Torque Distribution Algorithm for Stability Control of Electric Vehicle Driven by Four In-Wheel Motors under Emergency Conditions

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ABSTRACT With the rapid development of intelligent transportation system, the research on vehicle stability can be a theoretical basis for realizing autonomous driving technology. Previous stability control strategies have not taken into account the tire force saturation factor, the slip rate and the robustness of the control system sufficient. According to the characteristic that the torque of each wheel can be distributed independently, a torque distribution algorithm under emergency conditions is proposed. The proposed torque distribution algorithm is constructed using three hierarchical controllers. The upper controller attempts to judge whether the vehicle is in stable state using the phase plane method. Also, it judges whether the wheels are slipping. The middle controller aims to calculate the demands for the desired traction force and yaw moment, whereas the lower controller is designed to translate those virtual signals into actual actuator commands. When designing the middle controller, a sliding mode control method is utilized to guarantee system stability and robustness by taking into account various factors, including lateral wind and sensor noise. For the lower controller, the control allocation optimization method is utilized to determine an appropriate control input for each in-wheel motor by considering the road conditions, adhesion utilization and maximum output torque of the motor. Numerical simulation studies are conducted to evaluate the performance of the torque distribution algorithm. Comparison results indicate that the proposed algorithm presents better performance to distribute the appropriate torque for each wheel and ensure the stability of the vehicle under emergency conditions.

INDEX TERMS intelligent transportation system; stability control; swarm intelligence; torque distribution

1. INTRODUCTION

Enhancing the stability of the vehicle is the main research focus for the intelligent transportation system [1]. At the same time, an increasing number of vehicles worsens energy problems. Electric vehicles are environmentally friendly due to their zero emissions and low level of pollution [2]. Electric vehicles driven by four in-wheel motors have become a hot topic in the field of electric vehicle types due to advantages compared with traditional vehicles. For example, each wheel can be controlled independently. Moreover, the in-wheel motors are able to realize quick response with high accuracy, which presents the opportunity to apply many advanced control theories to improve the maneuverability and stability of the electric vehicles driven by four in-wheel motors [3-4]. For example, Demirci and Gokasan [5] proposed a stability control scheme for four-wheel drive four-wheel steer electric vehicle using Lagrangian neural networks.

Vehicle stability control (VSC) is an active safety control system that can assist the driver in maintaining directional control of the vehicle under emergency conditions. The function of VSC is to track the desired yaw rate and minimize the vehicle side slip. Because of the strong nonlinearity and uncertainty of the vehicle, it is not easy to design an active safety control system. In the past few years, researchers have made many contributions on VSC systems. Shino et al. [6] investigated the use of direct yaw moment control for improving the handling and stability of electric vehicles with two driving motors. Yoon et al. [7] proposed a unified chassis system for traditional vehicle rollover prevention and lateral stability control. Kang et al. [8] investigated a driving control algorithm for four-wheel-driven electric vehicles equipped with two motors at the front and rear driving shafts. Nam et al. [9] designed an adaptive sliding mode controller for robust yaw stabilization of in-wheel motor-driven electric vehicles. Zheng et al. [10] exploited the linear quadratic regulator theory for yaw control and developed a vehicle dynamics control (VDC) system for tracking the desired vehicle behavior. The control performance is good. While the computation cost is high,
which is not suitable to realize in real conditions. Kim et al. [11] designed a control algorithm based on wheel slip, yaw rate, and lateral acceleration to determine the motor torque for each wheel and enhance the stability of an independent motor-drive vehicle. Lian et al. [12] developed a braking force distribution strategy to solve the braking force distribution problem for electric vehicles driven by four independent motors based on regenerative braking strength continuity. Yue et al. [13] proposed a three-level structure to enhance the stability performance for a four-wheel independent drive electric vehicle. Maeda et al. [14] proposed a four-wheel driving force distribution method for instantaneous or split slippery roads. The current stability control strategies have not taken into account the tire force saturation factor, the slip rate and the robustness of the control system simultaneously.

The previous studies indicate that many attempts have been done in the field of stability control for the electric vehicles driven by in-wheel motors. How to distribute the torque of each wheel is complicated and many algorithms have been tested. But the torque distribution methods for electric vehicle driven by four in-wheel motors under emergency conditions still have some challenges remained to be solved, for the reason that the vehicle needs more control variables. For example, Yuan and Wang [15] presented a torque distribution method for a front and rear-wheel-driven electric vehicle with the aim to enhance its efficiency in a wide torque and speed range. In our previous study, the energy management issue is incorporated into the torque distribution problem for the four in-wheel motors driven electric vehicle [16]. In the situation of emergency conditions, the vehicle is easy to lose its stability. Current literatures mainly applied the direct yaw moment control, slip rate control and anti-sideslip control to ensure the vehicle stability. For example, Lin et al. [17] proposed a sliding mode approach based on a conditional integrator to control the vehicle yaw rate. Di Cairano et al. [18] proposed a model predictive control strategy to coordinate active front steering and differential braking to improve yaw stability and cornering control. To ensure vehicle stability under emergency conditions, both the vehicle side slip angle and the yaw rate should be considered. Most of current methods are in the phases of theory research and test by using intelligent control algorithms, result in low efficiency and control effect.

