The Hybrid Model for Lane-changing Detection at Freeway Off-ramps Using Naturalistic Driving Trajectories

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ABSTRACT In order to promote traffic safety at freeway off-ramps, this paper designed a hybrid model to identify a lane-changing with vision technology. The unmanned aerial vehicle was used to collect video stream data of five off-ramps for Xi’an Raoceng freeway during weekdays. The positional information of an individual vehicle is recorded at a frequency of 5Hz. Each trajectory is composed of 30 positional records and all trajectories are divided into lane-changing and lane-keeping units. Features such as lateral driving speed, lane departure and the lane deviation angle extracted from trajectory records are related to the lane-changing behaviors. We develop a hybrid model of the Gaussian Mixture Model and the Continuous Hidden Markov Model to identify lane-changing behaviors at off-ramps with these features. Basing on test set, we conduct a test for the hybrid model and the result shows that the prediction accuracy of proposed model is as high as 94.4% for lane-changing behavior and 93.6% for lane-keeping behavior.

INDEX TERMS Continuous Hidden Markov Model, Freeway Off-ramps, Hybrid model, Lane-changing, Naturalistic Driving Trajectories

I. INTRODUCTION
Numerous traffic accidents caused by improper driving behaviors, resulting in casualties and extensive property losses. More than 70% of road traffic accidents are caused by unsafe driving behaviors according to the annual road accidents report in China 2016 [1]. 4% to 7% of road traffic accidents in China are caused by improper lane-changing behavior [2]. However, this situation is more exiguous in the US, with 27% of accidents being the result of faulty lane-changing [3]. Freeway off-ramps are the sites of more crashes than other segments, due to the interference of lane-changing [4-5].

Therefore, many researchers focus on the research of lane-changing behavior for various types of road. The arrangement of off-ramps affects driving behaviors including the number of lanes, length of the speed-change lane, geometry design, signs placement, gaps acceptance and so on [6-9]. Laval et al. proposed a multilane hybrid model with the speed difference across lanes [10-11]. Xuan et al. proposed a lane-mark extraction algorithm based on computer vision detection technology [12]. Speed, speed difference and density difference are introduced to address the safety effect. Psychological indicators such as eye movement, head movement and electroencephalogram are also used to describe the lane-changing behavior. Yuan et al. explored a new identification method for lane changing intention by analyzing eye movement behavior [13-14]. The electroencephalogram is used to estimate the driver’s intention and lane departure [15-17]. The driver’s head motion is also used to identify the lane changing behaviors [18-19]. The driver perception characteristics are used to develop a theoretical warning model of lane change [20]. The 3D body posture of the driver is used to study the lane changing behaviors [21-22]. Vehicle trajectories are collected from vehicles and used to develop the lane-changing behavior [23]. The Next Generation Simulation (NGSIM) trajectory data sets are frequently employed to explore lane-changing behavior. Przybyla et al. employed NGSIM to develop a car-following model under natural driving data [24-27]. Steering features are extracted from the trajectories and compared with NGSIM to describe the lane-changing behavior [28-30].

Various models are developed to improve identification accuracies, such as Hidden Markov Model (HMM), logistic regression model (LR), support vector machine (SVM), and Bayesian model. Li et al. built the HMM to identify lane-changing behavior with different variables data [31-34]. Puan et al. used the support vector machine to model lane-changing using loop detector data [35-37]. Balal et al. used parameters extracted from the vehicle trajectories data to...
The current researches have some inadequacies. Some inputs of identification model are hard to be recorded during driving. For example, the collection of visual data and electroencephalogram (EEG) data of driver affects the safety of driving. Unfortunately, some input parameters of identification models are proved to be difficult to quantify, resulting in the low identification accuracy. Some researchers only used vehicle operating parameters or driver psychology indicators, lowering the prediction precision of models. Vehicle trajectories are obtained to model the lane-changing behavior in certain road scenario. Most of the research of lane-changing is on the freeway, only focused on ordinary road sections, and ignore the potentially high risks segments, such as ramps and intersection. Besides, it is time-consuming and expensive to collect massive, natural driving data from real vehicles for lane-changing behavior with these conventional approaches.

It is significantly important to warn the lane-changing of other vehicles in advance. The objective of this paper is to explore features which may be able to identify lane-changing behaviors at off-ramps in the initial stage. This paper collected vehicle trajectories at off-ramps of freeway using an unmanned aerial vehicle (UAV). A hybrid model, combining the Gaussian Mixture Model (GMM) and Continues Hidden Markov Model (CHMM), is proposed to recognize lane changing behavior observed at freeway exits. The remaining part of this paper is organized as follows: Section 2 introduced the video data collection and features extraction process; Section 3 is the establishment of the hybrid model; Section 4 is the model training results for lane-changing and keeping; Section 5 gives the research conclusions.

