Quantifying Credit Risk of Supply Chain Finance: A Chinese Automobile Supply Chain Perspective

Min Zhang¹, Jiantong Zhang¹, Ruolin Ma², Xiaodong Chen¹

¹School of Economics and Management, Tongji University, Shanghai, China
²Commercial college, Michigan State University, MI 48824 USA

Corresponding author: Xiaodong Chen (e-mail: xiaodongc@hotmail.com).

This work was supported in part by National Natural Science Foundation of China (No. 71771179).

ABSTRACT Credit risk is a major risk of supply chain finance business, and it has recently gained increasing attention. Due to the high dependence between enterprises, the assessment of the supply chain finance risk will be more complicated. In the current study, the research subjects are almost the single financing enterprises, and the credit risk of financing enterprise portfolio in supply chain finance has rarely been discussed. This study aims to establish a complete risk assessment model, which is based on modified KMV model and Copula function, to quantify the credit risk of enterprises in supply chain finance. Based on the model, we can measure the credit risk of the single financing enterprise and financing enterprise portfolio (up- and down-stream) in supply chain. The results indicate that default contagion does exist in supply chain and the intensity of default contagion between enterprises in the financing enterprise portfolio is asymmetrical. Moreover, the conclusions about the joint expected default frequency and conditional expected default frequency of financing enterprise portfolio in supply chain, are of great significance to commercial banks and other financial institutions.

INDEX TERMS Supply chain finance, credit risk, default contagion, financing enterprise portfolio, Chinese automobile supply chain

I. INTRODUCTION

As an innovative way of financing, supply chain finance (SCF) has shown increasingly vitality in the enterprise financing market of China [1], which participants include supply chain node enterprises, financial institutions and other supportive parties [2]. For commercial banks and other financial institutions, SCF provides financial support for the up- and down-stream enterprises in the supply chain, supported by the credit of the core enterprises, which is different from the traditional credit granting mode to the single financing enterprise [1]-[4]. Because of the complexity of the business, commercial banks and other financial institutions inevitably face certain credit risks. The basic process of credit risk management in SCF includes risk identification, measurement, evaluation and control [4]. Risk measurement plays a vital role among all these processes from both theoretical and technical perspective.

Under the financing environment in China, there are a lot of difficulties in risk measurement. On the one hand, SCF in China started relatively late, and there are still a series of obvious periodical deficiencies. For instance, with the exception of Shenzhen Development Bank, the SCF business for most Chinese commercial banks is not yet independent of the risk control system of traditional liquidity loans [4]. Besides, laws on the security right of movable property in China is incomplete, which leads to uncertainty in many operations and causes unexpected losses of SCF business [5]. On the other hand, the relations between financing enterprises in SCF are not only in logistics, capital flow, information flow and business flow, but also include the financing guarantee relationship of mutual credit support and joint liability [4], [6], [7], [8]. Due to the existence of financing guarantee relationship, there is default dependence between enterprises in SCF, which is difficult to capture and is easily ignored. These deficiencies and the complex relationship between enterprises increase the difficulty of financing and make the credit risk more difficult to control. Hence, the potential
credit risk of SCF business is seriously underestimated or misplaced, resulting in access errors and risk control out of the question.

Automobile supply chain is the most representative industrial supply chain, which is considered to be the pioneer of SCF. The concept of automobile supply chain finance was first proposed by Ford Credit in the 1970s [6]. Until the beginning of this century, the research on automobile SCF in China started to bloom [6]. This paper proposes to establish a complete risk assessment model of automobile SCF in China, which can not only measure the credit risk of the single financing enterprise, but also quantify the credit risk of financing enterprise portfolio (up- and down-stream) in supply chain. Furthermore, the model has been proved to be of theoretical and practical significance.

The rest contents of this paper are arranged as follows: section II provides the literature about quantifying credit risk of SCF. Section III presents the risk assessment model, which will be used in Section IV. Section IV gives case study, from a Chinese-automobile-supply-chain perspective. And section V concludes the paper.

II. RELATED WORK

Appropriately control of credit risk of SCF indicates great benefits for the supply chain overall. Considering the importance of academic and practice, many researches have been done towards the credit risk of SCF [8]-[20]. There are two main streams of literature, i.e., the measurement of individual credit risk and the measurement of portfolio credit risk.

A. INDIVIDUAL CREDIT RISK

As for the measurement of individual credit risk, Z scoring is the first statistical model adopted to predict the default probability of enterprise, which is calculated and modified by standard discriminant model [21], [22]. Besides Z scoring, a series of more accurate methods, such as logistic regression, neural network, smooth nonparametric method and expert system method, have been developed, which have been widely used in the field of credit evaluation [23], [24]. In recent years, there have been many new combination methods. Ma and Zhao [1] used Analytic Hierarchy Process (AHP) and fuzzy cluster analysis to evaluate the risk of supply chain business, risk of quality and risk of banks. Sun [25], [26] combined Data Envelopment Analysis cross model with cluster analysis and principal component analysis to evaluate the relative credit score and relative credit rating of small and medium-sized enterprises. In addition to the above empirical models, scholars have also established a variety of theoretical models to measure credit risk, such as KMV model, VaR model, Credit Metrics model, Credit Risk+ model and Credit Portfolio View model, etc. Among them, KMV model is the most successful and widely used model, which validity has been proved by many scholars [27], [28]. The classical KMV model is obtained by KMV Company based on a large number of empirical researches on American financial market. There is a large difference between Chinese financial market and American financial market, for this reason, it is imprecise to apply the classical KMV model to Chinese financial market directly. Therefore, many Chinese scholars have modified the parameters in KMV model to accommodate the characteristics of financial market in China, such as value of stock right [29], stock price volatility [30], and default point [6].

B. PORTFOLIO CREDIT RISK

As for the measurement of portfolio credit risk, it is mainly to implant default dependence into the single enterprise credit risk measurement model, and Copula functions are widely applied. Embrechts et al. [31] introduced the Copula function into the field of financial risk management for the first time, and then it was widely used in the fields of asset-related structure characterization, portfolio selection and optimization, risk measurement and capital allocation. Many scholars have studied the correlation and portfolio risk between stock indices by Copula function [32]-[38]. Besides, many scholars have established hybrid models based on Copula function, such as Copula-CVaR [39], Copula-CVaR-EVT [40] and CGARCH-EVT-Copula [41]. In the area of SCF, most of the literature is devoted to the optimization of pledge combination [41], and there is no mature research about credit risk of financing enterprise portfolio using Copula function. Scholars also characterize default dependence by many other methods [42], [43], such as binomial expansion technique, macroscopic pressure test and hybrid hop-diffusion model, which are not as widely used as Copula function.