In this study, a hierarchical torque distribution strategy is put forward for an electric vehicle driven by four in-wheel motors to enhance its maneuverability and stability under emergency conditions. The upper controller is designed to judge whether the vehicle is in the stable state and whether the wheels are slipping; it then transmits the results to the middle controller. The active intervention control is easy to trigger off once the vehicle falls into longitudinal and lateral unstable situation, which helps to ensure high sensitivity and accuracy. For the middle controller, a sliding mode control methodology is utilized to enhance system anti-interference performance. Considering that the vehicle will be affected by the side wind, sensor noise and other unmeasured signals, robust sliding control is adopted in designing the middle controller. The yaw rate controller is designed to control the yaw rate directly. While the slip rate controller is proposed to control the vehicle side slip angle indirectly through preventing a saturated tire force. The lower controller is designed to translate the virtual signals from the middle controller into actual actuator commands; it then transmits the commands to four wheels. Here, the tire force saturation, ground constraints and other factors are taken into account. Finally, the performance of the proposed torque distribution algorithm is evaluated using several typical simulation studies.

The rest of this paper is organized as follows. Section II presents the vehicle and the tire models. The torque distribution algorithm to improve the maneuverability and stability of electric vehicles driven by four in-wheel motors under emergency conditions is introduced in detail in Section III. Numerical case studies are conducted, and comparison results are analyzed in Section VI. Finally, conclusions and future work are drawn in Section V.

II. THE VEHICLE AND TIRE MODEL

This section presents the vehicle and tire models used for designing the torque distribution algorithm.

A. The planar vehicle model

Fig. 1 shows the relevant vehicle states and parameters for a planar vehicle model. The influence of the aerodynamic drag force, rolling resistance, steering system and suspension system are ignored.

![Fig. 1. The planar vehicle model](image)

The following three dynamic equations are constructed to balance the lateral forces, longitudinal forces, and moments about the vertical axis, respectively:

$$m(\ddot{v}_y - v_y \dot{\gamma}) = (F_{x1} + F_{x2}) \cos \delta_y + F_{x3} + F_{x4} - (F_{y1} + F_{y2}) \sin \delta_y$$

(1)
\( m \left( \ddot{v}_x + v_y \gamma \right) = (F_{x1} + F_{x2}) \sin \delta_f + (F_{y1} + F_{y2}) \cos \delta_f + F_{y3} + F_{y4} \)
\( I_z \gamma = l_f \left( F_{x1} + F_{x2} \right) \cos \delta_f - l_r \left( F_{y3} + F_{y4} \right) + \frac{l_w}{2} \left( F_{y1} - F_{y2} \right) \sin \delta_f + M_s \)

where \( v_x, v_y \) and \( \gamma \) are the longitudinal velocity, lateral velocity and yaw rate of the vehicle, respectively. \( F_{xi} \) and \( F_{yi} \) denotes the longitudinal and lateral tire force, respectively. \( i = 1, 2, 3 \) and \( 4 \), which represents the front left, front right, rear left and rear right wheel, respectively. \( m \) is the total mass of the vehicle, \( I_z \) is the moment of inertia about the yaw axis, \( \delta_f \) denotes the front wheel steering angle of the vehicle. \( l_f \) and \( l_r \) represent the distance from the center of gravity of the vehicle to the front and rear axles, respectively. \( l_w \) is the wheel tread of the vehicle. \( M_s \) is the direct yaw moment, which is generated by motor torque difference between the left and right wheels.

The direct yaw moment can be considered as an additional input variable to stabilize the vehicle motion and can be calculated as follows:
\[ M_s = \frac{l_w}{2} \left( F_{x2} - F_{x1} \right) \cos \delta_f + l_f \left( F_{x1} + F_{x2} \right) \sin \delta_f + \frac{l_w}{2} \left( F_{y4} - F_{y3} \right) \]

**B. The planar bicycle model**

The planar bicycle model represents the longitudinal and lateral stability characteristics of the vehicle and provides the basis for a dynamic analysis, as shown in Fig. 2 [19]. The motion of the vehicle can be expressed as follows:
\[ m(v + \gamma \dot{v}) = (F_{x1} + F_{x2}) \sin \delta_f + (F_{y1} + F_{y2}) \cos \delta_f + F_{y3} + F_{y4} \]
\[ I_z \dot{\gamma} = l_f (F_{x1} + F_{x2}) \cos \delta_f - l_r (F_{y3} + F_{y4}) + \frac{l_w}{2} (F_{y1} - F_{y2}) \sin \delta_f + M_s \]

where \( \dot{\beta} \) and \( \dot{\gamma} \) denote the side slip angular velocity and the yaw angular acceleration of the vehicle, respectively. \( F_{yi} = F_{x1} + F_{x2} \) and \( F_{yi} = F_{y3} + F_{y4} \) denote the lateral tire forces of the front and rear wheels, respectively.

