the AS and MEC. The overhang effects of SPM machines can be estimated accurately and efficiently using this procedure.

A. ANALYTICAL SOLUTION FOR SPM MACHINE WITHOUT OVERHANG STRUCTURE

The back EMF is an essential quantity when predicting machine performance levels. Because the back EMF can be directly calculated from the air-gap magnetic flux density distribution, it is essential to evaluate the air-gap magnetic flux density distribution accurately.

The air-gap magnetic flux density distribution of a slotless SPM machine without overhang in the case of radial magnetization can be precisely calculated via the AS presented in an earlier study [29] as follows:

\[
B_g = \frac{4}{\mu_r} \sum_{n=1,3,5,...}^{\infty} \frac{B_n}{n\pi} \sin \left( \frac{n\pi p}{2} \right) \frac{2n \cos(n\theta)}{(np)^2 - 1} \times \left[ \left( \frac{R_e}{R_i} \right)^{np+1} \left( \frac{R_i}{R_e} \right)^{np+1} + \left( \frac{R_m}{R_e} \right)^{np+1} \right] \times \left[ \frac{np + 2 \left( \frac{R_i}{R_e} \right)^{np+1} \left( \frac{R_m}{R_e} \right)^{np+1} - (np + 1) \left( \frac{R_i}{R_e} \right)^{2np} \left( \frac{R_m}{R_e} \right)^{2np} \right] \right]
\]

where \(B_n\) is the magnet remanence, \(\mu_r\) is the relative recoil permeability of the magnet, \(\rho_p\) is the pole-arc-to-pole-pitch ratio, \(r\) is the radius in the middle of the air gap, and \(p\) is the number of pole pairs. \(R_m\), \(R_e\), and \(R_i\) are the radii of the magnet’s outer surface, rotor’s outer surface, and stator’s inner surface, respectively.
Although most practical SPM machines have a slotted stator structure, slot effects cannot be considered using (1). In this study, the relative air-gap permeance that was calculated from the conformal mapping (CM) of the slot geometry as presented in [32] was combined with (1) to estimate the slot effects of SPM machines accurately. In other words, the air-gap flux density and back EMF in a slotted SPM machine without overhang were calculated accurately by using both the AS and CM, as shown in Fig. 2.

### B. MEC FOR SPM MACHINE WITH OVERHANG STRUCTURE

In this section, we propose a simple and effective MEC for calculating the air-gap flux density and back EMF of SPM machines with overhang structures. Some assumptions are made for the simplicity of the proposed MEC: 1) PMs are radially magnetized, 2) no magnetic saturation occurs in the rotor and stator cores, 3) the stator core is slotless.

The main flux path and two leakage flux paths of an SPM machine are shown on the left side of Fig. 3. $\Phi_s$ is the main flux flowing from the rotor core to the stator core through the air gap. One of the leakage fluxes is the magnet-to-magnet leakage flux $\Phi_{mm}$, which travels between two adjacent magnets. The other leakage flux is the magnet-to-rotor leakage flux $\Phi_{mr}$, which travels from the magnet to the rotor core.

An SPM machine with overhang is divided into non-overhang and overhang parts as shown in Fig. 4. The air-gap flux density of the non-overhang part can be calculated accurately using an MEC and considering the aforementioned flux paths. The proposed MEC for the non-overhang part is shown in Fig. 5(a); it is based on [11]. $\Phi_r$ is the flux source of one magnet pole and $\Phi_m$ is the flux leaving the magnet. $\mathcal{R}_g$ is the reluctance of the air gap, and $\mathcal{R}_{mo}$ is the reluctance of the magnet. $\mathcal{R}_{mm}$ and $\mathcal{R}_{mr}$ are the reluctances corresponding to $\Phi_{mm}$ and $\Phi_{mr}$, respectively. $\mathcal{R}_r$ and $\mathcal{R}_s$ are the reluctances of the rotor and stator cores, which are neglected owing to the assumption that no magnetic saturation occurs in the cores. These reluctances are calculated as follows:

\[
\mathcal{R}_g = \frac{g}{\mu_0 (w_{mst} + 2g) l_{st}}, \quad (2)
\]

\[
\mathcal{R}_{mo} = \frac{l_m}{\mu_0 \mu_r w_{mst} l_{st}}, \quad (3)
\]

