Guaranteed Cost Control for A Class of Nonlinear Discrete Time-Delay Systems

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ABSTRACT The guaranteed cost control problem for a class of nonlinear discrete time-delay systems is investigated. Based on the Lyapunov matrix, a complete-type Lyapunov–Krasovskii functional is constructed. Thereby, the Lyapunov stability theory is employed to design the exact form of the controller to ensure that the resultant closed-loop system is asymptotically stable and the cost function is bounded. A numerical example is presented to illustrate the usefulness of the theoretical results.

INDEX TERMS Guaranteed cost control, nonlinear discrete time-delay systems, Lyapunov matrix, asymptotic stability.

I. INTRODUCTION

A s is well known, time delays frequently occur in various practical systems and often result in poor performance and/or instability. Therefore, the study of the stability analysis and controller design for time-delay systems is very important and a hot research topic in recent years [1]–[13].

Guaranteed cost control problem, which is first proposed by Chang et al. [14] for uncertain systems, is to design a controller such that the resulting closed-loop system not only is asymptotically stable but also guarantees an adequate level of performance function. By using Lyapunov–Krasovskii functionals, an upper bound on the closed-loop value of a quadratic cost function is established. Based on this idea, many significant results on guaranteed cost control for continuous-time delay systems have been put forward in [15]–[24] and references therein. In discrete-time case, Shi, Boukas, Shi and Agarwal [25] investigated robust quadratic stability and robust quadratic guaranteed cost control for a class of linear uncertain systems with constant delay. The parameter uncertainty under consideration is time-varying and norm-bounded. In the work of Zou and Wang [26], the problem of the guaranteed cost control for a class of uncertain discrete-time systems with both state and input delays is studied. A novel LMI-based approach is proposed for the existence of a state feedback controller which guarantees not only the asymptotic stability of the closed-loop system, but also an adequate performance bound over all the possible parameter uncertainties. By choosing an appropriate Lyapunov functional and utilizing the free-weighting matrix technique, the authors proposed a delay-dependent condition and a guaranteed cost controller based on linear matrix inequalities [27]. Applying bilinear matrix inequality approaches, the problem of robust guaranteed cost control problem for uncertain discrete-time delay systems was considered in [28] and [29]. The guaranteed cost control laws are given via the solution of the bilinear matrix inequalities. In addition, Gyurkovics [30] studied the guaranteed cost control problem for discrete-time uncertain systems with both state and input delays. The delays are assumed to be time-varying and bounded. Sufficient conditions for the existence of the guaranteed cost controller were proposed in bilinear matrix inequalities. However, to the best knowledge of authors, the problem of guaranteed cost control for nonlinear discrete time-delay systems is still in the early stage and needs further research. It is worth noting that some attempt has been made in [31], which investigated guaranteed cost control problem for a class of uncertain nonlinear discrete-time systems with the discrete time-varying delay and the distributed delay. By constructing an appropriate Lyapunov–Krasovskii functional, a sufficient condition was established and an explicit form of the guaranteed cost controller was presented.

The above-mentioned works on guaranteed cost control are mainly performed in the Lyapunov–Krasovskii functional method, which is the most common research method of sta-
bility analysis and controller design for time-delay systems. Furthermore, there is another method, which is Lyapunov matrix method. Using the Lyapunov matrix method, Santos et al. [32] addressed the robust stabilization and guaranteed cost control problem for nonlinear continuous-time delay systems. Most recently, based on Lyapunov matrix, necessary and sufficient conditions of exponential stability for delayed linear discrete-time systems were proposed in [33]. Motivated by [32] and [33], this paper studies the guaranteed cost control problem for a class of nonlinear discrete time-delay systems by proposing a Lyapunov matrix method. Firstly, a complete-type Lyapunov–Krasovskii functional is constructed based on the Lyapunov matrix. Then according to the Lyapunov stability theory, the exact expression of the controller is designed to ensure that the asymptotic stability of the closed-loop system, and its cost function is bounded. Furthermore, an algorithm of guaranteed cost controller is given. Finally, a numerical example is given to demonstrate the effectiveness of the proposed approach.

