Minimum Parameters Learning Based Dynamic Surface Control for Advanced Aircraft at High Angle of Attack

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ABSTRACT Aiming at the difficulty of post-stall maneuvering control modeling and control of advanced aircraft under unsteady aerodynamics, a control method with high control accuracy and fast computation speed is proposed based on Radial Basis Function (RBF) network with minimum parameters learning (MPL) and dynamic surface control (DSC) method. Firstly, the aerodynamic characteristics of post-stall maneuvers are analyzed based on the experimental data of large-scale oscillation wind tunnels, and the key factors affecting the unsteady aerodynamic forces are obtained. Then, an accurate unsteady aerodynamic model is established based on the improved extreme learning machine (ELM) method. Secondly, the influence of unsteady aerodynamic forces on the control of post-stall maneuvers is considered. For the uncertainty of advanced aircraft model, high angle of attack flight control laws based on RBF-DSC are designed. In order to improve the calculation speed of the above control law and optimize the parameters, a post-stall maneuver control law method based on MPL-RBF-DSC is designed, and the stability of the method is proved. The coordinated allocation of the conventional aerodynamic surfaces and thrust vectors is realized based on the daisy chain method. Finally, the typical maneuver simulation of “Cobra” is carried out, which highlights the advantages of the design method in this paper, such as high control accuracy, short calculation time and strong robustness.

INDEX TERMS flight control; high angle of attack; wind tunnel test; unsteady aerodynamics modeling; MPL-RBF-DSC method.

I. INTRODUCTION

Air combat simulation shows that advanced fighters with high maneuverability and agility can achieve rapid occupancy, firing of predecessors and effective evasion in the process of short-range air combat, which provides important technical support for enhancing operational efficiency and improving survival probability. Therefore, modern advanced fighters and tactical missiles generally regard high maneuverability and agility as one of their key characteristics. For example, the fourth generation fighter (such as F-22) has post-stall maneuverability, requiring rapid nose pointing ability [1]. The future unmanned combat aircraft will have much higher mobility and agility than the present manned combat aircraft due to the removal of physiological overload restrictions.

When an advanced fighter is maneuvering at high angle of attack, the airflow on the fuselage and wing surface undergoes the process of attachment flow, eddy flow and eddy breakdown [2]. These special gas flow phenomena make the aerodynamic nonlinearity, hysteresis and coupling of the aircraft more prominent, and the aerodynamic force is no longer conventional or quasi-steady aerodynamic force. Hence, it is impossible to obtain the accurate aerodynamic model at high angle of attack from the conventional aerodynamic model. How to obtain aerodynamic data at high angle of attack and establish accurate unsteady aerodynamic model is the primary challenge for advanced fighter during the post-stall maneuver [3][4].

With the development of wind tunnel experimental equipment technology, the biaxial coupled oscillation wind
tunnel experimental equipment at home and abroad has become an important source of aerodynamic data for aircraft at high angle of attack, and the corresponding unsteady aerodynamic modeling methods have become the research hotspot for aircraft flying at high angle of attack [5]. Throughout the development of unsteady aerodynamic modeling methods, unsteady aerodynamic models have evolved from simple algebraic polynomial models to high-precision intelligent models. Typical unsteady aerodynamic modeling methods mainly include algebraic polynomial model, step function model, state space model, Fourier function analysis model, differential equation model and fuzzy logic model, and so on [6]. Based on above methods, the unsteady aerodynamics modeling method has been developed. A novel unsteady model based on indicial function is established for aircraft longitudinal dynamics in [7]. The model involves unsteady state expressed by first-order differential equation and squared terms of pitch rate, which effectively reflects the hysteresis characteristics and enhances the nonlinear representation ability. The multi-kernel neural networks and the modeling of the nonlinear unsteady aerodynamics at constant or varying flow conditions is proposed in [8], and the results indicate that the proposed multi-kernel neural networks outperform the single-kernel RBF neural networks in modeling noise-free and noisy aerodynamics at a constant Mach number, as well as in predicting the aerodynamic loads with varying Mach numbers. In [9], the paper presents an innovative unsteady aerodynamic modeling method based on the improved Extreme Learning Machine (ELM), which is further successfully applied into the biaxial coupled oscillation during the post stall maneuver flight. The simulation results demonstrate that the improved ELM method has characteristics of high precision, strong versatility and fast prediction in the unsteady aerodynamic modeling.

The aerodynamic force and the aerodynamic moments of the advanced fighters in post-stall maneuver have serious nonlinearity, hysteresis and coupling. Unsteady flow will cause severe changes in the state of advanced fighters, which will increase the inertial coupling moments of advanced fighters in post-stall maneuver and enhance the coupling between dynamics and kinematics [10]. The instability of the factors will be amplified. Under the influence of these strong coupling, nonlinearity and hysteresis factors, how to compensate or eliminate the influence of these factors and design a stable and reliable control law are the ultimate challenges we have to face.

