Effect of Imperfect Channel Estimation on the Performance of Cognitive Satellite Terrestrial Networks

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Abstract—The incorporation of cognitive radio (CR) techniques into satellite communication systems, has been considered as one of the most promising approaches to address the spectrum scarcity, which constitutes an advanced infrastructure known as cognitive satellite terrestrial network (CSTN). Due to the estimation error or feedback delay, perfect channel state information (CSI) of satellite and/or terrestrial links in CSTNs are normally unavailable. This paper investigates the effect of imperfect CSI on the performance of CSTN, where a secondary satellite network coexists with a primary terrestrial network by employing the underlay cognitive mechanism according to which the satellite user is allowed to access the licensed spectrum without deteriorating the operation of terrestrial user. Specifically, we derive the analytical expressions for the outage probability and ergodic capacity of the cognitive network, which provides an efficient approach to jointly evaluate the impacts of imperfect channel estimations for different links on the performance of considered network. Moreover, simple asymptotic outage probability formula in the high signal-to-noise ratio regime is presented to reveal the diversity order and coding gain of the CSTNs. Finally, numerical results are provided to confirm the validity of the theoretical analysis, as well as quantitatively analyze the effects of various system parameters on the performance of the CSTNs with CSI imperfection.

Index Terms—Cognitive satellite terrestrial networks, imperfect channel state information, outage probability, ergodic capacity.

I. INTRODUCTION

Satellite communication is capable of providing seamless connectivity and high speed broadband access for worldwide users based on an anywhere–anytime paradigm, especially for users who allocate in the disaster, desert, and/or suburb areas [1]-[3]. However, due to the explosive growing demand for broadband satellite services in recent years, the licensed spectrum of satellite networks seems to be insufficient to meet the forthcoming requirement. Given this fact, exploring new insights and architectures into the utilization of spectrum in satellite communications have challenged the traditional spectrum management approaches [4].

A. Background and Motivation

To alleviate the problem of spectrum scarcity, a cognitive radio (CR) technique, which can increase the spectrum utilization efficiency by adapting its transmission parameters according to the state of the environment [5], has been introduced in the satellite networks to form a new architecture networks, namely as cognitive satellite terrestrial network (CSTN) [6], [7]. In this network, satellite networks can perform as the primary/cognitive networks and share spectrum with terrestrial networks, which act as cognitive/primary networks, on the premise that the QoS of primary networks is ensured [8]. Thus, the spectrum utilization efficiency of primary networks can be enhanced as well as the problem of spectrum scarcity of cognitive networks can be alleviated.

Until now, several ways have been proposed to apply the CR in the CSTN, such as interweave or spectrum sensing [9], underlay [10], overlay [11], and database approaches [12]. Among those ways, the underlay strategy, with which cognitive user can utilize the licensed terrestrial spectrum resources as long as the harmful mutual interference is properly regulated below a predefined threshold, is the simplest and has been introduced in various networks, such as terrestrial networks [13] and CSTN [10]. On the subject of the underlay based CSTN, many works have been done from various performance metrics, such as outage probability[10], ergodic capacity [14], secure transmission rate [15] with a single antenna eavesdropper, and maximization transmission rate under the constraint of limited transmission power [16] and QoS of cognitive user [17]. Extension work of [15] to a multi-antennas eavesdropper was conducted in [18]. Moreover, some studies have explored the benefit of the underlay based CSTN in various scenarios. The authors in [19] studied the effects of practical hardware impairments on the outage probability of underlay based CSTN. The authors in [20] investigated the specific primary exclusion zone by employing the statistical modeling.

Although those aforementioned works have significantly improved our understanding on the performance of the underlay based CSTNs, they were restricted to the scenarios with perfect channel state information (CSI). In practical scenarios, however, the exact perfect CSI is normally unavailable for
satellite communications due to large latency [21], [22]. Moreover, when the CSI of interfering link between the cognitive transmitter and primary receiver is not perfect, the traditional interference power constraint can no longer guarantee the QoS of the primary user [23]. In this regard, channel estimation is a crucial issue for practical implementation of CR approach in the CSTNs, which needs further investigation to deal with the uncertainty of imperfect channel estimations, and thus guarantee the communication quality of the integrated networks.

B. Contribution and Novelty

This paper focuses on the uplink CSTN, where the satellite system acts as the cognitive network while the terrestrial system has the role of the primary one. In this work, perfect CSIs of both the cognitive satellite link and terrestrial interference link are considered unavailable due the following facts:

- For the cognitive satellite link, perfect CSI estimation is unavailable because of the high latency affected by the round trip propagation delay [21]. For a geostationary satellite, exact CSI experiences a quite delay, while for non-geostationary satellites, the high Doppler shifts may impair the availability of the CSI [22].
- For the terrestrial interference link, the acquisition of instantaneous CSI at the cognitive satellite user may be difficult due to the effect of delay and time-varying nature of the transmitter-receiver pair in different networks, which implies that the feedback overhead should be taken into account [24].
- Moreover, channel estimation is not simple and straightforward in CSTNs because of the inherent nature that the cognitive satellite link and terrestrial interference link may experience different signal latency. Hence, separate estimations are required at the satellite user for different links.

