ABSTRACT Hydropower, as a kind of clean energy, has the characteristics of rapid response to load changes, and is often used to undertake peak regulation and frequency regulation tasks. However, due to the uncertainties of reservoir inflow and runoff, the relative lag of power grid outgoing channels, and the interconnection of intermittent new energy, such as wind power and solar energy, the hydropower system is facing enormous peak regulation pressure and the continuous prominent problem of discarding water. In the study, aiming at the problem of risk control of discarding water in power generation process of hydropower station, a practical calculation method of risk of discarding water in power generation is proposed based on the analysis of frequency of runoff. Taking the Ertan hydropower station as an example, the corresponding water level processes of different discarding water risks under the control of pre-flood falling water level, flood limit water level and storage water level after flood season are deduced, and in the routine dispatching process, the water level operation control guidance corresponding to different discarding risk is introduced. The simulation results show that the proposed discarding water risk control method is practical and effective, and can provide guidance for discarding water risk control in the actual dispatching process of hydropower station.


I. INTRODUCTION
As a kind of renewable and clean energy with mature technology, hydropower energy has achieved comprehensive and rapid development. With the development of hydropower energy, rational allocation of water resources and optimization of operation and dispatch management of hydropower stations in river basins have gradually become the focus of research. However, due to the uncertainties of reservoir inflow, excessive concentration of hydropower development, relatively lagging construction of power grid supporting delivery channels, and intermittent new energy grid connection represented by wind power and solar energy, the hydropower system is facing enormous peak shaving pressure and continuous prominent water abandonment problems.

There are many uncertainties in the dispatching process of hydropower stations, such as the inflow of hydropower stations and the load of power grids. This not only limits the application of deterministic optimal dispatching results in hydropower stations, but also leads to certain risks in the dispatching process of hydropower stations. Therefore, it is necessary to study the risk dispatching of hydropower stations so as to provide dispatching decision-making reference for operators.

Scholars have done a lot of research on the risk of Hydropower Station Dispatching from the aspects of risk analysis[1-5], risk assessment[6] and risk decision-making[7]. Schmucke[8] proposed a systematic fuzzy risk description method and analyzed the risk sources in water resources management; Ruan[9] used risk rate, risk degree, vulnerability, restorability and recurrence period as risk evaluation indicators of water resources shortage, and used the fuzzy comprehensive evaluation method to determine the risk degree of water resources shortage; Based on probability
theory and grey system theory, Hu[10] proposed a grey-stochastic risk rate method for risk analysis. Feng[11] took Dongwushi Reservoir as the research object, constructed an index system to evaluate the risk caused by the change of flood limit water level, and proposed a fuzzy comprehensive evaluation method to determine the flood limit water level of reservoir. Liu[12] proposed a dynamic consistency risk measurement method based on conditional value of risk (CVaR) for short-term optimal dispatch of hydropower stations, which is used to calculate the space-time distribution of power generation. Yuan[13] introduced the risk preference factor and established the monthly decomposition model of contract electricity of cascade hydropower stations based on risk preference in the market environment, which provided the theoretical basis for hydropower in the power market competition. Liu Pan[14] defined the generation risk rate as the lower than the design value of the generation capacity or the generation guarantee rate. On this basis, a reservoir optimal dispatching chart compilation method considering the generation risk rate is proposed. Taking Qing river cascade hydropower station as an example, the optimized dispatching chart can obtain larger generation capacity and generation guarantee rate. Xu[15] takes Wanjiazhai Hydropower Station as the research object. Aiming at the risk of reservoir dispatching, a risk analysis model for medium and long-term dispatching of hydropower station is established by means of the Mean First Order Second Moment Method (MFOSM). The results of the model can provide reference for risk decision-making. Slobadan P[16] present a risk-based methodology developed to help a decision maker in selecting the appropriate weights, which define the relative importance of the two objectives—hydrologic forecast reliability and reservoir operation penalties.

Discarding water is an important assessment index for operation and management of hydropower stations, so risk control of discarding water is an important part of operation and dispatch of hydropower stations. In traditional reservoir optimal dispatching, the optimal dispatching results, which aim at maximizing power generation or generating benefits, mostly control reservoir operation at high water level to give full play to the head benefit of hydropower station. In theory, the optimal water level control process can be obtained, but the risk of abandoned water accompanied by high water level operation is not fully considered. However, due to the limitation of scientific and technological level, there is still a big difference between medium and long-term runoff forecast and actual runoff, which makes the deterministic optimal dispatching results of hydropower stations with the greatest power generation benefit accompanied by a certain risk of abandoning water in guiding the actual dispatching process of hydropower stations. Therefore, in the case of runoff uncertainty, the risk of discarding water in different water level control processes of hydropower stations is analyzed, which can provide a reference for coordinating the contradiction between generation benefit and discarding water risk in the actual operation and dispatch of hydropower stations.

At present, there are many achievements in the study of discarding water risk of hydropower stations. Xu Gang[17] calculated the actual discarding water amount in the period of time by the possible discarding water amount of historical data statistics, and established a long-term optimal dispatching model of cascade hydropower stations considering the risk of discarding water. In[18], a calculation model of discarding risk rate based on mid-term runoff forecast is established, and a reservoir pre-discharge dispatching method with discarding water risk control was proposed to reduce the abandonment of hydropower stations. The distribution pattern and the bound of forecast error which is a key source of risk were analyzed, and based on the definition of flood risk, the risk of dynamic control of reservoir flood limited water level within different flood forecast error bounds was studied in[19]. Diao[20] analyzed four uncertainties, i.e. hydrological, hydraulic, stage-storage uncertainty and time-delay uncertainty, as well as their probability distributions to obtain an integrated risk rate of FCOMFI, and on the basis of this analysis, an integrated risk analysis model of FCOMFI for reservoirs and its lower reach was established involving the above-mentioned four uncertainties. Xi and Huang[21]put forward the concept of the limit risk rate corresponding to the excess storage water level, expressed by the ratio of the frequency of the excess storage water level to the once-in-a-century flood limit water level, and obtained the expected water level of Shiquan Reservoir to make full use of the flood in flood season. Xu Wei and Peng Yong[22] coupled numerical rainfall forecast information with reservoir power generation dispatch, and established a risk and loss assessment model for hydropower dispatch decision-making, which assessed the risk and loss of water storage and abandonment for each frequency of water dispatch decision-making. Yuan[23] based on the energy conversion law of cascade reservoir hydropower station, constructs the minimum discarding water model; on the premise of satisfying the minimum discarding water quantity, the model maximizes the power head of hydropower station and indirectly obtains the maximum economic benefit of hydropower station by generating electricity under high water head. Wang and Liang[24] established multi-objective decision-making model of reservoir operation risk-benefit in flood season, and used the theory of game theory for reference to coordinate the benefit of hydropower station and the risk of discarding water, so as to provide a reference for decision makers in multi-objective decision-making. A dynamic control operation model that considers inflow uncertainty, i.e. the inflow forecasting error and uncertainty of the flood hydrograph shape was proposed and developed in[25]. Wu and Guo[26] improved the static discarding water strategy of hydropower stations, and proposed a dynamic discarding water strategy considering the relationship...
between water head and generation flow, and established an optimal dispatching model of cascade hydropower stations based on dynamic discarding water strategy. Xiao Yan [27] put forward an optimal dispatching concept based on discarding water probability by defining and classifying the optimization methods. In order to increase water storage of a reservoir while maintaining its security for flood control, two approaches, multiple duration limited water level and dynamic limited water level, were proposed in this study in [28].

