Compressed sensing image mapping spectrometer

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ABSTRACT This paper presents a novel snapshot imaging spectrometer based on the image mapping and compressed sensing concept named Compressed Sensing Image Mapping Spectrometer (CSIMS). The operation principle is to slice the input image to different directions and encode the strip pieces before dispersion by a prism. The detector obtains the mixture spatial-spectral data simultaneously. The datacube is reconstructed by the compressed sensing algorithm and combining all the pieces together. The mathematical model of CSIMS is established to describe the light wave propagation through the entire system based on the scalar diffraction theory. The simulations are conducted to prove the effectiveness of the CSIMS principle, and the results show that the reconstructed datacube reveals higher spatial resolution and more accurate spectral curves than that of the relative snapshot imaging spectrometer based on compressed sensing.

INDEX TERMS Snapshot imaging spectrometer, compressed sensing, datacube, reconstruction

I. INTRODUCTION Imaging spectrometer originated in 1980s, which can obtain two-dimension (2-D) spatial distribution and one-dimension (1-D) spectral radiation (3-D datacube) of the target. This technology can be used in astronomical observation, disease diagnosis, safety monitoring, remote sensing and military reconnaissance etc. Nowadays, the traditional types of imaging spectrometers including the whiskbroom, pushbroom, and tunable filtered imagers etc acquire 2-D or 1-D information from the datacube of the target during one integral time, so that another 1-D or 2-D information needs to be scanned to fill out the whole datacube. Thus, the data quality would be influenced by the working status of the instruments, the stability of the platform and the variation of the dynamic targets, which can not be recovered by postprocessing [1].

The snapshot imaging spectrometer (SIS) can collects the datacube in a single exposure period without scanning, which introduces the advantages of snapshot systems such as the compactness of the instruments, increasing the robustness, the lack of scanning component and high light collection. The snapshot instruments attract much attention from the researchers for the superiority of detecting dynamic targets [2]. Many kinds of SIS arisen in recent years such as the integral field spectroscopy (IFS) [3], image-replicating imaging spectrometry (IRIS) [4], imaging spectrometry using a light field architecture (IS-LF) [5], image mapping spectrometry (IMS) [6], computed tomographic imaging spectrometry (CTIS) [7], coded aperture snapshot spectral imaging (CASSI) etc. CASSI was proposed by D. J. Bray from Duke University [8]. The random coded aperture is placed on the first image plane of the fore optics, which encodes the input image to be dispersed by a prism. The mixed spectral-spatial data is measured by a large format detector [9][10][11]. The compressed sensing concept is employed to reconstruct the datacube from raw data, such as the Gradient Projection algorithm for Sparse Reconstruction (GPSR) [12], Two-step Iterative Shrinkage/Thresholding algorithm (TwIST) [13]. However, since CASSI is based on compressed sensing, it needs the input targets have piecewise smooth spatial structure to make the data be highly compressed in Fourier or wavelet or other basis. For improving the reconstruction data quality, some modified system structures are proposed [14]-[16]. For example, employing a varying mask or two cameras etc. These methods improve the data quality, but at the same time destroy the...
instantaneity and compactness of the system and reduce the optical throughput.

This paper proposes a novel snapshot imaging spectrometer named Compressing Sensing Image Mapping Spectrometer (CSIMS). CSIMS uses the concept from IMS and CASSI to slice the input image to pieces to ensure the piecewise smooth in spatial structure, and at the same time reduces the aliasing of the raw data substantially to make reconstructed datacube more accurate. CSIMS ensures the high optical throughput and the compactness of the structure with instantaneity detection. The system layout and mathematical model is illustrated in section 2. The reconstruction strategy is expounded in section 3. In section 4, a simulation is conducted and the results is analyzed to prove the improvement.

II. GENERAL PRINCIPLE

The operating principle of CSIMS is illustrated in Fig. 1. The image of a target is formed on the Coded Image Mapper (CIM) which is a custom made of strip mirrors with binary-coded mask. The CIM contains many long strip mirrors with different 2-D tilt angles for each mirror facet. The simplified layout of CIM is shown in Fig. 2, which contains 9 mirrors with 3 periods. The tilt angle is noted by \((\alpha_i, \beta_i)\) (i = 1, 2, 3; j = 1, 2, 3). In real system, the CIM may contains more strip mirrors and periods. The CIM slices the first image of the target to different directions and encodes each piece of image for spatial information modulation. Collimating lens collects the sliced coded images to be dispersed by a prism. Then the light signal is acquired by a format CCD detector. The reconstruction method is based on compressed sensing concept. Each sliced coded dispersed image is recovered to a piece of datacube by a sparse reconstruction method with the assumption that each sliced image is smooth in spatial domain. The datacube of each piece is remapped together to form the entire datacube.