Different from the previous works, we focus on the overall effects of imperfect channel estimations on the performance of uplink CSTNs, where the cognitive satellite system is allowed to access the spectral resources of the primary terrestrial system without deteriorating its communication quality. Specifically, our contributions are outlined as follows:

- By considering the inherent nature of both satellite and terrestrial links, an efficient method that separately estimates the cognitive satellite link and terrestrial interference link is employed at the satellite user, where the actual channel gain of the satellite link is estimated by using the training data, while the terrestrial interference channel is obtained by using a feedback link.
- We then derive the novel analytical expressions for the outage probability and ergodic capacity of the cognitive satellite system, which provide efficient means to evaluate the effects of key system parameters, such as number of training symbols and shadowing severities of satellite link, and feedback delay coefficient and interference constraints of terrestrial interference link.

The interference from terrestrial terminal to the satellite can be considered to be negligible due to large distance [10],[14]-[17].

- To gain further insights, the simple asymptotic expressions of outage probability at high SNR are developed to examine the asymptotic behavior of the considered system, which characterize the impact of key system parameters on two important performance metrics, namely the achievable diversity order and coding gain.

II. SYSTEM MODEL

As shown in Fig. 1, we consider an uplink cognitive satellite terrestrial network, where the terrestrial network acts as the primary system and shares the spectrum resource with the satellite network, which is the secondary system. To improve the overall spectrum efficiency, we assume that the underlay technique is employed, thus the satellite user utilizes the same spectrum as the terrestrial user simultaneously without deteriorating the communication quality of the latter. From a practical aspect of possible application, the terrestrial system can be a Long-Term Evolution (LTE) system while the satellite system may be a Digital Video Broadcasting-Return channel by Satellite (DVB-RCS) system [16], [17].

Let $h_s$ denote the channel coefficient between satellite and secondary user, and the received signal at the satellite is given by

$$y_s = h_s x + n_s,$$  \hspace{1cm} (1)

where $x$ denotes the transmitted signal with average power $P_s$, and $n_s \sim \mathcal{N}(0, \sigma_B^2)$ represents the zero mean additive white Gaussian noise (AWGN). Recall that in an underlay scheme, the interference at the BS caused by the cognitive user must not exceed a predefined interference power constraint $Q$. To achieve this, satellite user must adapt its transmit power based on the instantaneous CSI of the mutual interference link, namely, [10], [14]

$$P_s = \min\left(\frac{Q}{|h_p|^2}, P_t\right)$$  \hspace{1cm} (2)

where $P_t$ is the maximum allowable transmit power by the power amplifier, $h_p$ the terrestrial interference link from the satellite user to BS. In the specific scenario, we consider
that the satellite terminal is a mobile/portable terminal and therefore employ a widely-employed Shadowed-Rician (SR) fading model for the cognitive satellite network with closed form, which can be used either for mobile/portable terminals or for fixed terminals operating in various propagation environments [10], [14]-[17]. Moreover, without loss of generality, the Rayleigh fading channel is considered for the terrestrial interference link [10].

As discussed before, the perfect CSIs of both $h_s$ and $h_p$ are normally impossible to obtain at the cognitive satellite user. For the considered cognitive architecture, due to the inherent different nature of both links, the overall communication for channel estimation and data transmission can be summarized in the following protocols:

- **Channel Estimation of Satellite Uplink**: In this phase, the channel estimation is performed by using forward training, through which satellite user transmits pilots to the satellite, and then the satellite estimates the uplink channel and sends the estimated value over the downlink [21], [22]. However, due to the large propagation delay, the estimation of the uplink channel would become outdated.

- **Channel Estimation of Interference Link**: Due to the mobility of the nodes among different networks, the CSI of mutual interference link between the satellite user and BS would be outdated. Generally, it is considered that the channel knowledge of the interference link at the secondary transmitter is provided by using an existing feedback link [16], [17].

- **Data Transmission**: Finally, the secondary user transmits the desired signal to the satellite by employing a power control scheme without affecting the operation of the primary network. An underlay spectrum sharing approach is adopted, where the satellite user is allowed to access the licensed spectrum, simultaneously with the primary user, as long as it does not impose a harmful interference beyond a predefined threshold at the BS.

In what follows, these three phases will be described in detail.

A. Channel Estimation of Satellite Uplink

Let the satellite user transmit $L \in \mathbb{Z}^+$ training symbols to the satellite in $L$ time slots, therefore the receiver signal at the satellite can be expressed as [21]

$$r_i = h_s s_i + n_{s,i}, \quad i = 1, 2, \ldots, L$$

(3)

By employing an efficient joint detection processing along with (1) and (3) [22], the expression of the conditional probability density function (PDF) for the maximum likelihood (ML) detector is derived as

$$\ln f(y_s,r_1,r_2,\ldots,r_L|y_s,r_1,r_2,\ldots,r_L) = X - |y_s - h_s \hat{x}|^2 - \sum_{i=1}^{L} |r_i - h_s s_i|^2,$$

(4)

where $X = \ln \left( \frac{1}{\sqrt{2\pi\sigma^2}} \right)$. By taking the derivative of (4) with respect to $h_s$ and equating it to zero, it can further rewritten as

$$\hat{h}_s = \frac{x^* y_s + \sum_{i=1}^{L} s_i^* r_i}{L + 1}. $$

(5)