In the study of discarding water risk of hydropower stations, scholars pay more attention to the decision-making between power generation benefit and discarding water risk in flood season and the risk analysis of discarding water based on runoff forecast error, but the unified calculation method of discarding water risk of hydropower stations in different dispatching periods is seldom studied, and there is less research on the relationship between discarding water risk and operation level of hydropower stations. In view of the fact that the risk control of abandoned water is not fully taken into account in the current medium and long term optimal dispatching, and the optimal dispatching can’t be directly applied to practice, and the description of abandoned water control in the dispatching chart guiding the daily operation of hydropower stations is too general. The main contributions and novelty of this study are reflected as follows:

1) A refined method for calculating the expected output of water level of hydropower station is proposed.
2) Based on the analysis of the frequency distribution of runoff inflow of hydropower stations, a risk calculation method of discarding water is proposed.
3) Estimating the control process of the operation water level of a hydropower station corresponding to different discarding water risks under the key water level control objective of the hydropower station.

The remainder of this paper is arranged as follows: The runoff frequency is analyzed and a new method is proposed to estimate the runoff distribution parameters in Section II. Section III introduces how to quantify the risk of discarding water in hydropower stations and how to calculate the water level process based on risk control of discarding water, followed by the presentation of the details of experimental setup and results in Section IV. Finally, Section V summarizes the conclusions.

II. FREQUENCY ANALYSIS OF INFLOW OF HYDROPOWER STATION

The relationship between the value of a random variable and its probability is called the probability distribution. In hydrological research, in order to characterize the uncertainty of runoff, runoff is generally regarded as a hydrological random variable and its probability distribution is studied. Scholars often adopt distribution lines in hydrological frequency calculation, including normal distribution, lognormal distribution and Pearson III distribution. Based on the actual inflow process, the Pearson III distribution is generally used by scholars to describe the probability distribution of runoff and peak discharge [29].

A. PEARSON III DISTRIBUTION

Pearson III Distribution (P-III Distribution) is often called Gamma Distribution in mathematics. Its probability density function is shown in (1) and its graph is an asymmetric single peak curve, as shown in FIGURE 1.

\[
f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} (x-a_0)^{\alpha-1} e^{-\beta(x-a_0)}
\]

(1)

![FIGURE 1. P-III Distribution probability density distribution curve.](image)

Where \( \Gamma(\alpha) \) is a gamma function of \( \alpha \), and \( \alpha, \beta, a_0 \) are three parameters of the distribution function and satisfy \( \alpha > 0, \beta > 0 \). The three parameters of the distribution function can be determined by three statistical parameters of the random variable population: the mean \( (\bar{X}) \), coefficients of variation \( (C_v) \), coefficient of skewness \( (C_s) \). The specific transformational relations are as follows:

\[
\begin{cases}
\alpha = \frac{4}{C_v^2} \\
\beta = \frac{2}{\bar{X}C_v} \\
a_0 = \bar{X}(1-\frac{2C_v}{C_s})
\end{cases}
\]

(2)

In hydrologic calculation, the cumulative frequency value \( (P) \) not smaller than \( x_p \) can be obtained by integrating the probability density curve \( f(x) \). The calculation is as follows:

\[
P = P(x \geq x_p) = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_{x_p}^{\infty} (x-a_0)^{\alpha-1} e^{-\beta(x-a_0)} dx
\]

(3)

However, the integral calculation process is more complex, so variable transformation is generally used to simplify the calculation. Normalized variable \( \Phi \) with mean value of 0 and standard deviation of 1 is commonly used, i.e. coefficient of
deviation from mean is used as conversion variable, shown as follows:

\[
\Phi = \frac{x - \overline{x}}{\overline{x}C_v}, \\
X = \overline{x}(1 + C_v\Phi) \\
dx = \overline{x}C_v d\Phi
\] 

(4)

Based on variable transformation, the integral equivalence of (5) is as follows:

\[
P = P(\Phi > \Phi_p) = \int_{\Phi_p}^{\infty} f(\Phi, C_v) d\Phi
\] 

(5)

Therefore, as long as the value of \( C_v \) is known, the \( \Phi_p \) value under different \( P \) values can be obtained by querying the table of frequency factor \( \Phi_p \) for P-III frequency curve, and then using the known values of \( \overline{x} \) and \( C_v \), the \( x_p \) values corresponding to different \( P \) values can be calculated by (4), which is to say the value of random variable \( x_p \), corresponding to specified frequency \( P \) is gained.

B. PARAMETER ESTIMATION

In order to determine the line-type of the total frequency distribution of hydrological random variables, the parameters of the total frequency distribution are generally estimated with limited historical sample values, which are called parameter estimation. The commonly used parameter estimation methods of P-III distribution[30] are moment method, maximum likelihood method and weight function method and so on[30].

1) MOMENT ESTIMATION[32].