The side slip angles of the front and rear tires can be calculated as follows:
\[ \alpha_f = \arctan \left( \frac{\beta + \frac{l_f \gamma}{v_x}}{v_y} \right) - \delta_f \]
\[ \alpha_r = \arctan \left( \frac{\beta - \frac{l_r \gamma}{v_x}}{v_y} \right) \]

where \( \alpha_f \) and \( \alpha_r \) are the front and rear tire slip angles, respectively. \( \beta \) is the side slip angle of the vehicle.

**C. The tire model**

An accurate tire model is important when designing the torque distribution algorithm. To ensure the performance of the torque distribution algorithm, the magic tire model is adopted to calculate the tire force except when calculating the desired yaw rate. The magic formula model is given as [20]:
\[ F = D \sin \left( C \arctan \left( B \alpha - E \left( B \alpha - \arctan (B \alpha) \right) \right) \right) \]

where \( B, C, D, \) and \( E \) are coefficients depending on the tire characteristic and road conditions. \( F \) is either the longitudinal or lateral tire force, where \( \alpha \) is the tire side slip angle (if \( F \) in equation (9) represents the lateral force). Fig. 3 shows the relationship between the lateral tire force and the side slip angle under different vertical forces.

**III. TORQUE DISTRIBUTION ALGORITHM**

**A. Structure of the torque distribution algorithm**

Fig. 4 shows the overall structure of the proposed torque distribution algorithm with the assumption that the tire force, tire-ground adhesion coefficient, and side slip angle can all be estimated. The speed of each wheel, the yaw rate signal and the steering angle are taken as inputs to the torque distribution algorithm (as shown in the dotted square of Fig. 4). The state estimator sends the unmeasurable estimated signals to the torque distribution algorithm. The torque distribution algorithm then distributes the torque to each wheel based on the signals from the sensors and the state.
estimator. The torque distribution algorithm is divided into three layers: the upper controller, the middle controller, and the lower controller. The upper controller is designed to judge whether the vehicle is in the stable state and whether the wheels are slipping; it then transmits the results to the middle controller. The middle controller is designed to turn on or off the yaw rate controller and each slip rate controller based on the commands from the upper controller; it then calculates the desired traction force and yaw moment. A sliding mode control methodology is adopted to guarantee the system stability and enhance the robustness by taking into account factors such as the lateral wind and sensor noise. The lower controller is designed to translate the virtual signals from the middle controller into actual actuator commands; it then transmits the commands to the four wheels.

B. Upper controller

The upper controller is designed to judge whether the vehicle is in the stable state utilizing the phase plane method. The slip rate of each wheel is calculated and compared with a preset threshold to judge whether the wheels are slipping.

1) Stability judgement

The $\beta - \dot{\beta}$ phase plane analysis is more effective at representing the vehicle nonlinear stability characteristic than the conventional state plane method with the state variable $\beta$ and the yaw velocity $\gamma$ [21]. In this paper, we used a diamond shape to represent the stable area [22].

Fig. 5 presents how to determine the control threshold from phase plane analysis. When the vehicle runs at 100 km/h and the steering wheel angle is 80 degrees, the stable area is determined as a diamond shape by the saddle points (c1, c3) and the intersections with the longitudinal axis (c2, c4). Define a positive factor of the control threshold as $f_{\lambda i}$ and is $0 < f_{\lambda i} \leq 1$. The control threshold factor for different running conditions can be determined by a lookup table, which is obtained by simulation results for various initial conditions with the same vehicle speed and steering angle.

\[
\lambda_i = \frac{V_{ti} - \omega_i r}{V_{ti}}\cdot \text{driving} \quad \text{or} \quad \frac{\omega_i r - V_{ti}}{\omega_i r} \cdot \text{braking} \quad i = 1, 2, 3, 4
\]  

where $\lambda_i$ is the slip ratio of the $i$th wheel. $\omega_i$ is the $i$th wheel angular speed. $V_{ti}$ denotes the longitudinal speed of the $i$th wheel, and $r$ denotes the tire radius.

When the $i$th wheel slip ratio exceeds the preset threshold ($\lambda_i > \lambda_{\text{max}}$), the upper controller will send the command ‘$\text{index}=1$’ to the middle controller, and the $i$th slip ratio controller will be activated. When the $i$th wheel slip ratio is lower than the preset threshold ($\lambda_i < \lambda_{\text{max}}$), the upper controller will send the command ‘$\text{index}=0$’ to the middle controller, and the $i$th slip ratio controller will be closed.

