Fast Lyapunov Vector Field Guidance for
Standoff Target Tracking based on
Offline Search

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ABSTRACT To address the problem of the long convergence time of traditional Lyapunov Vector Field Guidance (LVFG), a fast convergent LVFG designed with guidance function is proposed. The necessary criteria of a fast convergent vector field are first illustrated through an analysis of the contraction and circulation components. Then, two types of guidance functions of the distance between aircraft and target are constructed to replace the original guidance parameter. Considering the saturation constraints from the aircraft performance, the optimal parameter in guidance functions based on offline search is estimated to achieve the best convergence. Meanwhile, standoff tracking for a moving target is solved with analysis of motion correction. Simulation results indicate the proposed vector field guidance converges faster to the standoff circle without traversing the orbit regardless of whether the aircraft’s initial position is inside or outside the circle, even for a moving target. In addition, selection of parameter in the guidance function is proved to make convergence speed approach the limit.

INDEX TERMS Vector Field Guidance, Lyapunov Function, Standoff Tracking, Fast Convergence, Moving Target, Offline Search.

I. INTRODUCTION

WITH the development of automatic control and navigation technologies, low-cost Unmanned Aerial Vehicles (UAVs) equipped with Flight Control System (FCS) are widely used in military reconnaissance [1], real-time target tracking [2], geographic mapping [3], forest firefighting [4], and power-line inspection [5].

An important application scenario is continuous tracking and monitoring for individual target, relying on fixed-wing UAV. Fixed-wing UAV, as an inexpensive, high-endurance automatic aircraft, maintains economical cruising speed and keeps the target observable at all times, so the standoff-target tracking approach that makes the UAV maintain a fixed standoff distance to the target came into being. In extended applications, the target may refer to an area of interest or the geometric centroid of dense targets in the group [6]. In addition, standoff distance is designed to protect the aircraft from detection and attack threats of hostile targets.

Extensive research works have been performed, including on “good helmsman” behavior-based approaches for constant line-of-sight orientation relative to the aircraft [7], [8], guidance law using a relative side-bearing angle [9], referenced point-based path following guidance methods [10]–[12], Lyapunov Vector Field Guidance (LVFG), and path planning based on neural networks [13], [14].

In path-following based on a referenced point, which can also be called a virtual target, movement control is achieved by reducing the angle between the course of the UAV and the line of sight to the virtual target to zero. The methods can be achieved simply and efficiently for stable standoff tracking. However, accuracy settling onto the standoff circle depends on relationship between aircraft speed, line of sight, and standoff distance.

LVFG has the advantages of small computational complexity and global convergence. The Lyapunov function was earlier used to design LVFG for standoff target tracking [15]. Asymptotically unbiased path-following approaches based on vector fields were proposed [16], [17]. Arbitrary curved path tracking using a vector field of course commands was further discussed in [18]. The above vector fields were stated
in two-dimensional space. Sensing and tracking performance can be improved by changing the altitude of the UAV with three-dimensional guidance vector fields [19], [20]. Tangent vector field guidance and LVFG were combined to track several discrete targets effectively and robustly [21], [22]. A nondimensional parameter, which is called as a guidance parameter in the circulation component here, was utilized to transfigure the Lyapunov vector field [23], and strategies for simultaneous cooperative multiple UAVs standoff tracking were presented. The above approach was extended to path following and achieved arrival-angle control [24], [25]. The circulation term in traditional LVFG was modified to make UAV faster settle onto the standoff orbit with a lower maximum curvature [26]. Standoff target tracking was achieved with unknown constant airspeed and target motion [27]. Co-operative standoff tracking methods of moving targets were presented to solve the phase separation problem [28]–[31]. Other innovative aspects for circumnavigation can be found in recent works [32], [33].

Although many significant and constructive research efforts have been performed, long convergence time is not developed to address recently. For the problem, this paper’s principal contributions are summarized as following:

1) The analysis of curvature problems and three criteria of fast convergence are illustrated to guide the design of fast LVFG in the Lyapunov vector field guidance framework.

2) Two types of guidance functions fulfilling the criteria are constructed to achieve fast convergence, and their performance and application scope are analyzed.

3) The optimal parameter estimations in guidance functions are proposed based on offline search, and the performance and the complexity of different solutions with rough or fine constrains are clarified.

The rest of the paper is organized as follows. In Section II, we describe the problem, and two guidance functions are presented and analyzed. The optimal parameter for a stationary and moving target was searched offline in Section III to design the guidance vector field with the proposed guidance functions. Simulation and analysis for standoff target tracking are presented in Section IV to demonstrate the advantage of the proposed algorithm, followed by conclusions in Section V.