II. DATA ANALYSIS

A. DATA COLLECTION

The video data was recorded from Xi’an Raocheng freeway, which is a six-lane freeway and the lane width is 3.75m. The total length of the road is 33.852 km. The horizontal radius of the road is greater than 600m. The design speed is 120km/h. The speed limit is 120km/h for passenger cars and 80km/h for trucks, respectively. There are 18 interchanges totally, and we selected five with heavy traffic flow for investigation. The road alignment is similar at five sites. The length of the deceleration lane is 240m. The warning signs are placed at 2km, 1km, and 500m, respectively, before exits. The vehicles are divided into passenger cars and trucks, according to “Technical Standard for Highway Engineering 2016”. Vehicles, of which the axis distance is greater than 3.8m, are trucks. Otherwise, the vehicles are defined as passenger cars.

The video of the traffic flow was captured with the DIY Inspire 2 drone from the manufacturer SZ DJI Technology Co., Ltd., and positioned at the five chosen off-ramp interchanges. Four batteries were prepared to support the drone to operate continuously during investigation. The flying altitude of the drone was 150 meters, and the size of the area to be surveyed at each off-ramp was 350×190 m². On weekdays from November 5th to November 9th, 2018, we collected video for 20 minutes at each site every day in the morning peak hours (7:00-9:00 am). During the investigation period, the air pollution index (API) was less than 100, and the weather was clear, and visibility was sufficient for HD video. 500 minutes of video was collected. Fig.1 is the investigation locations at Xi’an Raocheng freeway. Fig.2 is the video image collected by UAV.

B. VEHICLE TRAJECTORIES EXTRACTION

The off-ramp influence area in this study is 150m past the ramp and 200m before, and there is no on-ramp in this area. The slope of the road segment is less than 3%. The vehicle
trajectories were extracted from the video images at a frequency of 5Hz by the Tracker.

We used the software Tracker to obtain video trajectories automatically. Tracker is a free video analysis and modeling tool based on the Open Source Physics Java framework, and it consists of a Java™ runtime environment and Xuggle video engine. In our study, we can track the individual vehicle position, velocity, and acceleration speed automatically and manually.

The vehicle extraction process is as follows.

Step 1: Coordination setting. A coordinate reference system is set up in the video image as Fig. 3 to calculate the vehicle positions. In order to utilize the further analysis process, the X-axis is the central line of the outer lane closing to the exit. The Y-axis is perpendicular to the central line of the lane. The right direction is the positive direction of X-axis, and the upward direction is the positive direction of Y-axis.

![FIGURE 3. The coordinate system in the video image](image)

Step 2: Calibration. The spatial calibration is required to transform image space proportionately to the ground plane using a standard length. In this study, the 3.75m of lane width is used to calibrate the position of vehicles on the ground.

Step 3: A one-second video is composed of 25 frames. We recorded the positional of vehicles every five frames. A coordinate reference system is set up in the video to calculate positions of vehicles at different times. We set the target vehicle as a template image. The software recorded the features of the target vehicle and tracked it automatically as Fig. 4.

![FIGURE 4. The traveling trajectory of vehicles](image)

Therefore, the vehicle shadow does not affect the extraction of the vehicle trajectory.

Step 4: We calculated parameters for each vehicle. Parameters such as the coordinate’s values of X-axis and Y-axis, distance to the origin point, the vehicle speed in space, speed component in X-axis, speed component in Y-axis, acceleration speed in space, acceleration component in X-axis, acceleration component in Y-axis.

Fig. 5 is the position of each vehicle in the longitude direction. In Fig. 5, $t$ is the sampling time interval, and $x$ is the value of X-axis in the image, which multiple 100m is the real distance. The speed component in X-axis is calculated from the line. In Fig. 6, $t$ is the sampling time and $y$ is the value of Y-axis in the image, which multiple 100m is the real distance. The speed component in Y-axis is calculated from the line.

![FIGURE 5. The vehicle position at the longitude direction](image)

![FIGURE 6. The vehicle position at the lateral direction](image)

C. TRAFFIC FLOW COLLECTION

The images of video demonstrate that 17% of the vehicles enter the off-ramp after a lane-changing behavior. The drone also flies down 44m to collect the traffic flow features in this experiment. Four virtual loops have been placed in the target lanes as Fig. 7. The vehicles with length more than 6m are defined as trucks. The features such as traffic volume, mean speed and traffic density were calculated automatically for mainline by software Flux. Then we obtain truck percentage manually. Records of any vehicle speeds over 120km/h are eliminated. The summary of traffic features is as Table 1.
Table 1 is the traffic flow statistics of Xi’an Raocheng freeway. The traffic flow is between 3500 pcu/h–4700 pcu/h. The 85% speed is above 90km/h and the mean speed is 77km/h. The truck percentage is around 20% percentage, which shows that the prevailing vehicle type is the passenger car for this freeway.