Combining (1) and (5), we further get

$$\tilde{h}_s = h_s + \hat{n}_s,$$

(6)

where $\tilde{n}_s = (x^* n_s + \sum_{i=1}^{L} s_i^* n_{s,i})/(L + 1)$ is the estimation error component of exact channel $h_s$ satisfying $E[|\tilde{n}_s|^2] = \sigma^2/(L + 1)$. From (4), we can get that in case of perfect CSI estimation, the decision variable for symbol $x$ is

$$\Lambda = \arg \min_{\hat{x}} \left\{ |y_s - h_s \hat{x}|^2 + \sum_{i=1}^{L} |r_i - h_s s_i|^2 \right\}.$$  

(7)

Since perfect CSI is not available at the satellite, we replace $h_s$ with $\tilde{h}_s$ in (7). Thus, the data decision variable with estimated CSI can be written as

$$\Lambda = \arg \min_{\hat{x}} \left\{ |h_s (x - \hat{x}) - \tilde{n}_s \hat{x} + n_s|^2 + \sum_{i=1}^{L} |n_{s,i} - \tilde{n}_s s_i|^2 \right\}.$$  

(8)

Note that $\sum_{i=1}^{L} |n_{s,i} - \tilde{n}_s s_i|^2$ is independent of the $x$, so it does not contribute to the effective decision metric. Thus, the estimated SNR can be written as $\hat{\gamma}_s = \frac{|h_s|^2}{\sigma^2 (1 + \frac{\tilde{n}_s^2}{\sigma^2})}$ [21]. After estimating the uplink channel, the estimated value will be sent over the downlink [21], [22]. To this end, the instantaneous received SNR at the satellite can be written as

$$\gamma_s = \frac{P_s}{\sigma^2} |\hat{h}_s|^2,$$

(9)

where $|\hat{h}_s|^2 = \frac{|h_s|^2}{(1 + \frac{\tilde{n}_s^2}{\sigma^2})}$ and $P_s$ is the transmit power at the satellite user.

B. Channel Estimation of Terrestrial Interference Link

The channel gain of the terrestrial interference link may not be available to the satellite user. The outdated channel gains would lead to an overestimation or underestimation of the induced interference at the BS, which leads to the degradation either of the satellite’s ergodic capacity, or the terrestrial system’s communication quality [16]. For the scenario with overestimation, the available power resource at satellite user cannot be fully utilized. On the other hand, when the mutual interference is underestimated, the interference power constraint with perfect CSI assumption will cause an increase of outage probability of the terrestrial BS due to the higher transmit power adopted at the satellite user. Please note that this model can be used to access the impact of several factors on the CSI, including channel estimation error, mobility and feedback delay [23] and the references therein. Due to the imperfect feedback channel, the satellite user can only access...
the outdated CSI, which can be described using the correlation model as follows [23]
\[ \hat{h}_p = \rho h_p + \sqrt{1 - \rho^2} n_p, \]  
(10)
where \( \hat{h}_p \) denotes the delayed version of \( h_p \), \( n_p \) is the complex Gaussian variable having the same variance as \( h_p \). According to Jakes’ autocorrelation model [24], the feedback delay coefficients between \( \hat{h}_p \) and \( h_p \) is expressed as \( \rho = J_0(2\pi f \tau) \), and \( f \) is the maximum Doppler frequency, \( \tau \) is the delay coefficient between the actual instant and the transmission instant, \( J_0(\cdot) \) is the zeroth-order Bessel function of the first kind [28, eq. (9.210.1)].

With outdated CSI, it is not possible to meet the instantaneous interference power constraint. To eliminate the negative estimation. According to [23], the power margin factor can be calculated over the range [28, eq. (8.411)].

\[ \gamma_s = \min \left( \left( \frac{Q}{|\hat{h}_p|^2} \right), \left( \frac{\sigma^2}{\bar{Q}} \right) \right) = \min \left( \frac{\gamma_Q}{|\hat{h}_p|^2}, \frac{\bar{Q}}{\sigma^2} \right), \]  
(15)
where \( \gamma_Q = \frac{\bar{Q}}{\sigma^2} \) and \( \bar{Q} = P_s \).

C. Data Transmission

In the data transmission phase, by combining (9) and (11), the received end-to-end SNR at the satellite can be expressed as

\[ P_s = \min \left( \frac{Q}{|\hat{h}_p|^2}, \frac{\sigma^2}{\bar{Q}} \right), \]  
(11)

\[ \kappa \]

with \( \kappa \) being the power margin factor due to imperfect channel estimation. According to [23], the power margin factor can be numerically derived by solving the following equation as

\[ \epsilon = 1 - \exp \left( - \frac{Q}{\lambda P_t} \right) + \exp \left( - \frac{Q}{\lambda P_t} \right) \times Q \left( \frac{2\rho^2 Q}{\lambda P_t (1 - \rho^2)} \right)^\frac{1}{2} \left( \frac{2\kappa Q}{\lambda P_t (1 - \rho^2)} \right)^\frac{1}{2} + \frac{t}{r} Q \left( \frac{(u - r) Q}{2 P_t} \right)^\frac{1}{2} \left( \frac{(u + r) Q}{2 P_t} \right)^\frac{1}{2} - \frac{1}{2} \left( \frac{1 + t}{r} \right) \exp \left( \frac{u Q}{2 P_t} \right) \left( \frac{2\rho Q \sqrt{\kappa}}{(1 - \rho^2) \lambda P_t} \right), \]  
(12)

where \( \epsilon \) is the pre-selected outage probability required at the primary BS with

\[ u = 2 \lambda \left( 1 + \frac{\rho^2}{1 - \rho^2} + \frac{\kappa}{1 - \rho^2} \right), \]  
(13a)

\[ t = 2 \lambda \left( 1 + \frac{\rho^2}{1 - \rho^2} - \frac{\kappa}{1 - \rho^2} \right), \]  
(13b)

\[ r = \sqrt{\kappa^2 - \frac{16\rho^2 \kappa}{\lambda^2 (1 - \rho^2)^2}}, \]  
(13c)

and

\[ Q(a, b) = \int_0^\infty x \exp \left( - \frac{a^2 + x^2}{2} \right) I_0(ax) dx, \]  
(14)
is the Marcum Q-function of first-order, and \( I_0(\cdot) \) is the zeroth-order modified Bessel function of the first kind [28, eq. (8.431.1)].