C. SUMMARY REVIEW OF THE EXTANT LITERATURE

Tab. 1 comprehensively presents a list of the previous literature and highlights our contributions.
Through the related literature mentioned above, it can be noticed that there are many measurement models for individual credit risk. Among them, the KMV model based on option pricing theory has been widely used and confirmed to accord with the characteristics of Chinese financial market. As for the measurement of portfolio credit risk, methods are relatively fewer and Copula functions are used most frequently. However, from the perspective of Copula functions’ applied background, there is less mature application in SCF. What’s more, the research subject of most research is single financing enterprise, and the credit risk of financing enterprise portfolio in SCF has rarely been discussed. Therefore, from the perspective of financial institutions, this paper combines the modified KMV model with Copula functions to analyze the credit risk of both single financing enterprise and financing enterprise portfolio in SCF. Finally, the expected loss value of the credit risk is calculated and explored. It provides some managerial highlights for SCF, which are of great significance to financial institutions in selecting financing objects.

### III. RISK ASSESSMENT MODEL

The purpose of this paper is to study the credit risk of single financing enterprise and financing enterprise portfolio (up- and down-stream) in SCF. The KMV model is adopted to depict the credit risk of single financing enterprise, and the Copula function is applied to depict the credit risk of financing enterprise portfolio in supply chain.

#### A. KMV MODEL

KMV model is a credit measure model established by KMV Company in 1997 to estimate the default probability of loan enterprises. Its advantage is that we can predict the changes of enterprise credit status by observing the changes of stock market price at any time. The unique feature of KMV model is that it relies on standard European options. The value of an enterprise can be measured by the option pricing formula if the equity of an enterprise is regarded as a call option, the liability is regarded as a put option and the capital is regarded as underlying asset.

Black-Scholes Option Pricing Model is the most widely used credit model based on option pricing theory, which is the basis of KMV Model. It is not feasible to use the classical KMV model directly in Chinese.

### TABLE 1. Summary review of the extant literature.

<table>
<thead>
<tr>
<th>Literature</th>
<th>Research method</th>
<th>Research focus</th>
<th>Risk Quantification</th>
<th>Supply chain finance</th>
<th>Object of the study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altman [21], [22]</td>
<td>Z scoring/</td>
<td>Default probability of enterprise</td>
<td>√</td>
<td></td>
<td>Individual credit risk</td>
</tr>
<tr>
<td>Kurbat and Koralev [27]</td>
<td>ZETATM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peter and Jeffrey [28]</td>
<td>KMV model</td>
<td>The level of the EDFTM credit measure</td>
<td>√</td>
<td></td>
<td>Individual credit risk</td>
</tr>
<tr>
<td>Ma and Zhao [1]</td>
<td>KMV model</td>
<td>Measurement of default risk</td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zhang et al. [6]</td>
<td>AHP and fuzzy cluster</td>
<td>The risk of supply chain business</td>
<td></td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>Embrechts et al. [31]</td>
<td>Copula function</td>
<td>Financial risk management</td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jondeau et al. [32], Palaro et al. [33]</td>
<td>Copula function</td>
<td>The correlation and portfolio risk between stock indices</td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boubaker et al. [39]</td>
<td>Copula-CVaR</td>
<td>Portfolio optimization</td>
<td>√</td>
<td></td>
<td>Portfolio credit risk</td>
</tr>
<tr>
<td>He et al. [40]</td>
<td>Copula-CVaR-EVT</td>
<td>The optimization of pledge combination</td>
<td>√</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>Karmakar and Paul [41]</td>
<td>CGARCH-EVT-Copula</td>
<td>Portfolio forecasts</td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>This study</td>
<td>KMV model &amp;</td>
<td>The credit risk of enterprises in supply chain finance</td>
<td>√</td>
<td>√</td>
<td>Individual credit risk &amp; Portfolio credit risk</td>
</tr>
<tr>
<td></td>
<td>Copula function</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: “√” represents that the risk quantification method or the topic of supply chain finance is discussed in the corresponding literature.
financial market to evaluate the credit risk of financing enterprises. The reasons can be stated as follows. On the one hand, the classical KMV model is obtained by KMV Company based on a large number of empirical researches on American financial market. But at present, there is a large difference between Chinese financial market and American financial market. On the other hand, China still lacks the time statistical data of enterprise default, and has not construct the enterprise credit information database at present. Therefore, the classical KMV model should be modified to apply to the financial market in China better.

1) CORRECTION OF THE VOLATILITY OF EQUITY VALUE

According to the option pricing formula, the relationship between the equity value and the asset value of an enterprise can be expressed as:

\[
E = VN(d_1) - De^{-rt}N(d_2)
\]  

(1)

Where \(E\) represents the equity value of the enterprise, or the value of a call option; \(V\) represents the asset value of the enterprise; \(N\) represents the normal distribution; \(D\) represents the executive price, or the amount of debt due to be repaid in the \(T\) period. Among formula (1):

\[
d_1 = \frac{\ln\left(\frac{V}{D}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}
\]

\[
d_2 = d_1 - \sigma\sqrt{T-t}
\]  

(2)

Where \(\sigma_A\) represents the volatility of asset value.

The relationship between the volatility of equity value (\(\sigma_E\)) and the volatility of asset value (\(\sigma_A\)) can be expressed as:

\[
\sigma_E = \frac{\sqrt{V}}{E}N(d_1)\sigma_A
\]  

(3)

In empirical study, there are two main methods applied to estimate the volatility of equity value: historical data estimation and GARCH model. Historical data estimation is method of estimating fluctuation in the next period according to the fluctuation from historical data. However, the stock market is sensitive and changeable, and emergency can seriously affect the accuracy of the method [6]. GARCH model is an extension of ARCH model and a specialized regression model for financial data [6]. GARCH model is of great significance in analyzing the volatility of securities and is widely used in the risk assessment and prediction of financial asset returns. Many scholars in China think that GARCH (1,1) accords with the characteristics of Chinese stock market [6], [30], and GARCH (1,1) is expressed as:

\[
\sigma_t^2 = \alpha_0 + \alpha \sigma_{t-1}^2 + \beta \sigma_{t-1}^2
\]  

(4)

Where \(\sigma_t^2\) is the conditional variance of time series, and \(\sigma_{t-1}^2\) is the lag 1 value of it; \(\epsilon_{t-1}\) is the lag 1 value of residual; \(\alpha\) is the income coefficient, \(\beta\) is the lag coefficient, and \(\alpha \neq 0\), \(\beta \geq 0\), \(\alpha + \beta \leq 1\).