<table>
<thead>
<tr>
<th>Site</th>
<th>Traffic volume (pcu/h)</th>
<th>85% speed (km/h)</th>
<th>Mean speed (km/h)</th>
<th>Traffic speed density (pcu/km)</th>
<th>Truck percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3733</td>
<td>94.53</td>
<td>78.55</td>
<td>17.4</td>
<td>22.6%</td>
</tr>
<tr>
<td>B</td>
<td>3912</td>
<td>92.21</td>
<td>77.23</td>
<td>17.6</td>
<td>21.5%</td>
</tr>
<tr>
<td>C</td>
<td>3545</td>
<td>91.87</td>
<td>76.66</td>
<td>17.8</td>
<td>26.1%</td>
</tr>
<tr>
<td>D</td>
<td>4626</td>
<td>93.56</td>
<td>77.65</td>
<td>17.5</td>
<td>23.8%</td>
</tr>
<tr>
<td>E</td>
<td>3865</td>
<td>90.21</td>
<td>77.34</td>
<td>17.1</td>
<td>21.8%</td>
</tr>
</tbody>
</table>

**D. FEATURE EXTRACTION**

We divide drivers’ behaviors into lane-changing and lane-keeping. It is found that the lane-changing behavior lasts six seconds for analyzing the recorded data. Therefore, we preferred 6 seconds window to identify lane-changing and keeping behavior. A one-second record consists of 25 frames in this study. We recorded parameters of one point every five frames. Five records of each parameter are extracted, respectively, per second and therefore, we obtained 30 records for each trajectory sample. Finally, we extracted 205500 records, consisting of 3410 samples of lane-changing and 3440 keeping samples at freeway exits.

Correctness is a characteristics index, and it is one of the most important measurements for the quality of data. The correctness can be measured as (1):

\[ Q_c = 1 - \frac{1}{30} \left( \frac{\text{The number of incorrect data records}}{\text{Total number of incorrect data records}} \right), \]

In order to evaluate the quality of the driving trajectory data we extracted from the video stream, 50 samples were randomly selected from all trajectory samples, because the characteristics of the subsample set can represent the whole.

In this study, Grubbs outlier test is used to distinguish the outliers of the original data based on the speed of each sample point of each trajectory as (2). The outliers were treated as incorrect data records, and replaced by the mean of the adjacent points. Then correctness is calculated to evaluate the data based on the number of outliers.

\[ G_j = \frac{|v - \overline{v}|}{S} \geq G(p,n), \]

Where \( G_j \) denotes Grubbs’ test statistic. \( v \) denotes the speed of each sample calculated from one trajectory. \( \overline{v} \) denotes the mean of speed from one trajectory sample. \( S \) denotes the standard deviation of speed from one trajectory sample. \( G(p,n) \) denotes Grubbs’ critical value. \( p \) denotes confidence interval and generally, \( p \) is set to 0.95. \( n \) denotes the number of records of each sample and it is 30 in the study. The value of \( G(0.95, 30) \) is 2.745, found from the table of critical values for Grubbs’ test.

The correctness of each sample is calculated as (3):

\[ Q_c = 1 - \frac{1}{30} \sum_{i=1}^{50} Q^i, \]

Therefore, the correctness of the subsample is calculated as the following:

\[ Q_c = \frac{1}{50} \sum_{j=1}^{50} Q^j = 98.47\% \]

The correctness of the total sample is 98.47%. It shows that the quality of data we extracted is high enough for modeling.

Finally, we collected 205500 records including 3410 trajectory samples of lane-changing (L-C) and 3440 trajectory samples of lane-keeping (L-K). Features such as lane departure value, speed, acceleration speed, angle between moving direction and X-axis, angle between r and Y-axis are summarized as Table 2. Each parameter is interpreted as Fig. 8.