The effect of imperfect CSI on system performance were investigated in [25]-[27] by defining a confined region for the channel variations, which refers to the bounded CSI error model, and/or by assuming that the channel estimation error obeys the Gaussian distribution, namely gaussian CSI error model. These models are suitable for the robust system design within a certain ranges of uncertainty, which can be a potential research area in our future work.

A. Preliminary Results

In the Shadowed-Rician model, the amplitude of the shadowed line-of-sight (LoS) signal follows the Nakagami distribution, and the multi-path component of the signal envelope, that may occur because of the scatterers near the transmitter (e.g. trees, buildings), is characterised by Rayleigh distribution. This fading model can accurately describe the land mobile satellite (LMS) channel with significantly less computational burden [29], which is expected to play a prominent role in future satellite communication systems [10], [14]-[17].

Lemma 1. For scenarios with imperfect CSI estimation, the PDF of channel power gain \( \hat{h}_p \) in (15) is given by

\[ f_{|\hat{h}_p|^2}(x) = \alpha \exp \left( - \left( 1 + \frac{1}{L + 1} \right) \beta x \right) \times \frac{1}{x} F_1 \left( m_1; 1; \left( 1 + \frac{1}{L + 1} \right) \delta x \right), \]  
(16)

where \( F_1(a; b; c) \) represents the confluent hypergeometric function [28, eq. (9.210.1)], and \( \alpha, \beta \) and \( \delta \) are, respectively, given by

\[ \alpha = 2bm/(2bm (2bm + \Omega)), \]  
(17a)

\[ \beta = 1/2b, \]  
(17b)

\[ \delta = \Omega/2b (2bm + \Omega), \]  
(17c)

with \( \Omega \) being the average power of the LoS component, 2b the average power of the multipath component, and \( m \) the Nakagami parameter corresponding to the fading severity. According to [17], [29], the channel parameters of the satellite links closely depend on the elevation angle \( \theta \), which can be calculated over the range \( 20^\circ \leq \theta \leq 80^\circ \) by the following...
expressions

\[
b(\theta) = -4.7943 \times 10^{-8} \theta^3 + 5.5784 \times 10^{-6} \theta^2 \\
-2.1344 \times 10^{-4} \theta + 3.2710 \times 10^{-2},
\]
\[
m(\theta) = 6.3739 \times 10^{-5} \theta^3 + 5.8533 \times 10^{-4} \theta^2 \\
-1.5973 \times 10^{-1} \theta + 3.5156,
\]
\[
\Omega(\theta) = 1.4428 \times 10^{-5} \theta^2 - 2.3798 \times 10^{-3} \theta^2 \\
+ 1.2702 \times 10^{-1} \theta - 1.4864.
\]

**Lemma 2.** For scenarios with imperfect CSI estimation, the cumulative distribution function (CDF) of channel power gain \( \tilde{h}_s \) in (15) can be derived as

\[
F_{[\tilde{h}_s]}(x) = 1 - \alpha \exp \left( - \left( 1 + \frac{1}{L+1} \right) (\beta - \delta) x \right) \\
\times \sum_{k=0}^{m-1} \sum_{l=0}^{k} \Xi(k,l)x^l,
\]

where \( \alpha, \beta \) and \( \delta \) are defined in lemma 1 and

\[
\Xi(k,l) = \frac{(-1)^k}{k!} \frac{(1-m)_k}{k} \delta^k \sum_{l=0}^{k} \frac{1}{l!} \\
\times \left( 1 + \frac{1}{L+1} \right)^l (\beta - \delta)^{-k+l-1}.
\]

**Proof:** The proof can be found in [1].

Moreover, since the terrestrial interference link \( \tilde{h}_p \) follows the Rayleigh fading with exponential distribution, the PDF and CDF of the channel power gain \( \tilde{h}_p \) in (15) is defined as

\[
f_{[\tilde{h}_p]}(x) = \frac{1}{\lambda} \exp \left( - \frac{x}{\lambda} \right),
\]

(21)

\[
F_{[\tilde{h}_p]}(x) = 1 - \exp \left( - \frac{x}{\lambda} \right),
\]

(22)

where \( \lambda \) denotes the average power of terrestrial interference link.

In the following sections, based on these statistical properties of the fading channels, we will provide a comprehensive performance evaluation of the considered CSTNs with imperfect CSI.

**B. Outage Probability**

In wireless communications, the outage probability is an important QoS performance measure, which is defined as the probability that the instantaneous SNR \( \gamma_s \) falls below an acceptable threshold \( \gamma_{th} \), namely,

\[
P_{out} = \Pr \left( \gamma_s \leq \gamma_{th} \right) = F_{\gamma_s}(\gamma_{th}),
\]

(23)

where \( F_{\gamma_s}(x) \) denotes the CDF of \( \gamma_s \). In what follows, we will derive the exact CDF of \( \gamma_s \) in the following theorem.