![Figure 1: The principle of CSIMS.](image1)

A. SYSTEM DESIGN

The schematic layout of CSIMS is shown in Fig. 3. The fore optics (L1) is a telecentric lens imaging space for ensuring the chief ray from the L1 parallel with the optical axis. So that the direction of the reflected light from CIM is exclusively determined by the tilt angle of mirror facet. The NA of the collimating lens (L2) should be large enough to cover all the light from CIM. The pupil array is formed on the back focal plane of L2, which is the images of the aperture stop. An Amici prism is used to disperse the collimated light on the pupil array plane to obtain the mixed spectral-spatial data, meanwhile to make the optical axis straight. Then the light is re-imaged onto the CCD detector by a re-imaging lens array. Each re-imaging lens is coaxial with the relative sub-pupil to ensure the accurate position of each sliced spectral image.

![Figure 2: The layout of CIM. The black square is represented for no reflection, and the light-colored surface is represented for reflection mirror facet.](image2)

B. SYSTEM MODEL

In this section, we establish a mathematical model of CSIMS to describe the light wave propagating through the system, which can help to develop the operator matrix for datacube reconstruction. The model is derived based on scalar diffraction theory, meanwhile, all the elements are assumed to be ideal elements, which means that the aberration of the optical components is not considered.

The coordinates of each optical plane of the CSIMS system is marked in Fig. 3. The intensity of the object datacube is noted by \(I_o(x_o, y_o, \lambda)\). The point response function (PRF) of the fore optics is assumed to be \(h_1(x_i, y_i)\). Since the CSIMS is incoherent imaging system, the first image on the CIM is expressed as,

\[
I_i(x_i, y_i) = \int \int I_o(x_o, y_o) h_1(x_i, y_i; x_o, y_o) dx_o dy_o
\]

(1)

The CIM is composed of periodic mirrors with encoded facets, so that the reflection function of CIM is noted by \(l_{\text{cim}}(x_i, y_i)\), which is given as,
After the prism, the re-imaging lens array (L3) is used to re-collect the light for each sub-pupil and image on the detector. The L3 is assumed to be on the same plane with the prism for simplification, and the transmission function of L3 is:

\[
t_{L3}(x_{L3}, y_{L3}) = \sum_{m} \text{circ} \left[ \frac{\sqrt{(x_{L3} - \xi_m)^2 + (y_{L3} - \eta_m)^2}}{D_{L3}} \right] \cdot \exp \left[ -\frac{ik}{2f_3} \left( (x_{L3} - \xi_m)^2 + (y_{L3} - \eta_m)^2 \right) \right].
\] (7)

After that, the PRF on the detector plane is given as:

\[
h_d(x_d, y_d; x_o, y_o, \lambda) = \exp\left( \frac{jk}{f_3} \right) \int h_i(x_i, y_i; x_o, y_o, \lambda) t_{L3}(x_{L3}, y_{L3}) \cdot \exp \left[ \frac{jk}{2f_3} \left( (x_i - x_{L3})^2 + (y_i - y_{L3})^2 \right) \right] dx_{L3} dy_{L3}
\] (8)

Since it is assumed to be inherent system, the light intensity distribution on the detector is:

\[
I_d(x_d, y_d; x_o, y_o, \lambda) = \left\| \int h_i(x_i, y_i; x_o, y_o, \lambda) \cdot I_o(x_o, y_o, \lambda) dx_o dy_o \right\|^2
\] (9)

III. RECONSTRUCTION METHODS

The datacube reconstruction method for CSIMS is based on the compressed sensing concept. Each piece of the sliced image on the CIM is assumed to be piecewise smooth in spatial structure, which makes the datacube of each sliced parts can be compressed in the wavelet domain. So that the estimated datacube is recovered by calculating the optimization problem as follows:

\[
\tilde{f}_{(m,n)}(x, y, \lambda) = \text{arg min}_{f} \left\{ g(m,n) - H_{(m,n)} W_{(m,n)} f_{(m,n)} + \| f \|_0 \right\}
\] (10)