Moment estimation is a parameter estimation method proposed earlier. Its theoretical basis is: If the k-order moment of population \( X \) exists, then when the sample size tends to infinity, the k-order moment of sample converges to the corresponding k-order moment of the population in probability, and the continuous functions of sample moments converge to the corresponding continuous functions of population moments in probability. Therefore, the population moments and their continuous functions can be estimated by the sample moments and their continuous functions, respectively. Based on the limited historical runoff data, the moment estimation of the main parameters \( \overline{x}, C_v, C_s \) in the linearity of P-III distribution can be formulated as the following:

\[
\begin{align*}
\bar{X} &= \frac{1}{n} \sum_{i=1}^{n} x_i \\
C_v &= \sqrt{\frac{\sum_{i=1}^{n} (K_i - 1)^2}{(n-1)}} \\
C_s &= \frac{n^2 \sum_{i=1}^{n} (K_i - 1)^2 + \sum_{i=1}^{n} (K_i - 1)^3}{(n-1)(n-2)(n-3)}
\end{align*}
\]

(6)

Where \( n \) is the sample size, \( K_i \) is the modulus coefficient, which is the ratio of sample value to sample mean value.

2) MAXIMUM LIKELIHOOD ESTIMATION.

The principle of maximum likelihood estimation (MLE) in statistics can be briefly expressed as: the event with the greatest probability is the most likely to occur MLE is a statistical method based on this principle and the main purpose is to estimate the parameters which are most likely to lead to this result by deducing the known samples. Taking continuous random variable \( X \) as an example, the probability density function is known as \( f(x; \theta) \), \( \theta \in \Theta \), where \( \theta \) is the parameter and \( \Theta \) is the parameter range. Let \( X_1, X_2, \ldots, X_n \) be a sample of random variables, and let \( x_1, x_2, \ldots, x_n \) be the corresponding sample value. The likelihood function can be established as follows:

\[
L(\theta) = L(x_1, x_2, \ldots, x_n; \theta) = \prod_{i=1}^{n} f(x_i; \theta)
\]

(7)

If \( L(x_1, x_2, \ldots, x_n; \theta) = \max_{\theta \in \Theta} L(x_1, x_2, \ldots, x_n; \theta) \) exists, then \( \hat{\theta}(x_1, x_2, \ldots, x_n) \) is defined as the maximum likelihood estimate of the parameter \( \theta \), and \( \hat{\theta}(x_1, x_2, \ldots, x_n) \) is the maximum likelihood estimator of \( \theta \). Solving the maximum likelihood estimator problem can be transformed into solving the maximum mathematically. Generally, the parameters are obtained by solving the differential (8).

\[
\begin{align*}
\frac{d}{d\theta} L(\theta) &= 0 \\
\frac{d}{d\theta} \ln L(\theta) &= 0
\end{align*}
\]

(8)

For P-III distribution, the logarithmic likelihood function is established as the following:

\[
\ln L = n \alpha \ln \beta - n \ln \Gamma(\alpha) + (\alpha - 1) \sum_{i=1}^{n} \ln(x_i - a_0) - \beta \sum_{i=1}^{n} (x_i - a_0)
\]

(9)

The values of three parameters, \( \alpha, \beta \) and \( a_0 \) can be obtained by (9), then the parameters, \( \overline{x}, C_v, C_s \) can be deduced according to (2).

3) WEIGHTED FUNCTION ESTIMATOR.

In the case of small sample size, the error of parameters estimated by the moment method is large, especially \( C_s \). In
order to improve the accuracy of $C_i$ estimation, Maproposed the weight function method[31]. The method to estimate $\bar{X}$ and $C_i$ is the same as moment estimation, but the difference is that the method calculates moments in discrete form. The formulas are as follows:

$$E(x) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{X})\varphi(x_i)$$

$$H(x) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{X})^2 \varphi(x_i)$$

(10)

And the weight function, $\varphi(x)$, is generally represented as normal probability density function, as shown in (11), and the estimation of $C_i$ can be obtained by (12).

$$\varphi(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x - \mu}{\sigma}\right)^2}$$

(11)

$$C_i = -4\sigma \frac{E(x)}{H(x)}$$

(12)

C. GOODNESS OF FIT TEST (GFT)

When the population distribution of a random variable is unknown and the random variable is described by probability distribution, it is necessary to test various assumptions about the form of the population distribution according to the samples. This kind of problem is the distribution fitting test, which is a kind of non-parametric hypothesis test. The GFT method commonly used is the K-S test. The problem of fitting of distribution can be described as: assuming that $F (x)$ is the population distribution function of random variable $X$, $x_1, x_2, \cdots, x_n$ is the sample value of $X$, testing whether the population distribution $F (x)$ is $F_0 (x)$. With the above, the assumptions to be tested are as follows:

$$H_0 : F(x) = F_0 (x), \quad H_1 : F(x) \neq F_0 (x)$$

(13)

The K-S (Kolmogorov-Smirnov) test, also known as the D test, constructs statistic by comparing the differences between sample empirical distribution $F_0 (x)$ and population distribution $F (x)$. The statistic $D_n$ can be obtained by Eq. (15), and we can get the exact distribution of $D_n$, which does not depend on the population distribution.

$$D_n = \sup_{x} | F_n (x) - F(x) |$$

(15)

At a certain test level $\alpha$, the critical values of $D_n$ statistic, denoted as $D_{n, \alpha}$, can be taken by table lookup. If $D_n \geq D_{n, \alpha}$, we reject the null hypothesis, indicating that the distribution of the random variables is significantly inconsistent with the hypothesis distribution; otherwise, the null hypothesis can not be rejected, that is: at the certain test level, there is no sufficient evidence to indicate that the sample data does not come from the total distribution $F_0 (x)$. K-S test applies to discrete and continuous population distribution, and it does not depend on the partition of population value.