**Theorem 1.** The analytical expression of \( F_{\gamma_s}(x) \) for considered CSTNs with imperfect CSI can be expressed as

\[
F_{\gamma_s}(x) = 1 - \alpha \sum_{k=0}^{m-1} \sum_{l=0}^{k} \Xi(k,l) \left\{ \left( \frac{x}{\gamma_{th}} \right)^l \left[ 1 - \exp \left( -\frac{\tilde{\gamma}_Q}{\lambda\gamma_{th}} \right) \right] \right. \\
\times \exp \left( - \left( 1 + \frac{1}{L+1} \right) (\beta - \delta) x \right) \\
+ \frac{1}{\lambda} \left( \frac{x}{\tilde{\gamma}_Q} \right)^l \exp \left( -\frac{\tilde{\gamma}_Q}{\gamma_{th}} \right) \left( \left( 1 + \frac{1}{L+1} \right) (\beta - \delta) x + \frac{1}{\lambda} \right) \right. \\
\times \sum_{i=0}^{l} \frac{l!}{i!} \left( \frac{\tilde{\gamma}_Q}{\gamma_{th}} \right)^i \left( \left( 1 + \frac{1}{L+1} \right) (\beta - \delta) x \right)^{-l+i-1} \right\}. \]

(24)

**Proof:** The proof can be found in Appendix A.

**C. Ergodic Capacity**

The ergodic capacity is defined as the expected value of instantaneous mutual information of the end-to-end SNR \( \gamma_s \), namely

\[
C_s = \mathbb{E}[\log_2 (1 + \gamma_s)] = \int_0^\infty \log_2 (1 + x) f_{\gamma_s}(x) dx. \]

(25)

By applying the integration by parts approach, (25) can be further rewritten as

\[
C_s = \log_2 (1 + x) \left[ F_{\gamma_s}(x) - 1 \right]_{0}^{\infty} \\
- \frac{1}{\ln 2} \int_0^\infty \frac{1}{1 + x} \left[ F_{\gamma_s}(x) - 1 \right] dx \\
= \frac{1}{\ln 2} \int_0^\infty \frac{1}{1 + x} \left[ 1 - F_{\gamma_s}(x) \right] dx. \]

(26)

**Theorem 2.** The analytical expression of \( C_s \) for considered CSTNs with imperfect CSI can be computed as

\[
C_s = \frac{\alpha}{\ln 2} \sum_{k=0}^{m-1} \sum_{l=0}^{k} \Xi(k,l) \left\{ \Gamma \left( l + 1 \right) \left\{ 1 - \exp \left( -\frac{\tilde{\gamma}_Q}{\lambda\gamma_{th}} \right) \right\} \right. \\
\times U \left( l + 1, l + 1, 1 + \frac{1}{L+1} \right) (\beta - \delta) \right. \\
\times \frac{\Gamma_k \rho^{k+1-l} \tilde{\gamma}_Q^{k-l}}{\lambda \gamma_{th}^{k-l}} \sum_{i=0}^{l} \frac{l!}{i!} \\
\times \left( \left( 1 + \frac{1}{L+1} \right) (\beta - \delta) \right)^{-l+i-1} \\
\left. \times G_{1,1;1,1,1,1,1;0,1;1}^1 \left[ \begin{array}{c} \frac{\lambda \gamma_{th}}{\tilde{\gamma}_Q} \\
\frac{\lambda \gamma_{th}}{\tilde{\gamma}_Q} \end{array} \right| \begin{array}{c} l + 1 \\
-k + i; 0 \\
0; 0 \end{array} \right] \right\}, \]

(27)

where \( U(\cdot,\cdot,\cdot) \) denotes the confluent hypergeometric function, and \( G_{1,1;1,1,1,1}^1(\cdot;\cdot;\cdot) \) is the Meijer-G function of two variables [32].

**Proof:** The proof can be found in Appendix B.
D. Asymptotic Outage Probability at High SNR

Here, we study the asymptotic outage probability of the CSTNs with imperfect CSI and thereby reveal two important performance merits: diversity order and coding gain, where the diversity order specifies the slope of the decay, and the coding gain particularizes the relative horizontal shift at asymptotically high values of SNR.

We look into the asymptotic regime by considering two practical scenarios, namely, 1) proportional interference power constraint, where the interference power constraint \( Q \) is proportional to the maximum transmit power \( P_t \), i.e., \( \gamma_Q = \eta \gamma_t \) with \( \eta \) being a positive constant, and 2) fixed interference power constraint, where the peak interference power \( Q \) is fixed and independent of the maximum transmit power \( P_t \).

1) Proportional interference power constraint: Resorting to the same assumption as in [10], we consider the proportional interference power constraint scenario, which means that the BS is able to tolerate a high amount of interference from the satellite user.

