Dynamic Response Analysis for NW Planetary Gear Transmission used in Electric Wheel Hub

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ABSTRACT Taking the NW planetary gear transmission of an electric wheel hub as the research object, the mesh model was developed to calculate the mesh parameters using Masta. Then the lump parameter method was used to establish the dynamic model of the NW planetary transmission system considering the time-varying mesh stiffness, transmission error and bearing stiffness. The impacts of the torque load on the dynamic characteristics including the dynamic mesh force, center trajectory, offsets and load sharing coefficients were investigated. Results show that the dynamic mesh force waveform and spectral characteristics of each planetary gear at high and low speeds under the rated and maximum torque load are almost identical. Compared to the peak response at speed 485 rpm and 1770 rpm, both the magnitude and location of the third response at relative high speed about 4300 rpm are increased due to the load increasing. The mean and fluctuation range of the offsets in the radial direction are both larger than the tangential direction. As the load increases, the mean value of the radial offset increases and the magnitude decreases significantly. The mean value of the tangential offset decreases significantly, but the magnitude increases first and then decreases. The load sharing characteristics of the low-speed stage of the NW planetary gear is better than the first stage. With the increase of the load, the load sharing performance of a two-stage planetary transmission gets better and the difference in load sharing between the high-speed and low-speed stages also decreases.

KEYWORDS Electric wheel hub; NW planetary transmission; Dynamic characteristics; Load analysis; Offsets

I. INTRODUCTION

The electric wheel hub drive system refers to the vehicle drive that consists of a drive motor and a speed reducer or the drive motor alone. The wheel reducer is the final stage for speed reduction and torque amplification in the automotive driveline system. The electric wheel hub drive system greatly simplifies the complex power drive transmission of traditional vehicles with engine, gearbox and differential. However, the entire drive system consisting of the motor and reducer is assembled in a compact space, resulting in many unprecedented problems such as thermodynamic issues, electromechanical coupling and lightweight concerns, etc. The dynamic characteristics of the hub drive system also directly affect the motion and noise performance of the vehicle. Therefore, studying the dynamic characteristics of the wheel drive system of the electric vehicle is of great importance for improving the dynamic characteristics of the vehicle, shock absorption and noise reduction.

Many researchers investigated the dynamic characteristics of geared rotor system [1], [2]. Lin and Parker [3], [4] established a single-stage planetary transmission dynamic model and analyzed its natural frequencies and mode shapes. Chaari [5] investigated the dynamic characteristics of planetary gear transmission considering the influence of eccentricity and manufacturing error. Later, nonlinear dynamics of planetary gears was analyzed based on a finite element model [6]. After that, an experimental study determined the effects of certain types of manufacturing errors on the gear stress of the planetary gear set [7]. Furthermore, a generalized dynamic model of a multi-stage planetary gear train of an automotive transmission was proposed to study the forced vibration response to the gear mesh excitations [8]. The dynamic model was proposed considering rotational degree of freedom for a general compound planetary gear set [9]. The nonlinear dynamics of the gear-bearing system was analyzed by taking into consideration the nonlinear bearing force, oil film force and gear mesh force [10]. Studies performed by Gu and Velex [11], [12] revealed the component trajectory and load ratio of planetary gears considering eccentricity error and position error. Also, the dynamics of a wind turbine planetary gear set under gravity using a harmonic balance method with real-time excitation was studied [13]. The effects of axial eccentric load on the mesh
stiffness were analyzed for the planetary gear transmission system [14]. For the NW-type planetary transmission, the inherent characteristics of the NW-type spur planetary transmission was analyzed [15]. Based on loaded contact analysis, the mesh characteristics of NW planetary transmission was discussed by taking into consideration the influence of assembly error and gear tooth profile [16]. The load sharing characteristics of NW planetary gear trains was studied [17]. In conclusion, the aforementioned studies focused on single-stage or multi-stage planetary gear transmission and the inherent characteristics. Not much attention has been paid to the detailed dynamic response for NW planetary gears, especially for the electric wheel hub application.

In this paper, the NW planetary gear transmission of electric wheel hub is taken as the research object. The dynamic model was developed by the lumped parameter method. Then, the dynamic response of the system is calculated considering the time-varying mesh stiffness and transmission errors. The impacts of the torque load on the dynamic characteristics including the dynamic mesh force, center trajectory, offsets and load sharing coefficients were investigated.

II. STRUCTURE AND TRANSMISSION PRINCIPLE OF ELECTRIC HUB REDUCER

As shown in Figure 1, the electric wheel hub drive is mainly composed of a motor on the left side and a NW-type planetary gear set on the right side. The transmission principle diagram of the planetary reducer is shown in Figure 2, where \( s, r, c, na, \) and \( nb \) represent the sun gear, the inner ring gear, the planet carrier, the high-speed stage planetary gear, and the low-speed stage planetary gear of the NW planetary gear transmission, respectively.