II. LYAPUNOV VECTOR FIELD GUIDANCE

A. SCENARIO DESCRIPTION

Assuming the position vector of a target in the loiter plane is \( x_t = [x_t, y_t]^T \). The UAV located at \( x = [x, y]^T \) performs standoff target tracking with velocity vector \( \dot{x} = [\dot{x}, \dot{y}]^T \) and course \( \chi \). Without loss of generality, the position vector of the aircraft in the local coordinate system of the target can be presented as \( x_r = [x_r, y_r]^T \), or radial distance \( r \) and bearing angle \( \theta \) (in the local polar coordinate system).

A first-order course-hold loop control is used, and the constant-speed aircraft kinematic model can be presented as

\[
\begin{align*}
\dot{x} &= s \cos(\chi) \\
\dot{y} &= s \sin(\chi) \\
\dot{\chi} &= k(\chi_d - \chi)
\end{align*}
\]

where \( s \) is UAV speed, \( \chi_d \) is the desired course, and \( k \) is time constants of the first-order kinematics of the course.

B. LYAPUNOV VECTOR FIELD GUIDANCE FRAMEWORK

Various vector field methods based on the Lyapunov function were utilized to guide the fixed-wing aircraft to maintain a steady distance to the target for continuous tracking in the target local coordinate system. A series of Lyapunov vector fields were configured by different nondimensional parameters through modification of the above research [23], whose framework is

\[
\begin{bmatrix}
\dot{r} \\
\dot{\theta}
\end{bmatrix} = \frac{s}{\alpha} \begin{bmatrix}
-(r^2 - r_d^2) \\
c \cdot r_d \cdot r
\end{bmatrix}
\]

where \( c \) is a constant parameter defined as a guidance parameter here, \( \alpha \) is normalization term of speed and satisfies constrain \( r^2 + (c \dot{\theta})^2 = s^2 \), which leads to \( \alpha = \sqrt{r_1^2 + (c^2 - 2) r_d^2 + r_1^2} \).

\( \Delta = |\dot{r}| \) denotes the radial convergence speed to the standoff circle, which can be called as a contraction component. Circulation component \( B = |\dot{\theta}| \) presents the tangent speed of the aircraft with respect to the standoff circle, which is the normal of the contraction component. Fig. 1 shows the above components for standoff target tracking in the described situation.

As can be seen from (2), the contraction component decreases and the circulation component increases as guidance parameter \( c \) becomes larger for the constant-speed kinematic model, where changes are nonlinear under the influence of normalization term \( \alpha \). A positive or negative guidance parameter decides between counterclockwise or clockwise standoff target tracking, so we only discuss the case where
parameter $c$ is positive. In other words, the UAV only tracks the target counterclockwise in the following discussion.

To improve the convergence performance of the aircraft to the standoff circle and ensure the UAV maintains standoff distance $r_d$ from the target, the contraction component should be as large as possible when the aircraft is far away from the standoff circle, and be zero while the aerial vehicle settles onto the loiter circle. However, it is impossible to design the above contraction component only by adjusting the value of the guidance parameter on the basis of [23]. Meanwhile, the guidance parameter was transfigured to a guidance function of distance $r$ to decrease convergence time [26], but it was not fast enough.

In the light of the above analysis and existing research, guidance term $c$ should fulfill the following criteria:

1) the guidance parameter should be a continuous function of radial distance to overcome the deficiency of the nondimensional parameter;
2) achieve maximum at $r = r_d$ for steady standoff tracking;
3) and be zero when $r \to 0$ or $r \to \infty$ to guarantee fast convergence.

A strictly mathematical notation is presented as

$$
\begin{cases}
  c = c(r) > 0, r > 0 \\
  r_d = \arg \max(c(r)) \\
  \lim_{r \to 0, \infty} c(r) \to 0
\end{cases}
$$

(3)

Transforming (2) into the Cartesian coordinate system, the desired velocity for the UAV relative to the target is

$$
\dot{x}_d = \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix} = \frac{s}{\alpha r} \begin{bmatrix} \left( r^2 - r_d^2 \right) x_r + cr r_d y_r \\ \left( r^2 - r_d^2 \right) y_r - cr r_d x_r \end{bmatrix}
$$

(4)

For a stationary target, the course of the UAV can be represented as

$$
\chi = \arctan(\dot{y}/\dot{x})
$$

(5)

whose derivative with respect to time is course rate $\dot{\chi}$

$$
\dot{\chi} = \frac{\dot{x} \ddot{y} - \ddot{x} \dot{y}}{x^2 + y^2}
$$

(6)

Then, the curvature can be calculated as the quotient of the course rate and the UAV speed

$$
\kappa = \frac{\dot{\chi}}{s} = \frac{s \ddot{y} - \dot{y} \ddot{x}}{s(x^2 + y^2)}
$$

(7)

Substituting (4) and its time derivative into (7) leads to a curvature [26] as

$$
\kappa = \frac{s \alpha r^3 \left( (c^2 - 2) r^2 + 2 r_d^2 \right) - \alpha c r_d r (r^2 - r_d^2)}{s \alpha r^5}
$$

(8)