**Theorem 3.** For the scenario with proportional interference power constraint at the BS, the analytical expression of \( P^\infty_{\text{out}} \) at high SNR for the considered CSTNs with imperfect CSI can be expressed as

\[
P^\infty_{\text{out}} \approx \Delta \left( \frac{\gamma_{th}}{\gamma_t} \right),
\]

where

\[
\Delta = \alpha \left( 1 + \frac{1}{L + 1} \right) \left\{ \left[ 1 - \exp \left( -\frac{\eta}{\lambda} \right) \right] + \frac{\lambda + \eta}{\eta} \exp \left( -\frac{\eta}{\lambda} \right) \right\}
\]

**Proof:** The proof can be found in Appendix C. Based on Theorem 3, we present the following corollary and remark.

**Corollary 1.** The diversity order \( G_d \) and coding gain \( G_c \) of the considered CSTNs with ICSI are given by

\[
G_d = 1, \quad \text{and} \quad G_c = \frac{\Delta^{-1}}{\gamma_{th}},
\]

**Proof:** By representing asymptotic OP in (28) as \( P^\infty_{\text{out}}(\gamma_{th}) \approx (G_c \gamma_t)^{-G_d} \), the diversity order and coding gain of the CSTNs can be easily extracted.

**Remark 1.** The key insights of Corollary 1 shows that when the interference power constraint \( Q \) is proportional to the maximum transmit power \( P_t \), the achievable diversity order \( G_d \) remains one, which suggests that the imperfect CSI of both the satellite link and terrestrial interference link does not affect the diversity order. That is to say, the number of training symbols \( L \) and feedback delay coefficient \( \rho \) have no impact on the diversity order of the cognitive satellite network. Instead, it degrades the outage performance by degrading the coding gain, which can be quantitatively evaluated through (29).

2) Peak interference power constraint: In this subsection, we focus on the peak interference power constraint, where \( \gamma_Q \) remain fixed while \( \gamma_t \) grows large in the high SNR regime.

**Theorem 4.** For the scenario with peak interference power constraint at the BS, the analytical expression of \( P^\infty_{\text{out}} \) at high SNR for the considered CSTNs with imperfect CSI can be given by

\[
P^\infty_{\text{out}} \approx \Theta \left( \frac{\gamma_{th}}{\gamma_t} \right) + \Xi \left( \frac{\gamma_{th}}{\gamma_Q} \right),
\]

where

\[
\Theta = \alpha \left( 1 + \frac{1}{L + 1} \right) \left[ 1 - \exp \left( -\frac{\gamma_Q}{\lambda \gamma_t} \right) \right],
\]

and

\[
\Xi = \alpha \left( 1 + \frac{1}{L + 1} \right) \exp \left( -\frac{\gamma_Q}{\lambda \gamma_t} \right) \left( \lambda + \frac{\gamma_Q}{\gamma_t} \right).
\]

**Proof:** The result can be obtained by following similar procedures as the proof of Theorem 3. From Theorem 4, we have the following remark.

**Corollary 2.** When \( \gamma_t \) approaches infinity, the first term in (31) turns to zero, namely,

\[
P^\infty_{\text{out}} \approx \Xi \left( \frac{\gamma_{th}}{\gamma_Q} \right).
\]

Apparently, the diversity order \( G_d \) of the considered CSTNs equals to zero. This means that an outage floor is presented, which is determined by the the second terms in (31).

**Remark 2.** The observation of Corollary 2 indicates that when the peak interference power constraint \( Q \) is employed, the outage performance becomes saturated at high SNR, which demonstrates that only zero diversity order can be achieved by the considered CSTNs regardless of the CSI concerning the satellite link and terrestrial interference link. This is because the interference temperature constraint \( Q \) at the BS becomes the dominant factor to determine the maximum allowed transmit power at the satellite user.

IV. SIMULATION RESULTS

In this section, we provide simulation and numerical results to examine the impacts of various parameters on the performance of the considered CSTNs with CSI imperfection. Without loss of generality, we set the predefined threshold \( \gamma_{th} = 1 \text{ dB} \), and \( \lambda = 1 \). The simulation results are obtained by performing \( 10^6 \) channel realizations, and the different values of satellite elevation angle \( \theta = 20^\circ, 40^\circ, 60^\circ \) are corresponding to the heavy, average and light shadowing severities of the satellite links [29], [33], [34].

To begin with, Fig. 2 plots the exact and asymptotic outage probability of the cognitive satellite network under proportional interference power constraint for different values of \( Q \) and \( L \). Here, the cases that satellite uplink has the knowledge of perfect CSI (denoted as \( L = \infty \)) are also provided as benchmarks for performance comparison. As shown in this figure, we see an excellent match between the simulation results and the analytical curves, while the asymptotic curves match well with the exact results in the high SNR regime. Besides, the outage performance with imperfect CSI of satellite uplink gradually approaches toward the perfect CSI case with the
increase of training symbol \( L \), which can be explained by the fact that more training symbols \( L \) means more precise channel estimation at the satellite. In addition, from the asymptotic analysis, we can find that the quality of channel estimation for satellite link does not affect the diversity order of the considered CSTNs.

Fig. 3 shows the outage probability of cognitive satellite network for different feedback delay coefficient of terrestrial interference link under proportional interference constraints, where \( \rho = 1 \) denotes the ideal case without feedback delay. From the figure, we can observe that the outage probability improves when \( \rho \) increases, this is quite intuitive since the larger \( \rho \) is, the smaller the feedback delay would be presented. Moreover, we see that the curves associated with outdated CSI achieve the same diversity order of one, which confirms the analytical findings of Theorem 3 that the feedback delay leads to the system degradation by reducing the coding gain.