The overall structure of the hub is attached to the suspension of the vehicle by the hollow stub axle on the left side. The motor stator is fixed to the stub axle, while the motor rotor and the wheel rim are mounted on the stub axle through bearings. The input power is generated by the outer motor rotor and is transited to the sun gear of NW planetary gear transmission through the spline. For the NW planetary gear transmission, the inner ring gear is fixed to the stub axle and the output power is transferred to the rim through two planetary gear stages.

![Figure 1. Structure schematic](image1)

![Figure 2. Transmission schematic](image2)

The main geometric parameters are shown in Table I. Then a professional gear design software MASTA was used to establish the mesh model for electric hub reducer. As shown in Figure 3, the main shaft is fixed and the wheel rim and the motor rotor are mounted on stub axle through bearings.

<table>
<thead>
<tr>
<th>Number of planetary wheels</th>
<th>Sun</th>
<th>Planetary na</th>
<th>Planetary nb</th>
<th>Inner ring</th>
<th>Carrier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure angle(°)</td>
<td>22.5</td>
<td>22.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modulus/(mm)</td>
<td>2.5</td>
<td>2.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Center distance/(mm)</td>
<td>145</td>
<td>145</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of teeth</td>
<td>49</td>
<td>67</td>
<td>18</td>
<td>91</td>
<td></td>
</tr>
<tr>
<td>Tooth width/(mm)</td>
<td>22.5</td>
<td>21.5</td>
<td>40</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Mass/(kg)</td>
<td>83</td>
<td>3.3</td>
<td>0.6</td>
<td>150</td>
<td>160</td>
</tr>
<tr>
<td>Moment of inertia/(kgm²)</td>
<td>3.7</td>
<td>0.0016</td>
<td>0.000066</td>
<td>3.5</td>
<td>9.6</td>
</tr>
</tbody>
</table>

The motor rotor and the sun gear are connected by a designed spline. The load operating parameters of the electric hub reducer are shown in Table II, in which the load condition 3 refers to the rated working condition and the load working condition 5 refers to the maximum torque working condition. The time-varying mesh stiffness, transmission error and the bearing support stiffness were calculated by MASTA software. The main mesh parameters for the NW planetary gear transmission are shown in Table III. Note that \( s-na, r-nb \) (\( n = 1, 2, 3 \)) represent the outer and the inner mesh pair of each planet gear, respectively.
The electric hub reducer adopts spur gear transmission without axial load. As shown in Figure 4, the translational degrees of freedom for the center wheels (sun gear, inner ring gear, planet carrier) in horizontal and vertical directions (x, y) and the rotational degree of freedom (θ) are considered.

### TABLE II
NW Planetary Transmission Load Operating Parameters

<table>
<thead>
<tr>
<th>Load condition</th>
<th>Motor torque (Nm)</th>
<th>Motor speed (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
<td>2600</td>
</tr>
<tr>
<td>2</td>
<td>750</td>
<td>870</td>
</tr>
<tr>
<td>3</td>
<td>1250</td>
<td>520</td>
</tr>
<tr>
<td>4</td>
<td>1750</td>
<td>380</td>
</tr>
<tr>
<td>5</td>
<td>2250</td>
<td>320</td>
</tr>
<tr>
<td>6</td>
<td>2750</td>
<td>220</td>
</tr>
</tbody>
</table>

### TABLE III
Mesh Parameters for NW Planetary Gear Transmission

<table>
<thead>
<tr>
<th>Tooth pairs</th>
<th>Mesh stiffness $K_{sn}$, $K_{rn}$</th>
<th>Transmission error $\epsilon_{sn}$ - $\epsilon_{rn}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean (10^6 N/m)</td>
<td>Amplitude (°)</td>
</tr>
<tr>
<td>x-1a</td>
<td>6.6</td>
<td>9.0</td>
</tr>
<tr>
<td>x-2a</td>
<td>6.6</td>
<td>9.9</td>
</tr>
<tr>
<td>x-3a</td>
<td>6.6</td>
<td>20.4</td>
</tr>
<tr>
<td>r-1b</td>
<td>11.4</td>
<td>2.0</td>
</tr>
<tr>
<td>r-2b</td>
<td>11.4</td>
<td>2.0</td>
</tr>
<tr>
<td>r-3b</td>
<td>11.4</td>
<td>2.0</td>
</tr>
</tbody>
</table>

### III. Dynamic Modeling for Electric Hub Reducer System

The purpose of the deriving of the relative displacement between components is to project different vibration displacements onto the line of action. Figure 5 is a relative displacement relationship diagram between the components, $\alpha_n$ and $\alpha_0$ denote the outer and inner mesh gear pressure angles. $r_{bn}$ and $r_{bn}$ represent the base circle radius of the sun gear and the inner ring gear. In addition, $r_c$ represents the upper mounting radius of the planet gear in the planet carrier. While, $r_{bn}$, $r_{bn}(n = 1, 2, 3)$ represent the base circle radius of the first and second planetary gears. $\psi_{sn}$, $\psi_{rn}(n = 1, 2, 3)$ represent the position angles of the outer and inner mesh lines relative to the central wheels. $\epsilon_{sn}$, $\epsilon_{rn}(n = 1, 2, 3)$ indicate the tooth error excitation of the external and internal mesh.
The dynamic transmission error of the external mesh between the sun gear and the high-speed stage planetary gear can be represented by

\[
\delta_{bn} = -x_l \sin \phi_{bn} + y_l \cos \phi_{bn} + x_{bn} \sin \alpha_a \\
-\gamma_n \cos \alpha_a + r_c \beta_l + r_{bat} \theta_{bat} + \epsilon_{bn}
\]  
(1)