There are few methods to solve above problems and realize the post-stall control. A robust finite-time maneuver control scheme for the longitudinal attitude dynamic of the aircraft with unsteady aerodynamic disturbances and input saturation is presented in [11]. To efficiently eliminate the influence of unsteady aerodynamic disturbances, nonlinear finite-time observers are developed. The proposed observers can still precisely estimate the unmeasurable unsteady aerodynamic disturbances in finite time. A hybrid NDI control method based on the angular acceleration feedback control is proposed to increase system robustness and performance in [12]. The hybrid NDI control method can not only increase system robustness, but also improve the system dynamic performance. A nonlinear flight control and a nonlinear state observer are designed in one unified framework for a high performance aircraft in [13]. The unified design method is applied to a highly maneuverable aircraft operating at high angles of attack. Simulation results show that the θ-D control-θ-D observer suite exhibits excellent performance.

Among these methods, the dynamic surface control method is widely used for its fast convergence speed and high efficiency [14][15][16]. In [17], the neural control for longitudinal dynamics of a generic hypersonic aircraft in presence of unknown dynamics and actuator fault is analyzed. For the attitude subsystem, direct adaptive design is presented with the dynamic surface design and the singularity problem is removed. For the actuator fault, the unknown dynamics caused by the fault is approximated by neural networks. The uniform ultimate boundedness stability is guaranteed via small-gain theorem. In [18], the paper addresses the composite neural tracking control for the longitudinal dynamics of hypersonic flight dynamics. Under the dynamic surface control with novel neural design, the neural system converges in a faster mode and better tracking performance is obtained.

Although the dynamic surface control method has been successfully applied in flight control system, the uncertainty of the aircraft should be considered [19-21]. The control law under large attack angle is designed combining the dynamic surface control method and the daisy chain allocation method. The RBF network is applied to model the uncertainty, and the stability of the proposed control law which considering the uncertainty is also proved. Simulation results verify the validity of the proposed control law under unsteady aerodynamics and the aerodynamics uncertainty [22]. However, the number of the parameters in the RBF network is too much, and the calculation time of this network is also very long. Therefore, how to improve the performance of the RBF based DSC method is important.

Minimum parameters learning method can be applied to solve that problem, and the parameter need to be adjusted is very few, and computational complexity of control law can be reduced [23][24]. A decentralized neural network (NN) output feedback fault tolerant control (FTC) problem is addressed for a class of multi-input multi-output systems with actuator fault in [25]. In order to establish a quick response to the fault, the fault tolerant controller with minimum learning parameters has been designed so that the semi-global uniform ultimate boundedness of all the variables in the resulting closed-loop systems can be guaranteed. The minimum parameters learning method has
not been studied in the unsteady aerodynamic modeling and the high angle of attack control. In this paper, the main work is to control the aircraft more accurately and more efficiently under the unsteady aerodynamics.

The structure of this paper is as follows: Section 2 gives the mathematical model of aircraft and the problems of modeling and control to be solved in high angle of attack flight. The accurate unsteady aerodynamic model based on the improved ELM method is established in Section 3. The high angle of attack flight control law based on MPL-RBF-DSC method for the uncertainty of the model is designed in Section 4. The simulation results and comparative analysis are given in Section 5, and the full paper is concluded in Section 6.

II. MATHEMATICAL PROBLEM DESCRIPTION

Aerodynamic force and aerodynamic moments have serious nonlinearity, coupling and hysteresis at the high angle of attack. Conventional aerodynamic models are no longer applicable, and the computational ability of existing flow field simulation software is limited. At present, wind tunnel experiments are still used to obtain unsteady aerodynamic forces and moments. The flight aerodynamic data in this paper are derived from the wind tunnel experiment as shown in the figure 1. The wind tunnel experimental device is composed of aircraft model, tail strut, turntable and supporting cutter. The flight state of aircraft at high angle of attack can be simulated by rotating different mechanisms, and more accurate aerodynamic parameters can be obtained. The model is a scaling model of typical aircraft layout, and an accurate six-degree-of-freedom nonlinear aerodynamic force are obtained. The experimental data of aircraft at high angle of attack were obtained. The conversion and processing of aerodynamic data can be found in reference [9].

![Image](image.png)

**FIGURE 1.** The wind tunnel device with the scaled aircraft model

The simplified mathematical model of the aircraft is as follows (just considering the longitudinal motion):

\[
\begin{align*}
\dot{\rho} &= (D - mg \sin(\gamma) + T_1 \cos(\alpha) + T_2 \sin(\alpha)) \\
\dot{\alpha} &= q + (L - mg \cos(\gamma) - T_1 \sin(\alpha) + T_2 \cos(\alpha)) \frac{mV}{\alpha} \\
\dot{\theta} &= q + (D - mg \sin(\gamma) + T_1 \cos(\alpha) + T_2 \sin(\alpha)) \\
\dot{q} &= (M + M_t) \frac{F}{J_i} \\
\dot{x} &= V \cos(\alpha) \cos(\theta) + V \sin(\alpha) \sin(\theta) \\
\dot{y} &= V \cos(\alpha) \sin(\theta) - V \sin(\alpha) \cos(\theta)
\end{align*}
\]

However, conventional aerodynamic model is only effective at small angle of attack flight, and is no longer applicable to post-stall maneuvers with strong nonlinearity, strong coupling and strong hysteresis at high angle of attack. It is necessary to establish an accurate unsteady aerodynamic model to analyze the post-stall maneuver.