Where \( \tilde{f}_{(m,n)} \) (m=1,2,...,M; n=1,2,...,N) is the estimated datacube of each sliced part. The \( g(m,n) \) is the measurement on the detector for the \( (m,n) \) sliced piece, and the \( H_{(m,n)} \) is the forward transmission matrix for No. \( (m,n) \) sub-pupil, \( W_{(m,n)} \) is the wavelet basis for \( (m,n) \) sliced piece. If the entire datacube is the size of \( X \times Y \times L \), where \( X \times Y \) represents the size of spatial domain and \( L \) represents the spectral domain. The \( \tilde{f}_{(m,n)} \) can be transformed to a column vector with the size of \( \{X/M/N\} Y L \times 1 \). The related detector measurement data \( g(m,n) \) can be represented by a vector of size \( \{X/M/N/L-1\} Y \times 1 \). In this situation, the forward matrix \( H_{(m,n)} \) for sub-pupil \( (m,n) \) is a matrix with the size of \( \{X/M/N/L-1\} Y \times (X/M/N)YL \). The size of \( W_{(m,n)} \) and \( \theta_{(m,n)} \) is determined by the wavelet basis chose for sparse representation.

The Eq. (10) can be solved by many methods such as the GPR [12], the Iterative Shrinkage/Threshold (IST) [17], and Orthogonal Matching Pursuit (OMP) [18] et al. In this paper, we choose the Two-step Iterative
Shrinkage/Thresholding (TwIST) reconstruction method. This method uses the last two estimate values to calculate the new estimate value, until the objective function is less than the threshold value, which consumes less time and has better convergence. The best estimated value is the most sparse value in the wavelet basis, i.e., the coefficient $\theta'$ contains the most zeros and a few nonzero values. After estimate the sliced datacube for each sub-pupil, the entire datacube is shown as, 

$$f = [f_{(0,1)} \ f_{(2,0)} \ \ldots \ f_{(M,1)} \ f_{(0,2)} \ f_{(2,2)} \ \ldots \ f_{(M,2)} \ \ldots \ f_{(M,N)}]$$  \hspace{1cm} (11)

IV. SIMULATION RESULTS AND DISCUSSION

To prove the CSIMS spatial-spectral information measurement performance compared with the CASSI system, we conduct imaging simulations for these two systems, and acquire the mixed spatial-spectral raw data. The CSIMS imaging simulation is conducted based on the model in Eq. (9), and the CASSI imaging simulation is based on the model established in the article [8]. To ensure the same size reconstructed datacube, the coded aperture used in CASSI and the coded image mapper used in CSIMS are designed to contain the same number of elements. At the same time, the fore optics, collimating lens, prism and reimaging lens used in both system share the same parameters. We use the TwIST algorithm for reconstruction. The parameters in CSIMS and CASSI for imaging simulation are shown in Table. 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>CSIMS</th>
<th>CASSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$, F/#</td>
<td>100mm, 2</td>
<td>100mm, 2</td>
</tr>
<tr>
<td>$f_2$</td>
<td>100mm</td>
<td>100mm</td>
</tr>
<tr>
<td>$f_3$</td>
<td>25mm</td>
<td>25mm</td>
</tr>
<tr>
<td>Coded component size</td>
<td>128×128, $d_e = 22\mu m$</td>
<td>64×128, $N = 1$, $M = 2$, $d_e = 22\mu m$</td>
</tr>
<tr>
<td>Wavelength, bands</td>
<td>500nm-620nm, 4</td>
<td>500nm-620nm, 4</td>
</tr>
</tbody>
</table>

The detector element size is design to be 5.5$\mu$m, and the coded element size $d_e$ is 22$\mu$m. Since the $f_2/f_2 = 1/4$, the size of the image of each coded element equal to that of detector. The F/# of the fore optics is small enough that the diameter of the Airy disk is 3.03$\mu$m, which is much smaller than the element on the coded aperture or CIM, so that the diffraction from the coded element is little enough to be ignored in this situation. The input high resolution datacube is obtained by the push hyperspectral imager (PHI) [19], which is shown in Fig. 4.