Fig. 4 illustrates the effect of pre-selected interference outage constraint factor \( \varepsilon \) on the performance of cognitive satellite network under the proportional interference constraint. As we can readily observe, the outage probability associated with the case \( \varepsilon = 10\% \) is strictly smaller than that of the case \( 1\% \), which indicates that allowing a less stringent interference outage constraint could significantly improve the outage performance of the cognitive satellite user. Moreover, we can also find that, for the small feedback delay coefficient, i.e., \( \rho < 0.5 \), the improvement of the accuracy of the feedback CSI yields marginal outage probability enhancement. In contrast, for the large feedback delay coefficient, i.e., \( \rho > 0.7 \), a relatively small increase of \( \rho \) provides a substantial decrease of the outage probability.

Fig. 5 depicts the ergodic capacity (in bit/s/Hz) of cognitive satellite network for different training symbols \( L \) and feedback delay coefficient \( \rho \) under proportional interference constraint. Apparently, with the secondary satellite user employing partial CSIs of terrestrial interference link, the system capacity of cognitive satellite network degrades with the decrease of \( \rho \). In this case, the actual interference caused to the primary network exceeds the expected level. This is not acceptable from the BS’s point of view, and a possible solution is to demand an additional power margin factor \( \kappa \). Moreover, the CSI imperfection of satellite uplink would be alleviated with more transmitted training symbol at the ES.

Fig. 6 plots the exact and asymptotic outage probability of the cognitive satellite network under the peak interference power constraint, where the cases of no interference temperature constraint and the perfect CSI of satellite normal link (denoted as \( L = \infty \)) are plotted for comparison. As can be clearly seen, the outage probability of the cognitive satellite network under an interference temperature constraint is generally inferior to that of the network without such a constraint. Besides, the outage probability of the CR system becomes saturated due to the interference temperature con-
Fig. 5: Ergodic capacity of cognitive satellite network for different training symbols \( L \) and feedback delay coefficient \( \rho \) under proportional interference constraint with \( \theta = 40^\circ \), \( \eta = 1 \) and \( \gamma_t = \gamma_Q = 20 \text{ dB} \).

Fig. 7: Outage probability of cognitive satellite network for different feedback delay coefficient \( \rho \) under the peak interference constraint with \( \theta = 40^\circ \), \( \varepsilon = 10\% \), \( Q = 20 \text{ dB} \) and \( L = 2 \).

Fig. 6: Outage probability of cognitive satellite network for different different values of \( L \) under peak interference constraint with \( \theta = 40^\circ \), \( \varepsilon = 10\% \) and \( \rho = 1 \).

Fig. 8: Outage probability of cognitive satellite network for different elevation angle \( \theta \) under peak interference constraint with \( \varepsilon = 10\% \), \( \rho = 1 \) and \( L = 2 \).

outage performance can be achieved. This is an expected result since higher elevation angles correspond to better propagation conditions, thus resulting in an enhanced outage performance. Besides, it can be seen that with the increase of \( \theta \), although the diversity order remains one, a noticeable decrease of outage occurs. This is because the primary interfering satellite link degrades the system performance by reducing the code gain. Furthermore, the outage error saturation can be improved by relaxing the interference temperature constraint, i.e., \( Q \) gets larger. Finally, Fig. 9 illustrates the ergodic capacity (in bit/s/Hz) of the cognitive satellite network for different values of \( Q \) and a wide range of elevation angle \( \theta \) under the peak interference constraint. Similar to the outage performance, an improved satellite system’s capacity can be obtained for increasing values of \( Q \) and \( \theta \).
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Fig. 9: Ergodic capacity of cognitive satellite network for different values of $Q$ and elevation angle $\theta$ under peak interference constraint with $\varepsilon = 10\%$, $\rho = 1$ and $L = 2$.

V. CONCLUSIONS

In this paper, we have investigated the impact of imperfect CSI on the performance of cognitive satellite terrestrial network with respect to both proportional and peak interference constraints. Specifically, exact analytical expressions for the outage probability and ergodic capacity of the cognitive satellite network have been derived, whose validity has been confirmed by Monte Carlo simulations. Furthermore, simple asymptotic expressions for the outage probability at high SNR was also presented, which give useful insights on the achievable diversity order and coding gain. Our findings reveal that under the proportional interference constraint, the achievable diversity order remains one, regardless of imperfect CSI for both the cognitive satellite link and terrestrial interference link. However, under the peak interference power constraint, the outage error floor appears at the high SNR and the achievable diversity order reduces to zero regardless of the CSI imperfections concerning both links.

APPENDIX A

PROOF OF THEOREM 2

Based on (15) and the concept of conditional probability, the $F_{\gamma_t}(x)$ can be reexpressed as the sum of the following probabilities,

$$F_{\gamma_t}(x) = \Pr \left( \frac{\gamma_t}{\gamma_p} \left| \frac{\gamma_t}{\gamma_p} \right| \leq x, \frac{\gamma_t}{\gamma_p} \leq \frac{\gamma_Q}{\gamma_p} \right)$$

$$+ \Pr \left( \frac{\gamma_Q}{\gamma_p} \left| \frac{\gamma_t}{\gamma_p} \right| \leq x, \frac{\gamma_t}{\gamma_p} > \frac{\gamma_Q}{\gamma_p} \right).$$