In the high angle of attack flight, the longitudinal aircraft model can be simplified as follows:

\[
\begin{align*}
\dot{x}_i &= f_i(x) + g_i(x) x_i \\
\dot{x}_i &= \tilde{f}_i(x) + g_i(x) u
\end{align*}
\]

where

\[
\begin{align*}
x &= [V \quad \gamma \quad \alpha \quad \theta \quad q]^T \\
x_i &= \alpha \\
f_i(x) &= \left[-L + mg \cos(\gamma) - T_1 \sin(\alpha) + T_2 \cos(\alpha)\right] \frac{mV}{\alpha} \\
g_i(x) &= 1 \\
x_2 &= q \\
f_i(x) &= \frac{\tilde{F}}{J_i} \\
g_i(x) &= QS \sum_i C_{\alpha} \left[C_{\alpha}^\alpha - C_{\alpha}^\theta - C_{\alpha}^\alpha - C_{\alpha}^\theta \right]
\end{align*}
\]

\[u = \delta\]

The variables \(L\) and \(\tilde{F}\) represent the remaining force and moment obtained by removing the influence of the control inputs.

The whole process shown in figure 2 is divided into two parts: off-line calculation and on-line calculation. In off-line calculation, the experimental data of wind tunnel are analyzed, and the key factors affecting the unsteady aerodynamic force are obtained. The experimental data of wind tunnel are divided into training data and prediction data. The initial state and parameters are determined by using training data. An unsteady aerodynamic model is established on the basis of the improved ELM method. The aerodynamic model with high modeling accuracy is obtained by optimizing the model with prediction data. In the on-line calculation, based on the current flight state information of the aircraft, the unsteady aerodynamic force of the current step is calculated by using the established aerodynamic model, and an accurate six-degree-of-freedom nonlinear...
mathematical model of the aircraft is established. In order to verify the validity of the control law, the conventional aerodynamic force is considered in the solution of the control law. The model errors and uncertainties caused by unsteady aerodynamic force are eliminated based on MPL-RBF-DSC method, and the coordinated distribution of conventional aerodynamic surfaces and vector nozzle is accomplished by daisy chain method, so as to achieve high precision and strong robustness of post-stall maneuver control.

III. UNSTEADY AERODYNAMICS MODELING

A. THE DESCRIPTION OF ELM MODEL

Figure 3 gives the single hidden layer of feed-forward neural network structure. In the network, there are \( n \) input variables and \( m \) output variables. The number of neurons in the hidden layer is \( l \).

The connect weights \( \mathbf{w} \) between the input layer and the hidden layer is given by

\[
\mathbf{w} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1l} \\ w_{21} & w_{22} & \cdots & w_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \cdots & w_{nl} \end{bmatrix}_{n \times l}
\]

where \( w_{ij} \) is the connect weight value between the \( i \) th neuron in the input layer and \( j \) th neuron in the input layer.

The connect weights \( \mathbf{\beta} \) between the hidden layer and the output layer is

\[
\mathbf{\beta} = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1m} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{l1} & \beta_{l2} & \cdots & \beta_{lm} \end{bmatrix}_{l \times m}
\]

where \( \beta_{ij} \) is the corresponding connect weight value.

The bias of the hidden layer neurons is as follows

\[
\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_l \end{bmatrix}_{l \times 1}
\]

The input matrix and the output matrix of training set with \( Q \) samples are shown respectively as follows

\[
\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1Q} \\ x_{21} & x_{22} & \cdots & x_{2Q} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nQ} \end{bmatrix}_{n \times Q}
\]

\[
\mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1Q} \\ y_{21} & y_{22} & \cdots & y_{2Q} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{nQ} \end{bmatrix}_{n \times Q}
\]

The activation function of the hidden layer neurons is expressed as \( g(x) \), and the output \( T \) of the net is obtained from figure 3.

\[
\mathbf{T} = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_Q \end{bmatrix}_{Q \times 1}
\]

And \( \mathbf{H} \) is the output matrix of hidden layer neural network, and it can be written as

\[
\mathbf{H} = \begin{bmatrix} g(w_{11}, b_1, x_1) & g(w_{12}, b_2, x_1) & \cdots & g(w_{1l}, b_l, x_1) \\ g(w_{21}, b_1, x_2) & g(w_{22}, b_2, x_2) & \cdots & g(w_{2l}, b_l, x_2) \\ \vdots & \vdots & \ddots & \vdots \\ g(w_{n1}, b_1, x_n) & g(w_{n2}, b_2, x_n) & \cdots & g(w_{nl}, b_l, x_n) \end{bmatrix}_{Q \times l}
\]

The value of the parameters \( \mathbf{w} \) and \( \mathbf{b} \) can be set randomly at the first time, and these parameters will remain unchanged. The connect weights \( \mathbf{\beta} \) can be calculated by the least squares solution of the following equation

\[
\min_{\mathbf{\beta}} \| \mathbf{H} \mathbf{\beta} - \mathbf{Y} \|^2
\]

The solution of the above equation is \( \hat{\mathbf{\beta}} = \mathbf{H}^+ \mathbf{Y} \), and \( \mathbf{H}^+ \) is called the Moore-Penrose generalized inverse.