The main contributions of this paper can be summarised as follows: (i) based on the Lyapunov matrix, a complete-type Lyapunov–Krasovskii functional is constructed, which is different form the most existing ones; (ii) a sufficient condition for the existence of state-feedback guaranteed cost controller is derived; (iii) an explicit guaranteed cost controller is designed such that the closed-loop system is asymptotically stable as well as a specific quadratic cost function has an upper bound, and an explicit expression of the controller is given.

The organization of the rest of this article is as follows: Section II introduces Lyapunov matrix and functional. In Section III, an exact form of controller is proposed, which asymptotically stabilizes the close-loop system. Section IV shows that the cost function is bounded, and gives an algorithm of guaranteed cost controller. A numerical example is provided in Section V; and finally, we conclude this paper in Section VI.

Notation: The following notations will be used throughout this paper. \( \mathbb{Z} \) denotes the set of all integer numbers. \( \mathbb{Z}[a,b] \) refers to the set containing all integers \( c \) with \( a \leq c \leq b \). The set of real numbers will be denoted by \( \mathbb{R} \). Let \( \mathbb{R}^{n \times m} \) represent the set of all \( n \times m \) matrices over \( \mathbb{R} \), and denote \( \mathbb{R}^n = \mathbb{R}^{n \times 1} \).

II. LYAPUNOV MATRIX AND FUNCTIONAL

Consider the following linear discrete time-delay system described by

\[
x(k + 1) = Ax(k) + Bx(k - h), \quad k \in \mathbb{Z}[0, \infty),
\]

\[
x(s) = \phi(s), \quad s \in \mathbb{Z}[-h, 0],
\]

where \( x : \mathbb{Z}[-h, \infty) \rightarrow \mathbb{R}^n \) is the state vector, \( A \) and \( B \) are given real \( n \times n \) matrices, \( h > 0 \) is the delay, and \( \phi \in M(\mathbb{Z}[-h, 0], \mathbb{R}^n) \) is the initial function. Here \( M(\mathbb{Z}[-h, 0], \mathbb{R}^n) \) is the set of all maps from \( \mathbb{Z}[-h, 0] \) to \( \mathbb{R}^n \).

Let \( x(k, \phi) \) be the solution of system (1) starting from the initial function \( \phi \). For any \( k \in \mathbb{Z} \), we define the function \( x_k : \mathbb{Z}[-h, 0] \rightarrow \mathbb{R}^n \) by \( x_k(\theta) : \theta \mapsto x(k + \theta, \phi) \) for all \( \theta \in \mathbb{Z}[-h, 0] \), where \( x(\theta, \phi) = \phi(\theta), \theta \in \mathbb{Z}[-h, 0] \).

First, we introduce the Lyapunov matrix.

Definition 2.1: A matrix series \( \{U(\tau)\}_{\tau=-h}^{\tau} \) is called a Lyapunov matrix of system (1) associated with a symmetric matrix \( W \), if it satisfies the following properties:

\[
U(\tau + 1) = U(\tau)A + U(\tau - h)B, \quad \tau \in \mathbb{Z}[0, h - 1],
\]

\[
U(\tau) = U_T(-\tau), \quad \tau \in \mathbb{Z}[0, h],
\]

\[
-W = A^TU(0)A + A^TU(-h)B + B^TU(h)A + B^TU(0)B - U(0).
\]

Next we create a quadratic functional \( v(\phi) \) of the form:

\[
v(\phi) = v_1(\phi) + v_2(\phi),
\]

\[
v_1(\phi) = \phi^T(0)U(0)\phi(0) + 2\phi^T(0)U(0)\phi(-h - 1)B\phi(-h) + \sum_{\theta = -h}^{h-1} \phi(\theta_1)B^TU(\theta_1 - \theta_2)B\phi(\theta_2),
\]

\[
v_2(\phi) = \sum_{\theta = -h}^{h-1} \phi^T(\theta)V\phi(\theta) + \sum_{\theta = -h}^{h-1} \phi^T(\theta)P_{-\theta}\phi(\theta),
\]

where \( V \) and \( P_j \in \mathbb{R}^{n \times n} (j = 0, 1, \ldots, h) \) are real symmetric positive definite matrices. It can clearly get that