The traditional vector field is a special case where the guidance parameter is constant at 2, and its curvature is $\kappa_2$

$$
\kappa_2 = \frac{4 r_d^3}{(r^2 + r_d^2)^2}
$$

(9)

It is worth noting that $\kappa_2$ is a monotone decreasing function of radial distance $r$. When the UAV is near the center of the loiter circle, or $r$ is close to 0, curvature $\kappa_2$ obtains a maximum, which results in much redundant turning and time wasted on the path toward the loiter circle. Otherwise, when the aircraft is out of the loiter circle, or $r > r_d$, curvature $\kappa_2$ is always positive, which causes the left turning of the aerial vehicle to achieve counterclockwise standoff tracking. This characteristic further demands the UAV turn early even if it is far away from the standoff circle, so a massive amount of time is consumed as well. When $r=r_d$, curvature is a constant $\frac{1}{r_d^2}$ independent of the guidance parameter, which ensures the UAV flies on the circle with radius $r_d$.

### C. THE PROPOSED GUIDANCE FUNCTIONS

According to the analysis in the previous section, two guidance functions satisfying the criteria of fast convergence in the Lyapunov vector field framework are structured as

$$
c^n_1(r) = \begin{cases} 
\left( \frac{r}{r_d} \right)^n, & r < r_d \\
\left( \frac{r}{r_d} \right)^n, & r \geq r_d
\end{cases}
$$

(10)

$$
c^n_2(r) = \begin{cases} 
\exp \left[ 1 - \frac{r}{r_d^n} \right], & r < r_d \\
\exp \left[ 1 - \frac{r}{r_d^n} \right], & r \geq r_d
\end{cases}
$$

(11)

where $n \geq 0$ is an undetermined parameter, and its effect on the performance of the guidance vector field will be discussed in detail later. It is worth noting that $c^n_1=0=c^n_2=1$ for $n=0$. The proposed guidance functions can be regarded as the approximation of a traditional vector field [28]. The Lyapunov vector field modified in [26] is a special case of $n=1$ for $c^n_1(r)$, and is only compared with traditional vector fields considering different guidance parameters but does not further enhance convergence performance to the limit.

The curves of the two guidance functions as $r$ changes are shown in Fig. 2, where the proposed guidance functions are denoted with red and blue lines, the solid line is a case of $n = 2$, and the dash-dot line is $n = 15$; the same as below. As shown in Fig. 2, both the proposed guidance functions satisfy the fast convergence criteria analyzed in (4). Specifically, the larger $n$ is, the longer the distance between the two functions is attached to the $x$-axis and steeper near $r_d$. The second guidance function is less than the first for the same $r$ and $n$, which indicates the second guidance function $c^n_2(r)$ have a faster convergent performance.

Curves of contraction and circulation components with parameters $n = 2, 15$ are compared with the case of $c = 2$ (denoted with a black line, the same as below) as shown in Fig. 3. The black line is lower than the other curves for the contraction component in Fig. 3(a) and the opposite situation is shown in Fig. 3(b), except $r = 0$ and $r = r_d$, which indicates that the proposed guidance functions increase the speed of the aircraft toward the loiter circle as much as possible, thus greatly improving the convergence speed of standoff tracking. With the increase of parameter $n$, there are more ones in the contraction component and more zeros in
FIGURE 2. Proposed guidance functions with \( n = 2, 15 \).

the circulation component, which implies better convergence performance of the vector field.

Note: the contraction and circulation components are not strictly symmetric about \( r = r_d \), even if the larger \( n \) is, the stronger the symmetry is. Therefore, different \( n \) should be determined in the case whether the initial aircraft position is in or out of the standoff circle.

Fig. 4(a) presents the traditional Lyapunov vector field in [28], and new Lyapunov vector fields with proposed guidance functions are shown in Fig. 4(b) and Fig. 4(c) where \( n = 2 \). Through comparison with the traditional vector field, we can see that, in the proposed vector fields, the vectors far away from the standoff circle point to the circle more radially, regardless of whether they are inside or outside the circle, which further confirms the contribution of the proposed guidance functions to tracking efficiency, as per previous analysis.

D. CURVATURE ANALYSIS

To further analyze the influence of the proposed guidance functions on the curvature, the time derivative of the guidance functions can be obtained as

\[
\dot{c}_1^n(r) = \begin{cases} 
-n \frac{n-1}{\alpha r_d} (r^2 - r_d^2), & r < r_d \\
-\frac{n}{\alpha r^2} (r^2 - r_d^2), & r \geq r_d 
\end{cases} \tag{12}
\]

and

\[
\dot{c}_2^n(r) = \begin{cases} 
-c_2 n \frac{r^2}{\alpha r_d} (r^2 - r_d^2), & r < r_d \\
c_2 n \frac{r^2}{\alpha r_d} (r^2 - r_d^2), & r \geq r_d 
\end{cases} \tag{13}
\]