B. THE UNSTEADY AERODYNAMICS MODELING BASED ON THE IMPROVED ELM METHOD
The unsteady aerodynamics modeling contents based on the improved ELM method is given in [9]. In the longitudinal flight model, the input variables are selected as \( x = [\alpha, \dot{\alpha}, k] \), and the kernel function is 
\[
K(x, x_i) = \exp \left( -\frac{\|x - x_i\|^2}{2\sigma^2} \right).
\]

The unsteady aerodynamics modeling curves of force and moment coefficients under pitch large amplitude oscillation situation are as follows:

- Figure 4 shows the modeling result of \( C_{my} \) with tunnel data and ELM modeling.
- Figure 5 shows the modeling result of \( C_{fx} \) with tunnel data and ELM modeling.
- Figure 6 shows the modeling result of \( C_{fz} \) with tunnel data and ELM modeling.

Figure 4 - Figure 6 show the results of unsteady aerodynamic modeling based on the improved ELM method, in which the arrow direction represents the change direction of aerodynamic coefficients and aerodynamic moment coefficients with the changing of the attack angle. From these figures, it can be seen that the method has high accuracy in modeling different aerodynamic coefficients, and the consistency between the modeling curve and wind tunnel test data is preferably. For pitching moment coefficient, the modeling error is small even if there are two hysteresis loops. For the axial force coefficient with strong nonlinearity, the accuracy of the modeling is still high. By establishing an accurate unsteady aerodynamic model, it can provide an accurate model reference for subsequent high angle of attack flight aerodynamic characteristics analysis and post-stall maneuver control.

C. THE MATHEMATICAL EXPRESSION OF THE REDUCED FREQUENCY

The signal of the angle of attack in the wind tunnel test is shown as follows
\[
\alpha = \alpha_i + \alpha_s \sin(\omega t) \\
\dot{\alpha} = -\omega \alpha_s \cos(\omega t)
\]

where \( \alpha_i \) is the initial value; \( \alpha_s \) is the amplitude value. And the angular frequency \( \omega \) can be derived as
\[
\omega = \sqrt{\frac{|\rho_k|}{\alpha_s^2 - (\alpha_k - \alpha)}}
\]

With the following definition
\[
k = \frac{2\pi f_c}{V}
\]

Substitute (15) to (16) and combine \( \omega = 2\pi f \), the derivation of \( k \) in this paper can be expressed as
\[
k = \frac{|\rho_k|}{\sqrt{\alpha_s^2 - (\alpha_k - \alpha)}}
\]

According to the above deduction, the reduced frequency can be given by equation (17) in the actual high angle of attack flight. It can be seen that the reduced frequency is related to the mean aerodynamics chord, angle of attack, the rate of angle of attack and the flight speed. At the same time, the calculation of reduced frequency is also simple and feasible in actual flight.

IV. MINIMUM PARAMETERS LEARNING BASED DYNAMIC SURFACE CONTROL

A. CONTROL LAW DESIGN BASED ON MPL-RBF-DSC

In longitudinal post-stall maneuver, the control system includes the subsystem of \( \alpha \) and the subsystem of \( q \). The control system can be expressed as
\[
\dot{x}_1 = f_1(x) + g_1(x)x_2 + \Delta_1 \\
\dot{x}_2 = f_2(x) + g_2(x)x_2 + \Delta_2
\]

where \( \Delta_1 \) and \( \Delta_2 \) is the uncertainty. The RBF network is applied to establish the model of the above uncertainty.
\[
\Delta = \omega^T \rho(x) + \varepsilon_i, \quad i = 1, 2
\]

where \( \omega \in \mathbb{R}^{n \times d} \), \( \rho(x) = [\rho_1(x), \rho_2(x), \ldots, \rho_n(x)]^T \) is the radial basis function. \( \varepsilon_i \) is the approximation error.
\[ f_i(x) = \frac{1}{\sqrt{2 \pi} \sigma_i} \exp \left( \frac{-|x - \xi_i|^2}{2 \sigma_i^2} \right), \quad i = 1, 2, \ldots, N \]  

(20)

where \( \xi_i \) is the center of the i-th radial basis function and \( \sigma_i \) is the corresponding bandwidth. In the normal dynamic surface control, the parameters \( w_i \in \mathbb{R}^{n_u} \) can be obtained through the adaptive update control law. However, if the training data of RBF network is large and the dimension \( N \) is also big, the update of the \( w_i \) is difficult to compute on reality.