2) Slip rate calculation

A lower slip ratio leads to a greater lateral force coefficient for a tire and improves the vehicle stability. The slip ratio controller is designed to maintain the slip ratio of each wheel in a small value range. The slip ratio is defined as:

\[
\lambda_i = \frac{V_{ti} - \omega_i r}{V_{ti}}\cdot \text{driving} \quad \text{or} \quad \frac{\omega_i r - V_{ti}}{\omega_i r} \cdot \text{braking} \quad i = 1, 2, 3, 4
\]  

C. Middle controller

The middle controller consists of a yaw rate controller and four slip ratio controllers. The middle controller will activate or close the yaw rate controller and the slip ratio controller
according to the commands from the upper controller.

1) Yaw rate controller

The planar bicycle model represents the longitudinal and lateral stability characteristics of the vehicle and provides the basis for dynamic analysis. The target yaw rate is obtained by analyzing the planar bicycle model. A sliding mode control method is used to achieve the additional input \( M_d \) for tracking the target yaw rate.

The relationship between the steady-state steering angle and the road radius can be explained using the following equation:

\[
\frac{1}{R} = \frac{\delta_f}{L + K}
\]

(11)

where \( R \) is the turning radius of the vehicle. \( L = l + l_0 \) represents the wheel base of the vehicle from the front axle to the rear axle. \( K \) is the coefficient that can be determined using the following equation, where \( C_f \) and \( C_r \) denote the front and rear tire cornering stiffness, respectively.

\[
K = \frac{m v_s^2 (l C_f - l_0 C_f)}{2 C_f C_r L}
\]

The target yaw rate can be calculated based on the driver’s input signal using the tire model as:

\[
\gamma_i = \frac{v_s}{R} = \frac{v_s}{L + K} \delta_f
\]

(12)

where \( \gamma_i \) is the target yaw rate.

The desired yaw rate value is not always available and is influenced by many factors, such as the friction coefficient of the road. Thus, the desired value should not be directly used as the target yaw rate. Instead, the desired yaw rate can be calculated as follow [23]:

\[
\gamma_{des} = \begin{cases} 
\gamma_i, & \text{if } |\gamma_i| \leq 0.85 \frac{\mu g}{v_s} \\
0.85 \frac{\mu g}{v_s} \text{sign}(\gamma_i), & \text{if } |\gamma_i| > 0.85 \frac{\mu g}{v_s}
\end{cases}
\]

(13)

where \( \gamma_{des} \) is the desired yaw rate. \( \mu \) is the tire ground friction coefficient, and \( g \) is the gravitational acceleration.

In general, the tire cornering stiffness will change with the vertical force. To acquire a more precise target yaw rate, a follow-up cornering stiffness is defined as:

\[
C_f = \frac{C_1(F_{z1}) + C_2(F_{z2})}{2}
\]

(14)

\[
C_r = \frac{C_3(F_{z3}) + C_4(F_{z4})}{2}
\]

(15)

where \( C_i(F_{zi}) \) is the tire cornering stiffness of the \( i \)th tire under the vertical force \( F_{zi} \).

The sliding mode control is a robust control method to stabilize nonlinear and uncertain dynamical systems. The sliding mode control consists of the equivalent control and the switching control. The equivalent control keeps the state of system on the sliding surface, whereas the switching control forces the system to slide on the sliding surface [24].

By considering the external disturbance, the dynamic equation for the yaw motion can be expressed as follows:

\[
I \ddot{\gamma} = I_f (F_{yi} + F_{yi}) \cos \delta_f - l_r (F_{yi} + F_{yi})
\]

(16)

\[
+ \frac{l_2}{2} (F_{yi} - F_{yi}) \sin \delta_f + M_g + M_d
\]

where \( M_d \) is the external disturbance caused by several factors, such as the lateral wind, road roughness and sensor noise. At the same time, \( M_d \) is bounded and satisfies the following inequality function:

\[
|M_d| \leq D_1
\]

where \( D_1 \) is the upper bound of the external disturbance.

Ignoring the external disturbance, the sliding surface is defined as

\[
s = \gamma - \gamma_i
\]

Setting \( \dot{s} = 0 \), the following equation can be determined:

\[
\dot{s} = \gamma - \gamma_i = \frac{1}{I_z} \left\{ I_f (F_{yi} + F_{yi}) \cos \delta_f - l_r (F_{yi} + F_{yi}) \right\} - \frac{l_2}{2} (F_{yi} - F_{yi}) \sin \delta_f - \dot{M}_s
\]

(17)

Then, the equivalent control law is designed as

\[
M_{seq} = I \ddot{\gamma} + \frac{l_2}{2} (F_{yi} - F_{yi}) \sin \delta_f
\]

(18)