Substituting (10)–(13) into (8), two curvature functions based on the proposed guidance functions can be presented as \( \kappa_{c_1}^n \) and \( \kappa_{c_2}^n \). The results are too complicated to quantitatively analyze, so curvature curves were plotted in Fig. 5 when \( n \) was equal to 2 and 15, as well as the curvature of the traditional vector field, where the horizontal and vertical dashed lines denote \( \kappa = \frac{1}{r_d} \) and \( r = r_d \), respectively. Considering the operational and physical performance of the aircraft, its maximum turning rate is limited due to the maximum bank angle, so the curvature is subject to a saturation constraint as

\[
|\kappa| \leq \frac{\dot{\chi}_{\text{max}}}{s}. \tag{14}
\]

where the constraint on the curvature was set as \( \kappa_{\text{max}} = 0.01 \) in Fig. 5.

In Fig. 5, when the aircraft was located in the center of the standoff circle or radial distance \( r \) was far greater than \( r_d \), curvature curves based on the proposed guidance functions were all close to zero, which is significantly better than the curvature of the traditional method whose guidance parameter is a constant at 2. The more points on both \( \kappa_{c_1}^n \) and \( \kappa_{c_2}^n \) closing to the x-axis as \( n \) increases, the more directly the UAV flies onto the standoff circle, thereby speeding up convergence. When \( r > r_d \), there is a negative part of the curvature except for the black solid line. It is exactly the negative values that allow the aircraft to turn right at a position closer to the loiter circle and then turn left to
achieve counterclockwise standoff tracking. Compared with the traditional method whose curvature is always positive, the proposed vector field can greatly shorten convergence time. When $n$ becomes larger, the range of the curvature near zero also becomes bigger, and the extremes of the curvature are closer to the circle, leading to faster convergence speed. However, high requirements for the turning rate for UAV are put forward due to a large $n$. On the one hand, due to the saturation constraint on the bank angle, the turning of the aircraft cannot complete the requirements of the control command, which leads to course lag or even exceptional circumstances; on the other hand, when $r$ is disturbed near $r_d$, the curvature fluctuates violently around $\frac{1}{r_d}$, which causes the aircraft to sway left and right, and unstable standoff tracking quickly and directly. In brief, the choice of parameter $n$ needs to be related with specific scenarios, which is the focus of the next section.

III. DESIGN OF FAST VECTOR FIELD GUIDANCE

As per the above analysis, the proposed vector field constructed with a small $n$ has relatively long convergence time but can guarantee directly stable tracking. On the contrary, the extremely large $n$ makes the vector field more radial toward the standoff circle; although this leads to a much faster arrival at the circle, this situation may need more time on stabilization by means of the aircraft repeatedly traversing the loiter circle. This makes UAV bear the maximum load for a long time and limits applications of standoff target tracking.

The ideal track should take into account both fast approaching target and stable tracking target. Therefore, the selection of $n$ should handle the trade-off between fast convergence and stability, which requires parameter $n$ to be large enough, and allows no traversing the standoff circle.

A. PARAMETER CHOICE FOR STATIONARY TARGETS

According to the operational and physical performance of the aircraft, the maximum bank angle is restricted for UAV; thus, the maximum curvature of the track is limited. When the curvature function is always within the saturation constraint, the UAV can follow the desired course determined by the proposed LVFG. In this case, optimal parameter $n_{opt}$ should be selected to be as large as possible.

$$n_{opt} = \arg \max_n \left( \max_r \left| \kappa_{c_i} \right| \leq \kappa_{max} \right)$$

where $\kappa_c$ generally refers to the curvature generated by proposed guidance function $c_{n_i}$, $i = 1, 2$.

When the curvature function exceeds its threshold of a curvature constraint, the UAV turns at the minimum radius within a voyage under the influence of the maximum bank angle. The schematic diagram of the trajectory is under the above condition, shown as Fig. 6, where $r_{min}$ is the minimum turning radius whose corresponding circle is expressed as a solid line and its center is at point $o$. Points $p_{in}$ and $p_{out}$ represent the tangent points when the aircraft enters and exits the circle, respectively.

It can be seen from Fig. 6 that, when the minimum turning radius circle and the standoff circle are separated, it takes additional time for the aircraft to move from point $p_{out}$ to steady standoff tracking. Otherwise, when the $n$ exceeds a value, the voyage formed by the UAV turning at the maximum bank angle intersects with the standoff circle, which leads to steady tracking after crossing over the standoff circle several times, increasing convergence time, and even threatens the safety of
the UAV in some cases. Therefore, the optimal solution is that the aircraft should directly perform steady standoff tracking after turning at the maximum bank angle, that is, optimal parameter \( n \) should make the minimum turning radius circle of UAV tangent to the standoff circle.