\[
v(\phi) = \Upsilon_0^TF_1^T\Upsilon_\phi, \quad \forall \phi \in M(\mathbb{Z}[-h, 0], \mathbb{R}^n),
\]

\[
\Upsilon_\phi = \begin{bmatrix} \phi^T(0) & \phi^T(-1) & \cdots & \phi^T(-h) \end{bmatrix}^T,
\]

\[
F_U = \begin{bmatrix} U(0) & F_1^T \end{bmatrix},
\]

\[
F_1 = \begin{bmatrix} U(-h)B & \cdots & U(-2)B & U(-1)B \end{bmatrix},
\]

\[
F_2 = \begin{bmatrix} f_0 & f_1 & \cdots & f_{h-2} & f_{h-1} \\ * & f_0 & \cdots & f_{h-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & f_0 \\ * & * & \cdots & * \end{bmatrix},
\]

\[
f_0 = B^TU(0)B + V, \quad f_i = B^TU(i)B, \quad i \in \mathbb{Z}[1, h - 1],
\]

\[
F_2 = \text{diag}(f_1, f_2, \ldots, f_h), \quad f_i = \sum_{i=n}^h P_i, \quad i \in \mathbb{Z}[1, h].
\]
III. STABILIZATION

Consider the nonlinear discrete time-delay system of the form:

\[ y(k + 1) = Ay(k) + By(k - h) + Cu(k) + \lambda y(y(k), y(k - h), k), \]
\[ y(s) = \phi(s), s \in \mathbb{Z}_{[-h, 0]}, \quad (8) \]

where \( y : \mathbb{Z}_{[-h, 0]} \rightarrow \mathbb{R}^n \), \( u(k) \in \mathbb{R}^m \) is the control law, \( A \) and \( B \) are given real \( n \times n \) matrices, \( C \) is given real \( n \times m \) matrix, \( D \) is given real \( p \times p \) matrix. \( h > 0 \) is the delay, and \( \phi \in \mathcal{M}(\mathbb{Z}_{[-h, 0]}, \mathbb{R}^n) \) is the initial function. \( g(y(k), y(k - h), k) \) \( \in \mathbb{R}^p \) is an unknown nonlinear function and satisfies the following assumption:

Assumption 3.1: The nonlinear function \( g(y(k), y(k - h), k) \) satisfies

\[ ||g(y(k), y(k - h), k)||^2 \leq \alpha y(T)(k)y(k) + \beta y(T)(k)y(k - h), \forall k \in \mathbb{Z}_{[0, \infty)}, \]

where \( \alpha > 0 \) and \( \beta > 0 \).

Let \( y(k, \phi) \) be the solution of system (8) starting from the initial function \( \phi \). For any \( k \in \mathbb{Z} \), we define the function \( y_k : \mathbb{Z}_{[-h, 0]} \rightarrow \mathbb{R}^n \) by \( y_k(\phi) : \theta \mapsto y(k + \theta, \phi) \) for all \( \theta \in \mathbb{Z}_{[-h, 0]} \), where \( y(\phi, \theta) = \phi(\theta), \theta \in \mathbb{Z}_{[-h, 0]} \). Use \( y(k, \phi) \), \( y(k) \) and \( g(k) \) instead of \( y(k, \phi), y_k(\phi) \) and \( g(y(k), y(k - h), k) \), respectively, when no confusion may appear.

Then, the exact form of the controller to be designed is

\[ u(k) = -\lambda^{-1}(\Pi(k), k \in \mathbb{Z}_{[0, \infty)}, \quad (9) \]

where

\[ \lambda = \lambda_{\text{max}}(CTU(0)C), \]
\[ M = [CTU(1) 0 CT]^T, \]
\[ \Pi(k) = [y^T(k) y^T(k - h) \pi^T(k)]^T, \]
\[ \pi(k) = \sum_{\theta=-h}^{0} U(-h - \theta)By(k + \theta). \]