To avoid the large computing of RBF network parameters, the minimum parameters learning method is adopted to solve this problem. The minimum parameter learning method converts the weight vector of the neural network into a single parameter for on-line adjustment, and combines it with the dynamic surface control technology to design the post-stall maneuver control law of aircraft. The combination of this method not only reduces the computational complexity of the control law, but also avoids the “dimension disaster”.

The parameter \( \omega_1 \) and \( \omega_2 \) are used in the minimum parameters learning. Set \( \omega_1 = [w_1] \) and \( \omega_2 = [w_2] \). Then \( \hat{\omega}_1 \) and \( \hat{\omega}_2 \) are the estimate value of \( \omega_1 \) and \( \omega_2 \) respectively, and the error of estimate of \( \omega_1 \) and \( \omega_2 \) is \( \hat{\omega}_1 = \omega_1 - \hat{\omega}_1 \), \( \hat{\omega}_2 = \omega_2 - \hat{\omega}_2 \).

Step 1:

Define the dynamic surface \( s_1 \) as

\[ s_1 = x_1 - y_{1u} \]  

(21)

The derivation of \( s_1 \) is

\[ \dot{s}_1 = \dot{x}_1 - \dot{y}_{1u} = f_1(x) + g_1(x) s_1 + \Delta_1 - \dot{y}_{1u} = f_1(x) + g_1(x) s_1 + w_1^T \hat{\rho}_1(x) + \epsilon_{1u} - \dot{y}_{1u} \]  

(22)

Hence, desired signal \( \tau_1 \) can be given as

\[ \tau_1 = g_1^{-1}(x) \left( y_{1u} - f_1(x) - K_s \dot{s}_1 - \frac{\hat{\omega}_1}{2} \rho_1(x) \right) \]  

(23)

where, \( K_s \) is positive, and the control law of the parameter \( \hat{\omega}_1 \) is updated:

\[ \dot{\hat{\omega}}_1 = \xi_1 \left( \frac{1}{2} \hat{\omega}_1^T \rho_1(x) \rho_1(x) - \eta_1 \dot{\hat{\omega}}_1 \right) \]  

(24)

where \( \xi_1 > 0, \eta_1 > 0 \).

In order to solve the problem of the “differential explosion”, the first order filter \( z_2 \) is applied in (25) and the time constant is \( \tau_2 \).

\[ \tau_2 \dot{z}_2 + z_2 = \tau_2, \quad z_2(0) = \tau_2(0) \]  

(25)

Step 2:

Define the dynamic surface \( s_2 \) as

\[ s_2 = x_2 - z_2 \]  

(26)

Then the derivation of \( s_2 \) is

\[ \dot{s}_2 = \dot{x}_2 - \dot{z}_2 = f_2(x) + g_2(x) u + \Delta_2 - \dot{z}_2 = f_2(x) + g_2(x) u + w_2^T \hat{\rho}_2(x) + \epsilon_2 - \dot{z}_2 \]  

(27)

The control law of inner loop can be written as

\[ u = g_1^T(x) \left( \dot{z}_2 - f_2(x) - K_s s_2 - \frac{\hat{\omega}_1}{2} \rho_1(x) \right) \]  

(28)

where, \( K_s \) is positive, and the control law of \( \hat{\omega}_1 \) is updated:

\[ \dot{\hat{\omega}}_2 = \xi_2 \left( \frac{1}{2} \hat{\omega}_2^T \rho_2(x) \rho_2(x) - \eta_2 \dot{\hat{\omega}}_2 \right) \]  

(29)

where \( \xi_2 > 0, \eta_2 > 0 \).

B. DERIVATION AND PROOF OF STABILITY

The following error dynamic is defined

\[ y_{1u} = \zeta_1 - \tau_1 \]  

(30)

\[ \hat{\omega}_1 = \omega_1 - \hat{\omega}_1 \]  

(31)

\[ \hat{\omega}_2 = \omega_2 - \hat{\omega}_2 \]  

(32)

With formula (25) and (30), we have

\[ \dot{z}_2 = (\tau_2 - z_2)/\tau_2 = -y_{1u}/\tau_2 \]  

(33)

Substitute (33) into (31), and obtain the derivative of \( y_{1u} \)

\[ \dot{y}_{1u} = -y_{1u}/\tau_2 + B_1(\cdot) \]  

(34)

where

\[ B_1(\cdot) = \left( \left( y_{1u} - f_1(x) - K_s \dot{s}_1 \right) g_1(x) - (y_{1u} - f_1(x) - K_s \dot{s}_1) g_1(x) \right) \]  

(35)

\[ \frac{1}{2} \hat{\omega}_1^T \rho_1(x) \rho_1(x) \]  

where, \( f_1(x) \) includes the unsteady aerodynamics \( L \). With the knowledge of the ELM model, \( L \) can be expressed as

\[ L = H \beta \]  

(36)

It is easy to see that \( L \) is differentiable and \( f_1(x) \) is bounded and continuous. Therefore, \( B_1(\cdot) \) is continuous, and the maximum value of \( B_1(\cdot) \) can be set as

\[ B_1(\cdot)|_{\text{max}} = M_2 \]  