D. AN IMPROVED PARAMETER ESTIMATION METHOD—LM-MLE

In the case of short historical data series, if only L-moment estimation (LM)[32, 33] or maximum likelihood method (MLE) is used to estimate the parameters, the parameter $\alpha_0$ of P-III distribution is easy to be less than 0, so MLE is used to modify the $\alpha_0$ obtained by LM in the study.

$$[\alpha, \beta, \alpha_0] = [[\alpha, \beta, \alpha_0]_{LM}, \alpha_0 > 0$$

(16)

III. RISKS OF DISCARDING WATER BASED ON FREQUENCY RUNOFF

Dispatching of discarding water is continuous in time, and decision-making in current period may cause water to be discarded in the subsequent scheduling period. At present, the accuracy of medium and long-term runoff forecasting is poor, and the forecasting results are uncertain. Therefore, the water level control process of deterministic optimal dispatching is limited in practical application, while the dispatching chart guiding the daily operation and dispatching of hydropower stations has limitations on water abandonment control. In order to balance the benefits of power generation and risk control of discarding water in operation and dispatch of hydropower stations, a calculation method of discarding water risk of hydropower stations is proposed in this study, which is based on different frequencies of runoff. At the same time, it deduces that different risk of discarding water corresponds to the control process of operation level of hydropower stations, and applies it to risk control of discarding water in power generation dispatch.

A. DEFINITION AND CALCULATION OF RISKS OF DISCARDING WATER

Discarding water of hydropower station generally refers to the amount of reservoir water lost from discharge facilities of hydropower stations without generating electricity by hydropower units. Economic operation of hydropower stations requires minimizing the generation of discarding water. The risk of discarding water in hydropower station is usually defined as the probability that the water level will exceed the normal storage level of the reservoir when the hydropower station generates electricity as expected. This definition does not fully take into account of the different characteristics of the dry season, pre-flood falling stage, the flood season and the storage period, so it does not conform to the actual water level control rules of the hydropower station. From a qualitative point of view, the higher the operating water level of the reservoir, the greater the risk of discarding water; conversely, the lower the operating water level, the smaller the risk of discarding water. Therefore, the relationship between water level and abandonment risk can be used to quantify the abandonment risk of hydropower station from the point of view of water level control.
During the current operation of hydropower stations, the control constraints of key water levels (normal storage level, flood control high level and flood limit level) must be met in different operation periods. Water level restriction is an important cause of discarding water in power generation process of hydropower stations, but the key water level control associated with the scheduling period is too long, runoff prediction accuracy can’t meet the scheduling requirements. Therefore, the risk of discarding water based on the key water level control and runoff frequency of hydropower station can be defined as the runoff frequency corresponding to the maximum inflow without discarding water and meeting the water level control constraints under the maximum power generation scenario of hydropower station. The risk of discarding water is defined as follows:

$$\theta(Z_i, Z_{end}, Q, Q_i) = f\left(Q_i + \Delta(Z_i, Z_{end})/T - Q_i\right)$$  \hspace{1cm} (17)

where \( \theta \) is the risk of discarding water, \( Z_i \) is the level of the station in the current period, \( Z_{end} \) is the controlled water level at the end of time period, \( Q \) is the generation flow under the maximum output scenario during the risk assessment period of hydropower station, \( Q_i \) is the average discharge of upstream reservoir during the period, \( \Delta(Z_i, Z_{end}) \) is the difference of reservoir capacity between the water level of reservoir in t-period and the controlled water level at the end of time-period, \( T \) is the length of the corresponding risk assessment period, \( f(*) \) is runoff frequency curve or flood frequency curve.

The specific calculation steps of the risk of discarding water are as follows:

**Step 1:** Select long series historical inflow data of hydropower stations with high reliability;

**Step 2:** Fit the frequency curve of runoff in the corresponding calculation period and analyze the rationality of frequency distribution;

**Step 3:** According to the current water level and the controlled water level at the end of the dispatching period of the hydropower station, the difference of reservoir capacity between the two water levels, \( \Delta(Z_i, Z_{end}) \), is calculated;

**Step 4:** Calculate the generation flow \( Q \) under the maximum output scenario, and the maximum permissible inflow for hydropower stations without discarding water can be obtained by (17):

$$Q_{max} = Q_i + \Delta(Z_i, Z_{end})/T - Q_i$$  \hspace{1cm} (18)

**Step 5:** By inquiring the corresponding runoff frequency curve derived by Step2, the corresponding frequency of the maximum permissible inflow can be obtained, that is the risk of discarding water.

**B. WATER LEVEL-MAXIMUM OUTPUT ESTIMATION OF HYDROPOWER STATION**

When water level of upstream reservoir is known, it is impossible to directly and quickly inquire the maximum output and corresponding discharge of a hydropower station based on the information of the basic curve of a hydropower station. Therefore, a method of finely calculating the level-maximum output curve of the hydropower station is proposed in this study.

Suppose a hydropower station has two types of units, A and B, respectively, and the number of units is \( N_A \) and \( N_B \). The NHQ curves of each type of units are known, the Head-Maximum output curves of units, the reservoir-related characteristic curves and the maximum and minimum discharge \( Q_{max} \) and \( Q_{min} \) of the station are given. These curves are represented by the following formulas:

1) Storage capacity versus and elevation curve \( L_{vi} \):

$$V = f_{vi} (Z_{wp})$$  \hspace{1cm} (19)

2) Outflow and downstream water level curve \( L_{vc} \):

$$Z_{down} = f_{vc} (Q)$$  \hspace{1cm} (20)

3) Outflow and head loss curve \( L_{vh} \):

$$\Delta h = f_{vh} (Q)$$  \hspace{1cm} (21)

4) Head-Maximum output curve of unit \( L_{mn} \):

$$N_{max} = f_{mn} (H)$$  \hspace{1cm} (22)

5) NHQ curve of unit \( L_{nhq} \):

$$q = f_{nhq} (H, N)$$  \hspace{1cm} (23)

Suppose water level of upstream reservoir \( Z_s \), the specific steps for calculating the corresponding maximum output of the proposed approach are as follows and FIGURE 2.

**Step 1:** Let \( Q = (Q_{max} + Q_{min})/2 \), then calculate downstream water level \( Z_{down} \), curve \( L_{vc} \), and head loss \( \Delta h \) can be obtained by curve \( L_{vh} \), in this way \( H = Z_s - Z_{down} - \Delta h \); Query curve \( L_{mh} \) to get the maximum output of each type of unit, \( N_{max}^A \), \( N_{max}^B \) and \( Q_{max}^A \), \( Q_{max}^B \), and then maximum output and outflow of the station \( N_{max} = N_A \cdot N_{max}^A + N_B \cdot N_{max}^B \), \( Q_{max} = Q_A \cdot Q_{max}^A + Q_B \cdot Q_{max}^B \).