**TABLE 2**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Interpretation</th>
<th>Behavior</th>
<th>N</th>
<th>Mean</th>
<th>Std. Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta r )</td>
<td>lane departure(m)</td>
<td>L-C</td>
<td>3410</td>
<td>1.10</td>
<td>0.76</td>
</tr>
<tr>
<td>( \Delta v )</td>
<td>horizontal speed(m/s)</td>
<td>L-C</td>
<td>3410</td>
<td>-18.18</td>
<td>0.29</td>
</tr>
<tr>
<td>( v )</td>
<td>vertical speed(m/s)</td>
<td>L-C</td>
<td>3410</td>
<td>-16.88</td>
<td>0.25</td>
</tr>
<tr>
<td>( \Delta v )</td>
<td>vehicles speed in space(m/s)</td>
<td>L-C</td>
<td>3410</td>
<td>18.19</td>
<td>0.29</td>
</tr>
<tr>
<td>( a_r )</td>
<td>horizontal acceleration(m/s²)</td>
<td>L-C</td>
<td>3410</td>
<td>0.07</td>
<td>0.83</td>
</tr>
<tr>
<td>( a_r )</td>
<td>vertical acceleration(m/s²)</td>
<td>L-C</td>
<td>3410</td>
<td>0.19</td>
<td>0.99</td>
</tr>
<tr>
<td>( a )</td>
<td>acceleration in space(m/s²)</td>
<td>L-C</td>
<td>3410</td>
<td>1.07</td>
<td>0.72</td>
</tr>
<tr>
<td>( \theta_r )</td>
<td>angle between r and lane line(°)</td>
<td>L-C</td>
<td>3410</td>
<td>2.99</td>
<td>0.55</td>
</tr>
<tr>
<td>( \theta_r )</td>
<td>lane deflection angle(°)</td>
<td>L-C</td>
<td>3410</td>
<td>2.02</td>
<td>0.78</td>
</tr>
<tr>
<td>( \theta_r )</td>
<td>acceleration deflection angle(°)</td>
<td>L-C</td>
<td>3410</td>
<td>9.95</td>
<td>51.86</td>
</tr>
</tbody>
</table>

Table 2 shows that the parameters such as lateral driving speed, lane departure, and the lane deviation angle can be used to explain lane-changing behavior at the freeway off-ramps. Therefore, we use the vector \((\Delta r, \Delta v, \theta_r)\) as a feature vector to develop identification model of trajectories.

**FIGURE 8.** The demonstration of each parameter

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The independent \( T \)-test is a statistical test, which determines whether there is a statistically significant difference between the means in the above two unrelated groups, for the same continuous and dependent variable. Table 3 is the independent test results sample.

### Table 3

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equal Variances Assumed</th>
<th>Equal Variances Not Assumed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta y )</td>
<td>38.431 0.000 0.000</td>
<td>Equal variances assumed</td>
</tr>
<tr>
<td>( \Delta y )</td>
<td>11.851 0.001 0.000</td>
<td>Equal variances assumed</td>
</tr>
<tr>
<td>( \theta_0 )</td>
<td>5.840 0.019 0.000</td>
<td>Equal variances assumed</td>
</tr>
</tbody>
</table>

Table 3 shows that there are only three variables significant which can represent the driving behaviors including \( \Delta y \), \( \Delta y \), and \( \theta_0 \). \( \Delta y \) means lane departure at time \( t \). \( y \) means coordinate of \( Y \)-axis at time \( t \) and \( y_0 \) means coordinate of \( Y \)-axis at time 0.

\[
\Delta y = y_t - y_0. \tag{4}
\]

In Table 3, the lateral driving speed of the vehicle, lane departure, and lane deviation angle are with a statistically significant difference between the means in lane-changing and keeping. As shown in Table 2, the mean values of driving lateral speed for lane-changing and keeping are 0.54 m/s and -0.05 m/s respectively. The mean values of lane departure for lane-changing and keeping are 1.10 m and -0.15 m respectively. The mean values of lane deviation angle for lane-changing and keeping are 2.02° and 0.36° respectively.

## III. METHODOLOGY

In this paper we establish a hybrid model to identify and predict lane-changing behavior at the freeway off-ramps. It is important to recognize driving behavior in the initial stage, which can be implemented by continuous recognition. Therefore, CHMM is employed in this study.

### A. GAUSSIAN MIXTURE MODEL

GMM is employed to describe the probability distribution of all continuous variables we extracted and described the correlation between every two different variables. The GMM consists of \( K \) Gaussian models as (5).

\[
G(x) = \sum_{k=1}^{K} C_k g(x; \mu_k, \Sigma_k). \tag{5}
\]

Where \( g(x; \mu_k, \Sigma_k) \) indicates the \( k^{th} \) sub-Gaussian model. The probability density function of the Gaussian model is shown as (6). \( x \) is the lane-changing samples for training and it consists of \( d \) continuous variables. The value of \( C_k \) denotes the weight of the \( k^{th} \) sub-Gaussian model, and they are related to the composition of \( x \), \( \mu_k \) and \( \Sigma_k \) are the mean matrix and covariance matrix of all the continuous variables in the \( k^{th} \) sub-Gaussian model, respectively.

\[
g(x; \mu, \Sigma) = \frac{1}{\sqrt{(2\pi)^d \Sigma}} \exp \left[ -\frac{1}{2} (x - \mu)^T \Sigma^{-1} (x - \mu) \right], \tag{6}
\]

In order to get all the parameters of GMM from lane-changing samples, the Maximum Likelihood Estimation and the Expectation-Maximization algorithm (E-M) can be combined.