(37)

With the above formula, we can get

\[ \dot{s}_1 = f_1(x) + g_1(x) s_1 + w_1^T \hat{\rho}_1(x) + \epsilon_1 - \dot{y}_{1u} = -K_s s_1 - \frac{\hat{\omega}_1^T \rho_1(x) \rho_1(x)}{2} s_1 + w_1^T \hat{\rho}_1(x) + \epsilon_1 \]  

(38)

\[ \dot{z}_2 = f_2(x) + g_2(x) u + w_2^T \hat{\rho}_2(x) + \epsilon_2 - \dot{z}_2 = -K_s \dot{s}_2 - \frac{\hat{\omega}_2^T \rho_2(x) \rho_2(x)}{2} s_2 + w_2^T \hat{\rho}_2(x) + \epsilon_2 \]  

(39)

The Lyapunov function is defined as

\[ V = \sum V_i \]  

(40)

where

\[ V_i = \frac{1}{2} s_i^2 + \frac{1}{2} y_{1u}^2 + \frac{1}{2\xi_1} \hat{\omega}_1^2 \]  

(41)

\[ V_i = \frac{1}{2} s_i^2 + \frac{1}{2\xi_2} \hat{\omega}_2^2 \]  

(42)

In the following section, \( g_i(x) \) is written as \( g_i \). The Frobenius norm of \( A \) is defined as \( |A| = \sqrt{\text{tr}(A^T A)} \).

HYPOTHESIS 1: The reference signal \( y_{1u} \) is a smooth function. In the set \( D_s \subset R^i \), \( y_2 \), \( y_{1u} \) and \( y_{1u} \) are bounded. And \( s_i = [s_1, s_2]^T \), \( y_i = y_{1u} \) are also belong to the set \( D_s \subset R^i \).

Proof

Get the derivative of \( V_i \)
\[ \dot{V}_1 = s_i^2 \dot{s}_i + y_i^2 \dot{y}_i + \frac{1}{\sigma_i} \ddot{\theta}_i \dot{\theta}_i \]

From (31) we can get
\[ \ddot{\theta}_i = -\ddot{\theta}_i \]

Then, formula (43) can be written as
\[ \dot{V}_1 = s_i \dot{s}_i + y_i \dot{y}_i - \frac{1}{\sigma_i} \ddot{\theta}_i \dot{\theta}_i \]

Take the derivative of \( V_1 \)
\[ \dot{V}_2 = s_i \ddot{s}_i + \frac{1}{\sigma_i} \ddot{\theta}_i \dot{\theta}_i \]

Based on the Young’s inequality, we can get
\[ s_i \dot{\omega}_i \dot{\phi}_i \leq \frac{1}{2} \dot{\omega}_i^2 \| \dot{\phi}_i \| + 1 \]

With matrix inequality, we have
\[ \ddot{\theta}_i \dot{\theta}_i = \ddot{\theta}_i (\dot{\theta}_i - \dot{\theta}_i) \]

With (47) and (48), (46) can be continued as
\[ \dot{V} = \dot{V}_1 + \dot{V}_2 \]

The range of the error \( s_i \) is
\[ |s_i| \leq 2V \]

Therefore, let \( \epsilon_i > 0 \) by the proper parameters, the control error can be uniformly converged in bounds of above formula.

This completes the proof.

C. ALLOCATION DESIGN OF CONVENTIONAL AERODYNAMIC SURFACES AND THRUST VECTORS

For advanced layout aircraft with thrust vectoring, the coordinated deflection of conventional surfaces and thrust vectoring nozzle is the key point of post-stall maneuver control, and the timing of starting thrust vector is also the difficulty of research. In order to prolong the service life of the thrust vector engine, the thrust vector engine should not be opened for a long time, but only when the angle of attack is large. In this paper, daisy chain method will be used to turn on the thrust vector when the torque generated by the conventional surfaces cannot meet the post-stall maneuver requirements.

The deflections of the aircraft in longitudinal are
\[ u = [\delta_\alpha, \delta_\delta, \delta_\beta, \delta_Y]^T \]

The control output includes two parts:
\[ u = [u', u]^T \]
\[ u = [\delta_\alpha, \delta_\delta, \delta_\beta]^T \]

For \( g_s(x) \), it can be divided as:
\[ g_s(x) = g_s(x) \]

Hence, the output can be expressed
\[ u_s = g_s(x) \dot{x}_s - K_s \dot{x}_s - w_s^2 \dot{x}_s - g_s(x) \dot{x}_s \]

In order to simulate the actual control surface of the aircraft, the second-order prototype is used to model the
control surface.

\[
sat(u(s)) = \frac{u(s)}{a_1(s)} = \frac{\omega_0^2}{s^2 + 2\xi \omega_0 s + \omega_0^2}
\]  
(58)

where \(\omega_0\) and \(\xi\) are natural frequency and damping ration respectively.