**Step 2:** If \( |Q - Q_s| \leq \epsilon \) , then \( Q_s = Q_{max}^A \cdot Q_A = Q_{max}^B \cdot Q_B \) and \( N_{max} = N_{max}^A \), jump to Step 4, otherwise, jump to Step 3;

**Step 3:** If \( Q > Q_s \) , then \( Q_{max} = Q \), otherwise \( Q_{max} = Q_s \), and jump to Step 1;

**Step 4:** Further, in order to verify whether reducing discharge and raising head will increase the maximum output of the station, suppose outflow steps \( \Delta Q \), and let

$$Q = n_A \cdot (Q_A - \Delta Q) + n_B \cdot (Q_B - \Delta Q)$$  \hspace{1cm} (24)

According to the method described in Step 1, calculate the head \( H \) of station, Query curve \( L_{nhq} \) to get the output of each type of unit, \( N_{max}^A \), \( N_{max}^B \), and then \( N_{max} = n_A \cdot N_{max}^A + n_B \cdot N_{max}^B \), if \( N_{max} = N_{max}^A \), the calculation is completed, otherwise, go to Step 5.
**Step 5:** Let $N_{\text{max}} = N'_{\text{max}}$ and repeat Step 4.

**FIGURE 2.** Flow chart of water Level-Maximum Output estimation of Hydropower Station.

**C. WATER LEVEL OPERATION CONTROL BASED ON RISKS OF DISCARDING WATER OF HYDROPOWER STATION**

Based on the benefit of hydropower station, the process control of optimal water level, which is also the optimal criterion of hydropower station dispatching, is to control the operating water level as the upper limit of permissible operating water level in the period of time without discarding water. But when discarding water occurs, the optimal water level control of hydropower station should increase the output to reduce the water level before discarding water occurs. Therefore, it is necessary to reasonably assess the risk of discarding water corresponding to the operating water level of the power station in order to reduce the operating water level of the power station in time. However, the operation chart widely used in the daily operation of the reservoir is drawn from historical statistics, of which the main guiding principle is to maintain high-head power generation on the basis of guaranteeing the power generation rate of the hydropower station, and does not indicate the risk value of discarding water under different operating water levels. In order to reduce the risk of water abandonment of hydropower stations and rationally select the operation level to improve the benefits of hydropower stations, the calculation method of water abandonment risk proposed in section 3.1 is applied to the operation and dispatch of hydropower stations, and the operation control process of water level of hydropower stations under different risk of water abandonment is deduced.

Because of the time continuity of hydropower station dispatching, the risk control of discarding water must consider the whole dispatching process during the
dispatching period. Therefore, the discarding water risk calculation method aims at the critical control water level at the end of the dispatching period to calculate the risk of discarding water at the current operating water level. Taking

FIGURE 3 as a schematic diagram, this paper illustrates how to convert risk control of discarding water into water level control of hydropower station, so as to direct the actual operation and dispatch of hydropower station intuitively.

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The dispatching period is divided into T periods, with the key water level $Z_{T+1}$ at the end of the dispatching period as the water level control objective, and the initial water level of each dispatching period is discretized into L points in the feasible region. From time T, the risk of discarding water under discrete water levels from $Z_T^L$ to $Z_T^L$ is calculated separately. The representative risk probabilities, $P_T^1$, $P_T^2$ and $P_T^3$, are selected as control risks, and the corresponding water levels are assumed to be $Z_T^1$, $Z_T^2$ and $Z_T^3$, respectively, and then taking $Z_T^1$, $Z_T^2$ and $Z_T^3$ as control water levels, and taking $P_T^1$, $P_T^2$ and $P_T^3$ as control water risks, the water level process line corresponding to the risk of discarding water is calculated by inverse calculation, that is, the control process of operating water level. The specific steps of the proposed approach are as follows:

Step 0: Let $t = T$;

Step 1: Get the control water level $Z_{t+1} = [Z_{t+1}^1, Z_{t+1}^2, \ldots, Z_{t+1}^L]$ at time $t + 1$;

Step 2: In the water level interval at time t, the water level is discretized and L discrete points are obtained, just as $[Z_t^1, Z_t^2, Z_t^3, \ldots, Z_t^L]$;

Step 3: Let $t = 1$;

Step 3.1: Let $m = 1$;

Step 3.1.1: By using the method in Section 3.2, the maximum output $P_{t_{max}}^m$ of the power station in t-period is calculated when the initial water level is $Z_t^m$;

Step 3.1.2: In Step 3.1.1, the generation flow $Q_t$ is calculated under the condition of maximum output $P_{t_{max}}^m$ of the power plant;

Step 3.1.3: Under the condition of satisfying water level control constraints $Z_{t+1}^m$ and not abandoning water, the maximum inflow $Q_{t_{max}}^m$ is calculated corresponding to the maximum output $P_{t_{max}}^m$ of hydropower station and the risk probability of the discarding water is obtained by using the runoff frequency curve.;

Step 3.1.4: If $m \leq M$, $m = m + 1$ and jump to Step 3.1.1, Otherwise, continue to execute Step 3.2;

Step 3.2: If $l \leq L$, $l = l + 1$ and jump to Step 3.1, Otherwise, continue to execute Step 4;

Step 4: $t = t - 1$ and if $t \geq 0$, jump to Step 1, otherwise, return the result, that the operating water level processes under different risks of discarding water.

The flow chart of the detailed calculation process is shown in FIGURE 4.