Because the probability of each sample generated by GMM is very small, we use the log-likelihood function as (7).

\[
p(x | C_k, \mu, \Sigma_k) = \sum_{i=1}^{N} \log \left\{ \sum_{k=1}^{K} C_k g(x; \mu_k, \Sigma_k) \right\}, \tag{7}
\]

\( p(x|C_k, \mu_k, \Sigma_k) \) denotes the probability of all the sample points generated by the GMM. Generally, we get the last parameters when \( p(x|C_k, \mu_k, \Sigma_k) \) reaches a maximum.

The Expectation-Maximization (E-M) algorithm consists of an E-step and an M-step. Expectation step is to calculate the expectation \( \phi(i, k) \) as (8).

\[
\phi(i, k) = \frac{C_i g(x; \mu_k, \Sigma_k)}{\sum_{k=1}^{K} C_k g(x; \mu_k, \Sigma_k)}. \tag{8}
\]

Where \( \phi(i, k) \) denotes the probability of the \( k^{th} \) sub-Gaussian model generating the \( i^{th} \) lane-changing sample point.

The M step is to recalculate \( C_k, \mu_k \) and \( \Sigma_k \) based on \( \phi(i, k) \). The iterating E and M step stop until the log-likelihood converges.

### B. CONTINUOUS HIDDEN MARKOV MODEL

A Hidden Markov Model is a stochastic model consisting of a Markov chain and stochastic process for hidden state prediction. In this study, the hidden states are lane-changing and lane-keeping. Each hidden state cannot be observed, but it can generate observation vectors, consisting of several variables. In this study, these variables are lateral driving speed, lane departure, and the lane deviation angle. Fig. 9 shows the Markov process.

Proof of Theorem 1

Using the observed data in Table 3 and the above E-M algorithm, the last parameters of GMM can be gotten as shown in Table 4.
suitable for discrete variables. Then we choose the Continuous Hidden Markov Model (CHMM) to identify driver behaviors.

A basic HMM consists of three parameters: \( \boldsymbol{\pi}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\pi} \) is the hidden state probability distribution at the initial time. \( \boldsymbol{A} \) is the state transition probability matrix, which is made up of the transition probability between different hidden states, such as the lane-changing state and lane-keeping state. \( \boldsymbol{B} \) is the observation probability matrix, which is made up of the probability that the observation vector observed at each hidden state. In a CHMM, parameter \( \boldsymbol{B} \) has some changes that \( \boldsymbol{B} \) is divided into three parts, and they are \( \boldsymbol{C}, \boldsymbol{\mu}, \) and \( \boldsymbol{\Sigma}. \) They are generated by GMM as (9)–(10). Therefore the general model of CHMM is \( \lambda= (\boldsymbol{\pi}, \boldsymbol{A}, \boldsymbol{C}, \boldsymbol{\mu}, \boldsymbol{\Sigma}). \)

\[
b_j (O) = \sum_{k=1}^{K} C_{jk} g (O; \mu_j, \Sigma_j),
\]

\[
g(O; \mu_j, \Sigma_j) = \frac{1}{\sqrt{(2\pi)^j | \Sigma_j |}} \exp \left[ -\frac{1}{2} (O - \mu_j)^T \Sigma_j^{-1} (O - \mu_j) \right],
\]

Where \( b_j (O) \) denotes the probability of the observation vector \( O \) generated by state \( j. \) \( g \) is the \( k \)-th sub Gaussian model of GMM. The GMM is used to describe the probability distribution of three continuous variables and we can get the probability of any observation vector made up of these three continuous variables by putting them into GMM. \( C_{jk} \) is the weight value of the \( k \)-th sub-Gaussian model at hidden state \( j. \) \( \mu_j \) is the mean matrix of the \( k \)-th sub-Gaussian model at hidden state \( j. \) \( \Sigma_j \) is the covariance matrix of the \( k \)-th sub-Gaussian model at hidden state \( j. \)

In order to estimate the parameters \( \boldsymbol{\pi}, \boldsymbol{A}, \boldsymbol{C}, \boldsymbol{\mu}, \) and \( \boldsymbol{\Sigma} \), the Baum-Welch algorithm has been applied. It combines the Forward algorithm, Backward algorithm and the E-M algorithm together. The Forward algorithm is used to calculate the forward variable that is indicated by \( a_t (i) \) as (11)–(12) and it is the joint probability of the partial observation sequence \( (O_t, t=1,2, \ldots, T) \), consisting of \( t \) observation vectors, besides given that the hidden state is \( i \) at time \( t. \)

\[
a_t (i) = p ((O_1, O_2, \ldots, O_t, s_t = i) | \lambda = (\boldsymbol{\pi}, \boldsymbol{A}, \boldsymbol{C}, \boldsymbol{\mu}, \boldsymbol{\Sigma})), \quad (11)
\]

\[
a_t (i) = \sum_{j=1}^{N} a_{t-1} (j) b_j (O_t).
\]