To satisfy the reaching conditions of the sliding mode control of \( s \leq -\eta |s| (\eta > 0) \), the switching control law is chosen as

\[
M_s = I_s K_1 \text{sgn}(s)
\]

(19)

where

\[
K_1 = \frac{D_1}{I_z} + \eta
\]

Because the sliding mode control is constituted by the equivalent control and the switching control, the sliding mode control law can be expressed as follows:

\[
M_s = I \ddot{\gamma} - \frac{l_2}{2} (F_{yi} - F_{yi}) \sin \delta_f + I_s K_1 \text{sgn}(s)
\]

(20)

To restrain the chattering phenomenon, the sign function, \( \text{sgn}(s) \), is substituted by the saturated function, \( \text{sat}(s) \), which is defined as:

\[
\text{sat}(s) = \begin{cases} 
1, & s > \phi_i \\
0, & -\phi_i \leq s \leq \phi_i \\
-1, & s < -\phi_i
\end{cases}
\]

(21)

where \( \phi_i \) is the “boundary layer” and \( K_2 = 1/\phi_i \).

2) Slip ratio controller

Because the electric vehicle has four driving wheels, four slip ratio controllers are designed to realize the slip ratio control. When the \( i \)th slip ratio controller is activated, the

\[
I \ddot{\gamma} = I_f (F_{yi} + F_{yi}) \cos \delta_f - l_r (F_{yi} + F_{yi})
\]

(16)

\[
+ \frac{l_2}{2} (F_{yi} - F_{yi}) \sin \delta_f + M_g + M_d
\]

where \( M_d \) is the external disturbance caused by several factors, such as the lateral wind, road roughness and sensor noise. At the same time, \( M_d \) is bounded and satisfies the following inequality function:

\[
|M_d| \leq D_1
\]

where \( D_1 \) is the upper bound of the external disturbance.

Ignoring the external disturbance, the sliding surface is defined as

\[
s = \gamma - \gamma_i
\]

Setting \( \dot{s} = 0 \), the following equation can be determined:

\[
\dot{s} = \gamma - \gamma_i = \frac{1}{I_z} \left\{ I_f (F_{yi} + F_{yi}) \cos \delta_f - l_r (F_{yi} + F_{yi}) \right\} - \frac{l_2}{2} (F_{yi} - F_{yi}) \sin \delta_f - \dot{M}_s
\]

(17)

Then, the equivalent control law is designed as

\[
M_{seq} = I \ddot{\gamma} + \frac{l_2}{2} (F_{yi} - F_{yi}) \sin \delta_f
\]

(18)

To satisfy the reaching conditions of the sliding mode control of \( s \leq -\eta |s| (\eta > 0) \), the switching control law is chosen as

\[
M_s = I_s K_1 \text{sgn}(s)
\]

(19)

where

\[
K_1 = \frac{D_1}{I_z} + \eta
\]

Because the sliding mode control is constituted by the equivalent control and the switching control, the sliding mode control law can be expressed as follows:

\[
M_s = I \ddot{\gamma} - \frac{l_2}{2} (F_{yi} - F_{yi}) \sin \delta_f + I_s K_1 \text{sgn}(s)
\]

(20)

To restrain the chattering phenomenon, the sign function, \( \text{sgn}(s) \), is substituted by the saturated function, \( \text{sat}(s) \), which is defined as:

\[
\text{sat}(s) = \begin{cases} 
1, & s > \phi_i \\
0, & -\phi_i \leq s \leq \phi_i \\
-1, & s < -\phi_i
\end{cases}
\]

(21)

where \( \phi_i \) is the “boundary layer” and \( K_2 = 1/\phi_i \).
corresponding controller will track the desired wheel speed according to the maximum slip ratio, $\lambda_{\text{max}}$. The following equation can be used to calculate the desired wheel speed of the $i$th wheel:

$$\omega_{i_{\text{des}}} = \begin{cases} \frac{v_{\text{ref}}}{r(1 - \lambda_{\text{max}})} & \text{driving} \\ \frac{v_{\text{ref}}}{r(1 - \lambda_{\text{max}})} & \text{braking} \end{cases}$$

(22)

Again, the sliding mode control method is used to calculate the control torque for each wheel according to its slip ratio.

The equation of motion for the $i$th wheel is shown as follows [25]:

$$J_i \ddot{\omega}_i = T_{ai} - r \cdot F_{si} + M_e$$

(23)

where $M_e$ is the external disturbance caused by several factors, such as the lateral wind, road roughness and sensor noise. $M_e$ is bounded and satisfies the following inequality function:

$$|M_e| \leq E_1$$

where $E_1$ is the upper bound of the external disturbance.