Maximum curvature \( \kappa_{\text{max}} \) can be obtained from the performance constraint of the aircraft. While the guidance functions and parameter \( n \) are given, the distance between point \( p_{in} \) and the target can be obtained by combining the curvature function. Angle \( \beta \) between aircraft speed and radial speed can be obtained by the following law.

\[
\cos \beta = \left| \frac{\hat{v}}{s} \right| \tag{16}
\]

According to the triangle cosine theorem, the distance between the center of the minimum turning radius circle and the target is

\[
r_{ou}^2 = r_{in}^2 + r_{in}^2 - 2r_{in}r_{in}\cos <r_{min},r_{in}> \tag{17}
\]

where \( <r_{min},r_{in}> \) denotes the angle between \( r_{min} \) and \( r_{in} \), \( r_{in} \) is distance vector from point \( p_{in} \) to the target. Substituting (2) and (16), the above equation can be rewritten as follows, based on the trigonometric function formula and the geometric relationship.

\[
r_{ou}^2 = \begin{cases} 
  r_{in}^2 + r_{in}^2 - 2r_{in}r_{in}\cos <r_{min},r_{in}> & \text{if } r > r_d \\
  r_{min}^2 + r_{in}^2 - 2r_{min}r_{in}\cos <r_{min},r_{in}> & \text{else}
\end{cases} \tag{18}
\]

It is easy to judge the positional relationship between the minimum turning circle and the standoff circle according to \( r_{ou} \). If \( r_{ou} < \delta_{\text{out}} \) (\( r_{ou} > \delta_{\text{in}} \)), the two circles intersect, and the UAV trajectory fluctuates near the loiter circle, where \( \delta_{\text{out}} = r_d + r_{min} \) and \( \delta_{\text{in}} = r_d - r_{min} \) are discriminant thresholds for the initial position of the aircraft inside and outside the circle, respectively. If \( r_{ou} > \delta_{\text{out}} \) (\( r_{ou} < \delta_{\text{in}} \)), the minimum turning circle and the standoff circle are separated (inclusion), and the convergence is slow. For \( r_{ou} = \delta_{\text{out}} \) (\( r_{ou} = \delta_{\text{in}} \)), the two circles are tangential internally (externally), and the aircraft can rapidly converge to the standoff circle and directly form stable tracking where corresponding parameter is optimal for the given guidance function.

Substituting \( r_{ou} = \delta_{\text{out}} \) or \( r_{ou} = \delta_{\text{in}} \) into (18), since both \( r_{in} \) and \( c_i \) are nonlinear functions related to parameter \( n \), it is difficult to directly obtain a closed-form solution about \( n \).

It can be seen from previous analysis that the larger \( n \) is, the closer the saturated points of curvature function are to \( r_d \), that is, the nearer the minimum turning circle is to the loiter circle. Therefore, \( r_{ou} \) is a monotone subtraction function for \( n \), and the maximum of parameter \( n \) when \( r_{ou} \) is no less than \( \delta_{\text{out}} \) or no greater than \( \delta_{\text{in}} \) can be selected as the optimal parameter, which is obtained based on the search method. The optimal parameter search algorithm developed for the stationary target is summarized in Algorithm 1.

**Algorithm 1 Optimal parameter search for stationary targets.**

**Input:** Standoff radius \( r_d \), Maximum turning rate \( \chi_{\text{max}} \), UAV cruising speed \( s \); Grid points of parameter \( n \), \( n_1, n_2, \ldots, n_m \); Given guidance function \( c_i^{\text{opt}}(r) \);

**Output:** Optimal parameter for given guidance function \( n_{\text{opt}} \);

1: Calculate the minimum turning radius \( r_{\text{min}} \);
2: Calculate threshold \( \delta_{\text{out}} \) or \( \delta_{\text{in}} \) according to the initial position of UAV.
3: for \( k = 1 \) to \( m \) do
4: Obtain curvature function \( \kappa_{c_i}^{\text{opt}}(r) \) in (8) by \( c_i^{\text{opt}}(r) \) and its time derivative;
5: if \( \max_{r} \left| \kappa_{c_i}^{\text{opt}}(r) \right| < \frac{\chi_{\text{max}}}{s} \) then
6: \( n_{\text{opt}}^{i} = n_k \) and Continue;
7: end if
8: Calculate \( r_{in} \) and \( r_{ou} \) according to (18);
9: if \( r > r_d \) & \( r_{ou} \geq \delta_{\text{out}} \) or \( r < r_d \) & \( r_{ou} \leq \delta_{\text{in}} \) then
10: \( n_{\text{opt}}^{i} = n_k \);
11: end if
12: end for

It is noted that, when the maneuverability of the UAV is weak or the motion space is limited under a certain initial position and course conditions, such as position closing to the standoff circle with the course pointing to the standoff circle, the proposed vector field guidance law is hard to design and satisfy both fast convergence and avoidance of fluctuation only by adjustment of parameter \( n \). Due to the limited motion range, when the aircraft initializes in the standoff circle, the above problems are particularly prominent, and can be resolved by reducing the speed of the airplane, reducing the turning radius, and improving maneuverability.