2) CORRECTION OF DEFAULT POINT

The enterprise will tear up the contract when the asset value of the enterprise is less than the book value of the enterprise’s debt. Based on a large number of empirical studies in American financial market, KMV model defines the default point (DP) as:

\[
DP = \text{cash liability} + 0.5 \times \text{fixed liability}
\]  

(5)

Financial scholars have conducted extensive explorations based on financial market environment in China. It is found that the KMV model has the strongest risk identification ability when the coefficient of cash liabilities is 1 and the coefficient of fixed liability is 0.75 [6], [29]. Consequently, in this paper, the formula for calculating DP is expressed as:

\[
DP = \text{cash liability} + 0.75 \times \text{fixed liability}
\]  

(6)

3) DEFAULT DISTANCE AND EXPECTED DEFAULT FREQUENCY

The default distance (DD) of an enterprise defined by KMV model is the relative distance between the market value of future assets and DP. That is, DD is the multiple that percentage of asset value to default point relative to standard deviation of asset value. DD can be expressed as:

\[
DD = \frac{E(V) - DP}{E(V)\sigma_A}
\]  

(7)

If the probability distribution of assets is known, the expected default frequency (EDF) can be calculated by DD. Generally, it can be assumed that the asset value is derived from normal distribution or logarithmic normal distribution, so that the theoretical probability of default can be calculated. Assuming that the value of assets is normally distributed, the theoretical formula for calculating the EDF is stated as follows:

\[
EDF = N\left(\frac{DP - E(V)}{E(V)\sigma_A}\right) = N(-DD)
\]  

(8)

B. COPULA FUNCTION

The Copula function describes the correlation between variables, and is actually a class of functions that link the joint distribution function with their respective edge distribution functions. The concept of Copula was originally introduced by Sklar in 1959 when he answered M.Frechet’s question about the relationship between multidimensional distribution functions and low-dimensional edges. In the late 1990s, the relevant theories and methods developed rapidly and applied to the related analysis in the fields of finance and insurance, portfolio analysis and risk management.
1) THE DEFINITION OF BINARY COPULA FUNCTION

Binary Copula functions \( C(\cdot, \cdot) \) have the following properties:

- The definition domain of \( C(\cdot, \cdot) \) is \([0,1]^2\);
- \( C(\cdot, \cdot) \) is grounded and is two-increasing;
- For any variable \( u, v \in [0,1] \), satisfying \( C(u,1)=u \), \( C(1,v)=v \).

Sklar’s theorem states that any multivariate joint distribution can be written in terms of univariate marginal-distribution functions, or a copula which describes the dependence structure between the variables. For example, let \( H(\cdot, \cdot) \) be a joint distribution function with edge distribution \( F(\cdot) \) and \( G(\cdot) \), then there exists a Copula function \( C(\cdot, \cdot) \), satisfying:

\[
H(x,y) = C(F(x), G(y)) \tag{9}
\]

If \( F(x) \), \( G(y) \) are continuous, \( C(\cdot, \cdot) \) is uniquely determined. If \( F(x) \), \( G(y) \) are univariate distribution functions, and \( C(\cdot, \cdot) \) is the corresponding Copula function, then the function \( H(x,y) \) defined by the formula above is the joint distribution function of \( F(x) \) and \( G(y) \).

2) BASIC TYPES OF BINARY COPULA FUNCTION

There are five basic types of binary Copula function, which are normal Copula, t-Copula, Gumbel Copula, Clayton Copula and Frank Copula. Among them, Gumbel Copula, Clayton Copula and Frank Copula also known as Archimedes Copula functions. The shape of their density function, and their features and applications are shown in Tab. 2.

### TABLE 2. Basic Types of Binary Copula Functions.

<table>
<thead>
<tr>
<th>Function</th>
<th>Shape of Density Function</th>
<th>Features and Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Copula</td>
<td>Symmetric</td>
<td>It can only capture the symmetry relation between objects.</td>
</tr>
<tr>
<td>t-Copula</td>
<td>Symmetric; It has a thicker tail than the normal Copula function.</td>
<td>It is better to capture the tail correlation between objects.</td>
</tr>
<tr>
<td>Gumbel Copula</td>
<td>Asymmetric; In the form of “I”.</td>
<td>It can capture the changes of upper-tail correlation quickly, and can be used to describe the correlation between objects with upper-tail correlation.</td>
</tr>
<tr>
<td>Clayton Copula</td>
<td>Asymmetric; In the form of “L”.</td>
<td>It can capture the changes of lower-tail correlation quickly, and can be used to describe the correlation between objects with lower-tail correlation.</td>
</tr>
<tr>
<td>Frank Copula</td>
<td>Symmetric; In the form of “U”.</td>
<td>It can describe the negative correlation between variables and cannot capture the asymmetric correlation between objects.</td>
</tr>
</tbody>
</table>

3) CORRELATION MEASURE BASED ON COPULA FUNCTION

The simplest and most direct way to determine the correlation between two variables is to determine whether their trends are consistent. It is assumed that \((x_1, y_1)\) and \((x_2, y_2)\) are two sets of observations of random vector \((X,Y)\). If \((x_1 - x_2)(y_1 - y_2) > 0\), we say that \((x_1, y_1)\) and \((x_2, y_2)\) are consistent; If \((x_1 - x_2)(y_1 - y_2) < 0\), we say that \((x_1, y_1)\) and \((x_2, y_2)\) are inconsistent.