Theorem 3.1: Under Assumption 3.1, if there exist real symmetric positive definite matrices \( W, V \) and \( P_j \in \mathbb{R}^{n \times n}(j = 1, \ldots, h) \) such that

\[ F_U > 0, E > 0, \quad (10) \]

where \( F_U \) is defined as in (7), and

\[ E := \lambda^{-1} \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ * & E_{22} & E_{23} \\ * & * & E_{33} \end{bmatrix}, \quad (11) \]

\[ E_{11} = \lambda(I - DTU(0)D), E_{12} = (NT - \lambda DT)U(1), \]
\[ E_{13} = \begin{bmatrix} a_1 & a_2 & \cdots & a_h \end{bmatrix}, \]
\[ a_i = (NT - \lambda DT)U(-h + i)B, i \in \mathbb{Z}_{[1, h]}, \]

then the control law (9) asymptotically stabilizes system (8). Proof 3.1: Let \( \xi = \theta + k + 1 \) and \( \xi_i = \theta + k + 1, i = 1, 2 \). Then, from (5),

\[ v(y_k) = v_1(y_k) + v_2(y_k) \quad (12) \]

with

\[ v_1(y_k) = y^T(k)U(0)y(k) + 2y^T(k) \sum_{\xi=k-h+1}^{k} U(-\xi - h + k)By(\xi - 1) \]
\[ + \sum_{\xi_1=k-h+1}^{k} \sum_{\xi_2=k-h+1}^{k} y^T(\xi_1 - 1) \times B^T U(\xi_1 - \xi_2)By(\xi_2 - 1), \]
\[ v_2(y_k) = \sum_{\xi=k-h+1}^{k} y^T(\xi - 1)V y(\xi - 1) \]
\[ + \sum_{\theta=-h}^{0} \sum_{s=-k+\theta}^{k-1} y^T(s)P_{-\theta}y(s). \]

The forward difference of \( v_1(y_k) \) along the solution of system (8) is given as follows:

\[ \Delta v_1(y_k) = y^T(k + 1)U(0)y(k + 1) \]
\[ + 2y^T(k + 1) \sum_{\theta=-h+1}^{0} U(-\theta - h)By(k + \theta) \]
\[ + \sum_{\theta_1=-h+1}^{0} \sum_{\theta_2=-h+1}^{0} y^T(k + \theta_1) \times B^T U(\theta_1 - \theta_2)By(k + \theta_2) - v_1(y_k) \]
\[ = \gamma_1(k) + \gamma_2(k) + \varepsilon(k) + \eta(k), \quad (13) \]

where

\[ \gamma_1(k) = y^T(k + 1)U(0)y(k + 1) \]
\[ + 2y^T(k + 1)(U(-h)By(k) - U(0)By(k - h)), \]
\[ \gamma_2(k) = 2y^T(k + 1)\pi(k) - 2y^T(k - h)B^T\pi(k), \]

\[ \varepsilon(k) = y^T(k)B^TU(0)By(k) - y^T(k)U(0)y(k) + y^T(k - h)B^TU(0)By(k - h) - 2y^T(k)B^TU(-h)By(k), \]

\[ \eta(k) = 2 \sum_{\theta=-h}^{-1} y^T(k + \theta)B^TU(\theta)By(k) - 2y^T(k) \sum_{\theta=-h}^{-1} U(-\theta - h - 1)By(k + \theta). \]

It follows from (2) and (3) that

\[ U(-\theta - h - 1) = U^T(\theta + h + 1) = A^T U^T(\theta + h) + B^T U^T(\theta), \quad \theta \in \mathbb{Z}[-h, -1]. \tag{14} \]

Substituting (14) into (13), we derive

\[ \Delta v_1(y_k) = \gamma_1(k) + \gamma_2(k) + \varepsilon(k) - 2y^T(k)A^T \pi(k). \]

This, together with (2), (3), (4) and (8), implies that

\[ \Delta v_1(y_k) = \gamma_3(k) + \gamma_4(k) + \varepsilon_2(k), \tag{15} \]

where

\[ \gamma_3(k) = -y^T(k)W y(k), \]

\[ \gamma_4(k) = g^T(k)D^T U(0)D g(k) + 2g^T(k)D^T \Phi(k) + 2u^T(k)CT U(0)D g(k), \]

\[ \Phi(k) = U(1) y(k) + \pi(k), \]

\[ \varepsilon_2(k) = u^T(k)CT U(0)Cu(k) + 2u^T(k)C^T \Phi(k). \]