In practical applications, for the situations with the initial position of the aircraft near the center of the standoff circle, it is usually applicable in non-hostile scenarios such as regional surveys and peripheral situation surveillance, therefore the convergence speed and stability of the vector field are not required strictly. On the contrary, when the initial position of the aircraft is farther away from the circle, especially for a dangerous region or hostile target, the standoff distance is usually a safe distance. Hence, it is necessary to ensure...
the aircraft safely completes the mission, which requires the aircraft to directly form stable standoff tracking.

Generally speaking, it is time-consuming to determine optimal parameter \( n_{opt} \) based on the grid-search method. At the same time, both the consumed time and the accuracy of the result rely heavily on the grid interval. Before the aircraft carries out a real-time online mission, for a given UAV type, its mobility (cruising speed, maximum turning rate) has been decided, as well as standoff distance determined by the mission. Therefore, the required parameter can be calculated and selected offline after the mission is confirmed and before the UAV performs the mission.

**B. PARAMETER CHOICE FOR MOVING TARGETS**

The correction factor was introduced to modify the desired course of the UAV for the case of a moving target, and well-behaved guidance were performed [28]. Both the above method and the proposed method are based on the same vector field guidance framework, therefore the correction factor can be used in the method mentioned in this paper. Consequently, the choice of optimal parameter \( n_{opt} \) should be modified for moving target standoff tracking as well.

If the UAV moves in the designed course of the vector field guidance in the local coordinate system of the target, the relationship between target velocity and UAV modified velocity is as follows:

\[
v_g - v_t = \lambda v_d
\]

where \( \lambda \) is the correction factor (denoted as \( \alpha \) in the original reference), \( v_t \) is the target velocity vector, \( v_g \) is the corrected UAV velocity vector in the global coordinate system, and \( s = |v_g| \), \( s_t = |v_t| \), \( s_r = |\lambda v_d| \).

Taking the modulus of the above equation to obtain the quadratic equation with respect to \( \lambda \), the impossible negative root is discarded, and the larger positive root is the desired correction factor.

The work of the correction factor on UAV velocity is shown in Fig. 7. Through the above modification, the UAV can follow a desired course in the local coordinate system of the target according to the guidance vector field and realize standoff target tracking. As we can see from Fig. 7, the range of the correction factor is \( \lambda \in [1 - \frac{s_t}{s}, 1 + \frac{s_t}{s}] \), where the extremum of \( \lambda \) is obtained when \( v_g \) and \( v_t \) are collinear. The difference of the UAV course before and after modification is denoted as angle correction term \( \Delta \chi \), whose range is \( \Delta \chi \in [0, \arcsin (\frac{s_t}{s})] \) regardless of its directionality.

In order to ensure sufficient time and space to veer when the UAV approaches the standoff circle, the minimum turning radius should be calculated according to the influence of relative velocity \( v_r = v_g - v_t = \lambda v_d \) on course and speed. Therefore, \( r_{min}, r_{in} > 0 \) in (17) can be corrected as

\[
< r_{min}, r_{in} >_{c} = < r_{min}, r_{in} >_{d} \pm \Delta \chi
\]

where \( < r_{min}, r_{in} >_{c} \) denotes the corrected angle between \( r_{min} \) and \( r_{in} \), \( < r_{min}, r_{in} >_{d} \) denotes the included angle based on guidance law before correction.

**FIGURE 7.** Work of correction factor on unmanned aerial vehicle (UAV) velocity.

1) Solution 1 with rough constraints

On the one hand, due to the relative speed between the aircraft and the target, turning time is greatly compressed; on the other hand, the large angle correction term may limit turning space for the aircraft. Accordingly, the rigorous case where maximum relative speed and maximum angle correction term are roughly selected is the simplest and most direct correction solution with the following rough constraints.

\[
\begin{align*}
\max (s_r) &= s + s_t \\
\max(\Delta \chi) &= \arcsin \left( \frac{s_t}{s} \right)
\end{align*}
\]

2) Solution 2 with fine constraints

As shown in Fig. 7, when the angle correction term equals its minimum value of zero, the relative speed reaches its maximum value, and \( v_g \) and \( v_t \) are collinear and reverse; when the angle correction term is the maximum, \( v_r \) and \( v_t \) are vertical. For these reasons, it is virtually impossible to make both the relative speed and the angle correction term approach the maximum at the same time, and the relationship between them is as follows.

\[
s^2 + s_r^2 - 2ss_r \cos \Delta \chi = s_t^2
\]

Based on the above fine constraints, optimal parameter \( n_{opt} \) can be searched offline according to the grid points of the relative speed and the corresponding angle correction term calculated by (23). Algorithm 2 presents the specific implementation flow of Solution 2.