- **a) Kendall rank correlation coefficient (\( \tau \))**

  It is assumed that \((x_1, y_1)\) and \((x_2, y_2)\) are random variables derived from independent identical distribution. Then Kendall rank correlation coefficient can be expressed as:

  \[
  \tau = P[(x_1 - x_2)(y_1 - y_2) > 0] - P[(x_1 - x_2)(y_1 - y_2) < 0] \tag{10}
  \]

- **b) Spearman rank correlation coefficient (\( \rho \))**

  It is assumed that \((x_1, y_1)\), \((x_2, y_2)\) and \((x_3, y_3)\) are random variables derived from independent identical distribution. Then Spearman rank correlation coefficient can be expressed as:

  \[
  \rho = \frac{3}{2}P[(x_1 - x_2)(y_1 - y_2) > 0] - P[(x_1 - x_2)(y_1 - y_2) < 0] \tag{11}
  \]

IV. CASE STUDY

A. DATA SOURCES

The research objective of this paper is Chinese automobile SCF, so we selected three representative enterprises: Anhui Zhongding Sealing Parts Co., Ltd. (component supplier), SAIC Motor Corporation Limited (automobile core manufacturing enterprise), and Sinomach Automobile Co., Ltd. (distributor). According to the calculation, this paper obtained stock closing price, total equity and liabilities from Wind and their annual reports, semi-annual reports and quarterly reports, which are from 2006Q1 to 2018Q1.

B. KMV MODEL FOR THE MEASUREMENT OF INDIVIDUAL CREDIT RISK

1) VOLATILITY OF EQUITY VALUE
First of all, we assumed that the daily stock yields of the enterprise are derived from logarithmic normal distribution. The daily stock yields were calculated using the following formula:

$$R_e = (\ln P_t - \ln P_{t-1}) \times 100$$  \hspace{1cm} (12)

Where $P_t$ and $P_{t-1}$ are stock prices on days $t$ and $t-1$.

Using Augmented Dicky-Fuller (ADF), daily stock yields autocorrelation function (ACF) and partial correlation function (PACF) to examine the daily stock yields of three enterprises, we can come to a conclusion that daily stock yields of the three enterprises are remarkably stable and there is no obvious autocorrelation.

According to the above conclusions of ADF test and autocorrelation test, the daily stock yields of SAIC is stable, and there is no autocorrelation relationship. Therefore, the mean value equation is set up as white noise, and the daily stock yields equation is established as follows:

$$R_t = C + \mu_t$$  \hspace{1cm} (13)

Where $\mu_t$ is the residual sequence of yield.

The GARCH (1,1) model was established by Eviews, and the estimation results of the related parameters were obtained in Tab. 3.

### TABLE 3. The parameter estimation results of GARCH (1,1).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.170915</td>
<td>0.045911</td>
<td>3.722735</td>
<td>0.0002</td>
</tr>
<tr>
<td>RESID(-1)^2</td>
<td>0.103156</td>
<td>0.023053</td>
<td>4.474627</td>
<td>0.0000</td>
</tr>
<tr>
<td>GARCH(-1)</td>
<td>0.829705</td>
<td>0.030546</td>
<td>27.16251</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

As shown in the Tab. 3, the $p$ value of the lag term and the residual term in the model is less than 5%, which indicates that the coefficient is not zero significantly. The GARCH (1,1) model is established as:

$$\sigma_t^2 = 0.190915 + 0.103156\mu_{t-1}^2 + 0.829705\sigma_{t-1}^2$$

Then the formula of long-term variance rate is

$$V_t = \frac{c}{1-\alpha-\beta} = 24.92\%$$

Therefore, the formula of daily volatility is

$$\sqrt{V_t} = 49.92\%$$

and the formula of annual volatility is

$$\sqrt{V_{365}} = 49.92\% \times \sqrt{365} = 11.04\%$$

The annual volatility of the three enterprises are calculated using the same method, and the results are shown in Tab. 4.

### TABLE 4. The volatility of equity value of the three enterprises (Take 2007Q1-2008Q4 as example).

<table>
<thead>
<tr>
<th>Financing Time</th>
<th>Due Date</th>
<th>Zhongding Sealing Parts</th>
<th>SAIC</th>
<th>Sinomach Automobile</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006Q1</td>
<td>2007Q1</td>
<td>43.31%</td>
<td>59.04%</td>
<td>54.57%</td>
</tr>
<tr>
<td>2006Q2</td>
<td>2007Q2</td>
<td>45.65%</td>
<td>54.88%</td>
<td>60.75%</td>
</tr>
<tr>
<td>2006Q3</td>
<td>2007Q3</td>
<td>47.92%</td>
<td>58.51%</td>
<td>72.86%</td>
</tr>
<tr>
<td>2006Q4</td>
<td>2007Q4</td>
<td>46.25%</td>
<td>50.58%</td>
<td>69.30%</td>
</tr>
<tr>
<td>2007Q1</td>
<td>2008Q1</td>
<td>50.56%</td>
<td>57.49%</td>
<td>72.26%</td>
</tr>
<tr>
<td>2007Q2</td>
<td>2008Q2</td>
<td>55.90%</td>
<td>46.33%</td>
<td>77.45%</td>
</tr>
<tr>
<td>2007Q3</td>
<td>2008Q3</td>
<td>59.35%</td>
<td>62.35%</td>
<td>78.26%</td>
</tr>
<tr>
<td>2007Q4</td>
<td>2008Q4</td>
<td>66.68%</td>
<td>64.24%</td>
<td>82.24%</td>
</tr>
</tbody>
</table>

### 2) DEFAULT POINT

We got the cash liabilities and fixed liabilities from the annual reports, semi-annual reports and quarterly reports of three enterprises, and substituted the formula (6) to get their default points (DP), which are shown in Tab. 5.