Ignoring the external disturbance, the sliding surface is defined as

$$s_{\omega} = \omega_i - \omega_{i_{\text{des}}}$$

Setting $\dot{s}_{\omega} = 0$, the following equation can be derived:

$$\dot{s}_{\omega} = \dot{\omega}_i - \dot{\omega}_{i_{\text{des}}} = \frac{1}{J_i} (T_{ai} - r \cdot F_{si} - \dot{\omega}_{i_{\text{des}}}) = 0$$

(24)

Then, the equivalent control law is designed as

$$T_{\text{eq}} = J_i \ddot{\omega}_{i_{\text{des}}} + r \cdot F_{si}$$

To satisfy the reaching conditions of the sliding mode control that $s_{\omega} \cdot \dot{s}_{\omega} = -\eta_i |s_{\omega}|, (\eta_i > 0)$, the switching control law is chosen as

$$T_{\text{switch}} = J_i \ddot{\omega}_{i_{\text{des}}} + r \cdot F_{si}$$

(25)

where

$$K_\omega = \frac{E_i}{J_i} + \eta_i$$

Finally, the sliding mode control law can be expressed as follows:

$$T_{\text{ms}} = J_i \ddot{\omega}_{i_{\text{des}}} + r \cdot F_{si} + K_\omega \text{sgn}(s_{\omega})$$

(26)

Similarly, the sign function $\text{sgn}(s)$ is substituted by the saturated function $sat(s)$, which is defined as:

$$sat(s_{\omega}) = \begin{cases} 1, s_{\omega} > \phi_1 \\ K_\omega s_{\omega}, |s_{\omega}| \leq \phi_2 \\ -1, s_{\omega} < \phi_2 \end{cases}$$

(27)

where $\phi_2$ is the “boundary layer” and $K_\omega = 1/\phi_2$.

D. Lower controller

The lower controller is designed to translate the virtual signals from the middle controller into actual actuator commands; it then transmits the commands to the four wheels. Control allocation problems can normally be addressed as optimization problems. If sufficient control power exists, the secondary objectives can be achieved. The optimization-based method is used to allocate the torque for each wheel of the electric vehicle.

1) Cost function

The design of the cost function takes two factors into consideration: the allocation error and the adhesion utilization.

The input of the lower controller is the total control effect, named the virtual control input $v \in R^2$. The output of the lower controller is the true control input, named the true control input $u \in R^4$. The relationship between $v(t)$ and $u(t)$ can be expressed as follows:

$$v = B_i u$$

(28)

where

$$B_i = \begin{bmatrix} \cos \delta_j & \cos \delta_j & 1 & 1 \\ r & r & 1 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} F_x \\ M_y \end{bmatrix}$$

$$u = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}$$

Because of the different physical conditions of each driving motor and different road adhesion conditions for each of the wheels, each driving motor may not generate enough torque to satisfy the requirements of the controller. In this case, it is common to seek a solution minimizing the torque allocation error. Minimizing the allocation error is the first control objective of the lower controller [26]. To simplify the calculation mathematically, the square of the allocation error is used to define the first cost function as follows:

$$J_1 = \|B_i u - v\|^2$$

(29)

Adhesion utilization can be defined as follows:

$$\eta_i = \frac{F_{si}^2 + F_{si}^2}{(\mu F_{si}^2)^2}$$

(30)

where $\eta_i$ is the adhesion utilization of the $i$th wheel. An adhesion utilization of 100% means that the tire reaches its adhesion limitation. Optimizing the adhesion utilization can improve the maneuverability and stability of the vehicle [27]. The longitudinal tire force can be controlled directly by allocating the torque of each driving motor. As a result, the following function is chosen as the second cost function:

$$J_2 = \sum_{i=1}^{4} \frac{T_{ui}^2}{(r \mu F_{si})^2}, (i = 1, 2, 3, 4)$$

(31)

Eq. (31) can be rewritten as follows:

$$J_2 = \|W_i u\|^2$$

(32)

where
\[ W_u = \text{diag}\left[ \frac{1}{(r \mu F_{s1})}, \frac{1}{(r \mu F_{s2})}, \frac{1}{(r \mu F_{s3})}, \frac{1}{(r \mu F_{s4})} \right] \]

The final cost function can be constructed as:
\[
J = J_1 + J_2 = \|W_u u\|^2 + \xi \|B(u - v)\|^2
\]
\[
= \left\| \frac{v^{0.5} B}{W_u} u - \left( \frac{v^{0.5}}{0} \right) \right\|^2 \tag{33}
\]
where \( \xi \) is a weighting factor.

2) Constraints

The design of the constraints considers three factors: the slip ratio constraint, the in-wheel motor constraint and the tire force constraint.