Let \( \lambda = \lambda_{\text{max}}(C^T U(0)C). \) Then based on Assumption 3.1 and substituting the controller (9) into the above equation (15), we obtain

\[ \Delta v_1(y_k) \leq \gamma_4(k) - y^T(k)W y(k) - \lambda^{-1} \Pi^T(k)M^T M \Pi(k) - g^T(k)g(k) + \alpha y^T(k)g(k) + \beta g^T(k - h)g(k - h). \]

On the other hand, the forward difference of \( v_2(y_k) \) is given as follows:

\[ \Delta v_2(y_k) = y^T(k) \left( V + \sum_{\theta=-h}^{-1} P_{\theta} \right) y(k) - \sum_{\theta=-h}^{-1} y^T(k + \theta)P_{\theta}y(k + \theta) - y^T(k - h)W y(k - h). \]

Then

\[ \Delta v(y_k) \leq -y^T(k) \left( W - V - \sum_{\theta=-h}^{-1} P_{\theta} - \alpha I \right) y(k) + 2g^T(k)(D^T - \lambda^{-1} D^T U(0)C C^T)\Phi(k) - \sum_{\theta=-h}^{-1} y^T(k + \theta)P_{\theta}y(k + \theta) - \lambda^{-1} \Pi^T(k)M^T M \Pi(k) + g^T(k)(D^T U(0)D - I)g(k) \leq -\Gamma^T(k)E \Gamma(k), \tag{16} \]

where

\[ \Gamma(k) = \left[ g^T(k) \ y^T(k) \ y^T(k - 1) \ \ldots \ y^T(k - h) \right]^T. \]

The proof is complete.

**IV. GUARANTEED COST CONTROL**

In this section, we determine an upper bound on the quadratic performance index:

\[ J = \sum_{k=0}^{\infty} [y^T(k)Q y(k) + u^T(k)R u(k)], \quad Q \geq 0, \quad R > 0. \tag{17} \]

**Theorem 4.1:** Under Assumption 3.1, if there exist positive definite matrices \( W, V \) and \( P_i \in \mathbb{R}^n \times n \ (i = 0, 1, \ldots, h) \), such that \( F_U > 0 \) and

\[ \tilde{E} := \lambda^{-1} \left[ \begin{array}{ccc} E_{11} & E_{12} & E_{13} \\ * & E_{22} & E_{23} \\ * & * & E_{33} \end{array} \right] > 0 \tag{18} \]

with

\[ E_{22} = E_{22} - \lambda Q - \lambda^{-1} U^T(1)CRC^T U(1), \]

\[ E_{23} = E_{23} - \lambda^{-1} U^T(1)CRC^T U(-h + i) B, \quad i \in \mathbb{Z}[1, h], \]

\[ \tilde{E}_{33} = E_{33} - \lambda^{-1} \left[ U(-h + 1) B \ldots U(0) B \right]^T \times CRC^T \left[ U(-h + 1) B \ldots U(0) B \right], \]

then the control law (9) asymptotically stabilizes system (8), and the performance index \( J \) defined in (17) satisfies \( J \leq v(\varphi) \), where the functional \( v(\varphi) \) is given by (12).

**Proof 4.1:** According to Theorem 3.1 and its proof, we can obtain that when \( y \to 0 \) as \( k \to \infty \), and

\[ \Delta v(y_k) \leq -\Gamma^T(k)E \Gamma(k). \tag{19} \]

The combination of (17) and (19) yields

\[ \Delta v(y_k) + y^T(k)Q y(k) + u^T(k)R u(k) \leq -\Gamma^T(k)E \Gamma(k). \tag{20} \]
It follows that
\[
\sum_{k=0}^{\infty} [y^T(k)Qy(k) + u^T(k)Ru(k)] \\
\leq -\sum_{k=0}^{\infty} \Delta v(y_k) \\
= -\sum_{k=0}^{\infty} (v(y_{k+1}) - v(y_k)) \\
= -v(y_{\infty}) + v(\varphi) \\
= v(\varphi),
\]
where \(v(\varphi)\) is obtained by (12). The proof is completed.