Discarding water risk control will have different considerations for reservoir hydropower stations with different regulation performance. For hydropower stations with weak regulation ability, such as no regulation, daily regulation and weekly regulation, the risk assessment period of abandoned water can be set as the corresponding regulation period length, and the control water level at the end of dispatching period can be set as the highest reservoir water level in the corresponding dispatching period to calculate the risk of abandoned water from hydropower stations.
Reservoirs with seasonal regulation or above have different water level control objectives in different dispatching periods. The risk of abandoned water can be calculated according to the key water level control objectives in different dispatching periods. In the pre-flood falling stage, taking the drawdown level as the control water level at the end of the dispatch period, the risk of abandoned water can be calculated according to the frequency runoff, and the corresponding operation water level process can be deduced, and the appropriate risk of abandoned water can be selected to control the operation water level process according to the size of the inflow in the dispatch process of the power station. In the dry season, the reservoir has fewer inflows, and the possibility of discarding water due to the uncertainty of inflow is less. Hydropower stations can control the alternating water level in the dry season and the pre-flood falling stage as normal storage water level, and keep the operating water level of the power station near the normal storage water level. In the main flood season, reservoir inflow is more, and it is the concentrated period of flood

FIGURE 4. Flow chart of operating water level process of hydropower station under different discarding water risks.
occurrence. The risk of discarding water can be calculated according to flood volume to control the water level and make rational use of flood. During the storage period after the flood season, the controlled water level at the end of the dispatching period can be set as the storage level, the corresponding water level process of different discarded water risks can be deduced according to the frequency runoff, and the time of starting and storing can be reasonably selected to avoid discarding water caused by premature storage and the failure of filling up caused by late storage.

### IV. CASE STUDY

In this section, the Ertan Hydropower Station in Yalong River Basin is taken as the research object. Firstly, the average runoff of the Ertan reservoir is analyzed; furthermore, 0.01, 0.05, 0.1, 0.3, 0.5, 0.7, 0.9 and 0.95 are selected as the control the risk of abandoned water, and the corresponding control process of operating water level is deduced; finally, the water level control process is applied to conventional power generation dispatching of the hydropower station and the correctness of the proposed discarded water risk calculation method and the risk control of discarded water for power generation are verified.

#### A. A Survey of Research Objects

Yalong River is the largest tributary of the Yangtze River, with a total length of 1571 kilometers, which originates from the Bayan Hara Mountains in Qinghai Province. The Ertan Hydropower Station is located in the lower reaches of Yalong River, Panzhihua City, Sichuan Province, 33 kilometers from the junction of Yalong River and Jinsha River. The Ertan Hydropower Station is the first hydropower station in the cascade development of Yalong River. It was officially constructed in 1991. It is a hydropower station with power generation as its main task. Since the Ertan Hydropower Station put into operation, the power quality and system stability of Sichuan and Chongqing power grids have been greatly improved, which has made a significant contribution to the economic development of Sichuan and Chongqing areas. The Ertan Hydropower Station has a normal storage level of 1200 meters, a checked flood level of 1203.5 meters, a dead water level of 1155 meters, its annual average runoff is 1670 m$^3$/s, which belongs to the seasonally regulated hydropower station. Some parameters of the hydropower station are shown in Table I.

#### B. Results and discussion

1) **RUNOFF ANALYSIS OF ERTAN RESERVOIR**

   (1) Characteristic analysis of ten-day runoff

   The annual average runoff of the Ertan Hydropower Station in the past 55 years from 1958 to 2012 is selected for calculation, which is shown in FIGURE 5, and the multi-year average runoff is 1670 m$^3$/s. The runoff series covers the wet and dry years, and the typical years occur alternately, which can be used as representative series of historical runoff analysis.

   To further analyze the characteristics of historical inflow in the Ertan station, the Maximum, Minimum, Average, Variation coefficient and Skewness coefficient are calculated as Table II.
**FIGURE 5.** The annual average flow process of the Ertan hydropower station from 1958 to 2012.

**TABLE II**

RESULT TABLE OF TEN-DAY RUNOFF STATISTICS IN THE ERTAN STATION

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Early¹</td>
<td>Mid</td>
<td>End</td>
<td>Early</td>
<td>Mid</td>
<td>End</td>
<td>Early</td>
<td>Mid</td>
<td>End</td>
</tr>
<tr>
<td>Max((m^3/s))</td>
<td>742</td>
<td>674</td>
<td>606</td>
<td>564</td>
<td>566</td>
<td>542</td>
<td>534</td>
<td>535</td>
<td>614</td>
</tr>
<tr>
<td>Min((m^3/s))</td>
<td>441</td>
<td>380</td>
<td>345</td>
<td>353</td>
<td>344</td>
<td>336</td>
<td>391</td>
<td>338</td>
<td>342</td>
</tr>
<tr>
<td>Avg((m^3/s))</td>
<td>548</td>
<td>507</td>
<td>478</td>
<td>459</td>
<td>443</td>
<td>438</td>
<td>432</td>
<td>432</td>
<td>444</td>
</tr>
<tr>
<td>(C_v)</td>
<td>0.13</td>
<td>0.12</td>
<td>0.11</td>
<td>0.11</td>
<td>0.10</td>
<td>0.10</td>
<td>0.09</td>
<td>0.09</td>
<td>0.12</td>
</tr>
<tr>
<td>(C_s)</td>
<td>0.54</td>
<td>0.61</td>
<td>0.20</td>
<td>0.17</td>
<td>0.24</td>
<td>0.01</td>
<td>0.03</td>
<td>0.03</td>
<td>0.66</td>
</tr>
<tr>
<td>Max((m^3/s))</td>
<td>660</td>
<td>921</td>
<td>993</td>
<td>1173</td>
<td>1165</td>
<td>1825</td>
<td>2660</td>
<td>3778</td>
<td>5390</td>
</tr>
<tr>
<td>Min((m^3/s))</td>
<td>380</td>
<td>405</td>
<td>459</td>
<td>408</td>
<td>440</td>
<td>483</td>
<td>523</td>
<td>795</td>
<td>1329</td>
</tr>
<tr>
<td>Avg((m^3/s))</td>
<td>478</td>
<td>519</td>
<td>607</td>
<td>701</td>
<td>813</td>
<td>996</td>
<td>1355</td>
<td>1942</td>
<td>2814</td>
</tr>
<tr>
<td>(C_v)</td>
<td>0.12</td>
<td>0.17</td>
<td>0.20</td>
<td>0.21</td>
<td>0.23</td>
<td>0.29</td>
<td>0.38</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>(C_s)</td>
<td>1.08</td>
<td>2.20</td>
<td>1.30</td>
<td>0.65</td>
<td>0.08</td>
<td>0.74</td>
<td>0.69</td>
<td>0.46</td>
<td>0.66</td>
</tr>
<tr>
<td>Max((m^3/s))</td>
<td>6550</td>
<td>7337</td>
<td>6506</td>
<td>7150</td>
<td>7760</td>
<td>7517</td>
<td>7407</td>
<td>7108</td>
<td>5985</td>
</tr>
<tr>
<td>Min((m^3/s))</td>
<td>1461</td>
<td>1702</td>
<td>1678</td>
<td>1395</td>
<td>1090</td>
<td>1547</td>
<td>1225</td>
<td>1538</td>
<td>1686</td>
</tr>
<tr>
<td>Avg((m^3/s))</td>
<td>3528</td>
<td>3783</td>
<td>3432</td>
<td>3414</td>
<td>3385</td>
<td>3578</td>
<td>3734</td>
<td>3577</td>
<td>3250</td>
</tr>
<tr>
<td>(C_v)</td>
<td>0.34</td>
<td>0.35</td>
<td>0.33</td>
<td>0.38</td>
<td>0.47</td>
<td>0.40</td>
<td>0.42</td>
<td>0.35</td>
<td>0.29</td>
</tr>
</tbody>
</table>