When the \( i \)th slip ratio controller is activated, the torque calculated by Eq. (26) is applied to the \( i \)th wheel directly. The equality constraint caused by the \( i \)th slip ratio controller is formulated as follows:
\[
T_i = T_{o\text{a}i} \tag{34}
\]

The equality constraint caused by the four slip ratio controllers can be written as follows:
\[
W_s u = T_o \tag{35}
\]
where
\[
W_s = \text{diag}\left[ \text{index1} \text{ index2} \text{ index3} \text{ index4} \right]
\]
\[
T_o = [T_{o\text{a}1} \ T_{o\text{a}2} \ T_{o\text{a}3} \ T_{o\text{a}4}]^T
\]

The in-wheel motor can generate different maximum torques at different speeds. Therefore, the speed-torque characteristic of the in-wheel motor should be considered when solving the allocation problem [28]. For simplification, the influence of the state of charge on the motor characteristic is ignored when developing the torque allocation algorithm. Fig. 6 shows the speed-torque characteristic curve of the in-wheel motor used in this paper.

![Speed-torque characteristic curve for the motors](Image)

The in-wheel motor constraint can be formulated as follows:
\[
T_{m_{\text{b, max}}} \leq T_i \leq T_{m_{\text{d, max}}} \tag{36}
\]
where \( M_{m_{\text{b, max}}} \) is the maximum braking torque of the \( i \)th in-wheel motor and \( M_{m_{\text{d, max}}} \) is the maximum driving torque of the \( i \)th in-wheel motor.

The tire-force ellipse has been used for many years to qualitatively illustrate the concept of tire-road force interactions and the force-limiting behavior of the combined tire forces [29]. Fig. 7 shows the relationship between the longitudinal and the lateral tire forces. The Fig. indicates that skidding would not occur and steering control would be maintained as long as the resultant force remains inside the enveloping line. The enveloping line can be formulated as follows:
\[
\left( \frac{F_{\|i}}{F_{\|\text{max}}} \right)^2 + \left( \frac{F_{\|i}}{F_{\|\text{max}}} \right)^2 \leq 1 \tag{37}
\]

where \( F_{\|\text{max}} \) is the maximum braking torque of the \( i \)th tire, and \( F_{\|\text{max}} \) is the maximum driving torque of the \( i \)th tire.

Assuming that \( F_{\|\text{max}} = F_{\|\text{max}} = \mu F_{\|} \), the above enveloping line can be rewritten as follows:
\[
(F_{\|})^2 + (F_{\|})^2 \leq (\mu F_{\|})^2 \tag{38}
\]

When the resultant tire force is sufficiently large, it is unreasonable to increase the motor torque. Otherwise, the tire would reach the limit of adhesion and the vehicle would be out of control. The tire force constraint can be formulated as follows:
\[
-r \sqrt{(\mu F_{\|})^2 - (F_{\|})^2} \leq T_i \leq r \sqrt{(\mu F_{\|})^2 - (F_{\|})^2} \tag{39}
\]

As a result, the constraints of this optimization problem are formulated as follows:
\[
W_s u = T_o, \quad T_i \geq \max \left\{ T_{m_{\text{b, max}}}, -r \sqrt{(\mu F_{\|})^2 - (F_{\|})^2} \right\}, \quad T_i \leq \min \left\{ T_{m_{\text{d, max}}}, r \sqrt{(\mu F_{\|})^2 - (F_{\|})^2} \right\} \tag{40}
\]

This optimization problem is a regular least squares problem and can be solved using active set methods.

VI. CASE STUDY
The performance of the proposed torque distribution algorithm is evaluated using a co-simulation of CarSim and Simulink software. Three simulation scenarios are conducted including the double lane change test, the slalom test and the fishhook test, as shown in Fig. 8. Steering with the closed-loop driver model is used in the double lane change and slalom tests. Open-loop steering is used in the fishhook test. The even distribution scheme was chosen as a comparison to show their performance. The even-distribution algorithm represents the vehicle distributes the desired longitudinal force and yaw moment to each executive wheel evenly. The proposed algorithm is evaluated by comparing the performances between vehicles with and without a controller. Table 1 shows the vehicle parameters used for the test.

**Table 1** Vehicle parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of vehicle</td>
<td>$m$</td>
<td>1429 kg</td>
</tr>
<tr>
<td>Distance from center of gravity to front axle</td>
<td>$l_f$</td>
<td>1.05 m</td>
</tr>
<tr>
<td>Distance from center of gravity to rear axle</td>
<td>$l_r$</td>
<td>1.569 m</td>
</tr>
<tr>
<td>Tire radius</td>
<td>$r$</td>
<td>0.357 m</td>
</tr>
<tr>
<td>Tread</td>
<td>$l_w$</td>
<td>1.565 m</td>
</tr>
<tr>
<td>Front tire cornering stiffness</td>
<td>$C_f$</td>
<td>-65520 N/rad</td>
</tr>
<tr>
<td>Rear tire cornering stiffness</td>
<td>$C_r$</td>
<td>-57200 N/rad</td>
</tr>
<tr>
<td>Moment of inertia about the yaw axis</td>
<td>$I_z$</td>
<td>1765 kg.m$^2$</td>
</tr>
<tr>
<td>Rated power of in-wheel motor</td>
<td>$P_{rated}$</td>
<td>18 kw</td>
</tr>
<tr>
<td>Peak power of in-wheel motor</td>
<td>$P_{peak}$</td>
<td>32 kw</td>
</tr>
<tr>
<td>Rated torque of in-wheel motor</td>
<td>$T_{rated}$</td>
<td>230 Nm</td>
</tr>
<tr>
<td>Peak torque of in-wheel motor</td>
<td>$T_{peak}$</td>
<td>400 Nm</td>
</tr>
</tbody>
</table>