### TABLE 5. The default points of the three enterprises (Take 2007Q1-2008Q4 of SAIC as example).

<table>
<thead>
<tr>
<th>Financing Time</th>
<th>Due Date</th>
<th>Cash Liability</th>
<th>Fixed Liability</th>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006Q1</td>
<td>2007Q1</td>
<td>3476837.73</td>
<td>656621.40</td>
<td>3969303.78</td>
</tr>
<tr>
<td>2006Q2</td>
<td>2007Q2</td>
<td>3796998.36</td>
<td>693946.08</td>
<td>4317457.92</td>
</tr>
<tr>
<td>2006Q3</td>
<td>2007Q3</td>
<td>3851871.90</td>
<td>756668.10</td>
<td>4419372.98</td>
</tr>
<tr>
<td>2006Q4</td>
<td>2007Q4</td>
<td>4869088.12</td>
<td>1011832.65</td>
<td>5627962.61</td>
</tr>
<tr>
<td>2007Q1</td>
<td>2008Q1</td>
<td>5145271.52</td>
<td>972852.93</td>
<td>5874911.22</td>
</tr>
<tr>
<td>2007Q2</td>
<td>2008Q2</td>
<td>5968959.00</td>
<td>1067153.37</td>
<td>6769324.03</td>
</tr>
<tr>
<td>2007Q3</td>
<td>2008Q3</td>
<td>5595101.42</td>
<td>1122946.63</td>
<td>6437311.39</td>
</tr>
<tr>
<td>2007Q4</td>
<td>2008Q4</td>
<td>5510236.86</td>
<td>1419953.83</td>
<td>6577002.23</td>
</tr>
</tbody>
</table>
3) DEFAULT DISTANCE AND EXPECTED DEFAULT FREQUENCY

The evaluation of credit risk usually takes one year as a phase, so this paper also sets \( T-t=1 \). In addition, for the risk-free rate of return in the KMV model, this paper selected the benchmark one-year deposit rate published by the People's Bank of China.

The KMV model calculation process can be reduced as follows:

\[
\begin{align*}
E &= VN(d_1) - De^{-\sigma^2}N(d_2) \\
\sigma_e &= \frac{V}{E} N(d_1) \sigma_A \\
d_1 &= \frac{\ln\left(\frac{V}{D}\right) + (r + \frac{\sigma_A^2}{2})(T-t)}{\sigma_A \sqrt{T-t}} \\
d_2 &= d_1 - \sigma_A \sqrt{T-t}
\end{align*}
\]

\[ (14) \]

Since the order of magnitude of \( V \) and \( \sigma_A \) in the equations is very different, it is necessary to standardize \( V \) according to the liability \( D \). Besides, we introduced the parameter \( \frac{E}{D} = \frac{E_tD}{E_d} \), so that it can be solved iteratively. We brought \( V = xE \) into the equations and got the simplified equations as follows:

\[
1 = xN(d_1) - e^{-\sigma^2}N(d_2) / E_tD \\
\sigma_e = xN(d_1) \sigma_A \\
\ln(\frac{E}{D}) + (r + \frac{\sigma_A^2}{2})(T-t) \\
d_1 = \frac{\ln(E_tD) + (r + \frac{\sigma_A^2}{2})(T-t)}{\sigma_A \sqrt{T-t}} \\
d_2 = d_1 - \sigma_A \sqrt{T-t}
\]

Taking all known data into the model, which was solved by the fsolve function of MATLAB. We got the default distance (\( DD \)) and the expected default frequency (\( EDF \)), which are shown in Tab. 6.

<table>
<thead>
<tr>
<th>Financing Time</th>
<th>Due Date</th>
<th>( r )</th>
<th>( DP ) (Ten thousand dollars)</th>
<th>( E ) (Ten thousand dollars)</th>
<th>Volatility of Equity Value</th>
<th>( DD )</th>
<th>( EDF )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006Q1</td>
<td>2007Q1</td>
<td>2.79%</td>
<td>3476837.73</td>
<td>8437725.47</td>
<td>59.04%</td>
<td>1.67</td>
<td>4.72%</td>
</tr>
<tr>
<td>2006Q2</td>
<td>2007Q2</td>
<td>3.06%</td>
<td>3796998.36</td>
<td>11863913.68</td>
<td>54.88%</td>
<td>1.80</td>
<td>3.56%</td>
</tr>
<tr>
<td>2006Q3</td>
<td>2007Q3</td>
<td>3.87%</td>
<td>3851871.90</td>
<td>18893167.90</td>
<td>58.51%</td>
<td>1.70</td>
<td>4.50%</td>
</tr>
<tr>
<td>2006Q4</td>
<td>2007Q4</td>
<td>4.14%</td>
<td>4869088.12</td>
<td>17078532.84</td>
<td>50.58%</td>
<td>1.95</td>
<td>2.54%</td>
</tr>
<tr>
<td>2007Q1</td>
<td>2008Q1</td>
<td>4.14%</td>
<td>5145271.52</td>
<td>9498992.18</td>
<td>57.49%</td>
<td>1.70</td>
<td>4.48%</td>
</tr>
<tr>
<td>2007Q2</td>
<td>2008Q2</td>
<td>4.14%</td>
<td>5968959.00</td>
<td>5627333.99</td>
<td>46.33%</td>
<td>2.06</td>
<td>1.96%</td>
</tr>
<tr>
<td>2007Q3</td>
<td>2008Q3</td>
<td>4.14%</td>
<td>5595101.42</td>
<td>4107495.24</td>
<td>62.35%</td>
<td>1.49</td>
<td>6.86%</td>
</tr>
<tr>
<td>2007Q4</td>
<td>2008Q4</td>
<td>2.25%</td>
<td>5512036.86</td>
<td>3747188.64</td>
<td>64.24%</td>
<td>1.47</td>
<td>7.10%</td>
</tr>
</tbody>
</table>

The \( DD \) and \( EDF \) of the three enterprises are shown in Fig. 1–2.

![Graph showing default distance for three enterprises](image-url)

**FIGURE 1.** The default distance of the three enterprises.
FIGURE 2. The expected default frequency of the three enterprises.

We can find that under the same macroeconomic background, the variation trend of DD and EDF of the three enterprises are consistent, which further verifies the accuracy and applicability of the modified KMV model. Among them, the EDF of Sinomach Automobile in 2014Q4 is very large, which is an abnormal value. By searching for relevant information, we found that this was due to a series of major assets restructuring, equity acquisitions, issuance of shares to buy assets and other events in 2014, resulting in a large volatility of the stock price. It shows that the abnormal fluctuation of the stock price will directly lead to an increase in EDF of the enterprise.

C. COPULA FUNCTION FOR THE MEASUREMENT OF

FIGURE 3. The frequency histogram of EDF of the three enterprises.

PORTFOLIO CREDIT RISK

1) DETERMINATION OF MARGINAL DISTRIBUTION

Let X, Y, Z represent the EDF of Zhongding Sealing Parts, SAIC and Sinomach Automobile respectively. And let E1, E2, E3 represent the edge distribution of X, Y, Z respectively.