\[
\mathcal{R}_{mm} = \frac{\pi}{\mu_0 l_{st} \ln \left(1 + \pi g/w_{fo}\right)}, \quad (4)
\]

\[
\mathcal{R}_{mr} = \frac{\pi}{\mu_0 l_{st} \ln \left(1 + \pi \min\left(g, \frac{w_{fo}}{2}\right)/l_m\right)}. \quad (5)
\]
where

\[
\begin{align*}
    w_{mi} &= \frac{(R_e + R_m)}{2} \frac{\pi \alpha_p}{p} \quad \text{(6)} \\
    w_{m} &= \frac{(R_e + R_m)}{2} \frac{\pi (1 - \alpha_p)}{p} \quad \text{(7)} \\
    w_{mo} &= \frac{(R_e + R_m)}{2} \frac{\pi \alpha_p}{p} \quad \text{(8)} \\
    w_{fo} &= \frac{(R_e + R_m)}{2} \frac{\pi (1 - \alpha_p)}{p} \quad \text{(9)}
\end{align*}
\]

In these equations, \( l_s \) is the stack length, \( l_m \) is the magnet thickness, \( \mu_0 \) is the permeability of the vacuum, and \( g \) is the air-gap length. \( w_{mi} \) and \( w_m \) are the pole-arc width and the width between two adjacent magnets in the middle of the magnet, respectively, and \( w_{mo} \) and \( w_{fo} \) are the pole-arc width and the

Because the MEC shown in Fig. 5(a) is symmetric, it can be simplified as shown in Fig. 5(b) and (c). Therefore, the air-gap flux can be determined by means of flux division as follows:

\[
\Phi_s = \frac{\Phi_{m} \Phi_{r}}{R_m + 4R_e} \quad \text{(10)}
\]

\[
R_m = 4R_{mo} || 2R_{mo} || R_{mo} \quad \text{(11)}
\]

Then, the air-gap flux density can be calculated as follows:

\[
B_g = \frac{w_{mi}/(w_{mo} + w_{fo})}{(R_m + 4R_e)/R_m} B_r \quad \text{(12)}
\]

To consider the overhang effects appropriately, the proposed MEC for the overhang part was as shown in Fig. 6(a).
FIGURE 7. Modeling of the magnetic flux in the air gap between the overhang and stator core. (a) Straight line permeance model for the first part of overhang. (b) Circular arc permeance model for the second part of the overhang.

FIGURE 8. Flux path modeling between magnet and rotor end of SPM machine.

The air-gap length in the non-overhang part is uniform, whereas the air-gap length in the overhang part varies according to the axial position. In [27], the straight line and circular arc permeance models were proposed to calculate the effective air-gap length between the rotor overhang and stator core as shown in Fig. 7. Therefore, the effective air-gap length was defined as follows:

\[
g_e = \begin{cases} 
\sqrt{g^2 + z^2}, & \text{if } 0 < z \leq g \\
\frac{\pi}{2} z, & \text{if } g < z \leq l_{oh}
\end{cases}, \quad (13)
\]

where \( z \) is the height in the overhang part. With the effective air-gap length, the reluctances of the overhang part can be obtained as follows:

\[
R_g = g_e \left( \frac{\mu_0}{2} w_{fo} \right) dz \\
R_{iso} = \frac{l_m}{\mu_0 \mu_i w_{iso} dz} \quad (14)
\]

\[
R_{eoh} = \frac{\pi}{\mu_0 \mu_i w_{eoh} dz} \ln \left( 1 + \pi \min \left( \frac{g_e}{w_{fo}}, \frac{w_{fo}}{2} \right) \frac{l_m}{w_{fo}} \right) \quad (15)
\]

\[
R_{mom} = \frac{\pi}{\mu_0 \mu_i w_{mom} dz} \ln \left( 1 + \pi \min \left( \frac{g_e}{w_{fo}}, \frac{w_{fo}}{2} \right) \frac{l_m}{w_{fo}} \right) \quad (16)
\]

\[
R_{mr} = \frac{\pi}{\mu_0 \mu_i w_{mr} dz} \ln \left( 1 + \pi \min \left( \frac{g_e}{w_{fo}}, \frac{w_{fo}}{2} \right) \frac{l_m}{w_{fo}} \right) \quad (17)
\]

where \( dz \) is the infinitesimal stack length in the overhang part.