**IV. SIMULATION AND ANALYSIS**

To illustrate the performance of the proposed Lyapunov vector field, a simulation environment was designed as follows. The control gain was set as \( k=30 \), the speed of the aircraft was \( s=20m/s \), and standoff distance was \( r_d=300m \).
Algorithm 2 Optimal parameter search for moving target

**Input:** Standoff radius $r_d$; Maximum turning rate $\dot{\chi}_{\text{max}}$; UAV cruising speed $s$; Target speed $s_t$; Given guidance function $c_i^n(r)$; Grid points of relative speed, $s_1^r, s_2^r, \ldots, s_p^r$

**Output:** Optimal parameter for given guidance function $n_{\text{opt}}^i$;

1. Initialize $n_{\text{opt}}^i$ to zero;
2. for $j = 1$ to $p$ do
3. Calculate minimum turning radius $r_{\text{min}}$;
4. Calculate angle correction term $\Delta \chi$ in (23);
5. Correct $< r_{\text{min}}, r_{\text{in}} >$ using $\Delta \chi$ as (20);
6. Calculate optimal parameter $n_j$ by Algorithm 1;
7. if $n_{\text{opt}}^i < n_j$ then $n_{\text{opt}}^i = n_j$;
8. else Break;
9. end if
10. end for

Assuming the maximum course rate was $\dot{\chi}_{\text{max}} = 0.2 \text{rad/s}$, and the stationary target was located at $(0 \text{ m}, 0 \text{ m})$.

**A. INITIAL UAV POSITION INSIDE STANDOFF CIRCLE**

The initial position of the aircraft was set as $(5 \text{ m}, 0 \text{ m})$, close to the target, and its initial course was 0 deg. Fig. 8 gives the standoff circle of the target and UAV trajectories based on different LVFGs for 50 s. It can be seen from the figure that the main component of aircraft speed based on the traditional Lyapunov vector field (solid black line) is used for veering instead of flying radially to the standoff circle, which forms stable standoff tracking after the aircraft turns about 360 deg. Using the proposed vector fields, the aircraft can quickly approach the standoff circle from the beginning on, and then turn left to directly perform stable tracking, which converges faster.

To more intuitively show convergence performance under different parameters, convergence error is defined as the absolute of the distance difference from the target and standoff distance.

$$e_r = |r - r_d|$$ (24)

Maximum distance $r$ to the target in standoff tracking, and time taken for the traditional and the proposed vector fields to converge to 0.5%$r_d$ were investigated, respectively, and the results are shown in Table 1. At the same time, integrated time absolute error (ITAE) was used to measure the tracking performance of a guidance law with different guidance functions and parameters. ITAE is defined as follows, and ITAE curves as time changes are shown in Fig. 9.

$$J_{\text{ITAE}} = \int_0^t \tau e_r(\tau) d\tau$$ (25)

It can be seen from Table 1 and Fig. 9, the traditional guidance method is of the slowest convergence speed, and even simulation time needs to be extended to 100 s before a time result can be obtained. Meanwhile, its ITAE is also always the largest and escalates, which indicates the aircraft never arrives to the loiter circle within 50 s. For the proposed guidance vector fields, the increase rate of ITAE significantly decreased after 8 s, which presents that the UAV can quickly...
converge to the circle. The larger parameter \( n \) is, the shorter the time to converge to 0.5\% \( r_d \). However, when parameter \( n \) is too large, such as \( n = 15.0 \), time to first reach 0.5\% \( r_d \) dramatically decreases, and the corresponding ITAE is also the smallest, temporarily near 15 s. The advantage of a large \( n \) in ITAE was not maintained to the end, and the aircraft traversed the loiter circle twice (the maximum of radial distance is larger than the standoff distance) before converging to steady tracking. The vector field adopted with the optimal parameter searched offline (highlighted in bold) appears to have no oscillation and could directly converge to the standoff circle. The corresponding ITAE became horizontal after 20 s because of continuous standoff target tracking on the standoff circle.

**B. INITIAL UAV POSITION OUTSIDE THE STANDOFF CIRCLE**

Assuming that the initial position of the aircraft is (-900 m, 0 m), which is located outside the standoff circle, an experiment was simulated for 100 s, and the other simulation conditions remained the same. Fig. 10 and Fig. 11 show the UAV trajectories and ITAE variation curves, and the corresponding ITAE became horizontal after 20 s because of continuous standoff target tracking on the standoff circle.

The target vehicle is assumed to move counterclockwise around a circle with a radius of 1000 m, and the target speed is 10 m/s. The relationship curves between relative speed and optimal parameter \( n_{opt} \) based on different solutions are given in Fig. 12, where Solutions 1 and 2 are denoted by the dotted line and solid line, respectively. The minimum of each curve in Fig. 12, where Solutions 1 and 2 are denoted by the dotted line and solid line, respectively. The minimum of each curve is 31.57, 300.17.