First of all, according to the frequency histograms of EDF of the three enterprises in Fig. 3, we can see that X, Y, Z are not derived from normal distribution, but right biased distribution.
Secondly, the skewness and kurtosis of EDF of the three enterprises were calculated, and the results are shown in Tab. 7. The skewness and kurtosis of normal distribution are 0 and 3 respectively. It can be seen from the Tab. 7 that the skewness of EDF of the three enterprises are all positive, indicating that all of them are right biased distribution, and the kurtosis are all greater than 3, indicating that the peak value is higher than the normal distribution, showing the characteristics of sharp peak and thick tail.

<table>
<thead>
<tr>
<th>Enterprises</th>
<th>skewness</th>
<th>kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhongding Sealing</td>
<td>1.3088</td>
<td>4.2463</td>
</tr>
<tr>
<td>Parts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAIC</td>
<td>1.2985</td>
<td>3.4602</td>
</tr>
<tr>
<td>Sinomach Automobile</td>
<td>1.9511</td>
<td>7.4397</td>
</tr>
</tbody>
</table>

Finally, according to the results of normal test of JB test, KS test and Lillie test functions in Tab. 8, the h values of the three tests are all equal to 1, and the p value are all less than 0.01. The results show that X, Y, Z are not from normal distribution, but the distribution of high kurtosis and fat tail. However, it is difficult to find this type of distribution in the usual distributions.

<table>
<thead>
<tr>
<th>Enterprises</th>
<th>JB test</th>
<th>KS test</th>
<th>Lille test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhongding Sealing Parts</td>
<td>1</td>
<td>1.36E-10</td>
<td>1</td>
</tr>
<tr>
<td>SAIC</td>
<td>1</td>
<td>1.36E-10</td>
<td>1</td>
</tr>
<tr>
<td>Sinomach Automobile</td>
<td>1</td>
<td>1.36E-10</td>
<td>1</td>
</tr>
</tbody>
</table>

In order to obtain the edge distribution of X, Y, Z, the nonparametric method of kernel density estimation is used. We used the ecdf function to find the empirical distribution function of the sample and used it to approximate the population distribution. Besides, we obtained the kernel distribution estimates of samples by ksdensity function. The illustrations of empirical distribution function and kernel distribution estimation are shown in Fig. 4, and they almost coincide with each other, which indicates it is accurate to use the kernel density estimation to determine the edge distribution of X, Y, Z.

2) SELECTING THE APPROPRIATE COPULA FUNCTION

According to the edge distribution, the binary frequency histograms of three combinations, i.e., \((E_1, E_2), (E_2, E_3), (E_1, E_3)(i=1,2,\cdots,n)\), can be drawn. Among them, \(E_1\), \(E_2\) and \(E_3\) represent Zhongding Sealing Parts, SAIC and Sinomach Automobile respectively. Take “SAIC & Sinomach Automobile” as an example, as shown in Fig. 5.
As seen from the diagram, the tail of the histogram is asymmetric. That is, the joint density function (Copula function) of \((E2, E3)\) has an asymmetric tail. According to the previous description of the five kinds of Copula functions, Gumbel Copula is asymmetric, and its density distribution is in the form of “J”, so we can preliminarily conclude that the Gumbel Copula function should be selected to describe the structure of default dependence of “SAIC & Sinomach Automobile”.

In order to select the Copula function more accurately, we calculated the parameters in the Copula by copulafit function and introduce the empirical Copula function. Then, through calculating the square Euclidean distance between the constructed Copula function and the empirical Copula function, we evaluated and selected the most appropriate Copula function.

The parameters of the three combinations in the five Copula functions were calculated and shown in Tab. 9. The corresponding expression of Copula functions can be obtained according to the parameter, which are not discussed here.

![Figure 5](image.png)

**FIGURE 5.** The binary frequency histogram of “SAIC & Sinomach Automobile”.

<table>
<thead>
<tr>
<th>Combinations</th>
<th>Normal Copula (\hat{\alpha} )</th>
<th>(t)-Copula parameter</th>
<th>Gumbel Copula</th>
<th>Clayton Copula</th>
<th>Frank Copula</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1 &amp; E2</td>
<td>0.8175</td>
<td>0.8662</td>
<td>2.8302</td>
<td>4.9830</td>
<td>10.2339</td>
</tr>
<tr>
<td>E2 &amp; E3</td>
<td>0.6782</td>
<td>0.7958</td>
<td>2.1628</td>
<td>2.5420</td>
<td>7.1327</td>
</tr>
<tr>
<td>E1 &amp; E3</td>
<td>0.5770</td>
<td>0.7197</td>
<td>1.8777</td>
<td>2.1899</td>
<td>5.6984</td>
</tr>
</tbody>
</table>

According to the parameters of each Copula function, the diagrams of density function can be drawn. Take “SAIC & Sinomach Automobile” as an example, as shown in Fig. 6.
FIGURE 6. The density function diagrams of five Copula functions.

From the above figures we can see that, Gumbel Copula is closer to the binary frequency histogram of “SAIC & Sinomach Automobile”.

The rank correlation coefficients of Kendall and Spearman corresponding to five kinds of Copula functions can be obtained by copulastat function. The calculated results are shown in Tab. 10.

<table>
<thead>
<tr>
<th>Combinations</th>
<th>Normal Copula</th>
<th>t-Copula</th>
<th>Gumbel Copula</th>
<th>Clayton Copula</th>
<th>Frank Copula</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau$</td>
<td>$\rho$</td>
<td>$\tau$</td>
<td>$\rho$</td>
<td>$\tau$</td>
</tr>
<tr>
<td>E1 &amp; E2</td>
<td>0.6093</td>
<td>0.8042</td>
<td>0.6669</td>
<td>0.8174</td>
<td>0.6467</td>
</tr>
<tr>
<td>E2 &amp; E3</td>
<td>0.4745</td>
<td>0.6607</td>
<td>0.5859</td>
<td>0.7628</td>
<td>0.5376</td>
</tr>
<tr>
<td>E1 &amp; E3</td>
<td>0.3916</td>
<td>0.5590</td>
<td>0.5114</td>
<td>0.6796</td>
<td>0.4674</td>
</tr>
</tbody>
</table>

From the results in Tab. 10, it can be seen that the Kendall and Spearman rank correlation coefficients of the five kinds of Copula functions satisfies the same relationship, which are $\tau(E1 & E2) > \tau(E2 & E3) > \tau(E1 & E3)$ and $\rho(E1 & E2) > \rho(E2 & E3) > \rho(E1 & E3)$. It shows that the correlation between the direct up- and down-stream enterprises tends to be stronger, and the correlation between the cross-tier enterprises tends to be weaker. In addition, the correlation between component supplier and core manufacturer is stronger than that between core manufacturer and distributor.