¹ Early: Early-Ten days; Mid: Middle-Ten days; End: End-Ten days.
From Table II, it can be seen that the annual distribution of runoff in Ertan Hydropower Station is uneven. The annual average runoff in January ~ April is between 430-600 m³/s, which belongs to dry season with \( C_s \) and relatively stable runoff distribution. After May, the inflow increases gradually, especially after June, and the inflow from June to October accounts for about three quarters of the total water in the year. Therefore, in the dry season, less water can be used, it is necessary to keep the high water head power generation as far as possible, and try to reduce the reservoir water level before June to make full use of the reservoir water for power generation operation.

(2) Fitting of 10-day average runoff distribution

Using P-III distribution to fit the frequency distribution of ten-day average runoff, the frequency distribution curve of 10-day average runoff can be obtained. The 10-day average runoff results corresponding to typical frequencies are queried by the frequency curves obtained above as shown in Table III.

<table>
<thead>
<tr>
<th>Level Period</th>
<th>Typical frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td>1</td>
<td>917.99</td>
</tr>
<tr>
<td>2</td>
<td>840.47</td>
</tr>
<tr>
<td>3</td>
<td>724.53</td>
</tr>
<tr>
<td>4</td>
<td>691.44</td>
</tr>
<tr>
<td>5</td>
<td>642.23</td>
</tr>
<tr>
<td>6</td>
<td>602.50</td>
</tr>
<tr>
<td>7</td>
<td>592.69</td>
</tr>
<tr>
<td>8</td>
<td>587.20</td>
</tr>
<tr>
<td>9</td>
<td>709.19</td>
</tr>
<tr>
<td>10</td>
<td>839.33</td>
</tr>
<tr>
<td>11</td>
<td>1134.74</td>
</tr>
<tr>
<td>12</td>
<td>1439.00</td>
</tr>
<tr>
<td>13</td>
<td>1353.67</td>
</tr>
</tbody>
</table>
(3) The goodness of fit test of ten-day average runoff distribution

To verify the rationality of Pearson type III distribution for fitting ten-day average runoff of the Ertao Reservoir, the K-S test is used to test the goodness of distribution fitting for ten-day average runoff distribution. The p-value is shown in FIGURE.6.
From FIGURE 6, it can be seen that the p-value of distribution fitting test is much larger than the significant level α (α=0.05), which shows that it is reasonable to use the P–III distribution to fit the 10-day average runoff distribution of the Ertan station.

2) ANALYSIS OF WATER LEVEL OPERATION CONTROL UNDER RISKS OF DISCARDING WATER

According to the characteristic water level control of reservoir operation and dispatching, the control target of the characteristic water level in key dispatching period is selected for risk control analysis of discarding water. Based on experience in optimal operation of the Ertan Hydropower Station, the water level of the selected hydropower station will fall to 1155 at the end of May, and be controlled between 1185 m and 1190 m at the end of June, and be stored to 1200 m at the end of September. The control process of the operation level of the hydropower station will be deduced under the control of the different typical risk of discarding water. According to the proposed idea of water level operation control, the process of water level control for three water level control objectives corresponding to different water level risk is shown in FIGURE. 7, FIGURE. 8 and FIGURE. 9.

1%~95% in FIGURE. 7, FIGURE. 8 and FIGURE. 9 are typical frequencies of runoff of the Ertan station, and their values correspond to the risk of discarding water. The water level process under different risks of discarding water is the highest water level process under the condition that the hydropower station operates to the key target water level at the end of dispatching period without discarding water under the corresponding frequency average inflow and maximum output scenario. According to the operating water level control line, the risk of discarding water corresponding to the operating water level of the power station in different periods of the dispatching period can be obtained.

On the whole, the higher the water level of the hydropower station, the greater the risk of discarding water; on the contrary, the lower the water level, the smaller the risk of discarding water. Because the controlled water level of the reservoir is dead water level at the end of the pre-flood falling stage, the discarding water can be avoided by reducing the depth of fluctuation, but this will increase the risk of discarding water in June with more inflow. Based on the analysis of FIGURE 8 and 9, when the inflow of water in late April is larger than the inflow corresponding to the typical frequency of 0.3, the process of water level control at the corresponding risk of discarding water can be selected to make full use of the reservoir water resource to generate electricity and avoid that the reservoir water can’t give full play to the benefit of the reservoir water resource when there is more inflow in the later period; otherwise, the station can maintain power generation operation under high water level and begin to reduce level in early May. In this way, the head benefit can be brought into full play, and the reservoir water resource can be fully utilized in the later period of the pre-flood falling stage. The water level process corresponding to the risk of discarding water is selected according to the high and low inflow of the station, which is used as the water level reduction mode of the station, and adjust it according to the inflow of the reservoir.

With the increase of inflow in June, the water level of the station should be raised gradually to bring the benefit of water head into full play. As shown in FIGURE.9, the risk of discarding water is small when the inflow water in the early flood season is less than the inflow water corresponding to the typical frequency of 0.5, so the excessive decline can be avoided under the premise of guaranteeing the output during the pre-flood falling stage. When the inflow water is larger than the inflow water with the typical frequency of 0.1, the risk of discarding water is high, and the inflow water can be fully utilized in the early stage; when the inflow water is less than the inflow water with the typical frequency of 0.3, the efficiency of using water can be increased after reaching the rated head of 165 meters in the first flood season to reduce
the risk of discarding water. In the early stage of the flood season, when the water head of the station reaches the rated head (165 m), the station can increase its output to reduce the risk of discarding water and give full play to the benefits of water volume.