**TABLE 2. Convergence performance when UAV initial position is (-900 m, 0 m).**

<table>
<thead>
<tr>
<th>Guidance term configuration</th>
<th>0.5% ( r_d )/s</th>
<th>min(( r_d ))/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 2 )</td>
<td>112.57</td>
<td>300.34</td>
</tr>
<tr>
<td>( n = 15.0 )</td>
<td>30.63</td>
<td>289.71</td>
</tr>
<tr>
<td>( n_{opt} = 12.0 )</td>
<td><strong>31.53</strong></td>
<td><strong>300.17</strong></td>
</tr>
<tr>
<td>( n = 2 )</td>
<td>55.17</td>
<td>300.17</td>
</tr>
<tr>
<td>( n = 15.0 )</td>
<td>30.03</td>
<td>254.84</td>
</tr>
<tr>
<td>( n_{opt} = 6.0 )</td>
<td><strong>31.60</strong></td>
<td><strong>300.17</strong></td>
</tr>
</tbody>
</table>

**FIGURE 10. UAV trajectories when UAV initial position is outside standoff circle.**

**FIGURE 11. ITAE results when UAV initial position is outside standoff circle.**

**FIGURE 12. Convergence performance when UAV initial position is outside standoff circle.**

Compared with the second guidance function, the first guidance function always has a weak advantage. The main reason may be that the first guidance function has a milder influence on the guidance law than the second for the same change on parameter \( n \). While the same grid interval was applied to search parameter, the final result for the first guidance function was perhaps closer to the real optimal parameter, so the performance of the proposed vector field could be improved. However, the influence of this difference on tracking performance is limited. The choice of the second guidance function can reduce the search scope and improve search speed. Therefore, the guidance function can be flexibly selected according to the above characteristics of different guidance functions in practice.

**C. STANDOFF TRACKING FOR MOVING TARGET**

The target vehicle is assumed to move counterclockwise around a circle with a radius of 1000 m, and the target speed is 10 m/s. The relationship curves between relative speed and optimal parameter \( n_{opt} \) based on different solutions are given in Fig. 12, where Solutions 1 and 2 are denoted by the dotted line and solid line, respectively. The minimum of each curve was selected to design the proposed vector field guidance law. For the same guidance function, and the parameter searched with Solution 2 was larger than Solution 1 to meet the fast convergence criteria.

Fig. 13 shows UAV trajectories according to different parameters determined by Solutions 1 and 2, where Fig. 13(a)
FIGURE 12. Relationship curves between relative speed and the optimal parameter based on different solutions.

TABLE 3. Convergence performance when UAV initial position is (−900 m, 0 m).

<table>
<thead>
<tr>
<th>Guidance term configuration</th>
<th>$0.5/\sigma _{rd}/s$</th>
<th>$\min (r)/m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 2$</td>
<td>120.13</td>
<td>300.19</td>
</tr>
<tr>
<td>$n_{d1} = 1.0$</td>
<td>49.43</td>
<td>300.08</td>
</tr>
<tr>
<td>$n_{d2} = 1.7$</td>
<td>48.53</td>
<td>300.08</td>
</tr>
<tr>
<td>$n_{s1} = 4.7$</td>
<td>51.33</td>
<td>300.08</td>
</tr>
<tr>
<td>$n_{s2} = 2.3$</td>
<td>50.20</td>
<td>300.08</td>
</tr>
<tr>
<td>$c_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_{d1} = 2.3$</td>
<td>49.43</td>
<td>300.08</td>
</tr>
<tr>
<td>$n_{d2} = 2.8$</td>
<td>48.53</td>
<td>300.08</td>
</tr>
<tr>
<td>$n_{s1} = 4.7$</td>
<td>51.33</td>
<td>300.08</td>
</tr>
<tr>
<td>$n_{s2} = 2.8$</td>
<td>50.20</td>
<td>300.08</td>
</tr>
</tbody>
</table>

V. CONCLUSION

This paper presented fast Lyapunov vector field guidance for standoff target tracking with short convergence time and no traversing the circle orbit. Two guidance functions were proposed to fulfill the criteria of fast convergence based on analysis of the contraction and circulation components in the Lyapunov vector field guidance framework. The parameter in the guidance function was optimized under the physical saturation constraints of the aircraft based on an offline search. For a moving target, fast Lyapunov vector field guidance was implemented by the introduction of a correction factor.
and angle correction term, and the corresponding optimal parameter search method was presented with a rough or fine constraint. A series of simulations were performed to demonstrate the lower time consumption and the stability of the guidance method designed with the proposed guidance functions and the searched optimal parameter estimates. Different guidance functions and solutions for the moving target can be used in combination according to mission requirements for tracking performance and search time.

There are many potential modifications for application considerations. For uncertain target speed and unknown background airspeed, they can be regarded as the disturbance speed with determined interval, and analysis of a moving target can be referenced to provide the optimal parameter with a loose constraint. Therefore, the proposed fast guidance method can be extended to wide application scenarios.

REFERENCES


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