We used the spline function to calculate the value of the empirical distribution function at the original sample point, and drew diagrams of empirical Copula functions of the three combinations, which are shown in Fig. 7.

FIGURE 7. The diagrams of empirical Copula functions of the three combinations.

The results of the calculation of the square Euclidean distance between the five Copula functions and the empirical Copula functions are shown in Tab. 11.

The results in the Tab. 11 show that the Copula functions corresponding to the least square Euclidean square distance of the three combinations are Clayton Copula, Gumbel Copula and Gumbel Copula, respectively. Among them, “SAIC & Sinomach Automobile” exactly confirmed our preliminarily conclusion based on binary histogram.
TABLE 11. The square Euclidean distance.

<table>
<thead>
<tr>
<th>Combinations</th>
<th>Normal Copula</th>
<th>t-Copula</th>
<th>Gumbel Copula</th>
<th>Clayton Copula</th>
<th>Frank Copula</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1 &amp; E2</td>
<td>0.0319</td>
<td>0.0240</td>
<td>0.0345</td>
<td><strong>0.0116</strong></td>
<td>0.0187</td>
</tr>
<tr>
<td>E2 &amp; E3</td>
<td>0.0184</td>
<td>0.0185</td>
<td><strong>0.0128</strong></td>
<td>0.0395</td>
<td>0.0204</td>
</tr>
<tr>
<td>E1 &amp; E3</td>
<td>0.0228</td>
<td>0.0210</td>
<td><strong>0.0173</strong></td>
<td>0.0420</td>
<td>0.0229</td>
</tr>
</tbody>
</table>

3) CALCULATION OF ALL KINDS OF PROBABILITY OF DEFAULT

We can use KMV model to calculate the EDF of a single financing enterprise, but because of the existence of default contagion, if we only take the result of the KMV model as the EDF of the financing enterprise, we will certainly underestimate the credit risk of the financing enterprise.

Therefore, we need to introduce the appropriate Copula function to calculate the EDF of the enterprises in the financing portfolio when they default at the same time, and then calculate the joint EDF of the financing enterprise portfolio.

For financing enterprise portfolio, default is generally divided into four types. Assuming that the indicator variable \( Y_n \) represents the default state of the financing enterprise, that is:

\[
Y_n = \begin{cases} 
1, \text{default} \\
0, \text{not-default} 
\end{cases} \quad n = 1, 2 \tag{16}
\]

TABLE 12. Four default types.

<table>
<thead>
<tr>
<th>Default Types</th>
<th>SAIC</th>
<th>Sinomach Automobile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>default</td>
<td>DD \leq 0</td>
</tr>
<tr>
<td>Type 2</td>
<td>default</td>
<td>DD \leq 0</td>
</tr>
<tr>
<td>Type 3</td>
<td>not default</td>
<td>DD &gt; 0</td>
</tr>
<tr>
<td>Type 4</td>
<td>not default</td>
<td>DD &gt; 0</td>
</tr>
</tbody>
</table>

When the asset value of the financing enterprise is lower than the default point, that is, the default distance is zero, the enterprise will choose to default. Suppose that index 2 and 3 represent SAIC and Sinomach Automobile respectively, and C denotes Copula function.

a) The probability of both SAIC and Sinomach Automobile defaulting can be expressed as follows:

\[
P[V_2 \leq DP_2, V_3 \leq DP_3] = P[DD_2 \leq 0, DD_3 \leq 0] = P[Y_2 = 1, Y_3 = 1] = C(p_2, p_3) \tag{17}
\]

b) The probability of SAIC defaulting and Sinomach Automobile not defaulting can be expressed as follows:

\[
P[V_2 \leq DP_2, V_3 > DP_3] = P[DD_2 \leq 0, DD_3 > 0] = P[Y_2 = 1, Y_3 = 0] = p_2 - C(p_2, p_3) \tag{18}
\]

c) The probability of SAIC not defaulting and Sinomach Automobile defaulting can be expressed as follows:

\[
P[V_2 > DP_2, V_3 \leq DP_3] = P[DD_2 > 0, DD_3 \leq 0] = P[Y_2 = 0, Y_3 = 1] = p_3 - C(p_2, p_3) \tag{19}
\]

d) The probability of both SAIC and Sinomach Automobile not defaulting can be expressed as follows:

\[
P[V_2 > DP_2, V_3 > DP_3] = P[DD_2 > 0, DD_3 > 0] = P[Y_2 = 0, Y_3 = 0] = 1 - p_2 - p_3 + C(p_2, p_3) \tag{20}
\]

As for the credit risk analysis of the financing enterprise portfolio in SCF, commercial banks and other financial institutions generally think that any financing enterprise default in the portfolio is regarded as the trigger condition for all the financing enterprises in the portfolio to default. Then the formula for calculating the joint EDF is as follows:

\[
P = 1 - \left[1 - p_2 - p_3 + C(p_2, p_3)\right] \tag{21}
\]

\[
= p_2 + p_3 - C(p_2, p_3) \tag{21}
\]

If default contagion is considered, the EDF of Sinomach Automobile under the condition that SAIC defaults can be expressed as follows:

\[
P[V_2 \leq DP_2 | V_2 \leq DP_2] = \frac{P[V_2 \leq DP_2, V_3 \leq DP_3]}{P[V_2 \leq DP_2]} \tag{22}
\]

Likewise, the EDF of SAIC under the condition that Sinomach Automobile defaults can be expressed as follows:

\[
P[V_2 \leq DP_2 | V_3 \leq DP_3] = \frac{P[V_2 \leq DP_2, V_3 \leq DP_3]}{P[V_3 \leq DP_3]} \tag{23}
\]
According to the results, we drew the graphs including the simultaneous EDF, the joint EDF and the EDF of single enterprise of the financing portfolio, as shown in Fig. 8.