The Backward algorithm is used to calculate the backward variable that is indicated by \( \beta_t (i) \) as (13)–(14) and it is the joint probability of the partial observation sequence \( (O_t, t=t+1, t+2, \ldots, T) \), consisting of \( T-t \) observation vectors, besides given that the hidden state is \( i \) at time \( t. \)

\[
\beta_t (i) = p ((O_{t+1}, O_{t+2}, \ldots, O_T, s_T = i) | \lambda = (\boldsymbol{\pi}, \boldsymbol{A}, \boldsymbol{C}, \boldsymbol{\mu}, \boldsymbol{\Sigma})), \quad (13)
\]

\[
\beta_t (i) = \sum_{j=1}^{N} a_j (O_{t+1}) \beta_{t+1} (j).
\]

The E-M algorithm is used to iteratively recalculate the parameters and we obtain a maximum likelihood estimate of the CHMM. By dividing likelihood estimate function \( p(O|\lambda) \) into five parts, we can get the formats of \( \boldsymbol{\pi}, \boldsymbol{A}, \boldsymbol{C}, \boldsymbol{\mu}, \) and \( \boldsymbol{\Sigma} \) respectively and \( p(O|\lambda) \) is the conditional probability of the entire observation sequence \( O \) generated by the model.

The E-M algorithm consists of the Expectation step and the Maximization step. The E step is used in estimation of two probability variables \( \gamma_t (i), \xi_t (i,j) \) as (14)–(15) and initialize the values of \( \boldsymbol{\pi}, \boldsymbol{A}, \boldsymbol{C}, \boldsymbol{\mu}, \) and \( \boldsymbol{\Sigma}. \) The M step is to re-estimate them based on \( \gamma_t (i), \xi_t (i,j) \), until they all reach convergence.

\[
\gamma_t (i) = \frac{a_t (i) \beta_t (i)}{\sum_{i=1}^{N} a_t (i) \beta_t (i)},
\]

\[
\xi_t (i,j) = \frac{a_t (i) a_j (O_{t+1}) \beta_{t+1} (j)}{\sum_{i=1}^{N} \sum_{j=1}^{N} a_t (i) a_j (O_{t+1}) \beta_{t+1} (j)},
\]

Where \( \gamma_t (i) \) is the probability that the hidden state is \( i \) at time \( t, \) besides given the observation sequences \( O. \) \( \xi_t (i,j) \) is the probability that the hidden states are \( i \) and \( j \) at time \( t \) and \( t+1, \) respectively, besides given the observation sequences \( O. \)

IV. LANE-CHANGING MODEL

A. MODEL TRAINING

In our study, lane-changing detection and identification are achieved in a continuous manner. In a continuous identifier, each complete maneuver is modelled as a Markov model. The general model of the CHMM is \( \lambda = (\boldsymbol{\pi}, \boldsymbol{A}, \boldsymbol{C}, \boldsymbol{\mu}, \boldsymbol{\Sigma}). \) \( \lambda_C \) and \( \lambda_K \) has been trained by a set of trajectory samples for the complete lane-changing maneuver and lane-keeping maneuver, respectively.

In this training step, all the trajectories samples we recorded at freeway off-ramps are classified into lane-changing and keeping behavior. Each trajectory sample consists of 30 records and each record is just a feature vector, consisting of lateral driving speed, lane departure and the lane deviation angle. Each feature vector may be generated by the different hidden states, and the hidden states are the lane-changing and lane-keeping. Each lane-changing or keeping driving trajectory consists of 30 feature vectors and each vector is generated by lane-changing or lane-keeping hidden state.

In order to identify lane-changing behavior, \( \lambda_C \) and \( \lambda_K \) are trained, respectively. We trained \( \lambda_C \) and \( \lambda_K \) based on lane-changing trajectory samples and lane-keeping trajectory samples respectively. Training \( \lambda_C \) contains the following three steps.

Step 1: Initialization. Before iterating to determine all the parameters of \( \lambda_C, \) it is necessary to initialize all the parameters and they are \( \boldsymbol{K}, \boldsymbol{\pi}, \boldsymbol{A}, \boldsymbol{C}, \boldsymbol{\mu}, \) and \( \boldsymbol{\Sigma}. \) The GMM is the most classical method to extract the image from the background and basically 3 to 5 Gaussian models making up GMM to characterize the features of each pixel in the image. Therefore, in order to simplify the model, we set the initial value of \( \boldsymbol{K} \) to 3. Because we chose lane-changing trajectory to train \( \lambda_C, \) it is logical to consider that the initial hidden state is the same as the last state and it is a lane-keeping state. Therefore, the initial lane-changing state probability is set to zero and lane-keeping state probability is set to one,
respectively. We gave $A$, $C$, $\mu$, and $\Sigma$ initial values randomly at the same time.