The coded aperture size of CASSI is 128×128, and the spectral bands is design to be 4, so that the size of measurement raw data is 128×131, and reconstructed datacube size is 128×128×4. The CIM of CSIMS contains 2 mirrors for simplification in this simulation. Each mirror has 64×128 coded elements. The system forms 2 sub-pupils and 2 sub-imaging spectrometers. The simulations are conducted by MATLAB 2019a based on the model in Eq. (9) for CSIMS and in article [8] for CASSI. The results are shown in Fig. 5.

We use the TwIST algorithm to reconstruct the datacube. The method is conducted directly to the raw data shown in Fig. 5(b), and the raw data from CSIMS should be sliced to 4 pieces to form 64×67 matrix for each piece. The TwIST is conducted to each 64×67 matrix, and the results are combined together to form the entire datacube. The reconstructed spectral images for the two system are shown in Fig. 6.

We choose the Average Gradient (AG) and Entropy [20] to evaluate the spectral images. The results are shown in Table. 2.

<table>
<thead>
<tr>
<th>Wavelength, bands</th>
<th>CSIMS</th>
<th>CASSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>500nm</td>
<td>0.0535</td>
<td>0.5289</td>
</tr>
<tr>
<td>540nm</td>
<td>0.0575</td>
<td>0.5821</td>
</tr>
<tr>
<td>580nm</td>
<td>0.0603</td>
<td>0.6735</td>
</tr>
<tr>
<td>620nm</td>
<td>0.0927</td>
<td>1.0055</td>
</tr>
</tbody>
</table>

From the results, we can find that the AG and Entropy value of CSIMS are all larger than that of CASSI, which means that the reconstructed datacube from CSIMS reveal more detail of the target than that of CASSI in spatial
domain. The partial enlarged detail in shown in Fig. 7, and the Fig. 7(b) can obviously reveal more details of the object than Fig. 7(c).

In the spectral domain, we choose 3 points randomly to make the spectral curves, which are shown in Fig. 8. The Relative spectral Quadratic Error (RQE) and Spectral Angle (SA) [21] are chosen to evaluate the curves, and the results are summarized in Table 3. We can find that the SA and RQE calculated from CSIMS are all smaller than that of CASSI, which means that the spectral curves reconstructed from CSIMS are more accurate.

![Figure 6](image6.png)

**FIGURE 6.** The reconstructed spectral images, (a) is for CSIMS, and (b) is for CASSI

![Figure 7](image7.png)

**FIGURE 7.** The partial enlarged detail of reconstructed spectral image at 540nm, (a) is the input data, (b) is for CSIMS, (c) is for CASSI

<table>
<thead>
<tr>
<th>Points</th>
<th>CSIMS SA</th>
<th>CSIMS RQE</th>
<th>CASSI SA</th>
<th>CASSI RQE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.0189</td>
<td>0.0018</td>
<td>0.0215</td>
<td>0.0022</td>
</tr>
<tr>
<td>B</td>
<td>0.0245</td>
<td>0.0031</td>
<td>0.0429</td>
<td>0.0057</td>
</tr>
<tr>
<td>C</td>
<td>0.0380</td>
<td>0.0047</td>
<td>0.0739</td>
<td>0.0084</td>
</tr>
</tbody>
</table>

**TABLE III**

**The Evaluations for reconstructed spectral images**
VI. CONCLUSION

In this paper, we propose a new type of snapshot imaging spectrometer named Compressed Sensing Image Mapping Spectrometer (CSIMS). Compared with CASSI, CSIMS slices the input field to different parts, which can reduce the spatial-spectral mixture in the raw data and increase the number of useful data for reconstruction. We design the structure of the CIM, which can slice and reflect the input image of target to different directions, and encode the different pieces for dispersion. The mathematical model of CSIMS is established to describe the light wave propagation through the system based on the scalar diffraction theory. Then, the imaging simulations are conducted to prove the advantages of CSIMS compared with CASSI. The evaluations of the reconstructed spectral images and spectral curves all prove that the CSIMS can reveal more detail of the target, and the information is more accurate than that of CASSI in both spatial and spectral domain.

REFERENCES


Xiaoming Ding received the B.S. and Ph.D. degrees from Beihang University, Beijing, China, in 2012 and 2019, respectively. He is currently with Tianjin Normal University, Tianjin, China. His current research focuses on snapshot imaging spectrometer and computational imaging.