**A. Double lane change test**

The double lane change test is an important experiment to examine the stability of the vehicle. The double lane change test is performed at a constant vehicle speed of 80 km/h (22.2 m/s) for the road path shown in Fig. 8(a). The friction coefficient of the road is 0.3. The dynamic response is shown in Fig. 9.

![Double lane change test](image1)

(a) Double lane change test

![Lateral Offset vs Position](image2)

(b) Velocity of vehicle with controller

![Yaw rate vs Time](image3)

(b) Yaw rate of vehicle with controller
vehicle side slip angle responses. Compared with the vehicle without controller, the vehicle with controller shows an apparently smaller sideslip angle. Fig. 9(e) shows the maximum slip ratio of the four wheels, and it can be seen that the maximum value does not exceed 0.1. Fig. 9(f) shows the external yaw moment generated by the in-wheel motor. The results indicate that the proposed algorithm can control the slip ratio well and distribute the torque to the four wheels according to the command of the driver. The vehicle with the controller performs better than the vehicle without the controller.

B. Slalom test

The slalom test is another important experiment to examine the stability of the vehicle. The standard slalom test is conducted on dry road conditions (friction coefficient of the road is 0.8 or greater). To demonstrate the advantage of the proposed algorithm, the slalom test is performed at a constant vehicle speed of 80 km/h (22.2 m/s) for the road path shown in Fig. 8(b), and the friction coefficient of road is 0.5. The dynamic response is shown in Fig. 10.
Fig. 10. Results for the slalom test

C. Fishhook test

To begin the fishhook test, the vehicle is driven in a straight line at a constant speed of 80 km/s (22.2 m/s) for 1 sec; the driver then releases the throttle and triggers the steering wheel input described in Fig. 8(c). The test is performed on dry pavement, and the tire-ground adhesion coefficient is 0.85.

Fig. 11(a) shows the velocity of the vehicle with a controller, where the vehicle velocity decreases from 22.2 m/s to 14.2 m/s due to resistance from the air and the road. Fig. 11(b-c) shows the yaw rate responses of the vehicle with and without a controller. It can be seen intuitively that the yaw rate of the vehicle without controller substantially fluctuates. The maximum tracking error of the yaw rate of the vehicle without controller reaches 0.2 rad/s. The tracking ability of the yaw rate of the vehicle with controller is substantially improved. Fig. 11(d) shows a comparison of two vehicle side slip angle responses. The maximum side slip angle of the vehicle without controller is 0.095 rad, and the mean square error is 0.0261 rad. The maximum side slip angle of the vehicle with controller is 0.04 rad, and the mean square error is 0.0204 rad. This indicates that the side slip angle of the vehicle with controller is close to zero. Fig. 11(e) shows the maximum slip ratio of the four wheels, and it can be seen that the maximum value does not exceed 0.2. Fig. 11(f) shows the external yaw moment generated by the in-wheel motor. The results indicate that the proposed algorithm shows high performance in the open-loop fishhook test.
A torque distribution algorithm for electric vehicles driven by four in-wheel motors under emergency conditions was proposed to improve stability. The torque distribution algorithm was divided into three layers: the upper controller, the middle controller and the lower controller. The diamond shape method was used to define the stable area in the upper controller. In the middle controller, a sliding mode control method, which can guarantee system stability and enhance robustness, was used to control the yaw rate and the slip ratio. In the lower controller, an optimization-based method was used to allocate the torque to the four wheels to indirectly control the vehicle side slip angle, which considers tire force saturation. To investigate the performance of the control algorithm, three tests were conducted in the CarSim and Simulink simulation environments. The simulation results show that the proposed algorithm can improve the handling stability of electric vehicles driven by four in-wheel motors in emergency conditions and can provide accurate data and example for the real vehicle experiment.

In our future works, we will improve the handling stability of electric vehicles driven by four in-wheel motors by considering the braking and active steering systems. At the same time, the estimation accuracy of the vehicle states is strongly influenced by the performance of the observer, and

![Figure 11: Results for fishhook test](image)

![Figure 12: Comparison results for phase plane on different tests](image)
the estimation error always exists. Therefore, the performance analysis according to the error distribution of the vehicle states should be considered in the future work.

REFERENCES


Energies, 8(8): 8537-8561.