With the limitation, we can get

\[
u_{sat} = sat(u)
\]
(59)

The deflection angle of the vector nozzle can be expressed as

\[u = g(x)\left(\bar{z}_2 - f_2(x) - K_s \bar{z}_2 - \hat{w}_r \rho_s(x) - g_1(x) s_1 - g_2(x) u_{sat}\right)
\]
(60)

The final outputs are

\[u = [u_{sat} \ u_{sat}]^T
\]
(62)

The daisy chain method is applied in figure 7, where \(\bar{q} = \bar{z}_2 - f_2(x) - K_s \bar{z}_2 - \hat{w}_r \rho_s(x) - g_1(x) s_1\).

\[
\Delta t_1 \text{ is the period of rapid pitching from } \alpha_{sat} \text{ to } \alpha_{sat}.
\]

B. SIMULATION OF COBRA MANEUVER UNDER UNSTEADY AERODYNAMICS

The parameters of the DSC method are selected on the basis of the response speed of different control variables in different control loops. The selection criteria are similar to the dynamic inversion method. Other parameters are selected according to the accuracy and dynamics of the angle of attack control while ensuring stability. The parameters of the Cobra maneuver are \(\alpha_{sat} - \alpha_{sat} = 11^\circ\), \(\alpha_{sat} = 78^\circ\), \(\Delta t_1 = \Delta t_2 = 2s\) and \(t_1 = 3s\). The parameters of DSC method are \(r_2 = 1\), \(K_1 = 0.6\), \(K_2 = 25.0\). In the RBF network, \(\Gamma_1 = 20\), \(\Gamma_2 = 30\), \(\eta_1 = 1\), \(\eta_2 = 1\), and the initiate value is \(w_1 = \text{rand}(50,1)\), \(w_2 = \text{rand}(30,1)\). \(\zeta\) and \(\sigma\) are selected by the random value. In the MPL-RBF-DSC, set \(q_0 = \text{rand}(1,1)\) and \(\omega_0 = \text{rand}(1,1)\). In order to improve the accuracy of the simulation model, a second-order model of the control input mechanism is established. The specific parameters of the model can be seen in Table 1.

<table>
<thead>
<tr>
<th>(\delta) (deg)</th>
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<th>(\delta) (deg/s)</th>
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<th>(\alpha) (rad/s)</th>
<th>(\zeta)</th>
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<tbody>
<tr>
<td>(\delta_0) 20</td>
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V. SIMULATION

A. THE INDUCTION OF COBRA MANEUVER

“Cobra” maneuver is a typical post-stall maneuver shown in figure 8, in which the flight velocity can be reduced rapidly and the enemy aircraft can be thrown off and occupy a favorable position in air combat. The maneuver can be divided into two parts, which correspond to the process of increasing and decreasing the angle of attack. The desired signal of the angle of attack is given as below.

Increase of the angle of attack:

\[
\alpha_x(t) = \frac{\alpha_0 + \alpha_{sat}}{2} - \frac{\alpha_0 - \alpha_{sat}}{2} \cos\left(\frac{\pi}{\Delta t_1}(t - t_1)\right)
\]
(63)

where \(\alpha_0\) is the initial value; \(\alpha_{sat}\) is the maximum value; \(\Delta t_1\) is the time when \(\alpha = \alpha_{sat}\).

Decrease of the angle of attack:

\[
\alpha_x(t) = \frac{\alpha_{sat} + \alpha_0}{2} - \frac{\alpha_{sat} - \alpha_0}{2} \cos\left(\frac{\pi}{\Delta t_2}(t - t_1 - \Delta t_1)\right)
\]
(64)

where \(\alpha_{sat} = \alpha_0\) is the final value in “Cobra” maneuver, and

In figure 9, the Cobra maneuver control results are shown by three curves with different colors, and the results are obtained by the DSC control method. The green color is the desired signal of the attack angle. The blue one and the black one are the track angle of attack with RBF and MPL-RBF respectively. It is explicit that the control method by RBF and MPL-RBF can both get good performance, but the control accuracy of the MPL-RBF is higher than the normal RBF. The curve of the MPL-RBF is almost coincided with the desired signal. However, in the
computing of the control law, there are only two parameters
that need to be adjusted and refreshed in the MPL-RBF
method. The parameters need to be adjusted in the normal
RBF methods are more than that of the MPL-RBF. Hence,
the control accuracy and the computing speed of the MPL-
RBF are both well than the normal RBF method.

\[ \begin{align*}
\text{FIGURE 10(a). The curve of } C_m \\
\text{FIGURE 10(b). The curve of } C_x \\
\text{FIGURE 10(c). The curve of } C_z \\
\text{FIGURE 10(d). The curve of } V \\
\text{FIGURE 10(e). The curve of } h \\
\text{FIGURE 10(f). The curve of } \delta_e \\
\text{FIGURE 10(g). The curve of } \delta_f \\
\text{FIGURE 10(h). The curve of } \delta_b
\end{align*} \]
The longitudinal aerodynamics comparisons are depicted in figure 10(a) – figure 10(c). The related curves of the two methods change sharply during the Cobra maneuver. The changes of the coefficients of the unsteady aerodynamics are similar to the wind tunnel test data, and the nonlinearity and the hysteresis of the aerodynamics are serious. The ranges of the attack angle under the two conditions are similar. In the curve of axial force coefficient, the nonlinearity of the MPL-RBF are more serious. The reason is that the velocity decreases rapidly, which leads to better angle of attack control with the increase of drag coefficient. There are two hysteresis loops in the pitch moment coefficient curve.