From FIGURE 9, it can be seen that the risk of discarding water in flood season is very high, and the water level above 1185m can be maintained to reach the rated head, so the risk of discarding water in flood season is not controlled. When the inflow water is less than the inflow water with the typical frequency of 0.7, the water level can be raised appropriately after the flood season to fill the reservoir; the inflow water is larger than the inflow water with the typical frequency of 0.5, the risk of discarding water is higher, and the output can be increased appropriately in the process of water storage to reduce the operating water level, so as to avoid discarding water caused by premature full storage.
3) POWER GENERATION DISPATCHING OF HYDROPOWER STATIONS WITH RISK CONTROL OF DISCARDING WATER

In order to further illustrate and verify the effectiveness of the abandoned water risk control method proposed in this study, the discarding water risk control method is applied to the conventional power generation dispatching of the Ertan Hydropower Station, and in the typical years of wet, flat and dry water, the generation index statistics between the discarding water risk control dispatching of the station and the conventional dispatching of dispatching charts are compared. The power generation dispatching diagram of the Ertan Hydropower Station is shown in FIGURE 10.

Taking 1974 as a typical flood year, 1997 as a typical mean year and 2011 as a dry year respectively, conventional dispatching is carried out with actual historical inflow, and on this basis, water level operation control with different abandonment risk is combined with the conventional dispatching process, the corresponding dispatching results under different dispatching scenarios are shown in Table IV.

FIGURE 10. The power generation dispatching diagram of the Ertan Hydropower Station.

TABLE IV
ROUTINE DISPATCHING RESULTS AND RISK CONTROL DISPATCHING RESULTS OF ABANDONED WATER IN TYPICAL YEARS

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Power generation 10^8 kWh</td>
<td>Discarding water 10^8 m^3</td>
<td>Power generation 10^8 kWh</td>
</tr>
<tr>
<td>conventional</td>
<td>181.64</td>
<td>223.04</td>
<td>167.37</td>
</tr>
<tr>
<td>0.01</td>
<td>181.65</td>
<td>223.00</td>
<td>167.38</td>
</tr>
<tr>
<td>0.1</td>
<td>181.79</td>
<td>222.67</td>
<td>167.44</td>
</tr>
<tr>
<td>0.3</td>
<td>181.79</td>
<td>222.67</td>
<td>167.46</td>
</tr>
<tr>
<td>0.5</td>
<td>181.79</td>
<td>222.75</td>
<td>167.50</td>
</tr>
<tr>
<td>0.7</td>
<td>181.71</td>
<td>223.13</td>
<td>167.50</td>
</tr>
<tr>
<td>0.9</td>
<td>181.71</td>
<td>223.13</td>
<td>167.49</td>
</tr>
<tr>
<td>0.95</td>
<td>181.71</td>
<td>223.13</td>
<td>167.49</td>
</tr>
</tbody>
</table>
From Table IV, it can be seen that for the typical wet year 1974, power generation under discarding water risk control is higher than that under conventional dispatching, less water abandoned under smaller discarding water risk control, but there is a certain increase under larger discarding water risk control. When discarding water risk control is 0.1, the optimal control dispatching result of maximum power generation and minimum water abandoned can be achieved. For 1997, when the risk of water discarding is less than 0.5, the power generation increases with the increase of the risk of water abandonment, and the amount of water abandonment decreases compared with the conventional dispatching. When the risk of discarding water is 0.5, the optimal control dispatching result can be achieved. For the dry year 2011, the power generation increases with the increase of the risk of water abandonment, and the amount of water abandonment decreases with the increase of the risk of water abandonment, and the risk of water abandonment. The optimal control scheduling result can be achieved under the control, when the risk of discarding water is 0.5.

When the water level of discarding water risk control is higher than the water level of anti-abandoned water line in the dispatching chart, the head benefit can be effectively used to increase power generation and reduce discarding water accordingly. However, when the risk of discarding water is increased, the increase of operating water level will increase discarding water, and when the water level is higher than the control water level corresponding to the risk of discarding water 0.5, the increment of power generation decreases. The water level should be lowered appropriately to make full use of the benefit of water volume in a wet year.

In order to make full use of water volume and head benefit, the water level control process with discarding water risk of 0.1 can be used for power generation dispatch in wet year. For the mean year, the results are in good agreement with those in the wet year and the operation water level can be raised to the operation control water level corresponding to the risk of discarding water 0.7. In mean year, the water level control process with the risk of discarding water 0.5 can be selected for the power generation dispatch of the Ertan station. For dry years, with the increasing risk of discarding water, the operating water level of the station increases, which can make full use of the head benefit of dry years, and increase the power generation capacity of the station, and reduce the discarding water of the Ertan station accordingly.

V. CONCLUSION

In this study, Pearson-III distribution is used to describe the runoff distribution characteristics of the basin, and the parameters of Pearson-III distribution are estimated by L moment and maximum likelihood estimation. From the Kolmogorov-Smirnov test, it can be seen that the probability values of the Kolmogorov-Smirnov test are far greater than the significant level $\alpha (\alpha=0.05)$, which verifies the effectiveness of the above methods. On this basis, a calculation method of discarding water risk based on runoff frequency of hydropower station is proposed, and the risk control of discarding water is applied to the dispatching process of hydropower station. At the same time, a refined method for calculating the expected output of water level of hydropower station is proposed, and the control process of water level of hydropower station under different risk of discarding water based on predictive power is deduced. Finally, taking Ertan Hydropower Station in Yalong River Basin as the research object, the 10-day average runoff frequency distribution of Ertan is fitted and tested on the 10-day calculation scale. The proposed method is applied to the operation and dispatch of Ertan, and the corresponding water level control process of Ertan with different discarding water risks is deduced. At the same time, taking the historical runoff of typical wet, mean and dry water years as input, different water level control modes corresponding to the risk of typical discarding water are used for power generation dispatching, and the calculation results are compared with the dispatching results of conventional dispatching charts, and the operation water level control of the Ertan station in different water years is analyzed to provide reference for the actual operation of power stations.

REFERENCES