The end effects that resulted from the leakage flux from the magnet to the rotor end can be ignored in non-overhang SPM machines. However, the end effects become significant when an overhang structure is adopted. Therefore, the magnet-to-rotor-end leakage flux \( \Phi_{mre} \) was considered using the flux path model shown in Fig. 8. The reluctance corresponding to \( \Phi_{mre} \) is calculated as follows:

\[
R_{mre} = \frac{\pi}{\mu_0 \mu_i w_{mre} dz} \left( l_m + 3l_{oh} - 3z \right) \quad (18)
\]

Owing to its symmetry, Fig. 6(a) can be simplified as Fig. 6(b) and (c). As in the case of the non-overhang part, the air-gap flux of the overhang part can be obtained as follows:

\[
\Phi_g = \frac{\mu_0 (l_m + 3l_{oh})}{R_{eoh} + 4R_g} \quad (19)
\]

\[
R_m = 4R_{eoh} || 2R_{mre} || 2R_{mom} \quad (20)
\]

Therefore, the air-gap flux density of the overhang part can be calculated as

\[
B_g = \frac{w_{fo} \left( w_{eoh} + w_{fo} \right)}{(9R_m + 4R_g) R_m} B_r \quad (21)
\]
Table 1. Analysis model specifications of SPM machines without and with overhang.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
</tr>
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<tbody>
<tr>
<td>Number of poles/number of slots</td>
<td>8/24</td>
<td>8/24</td>
<td>8/24</td>
<td>8/24</td>
<td>8/9</td>
</tr>
<tr>
<td>Radius of rotor’s outer surface (R_r), mm</td>
<td>35.5</td>
<td>35.5</td>
<td>35.5</td>
<td>35.5</td>
<td>35.5</td>
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<tr>
<td>Radius of magnet’s outer surface (R_m), mm</td>
<td>38.5</td>
<td>38.5</td>
<td>38.5</td>
<td>38.5</td>
<td>38.5</td>
</tr>
<tr>
<td>Air-gap length (g), mm</td>
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<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Pole-arc-to-pole-pitch ratio (α_p)</td>
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<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Magnet remanence (B_r), T</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>1.07</td>
<td>0.4</td>
</tr>
<tr>
<td>Stack length (l_s), mm</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Overhang length (l_o), mm</td>
<td>0, 2, 4, 6</td>
<td>0, 4, 8, 12</td>
<td>0, 6, 12, 18</td>
<td>0, 4, 8, 12</td>
<td>0, 4, 8, 12</td>
</tr>
</tbody>
</table>

FIGURE 9. Analysis models of SPM machines with overhang structure. (a) Model 1. (b) Models 2 and 4. (c) Model 3. (d) Model 5.

FIGURE 10. Comparison of back EMF in analysis models without overhang. (a) Model 1. (b) Model 2. (c) Model 3. (d) Model 4. (e) Model 5.

The overall air-gap flux density of the SPM machine with overhang can easily be obtained by averaging the air-gap flux densities in the non-overhang and overhang parts. This average value is applied to the air-gap flux density distribution calculated using the AS, as presented in section II-A. In other words, the values of the air-gap flux density distribution are scaled to have the average value estimated by the MEC, as presented in this section. Finally, the scaled air-gap flux density distribution represents that of the SPM machines with overhang.
III. RESULTS AND VERIFICATION

We compared the values of the back EMF obtained using the proposed AM and 3D FEM using five analysis models to verify the validity of the proposed AM. The detailed parameters of the analysis models are listed in Table 1. Fig. 9 shows the analysis models with overhang in which the length of the overhang is equal to 20% of the stack length. Models 1, 2, and 3 are 8-pole/24-slot SPM machines with ferrite magnets of three different stack lengths. Model 4 has the same machine structure as model 2, as shown in Fig. 9(b), but employs a rare-earth magnet. The magnet remanences of the ferrite and rare-earth magnets are 0.4 and 1.07 T, respectively. Model 5 is an 8-pole/9-slot SPM machine with a ferrite magnet. Models 1, 2, 3, and 4 are integer-slot machines, whereas model 5 is a fractional-slot machine with a different stator structure and winding, as shown in Fig. 9(d). All analysis models adopt overhang structures with four different lengths, i.e., 0%, 10%, 20%, and 30% of the stack length. An overhang length equal to 0% of the stack length implies a non-overhang SPM machine.