Then, due to independent nature of $\tilde{h}_p$ and $\tilde{h}_s$, $I_1$ and $I_2$ can be rewritten as

$$I_1 = \int_0^{\frac{\gamma_Q}{\gamma_t}} F_{\tilde{h}_s} \left( \frac{\gamma_t}{\gamma_p} \right) f_{\tilde{h}_s} \left( \frac{\gamma_Q}{\gamma_p} \right) dx,$$

and

$$I_2 = \int_{\frac{\gamma_Q}{\gamma_t}}^{\infty} F_{\tilde{h}_s} \left( \frac{\gamma_t}{\gamma_p} \right) f_{\tilde{h}_s} \left( \frac{\gamma_Q}{\gamma_p} \right) dx.$$

APPENDIX B

PROOF OF THEOREM 3

Specifically, substituting (24) into (26), we get

$$C_s = \frac{\alpha}{\ln 2} \sum_{k=0}^{m-1} \sum_{l=0}^{k} \Xi(k, l) \left\{ \frac{1}{\gamma_t} \left[ 1 - \exp \left( -\frac{\gamma_Q}{\gamma_t \lambda} \right) \right] \right\}$$

By applying [30, eq. (2.3.6.9)], $I_3$ can be first obtained as

$$I_3 = \Gamma(l+1) U \left( l + 1, 1, \frac{1}{1 + \frac{L+1}{L+1} \frac{\beta - \delta}{\gamma_t}} \right).$$

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Then, according to [31, eq.(10)], we represent the functions $(1 + x)^{-1}$ and $((1 + \frac{1}{L+1})(\beta - \delta)x + 1)^{-\frac{k+i-1}{\gamma}}$ with respect to the Meijer-G function as

$$
(1 + x)^{-1} = G_{\infty,1}^{1,1}\left[ 1 \bigg| \begin{array}{c} \frac{\lambda \beta - \delta}{\gamma} \\bigg] \end{array} \right], (B.3)
$$

and

$$
((1 + \frac{1}{L+1})(\beta - \delta)x + 1)^{-\frac{k+i-1}{\gamma}} = \frac{\lambda^{k+i+1}}{\Gamma(k-i+1)} \times G_{1,1}^{1,1}\left[ 1 + \frac{1}{L+1} \begin{array}{c} \lambda (\beta - \delta) \\bigg| \begin{array}{c} -k+i \\ \bigg] \end{array} \right], (B.4)
$$

Subsequently, substituting (B.3) and (B.4) into (B.1) along with (C.1) yields the asymptotic CDF of $\mathbb{E}[\gamma_s]$ as

$$
I_4 = \frac{\lambda^{k+i+1}}{\Gamma(k-i+1)} \left( \frac{1 + \frac{1}{L+1}}{\gamma} \right)^{-(l+1)} \times G_{1,1}^{1,1}\left[ \frac{\lambda \beta - \delta}{\gamma} \bigg| \begin{array}{c} l+1 \\ \bigg] \end{array} \right]. (B.5)
$$

To this end, the desired result can be obtained by using (B.2) and (B.5) into (B.1).

**APPENDIX C**

**PROOF OF THEOREM 4**

We first apply [28, eq. (9.14.1)] to express the confluent hypergeometric function $pF_q (\cdot; \cdot; \cdot)$ as

$$
pF_q \left( m_1, \ldots, m_p; n_1, \ldots, n_q; x \right) = \sum_{k=0}^{\infty} \frac{(m_1)_k \cdots (m_p)_k}{(n_1)_k \cdots (n_p)_k} \frac{x^k}{k!}.
$$

By using (C.1) into (16) along with help of [28, eq. (3.381.1), (8.354.1)], the asymptotic CDF of $|\hat{h}_s|^2$ can be derived as

$$
F_{|\hat{h}_s|^2} (x) = \alpha \left( 1 + \frac{1}{L+1} \right) x. (C.2)
$$

Thus, substituting (C.2) into (A.1) yields the $I_1^\gamma$ and $I_2^\gamma$ as

$$
I_1^\gamma = \int_0^{\mu} F_{|\hat{h}_s|^2} \left( \frac{x}{\mu \gamma} \right) f_{|\hat{h}_s|^2} (y) dy = \alpha \left( 1 + \frac{1}{L+1} \right) \frac{x}{\gamma \mu} \left[ 1 - \exp \left( -\frac{\mu}{\gamma} \right) \right], (C.3)
$$

and

$$
I_2^\gamma = \int_{\mu}^{\infty} F_{|\hat{h}_s|^2} \left( \frac{xy}{\mu \gamma} \right) f_{|\hat{h}_s|^2} (y) dy = \alpha \left( 1 + \frac{1}{L+1} \right) \frac{x}{\gamma \mu} \exp \left( -\frac{\mu}{\gamma} \right) (\lambda + \mu). (C.4)
$$

Then, substituting (C.3) and (C.4) into (A.1), we have the asymptotic CDF of $\gamma_d$ as

$$
F_{\gamma_d} (x) = \alpha \left( 1 + \frac{1}{L+1} \right) \frac{x}{\gamma \mu} \left[ 1 - \exp \left( -\frac{\mu}{\gamma} \right) \right] + \alpha \left( 1 + \frac{1}{L+1} \right) \frac{x}{\eta \gamma} \exp \left( -\frac{\eta}{\gamma} \right) (\lambda + \eta). (C.5)
$$

Eventually, combining (23) and (C.5), one can obtain $D_{\gamma_d}$ as shown in Theorem 3.