Next, an algorithm of the guaranteed cost controller is given.

**Algorithm 4.1:**

**Input:** The time delay \(h\), matrices \(A, B, C\) and \(D\) of system (8), constants \(\alpha\) and \(\beta\), positive-definite matrices \(W, V\), \(P_i \in \mathbb{R}^{n \times n} (i = 0, 1, \cdots, h)\) and \(R\), and semi-positive definite matrix \(Q\).

**Output:** The guaranteed cost controller \(u(k)\).
1. Calculate the matrices \(U(\tau)\) for \(\tau \in \mathbb{Z}[-h, h]\).
2. Calculate the matrices \(F_U\) and \(\bar{E}\) according to (7) and (18).
3. Test the positive definiteness of \(F_U\) and \(\bar{E}\). If they are positive definite, then go to next step; otherwise, return to Input.
4. Calculate the guaranteed cost controller \(u(k)\) according to (9).

**Remark 4.1:** Algorithm 4.1 can be easily realized by MATLAB. In Step 1, detailed calculation methods of the Lyapunov matrix \(U(\tau)\) can be referred to [33].

**Remark 4.2:** When the time delay \(h\) is smaller, test of the positive definiteness of \(F_U\) can be realized by using one of equivalent conditions of positive definite matrices, for example, all order principal minor determinants of \(F_U\) is positive, or all eigenvalues of \(F_U\) is positive. While for increasing \(h\), one can employ some iterative methods, for example, utilize the power method to calculate the maximum eigenvalue of \(-F_U\), or utilize the inverse power method to calculate the minimum eigenvalue of \(F_U\). The symmetric property of \(F_U\) guarantees the computational reliability, although the dimension of \(F_U\) is large sometimes.

**V. ILLUSTRATIVE EXAMPLE**

In this section, we will present one numerical example to illustrate the effectiveness of the theoretical results.

**Example 5.1:** Consider the nonlinear discrete time-delay systems (8) with \(\alpha = 0.5, \beta = 0.5, h = 6\) and
\[
A = \begin{bmatrix} 0.5 & -0.4 \\ -0.2 & 0.31 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 & 0.3 \\ 0.2 & -0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 0.4 \\ 0.5 \end{bmatrix}, \\
D = \text{diag}(0.1, 0.1).
\]
The matrices \(Q\) and \(R\) in the cost function \(J\) are chosen as
\[
Q = \text{diag}(0.1, 0.1), \quad R = 0.2.
\]

**FIGURE 1.** The state responses of the open-loop system.

**FIGURE 2.** The state responses of the closed-loop system.

Clearly, \(Q > 0\) and \(R > 0\). Choose \(W = I\). By using a method similar to one in [33], the Lyapunov matrix \(U(\tau), \tau \in [-5, 5]\) can be easily computed.

By Theorem 4.1, it is concluded that the controller (9) is a guaranteed cost controller for the nonlinear discrete time-delay system (8). Moreover, the guaranteed cost \(J^* = 8482.9\), when the initial map \(\varphi\) is taken as
\[
\varphi(k) = \begin{bmatrix} 0.1 \\ 0.35 \end{bmatrix}, \quad k \in [-5, 0].
\]

The state responses of the open-loop and closed-loop systems are given in Figures 1 and 2, respectively. From the two figures, it is seen that the response curves of the open-loop system converge very slowly, while ones of the closed-loop system converge quickly. This indicates that our theoretical results are effective.

**VI. CONCLUSIONS**

This paper investigates the guaranteed cost control problem for a class of nonlinear discrete time-delay systems. Firstly,
we construct a complete-type Lyapunov–Krasovskii functional based on the Lyapunov matrix. Secondly, sufficient LMI conditions are derived to insure the positivity of the functional and the negativity of its forward difference along the trajectories of system, which can be solved by using softwares such as MATLAB’s Toolbox YALMIP. Thirdly, we get the precise form of a guaranteed cost controller and a guaranteed cost. Furthermore, an algorithm is proposed for the guaranteed cost controller. Finally, a numerical example demonstrates the effectiveness of the proposed design method.

REFERENCES