As shown in Fig. 10, the back EMF waveforms obtained by the proposed AM and 3D FEM are well-matched in all analysis models without overhang. Moreover, Fig. 11 shows that the proposed AM can accurately calculate the back EMF waveforms for all analysis models with large overhang structures, i.e., when the overhang length is equal to 20% of the stack length.

The results obtained using all analysis models are provided in Table 2. They reveal that the overhang effects vary according to the stack lengths of the SPM machines. In other words, the overhang structures that are as long as 20% of the

<table>
<thead>
<tr>
<th>Analysis model</th>
<th>( l_{oh} ) (mm)</th>
<th>3D FEM (V)</th>
<th>AM (V)</th>
<th>Diff* (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0</td>
<td>6.3</td>
<td>6.4</td>
<td>-1.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7.2</td>
<td>7.3</td>
<td>-0.8</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>7.8</td>
<td>7.8</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>8.3</td>
<td>8.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Model 2</td>
<td>0</td>
<td>12.7</td>
<td>12.8</td>
<td>-1.4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>14.2</td>
<td>14.3</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>14.9</td>
<td>14.9</td>
<td>0.1</td>
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<td></td>
<td>12</td>
<td>15.4</td>
<td>15.4</td>
<td>0.0</td>
</tr>
<tr>
<td>Model 3</td>
<td>0</td>
<td>19.0</td>
<td>19.3</td>
<td>-1.4</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>20.9</td>
<td>21.0</td>
<td>-0.6</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>21.7</td>
<td>21.8</td>
<td>-0.4</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>22.0</td>
<td>22.3</td>
<td>-1.3</td>
</tr>
<tr>
<td>Model 4</td>
<td>0</td>
<td>33.9</td>
<td>34.4</td>
<td>-1.4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>37.9</td>
<td>38.2</td>
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<tr>
<td></td>
<td>8</td>
<td>39.9</td>
<td>40.0</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>41.0</td>
<td>41.2</td>
<td>-0.5</td>
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<tr>
<td>Model 5</td>
<td>0</td>
<td>12.7</td>
<td>12.8</td>
<td>-0.8</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>14.3</td>
<td>14.3</td>
<td>0.0</td>
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</tr>
<tr>
<td></td>
<td>12</td>
<td>15.2</td>
<td>15.4</td>
<td>-1.1</td>
</tr>
</tbody>
</table>

Diff*: Difference = (3D FEM – AM)/3D FEM \times 100.
IV. CONCLUSION

Overhang is an effective structure for enhancing the torque and power densities of SPM machines. However, considering the overhang effects of SPM machines efficiently and accurately has been challenging. In this work, an analytical method for considering the overhang effects of SPM machines accurately was proposed. The validity of the proposed method was verified by a 3D FEM for different cases. Therefore, the proposed method can generally be used for the analysis and design of SPM machines with overhang. Further, it can significantly reduce computational time as compared to the 3D FEM.

REFERENCES


Computer used: Intel Core i7-7700K CPU@4.2 GHz, 32 GB RAM. h – hour, min – minute, s – second.

<table>
<thead>
<tr>
<th>Analysis model</th>
<th>$L_o$ (mm)</th>
<th>3D FEM (V)</th>
<th>AM (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0</td>
<td>2 h 4 min</td>
<td>10 s</td>
</tr>
<tr>
<td>Model 2</td>
<td>8</td>
<td>4 h 22 min</td>
<td>10 s</td>
</tr>
<tr>
<td>Model 3</td>
<td>12</td>
<td>6 h 43 min</td>
<td>10 s</td>
</tr>
<tr>
<td>Model 4</td>
<td>8</td>
<td>5 h 19 min</td>
<td>10 s</td>
</tr>
<tr>
<td>Model 5</td>
<td>8</td>
<td>18 h 56 min</td>
<td>10 s</td>
</tr>
</tbody>
</table>

Table 3. Comparison of computation time of back EMF of SPM machines without and with overhang using proposed AM and 3D FEM.