Step 2: Determining lane-changing trajectory samples as observation sequence $O_t$. Each sequence is a trajectory sample consisting of 30 feature vectors, and these feature vectors are in order. We randomly chose 66% of the lane-changing trajectory samples being set as the model training set (2251 samples) and the remaining 34% was set as a test set (1159 samples) based on the Holdout cross-validation method. Table 4 presents the partial lane-changing trajectory sample records.

<table>
<thead>
<tr>
<th>No.</th>
<th>Sample time(s)</th>
<th>Lane departure (m)</th>
<th>Lateral driving speed (m/s)</th>
<th>Lane deflection angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.144</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>0.209</td>
<td>0.067</td>
<td>0.342</td>
<td>1.7</td>
</tr>
<tr>
<td>3</td>
<td>0.417</td>
<td>0.142</td>
<td>0.563</td>
<td>2.3</td>
</tr>
<tr>
<td>4</td>
<td>0.626</td>
<td>0.302</td>
<td>0.353</td>
<td>2.2</td>
</tr>
<tr>
<td>5</td>
<td>0.834</td>
<td>0.29</td>
<td>0.353</td>
<td>1.1</td>
</tr>
<tr>
<td>6</td>
<td>1.043</td>
<td>0.449</td>
<td>0.615</td>
<td>0.1</td>
</tr>
<tr>
<td>7</td>
<td>1.251</td>
<td>0.546</td>
<td>0.463</td>
<td>2.2</td>
</tr>
<tr>
<td>8</td>
<td>1.460</td>
<td>0.643</td>
<td>0.480</td>
<td>3.3</td>
</tr>
<tr>
<td>9</td>
<td>1.669</td>
<td>0.746</td>
<td>0.235</td>
<td>1.2</td>
</tr>
<tr>
<td>10</td>
<td>1.877</td>
<td>0.741</td>
<td>0.362</td>
<td>2.2</td>
</tr>
<tr>
<td>11</td>
<td>2.086</td>
<td>0.897</td>
<td>0.488</td>
<td>2.3</td>
</tr>
<tr>
<td>12</td>
<td>2.294</td>
<td>0.944</td>
<td>0.117</td>
<td>1.8</td>
</tr>
<tr>
<td>13</td>
<td>2.503</td>
<td>0.946</td>
<td>0.218</td>
<td>1.8</td>
</tr>
<tr>
<td>14</td>
<td>2.711</td>
<td>1.035</td>
<td>0.48</td>
<td>2.2</td>
</tr>
<tr>
<td>15</td>
<td>2.92</td>
<td>1.146</td>
<td>0.775</td>
<td>3.0</td>
</tr>
<tr>
<td>16</td>
<td>3.128</td>
<td>1.359</td>
<td>0.488</td>
<td>1.7</td>
</tr>
<tr>
<td>17</td>
<td>3.545</td>
<td>1.344</td>
<td>0.636</td>
<td>2.5</td>
</tr>
<tr>
<td>18</td>
<td>3.754</td>
<td>1.615</td>
<td>1.108</td>
<td>1.6</td>
</tr>
<tr>
<td>19</td>
<td>3.963</td>
<td>1.806</td>
<td>0.413</td>
<td>1.4</td>
</tr>
<tr>
<td>20</td>
<td>4.171</td>
<td>1.787</td>
<td>0.246</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Step 3: Iterative optimization. The input of iteration is initial values containing $\pi$, $A$, $C$, $\mu$, $\Sigma$ and the observation sequence $O_t$ consisting of lateral driving speed of vehicle $v_y$, lane departure $\Delta y$ and speed deflection angle $\theta$. The identification model is the CHMM mixed with the GMM. The observation probability matrix $B$ consists of $C$, $\mu$, $\Sigma$, and they are generated by the GMM. Two identification models are lane-changing and lane-keeping. Each identification model contains six Gaussian models belonging to two GMM and each GMM has three.

Under the MATLAB 2014Ra platform, we utilized these steps based on the Naturalistic Driving Trajectories. Finally, the log-likelihood converges in forty-one iterations as Fig. 10. And we obtained the lane-changing identification model $\lambda_c$.