In figure 10(d), the velocity of the aircraft decreases from 30m/s to 6m/s in the Cobra maneuver, and the curves of the two conditions are close. When the velocity of the aircraft reaches minimum value, the attack angle reaches the peak value of the curve. It is clear that the smaller the velocity, the larger the attack angle during the Cobra maneuver.

The height curves can be seen in figure 10(e). The height of the two methods just increase less than 20m. In the initial stage of the Cobra maneuver, the height of the MPL-RBF control is more stable than that of the normal RBF control, and the MPL-RBF condition can meet the requirement of the Cobra maneuver.

The curves of control inputs are shown in figure 10(f) - figure 10(i). In the maneuver, the aerodynamics change sharply, resulting in the larger deflections of the control surfaces than that of level flight, and the deflection of the elevator is saturated when the attack angle is more than 40 degrees in the two methods. With the influence of the unsteady aerodynamics, the deflections of the control inputs are seriously different with the normal aerodynamics. Hence, the unsteady aerodynamics should be taken into account in the design and control of the post-stall maneuver.

Figure 10(j) presents the curve of reduced frequency. The reduced frequency changes from 0 to 0.08. From the results, it can be seen that the range of frequency reduction is consistent with the wind tunnel test data, which proves that the derivation of the equivalent formula of frequency reduction is correct. At the same time, it can be found in the simulation that the reduction frequency has a great influence on the accuracy of unsteady aerodynamic modeling. Therefore, in the subsequent flight test verification, attention should be paid to finding the accurate reduction frequency.

C. CONTROL LAW VERIFICATION WITH PARAMETERS UNCERTAINTY

To verify the validity of the designing control law with the MPL-RBF-DSC method, the uncertainty curves of axial force coefficient, normal force coefficient and pitching moment coefficient varying with time are given in figure 11 according to the range of aerodynamic parameters during flight. The amplitude and period of the uncertainty curves can be seen in the horizontal and vertical coordinates of figure 11. The parameters of DSC method are kept same in the last section.

Known from figure 12, in the case of large aerodynamic parameter uncertainties, the MPL-RBF-DSC method can still achieve good control effect, and the angle of attack can track the desired angle of attack signal well. Therefore, when the influence of unsteady aerodynamic force is considered in post-stall maneuver, the method proposed in this paper can be used with high control accuracy and strong robustness.
VI. CONCLUSION
In this paper, the high angle of attack control method under unsteady aerodynamic force has been investigated. A method based on MPL-RBF-DSC has been proposed to realize the post-stall maneuver control with high control accuracy, strong robustness and fast computation. During the high angle of attack flight, it is necessary to establish an accurate unsteady aerodynamic model to simulate the real aerodynamic characteristics of post-stall maneuver, and to fully understand and master the key factors affecting post-stall maneuver control. The method proposed in this paper can model the errors and uncertainties of the aircraft caused by unsteady aerodynamics. At the same time, the MPL-RBF method can optimize the post-stall maneuver control with fewer parameters, and the control accuracy is better than the conventional DSC method. The method presented in this paper provides a feasible technical approach for the practical flight test of post-stall maneuver control method.

APPENDIX

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$V$</td>
<td>Velocity</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Climb angle</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The angle of attack</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Pitch angle</td>
</tr>
<tr>
<td>$q$</td>
<td>Pitch rate</td>
</tr>
<tr>
<td>$X$</td>
<td>X position value</td>
</tr>
<tr>
<td>$h$</td>
<td>Height</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>$L$</td>
<td>Lift</td>
</tr>
<tr>
<td>$D$</td>
<td>Drag</td>
</tr>
<tr>
<td>$F_x$</td>
<td>Axial force</td>
</tr>
<tr>
<td>$F_y$</td>
<td>Normal force</td>
</tr>
<tr>
<td>$M$</td>
<td>Pitch moment</td>
</tr>
</tbody>
</table>

$C_r$ | Axial force coefficient |
$C_n$ | Normal force coefficient |
$C_m$ | Pitch moment coefficient |
$T_s$ | Axial engine force |
$T_e$ | Normal engine force |
$M_e$ | Engine pitch moment |
$Q$ | Dynamic pressure |
$S_o$ | Wing area |
$C_A$ | Mean aerodynamics chord |
$k$ | Reduced frequency |
$J_y$ | The inertia of the Y body-axis |
$\delta_e$ | Deflection angle of elevator |
$\delta_o$ | Deflection angle of leading edge flap |
$\delta_n$ | Deflection angle of trailing edge flap |
$\delta_z$ | Deflection angle of longitudinal nozzle |

REFERENCES


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