Based on the figures presented above, we can draw the following conclusions:

Firstly, the joint EDF is greater than the EDF of single enterprise, and the joint EDF is close to the EDF of the larger one in the financing enterprise portfolio. It indicates that ignoring default dependence or default contagion may underestimate the risk. Therefore, when measuring the credit risk of supply chain financing enterprise portfolio, we should analyze the default dependence between them, rather than treating the financing enterprises as isolated individuals. In addition, the joint EDF of the three combinations is close to the EDF of Zhongding Sealing Parts, Sinomach Automobile, Sinomach Automobile respectively, the larger one in financing enterprise portfolio.

Secondly, the simultaneous EDF is smaller than the EDF of single enterprise, and the simultaneous EDF is close to the EDF of the smaller one in the financing enterprise portfolio. The extreme credit risk event that the enterprises in the financing portfolio default simultaneously is only a special case of default jointly.

Thirdly, the trend of joint EDF of “SAIC & Sinomach Automobile” is almost same as “Zhongding Sealing Parts & Sinomach Automobile”, which both mainly depend on the EDF of Sinomach Automobile. It shows that if the financing portfolio includes Sinomach Automobile, the joint EDF is relatively high.

Then, we drew the graphs of the conditional EDF of
the three combinations, as shown in Fig. 9.

Based on the figures presented above, we can draw the following conclusions:

Firstly, the conditional EDF is obviously greater than the EDF of single enterprise. This suggests that default can be contagious. In other words, with one enterprise in the financing portfolio defaulting, the EDF of others will increase.

Secondly, in most cases, the EDF of Zhongding Sealing Parts under the condition that SAIC defaults is greater than the EDF of SAIC under the condition that Zhongding Sealing Parts defaults, the EDF of Sinomach Automobile under the condition that SAIC defaults is greater than the EDF of SAIC under the condition that Sinomach Automobile defaults, and the EDF of Sinomach Automobile under the condition that Zhongding Sealing Parts defaults is greater than the EDF of Zhongding Sealing Parts under the condition that Sinomach Automobile defaults. It shows that if the relationship between enterprises in the financing portfolio are up- and down-stream, the intensity of default contagion between enterprises is asymmetrical. The default contagion of core enterprise to up- and down-stream enterprises is more significant than up- and down-stream enterprises to core enterprise, and the default contagion of upstream enterprise to downstream enterprise is more significant than downstream enterprise to upstream enterprise.

In order to verify the authenticity and effectiveness of our conclusions, we selected two component suppliers, which are Anhui Zhongding Sealing Parts Co., Ltd. and Ningbo Huaxiang Electronic Co., Ltd., as the financing enterprise portfolio. We calculated the conditional EDF of the portfolio using the same method, and the results are shown in Fig. 10.

![Graph](image-url)

**FIGURE 9.** The conditional EDF of the three combinations.

As shown in Fig. 10, there is no obvious relationship between conditional EDF of Zhongding Sealing Parts and Ningbo Huaxiang Electronic. In other words, there is no default contagion and default dependency between them. For commercial banks and other financial institutions, choosing enterprises at the same tier in the supply chain as the financing enterprise portfolio can avoid the risk of default contagion effectively.

In addition, we have drawn the graphs of simultaneous EDF and joint EDF of “Zhongding Sealing Parts & Ningbo Huaxiang Electronic” and “Zhongding Sealing Parts & SAIC”, as shown in Fig. 11–12.
As shown in the figures, in most cases, the simultaneous EDF and joint EDF of “Zhongding Sealing Parts & Ningbo Huaxiang Electronic” are greater than that of “Zhongding Sealing Parts & SAIC”. We can draw two conclusions. On the one hand, the simultaneous EDF of financing enterprise portfolio, of which the enterprises are at the same tier of the supply chain, are always higher. On the other hand, if the financing enterprise portfolio includes the core enterprise of supply chain, the joint EDF of it can be effectively reduced.

Furthermore, we drew the graphs of the simultaneous EDF of the three combinations, considering the default contagion and assuming that they are independent of each other, as shown in Fig. 13.

It can be observed from the diagrams that the simultaneous EDF is always greater when considering default contagion than assuming that they are independent of each other, especially in 2008-2009 and 2015-2016, which are two special periods of economic fluctuation in China. This shows that ignoring the default contagion and treating the financing enterprises as isolated individuals will seriously underestimate the potential credit risk of the financing enterprise, especially in times of economic crisis.
V. CONCLUSIONS
Through the above modeling and empirical analysis, we analyze the credit risk of single financing enterprise and financing enterprise portfolio in supply chain. We find that the default distance (\(DD\)) and expected default frequency (\(EDF\)) of a single financing enterprise can be calculated by KMV model, and the relationship of default risk between financing enterprises can be compared simply. However, because of the existence of default contagion, if the financing enterprises are separated from each other, the default risk will be underestimated. Therefore, on the basis of the results of KMV model, the credit risk of the financing enterprise portfolio can be measured more accurately by adding the Copula function to describe the default dependence between the financing enterprises.

The study provides important theoretical and practical contributions. Through case analysis, we have come to the following conclusions.

Firstly, different financing enterprise portfolios apply to different Copula functions. Because of the intricate relationship between enterprises in the supply chain, their stock prices have different dependence relations. It mainly that Copula functions of different financing enterprise portfolios are different, and we should choose the most appropriate Copula function to describe the correlation between enterprises in financing portfolios.

Secondly, default contagion does exist, if it is ignored, the default risk of financing enterprises will be underestimated. There are four main bases: the joint \(EDF\) is greater than the \(EDF\) of single enterprise; the conditional \(EDF\) is obviously greater than the \(EDF\) of single enterprise; the simultaneous \(EDF\) is always greater when considering default contagion than assuming that they are independent of each other.

Thirdly, the intensity of default contagion between enterprises in the financing enterprise portfolio is asymmetrical. The default contagion of the core enterprise to the upstream and downstream enterprises is stronger, and the default contagion of the upstream enterprise to the downstream enterprise is stronger.

Finally, the joint \(EDF\) of the financing enterprise portfolio mainly depends on the enterprise with the larger \(EDF\) in the portfolio, so commercial banks and other financial institutions should choose the financing enterprises carefully.

In addition, this paper also has some limitations. For example, it has not considered more complex Copula functions, and considered only binary combinations. On the basis of this paper, the scholars can compare more types of Copula functions, generalize the binary to multivariate, and obtain more accurate and practical conclusions.
REFERENCES


[34] L. Chollete, A. Heinen, and A. Valdesoso, “Modeling international financial returns with a multivariate regime-