$\lambda_c$ is made up of $\pi$, $A$, $C$, $\mu$, and $\Sigma$ as follows.

\[
\pi = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

Matrix $\pi$ indicates that the probability of the initial hidden state of lane-changing is zero and lane-keeping is one.

\[
A = \begin{bmatrix} 0.8632 & 0.1368 \\ 0.1715 & 0.8285 \end{bmatrix}
\]

Matrix $A$ is the probability transition matrix. The transition probability from the current lane-changing to the next lane-changing state is 0.8632. The transition probability from the current lane-changing to the next lane-keeping is 0.1368. The transition probability from the current lane-keeping state to the next lane-keeping is 0.8285. The transition probability from the current lane-keeping to the next lane-changing is 0.1715.

\[
C = \begin{bmatrix} 0.4785 & 0.2399 & 0.2816 \\ 0.4871 & 0.3430 & 0.1699 \end{bmatrix}
\]

The first row of matrix $C$ indicates the weights for the three Gaussian models in the lane-changing state. The second row indicates the weights for the three Gaussian models in the lane-keeping state.
The first column of $\mu(:,:,m)$ indicates the mean value of vector $(\Delta y, v_y, \theta_v)$ of the $m^{th}$ Gaussian model in the lane-changing state. The second column of $\mu(:,:,m)$ indicates the mean value of vector $(\Delta y, v_y, \theta_v)$ of the $m^{th}$ Gaussian model in the lane-keeping state.

B. THE DISTRIBUTION FEATURE OF VARIABLES

According to the lane-changing model $\lambda_{C}$ we trained, we can extract the probability density and probability distribution of every continuous variable in lane-changing hidden state and in the lane-keeping state. They show the individual features of lane-changing behavior different from lane-keeping as Fig. 11-13.

Fig. 11 shows that the lateral speed of lane-changing mainly distributes in the range of $(1, 2)$, and distributes in the range of $(0, 1)$ of lane-keeping.

![Figure 11. The probability density distribution of lateral speed; (a) is the probability density distribution of lateral speed; (b) is the probability cumulative distribution of lateral speed.](image1)

Fig. 12 shows that the lane departure of lane-changing mainly distributes in the range of $(0.5, 1)$. And it mainly distributes in the range of $(0, 0.5)$ of lane keeping.

![Figure 12. The probability density distribution of lane departure; (a) is the probability density distribution of lane departure; (b) is the probability cumulative distribution of lane departure.](image2)

Fig. 13 shows that the lane deviation angle of lane-changing mainly distributes in the range of $(1, 2.6)$, and mainly distributes in the range of $(0, 1)$ of lane keeping.
They show that there are obvious distinctions in the distribution of lateral driving speed, lane departure, and speed deflection angle between lane-changing and lane-keeping behavior. It proves that it is reasonable to set these three variables as the feature of lane-changing behavior.

In order to test the accuracy of predicting the driving behavior of $\lambda_C$ and $\lambda_K$ we trained, 34% trajectory samples we collected were considered as a test set based on the Holdout cross-validation method. 1159 lane-changing trajectory samples and 1170 lane-keeping trajectory samples are set as a test set to validate the driving state of each sample. Table 5 presents the partial test results.

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>$\lambda_C$</th>
<th>$\lambda_K$</th>
<th>Prediction result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-75.23</td>
<td>-202.36</td>
<td>Lane-changing</td>
</tr>
<tr>
<td>2</td>
<td>-56.54</td>
<td>-196.56</td>
<td>Lane-changing</td>
</tr>
<tr>
<td>3</td>
<td>-137.25</td>
<td>-82.53</td>
<td>Lane-keeping</td>
</tr>
<tr>
<td>4</td>
<td>-84.52</td>
<td>-162.35</td>
<td>Lane-keeping</td>
</tr>
<tr>
<td>5</td>
<td>-59.21</td>
<td>-138.57</td>
<td>Lane-keeping</td>
</tr>
<tr>
<td>6</td>
<td>-49.87</td>
<td>-123.98</td>
<td>Lane-keeping</td>
</tr>
</tbody>
</table>

Table 6 shows the accuracy of predicting lane-changing and lane-keeping behavior at the freeway off-ramps, by the proposed model. The accuracy of predicting the lane-changing behavior is 94.4%. The accuracy of predicting the lane-keeping behavior is 93.6%.

<table>
<thead>
<tr>
<th>The Accuracy of Testing</th>
<th>Lane-changing trajectory</th>
<th>Lane-keeping trajectory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Testing samples</td>
<td>1159</td>
<td>1170</td>
</tr>
<tr>
<td>Correct identifications</td>
<td>1094</td>
<td>1095</td>
</tr>
<tr>
<td>Accuracy</td>
<td>94.4%</td>
<td>93.6%</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

The paper presents a hybrid model for the lane-changing prediction at freeway off-ramps. We collected 205500 trajectory records at five investigation sites. The experiment draws the following conclusions.

(1) The parameters such as lateral driving speed, lane departure and the lane deviation angle obtained from the driving trajectories can be applied to explain the lane-changing behavior at freeway off-ramps.

(2) The identification models can achieve 94.4% accuracy for lane-changing and 93.6% for lane-keeping.

(3) The trained models can predict driving behaviors without additional sensors on the vehicle. It can be integrated into traffic cameras to detect the unsafe driving behavior and give warning to drivers.

REFERENCES