Similarly, the dynamic transmission error of the internal mesh between the sun gear and the second-stage planetary gear can be represented by

\[
\delta_{bn} = -x_l \sin \phi_{bn} + y_l \cos \phi_{bn} + x_{bn} \sin \alpha_a \\
-\gamma_n \cos \alpha_a + r_c \beta_l + r_{bat} \theta_{bat} + \epsilon_{bn}
\]  
(2)

The relative displacement between the first-stage planetary gear and the planet carrier can be represented by

\[
\delta_{na} = x_s \sin \phi_n + y_c \cos \phi_n - x_{na} \\
\delta_{nby} = -x_s \cos \phi_n + y_c \sin \phi_n + r_c \beta_l - \gamma_n
\]  
(3)

The relative displacement between the second-stage planetary gear and the planet carrier can be represented by

\[
\delta_{na} = x_s \sin \phi_n + y_c \cos \phi_n - x_{na} \\
\delta_{nby} = -x_s \cos \phi_n + y_c \sin \phi_n + r_c \beta_l - \gamma_n
\]  
(4)

The relative displacement between the planet carrier (rim) and the inner ring gear (spindle) can be represented by

\[
\delta_{sex} = x_s - x_s \\
\delta_{sey} = y_c - y_c
\]  
(5)

The relative displacement between sun gear (outer rotor of the motor) and the inner ring gear (spindle) can be represented by

\[
\delta_{sex} = x_s - x_s \\
\delta_{sey} = y_c - y_c
\]  
(6)

In which \( \phi_{sn} = \phi_n - \alpha_s, \phi_{tn} = \phi_n + \alpha_r \).

**B. System Dynamics Equations**

Electromagnetic torque \( T_m \) is generated between the motor rotor and motor stator as input power, and the effect of the ground on the tire is transmitted to the rim (planetary) to form a torque load \( T_l \). The mass of each component is represented by \( m_i \) and its moment of inertia is represented by \( I_i \), where \( i = s, r, c, na, nb \). The kinetic differential equations can be obtained by the Lagrangian energy method. The differential equations of motion for sun gear can be represented by

\[
m_s \ddot{x}_s = (k_{rcx} \dot{\delta}_{rcx} + c_{rcx} \dot{\delta}_{rcx}) - \sum_{n} (k_{sn} \dot{x}_n + c_{sn} \dot{\theta}_n) \sin \phi_{sn} \\
= \sum_{n=1}^{N} k_{sn} \dot{x}_n + c_{sn} \dot{\theta}_n \sin \phi_{sn} \\
+ m_s \ddot{y}_c = (k_{rcy} \dot{\delta}_{rcy} + c_{rcy} \dot{\delta}_{rcy}) + \sum_{n} (k_{sn} \dot{x}_n + c_{sn} \dot{\theta}_n) \cos \phi_{sn} \\
= - \sum_{n=1}^{N} (k_{sn} \dot{x}_n + c_{sn} \dot{\theta}_n) \cos \phi_{sn} \\
J_s \ddot{\theta}_s + \sum_{n=1}^{N} r_n (k_{sn} \dot{x}_n + c_{sn} \dot{\theta}_n) = T_m - \sum_{n=1}^{N} r_n (k_{sn} \dot{x}_n + c_{sn} \dot{\theta}_n)
\]  
(7)

The differential equations of motion for planet carrier can be represented by

\[
\sum_{n=1}^{N} (k_{nrx} \dot{\delta}_{nrx} + c_{nrx} \dot{\delta}_{nrx}) \sin \phi_{sn} = - \sum_{n=1}^{N} (k_{nrx} \dot{x}_n + c_{nrx} \dot{\theta}_n) \sin \phi_{sn} \\
+ m_s \ddot{x}_s + (k_{rcx} \dot{\delta}_{rcx} + c_{rcx} \dot{\delta}_{rcx}) + \sum_{n=1}^{N} (k_{nrx} \dot{x}_n + c_{nrx} \dot{\theta}_n) \cos \phi_{sn} \\
+ m_s \ddot{y}_c + (k_{rcy} \dot{\delta}_{rcy} + c_{rcy} \dot{\delta}_{rcy}) + \sum_{n=1}^{N} (k_{nry} \dot{\theta}_n + c_{nry} \dot{\theta}_n) \cos \phi_{sn} \\
- \sum_{n=1}^{N} r_n (k_{sn} \dot{x}_n + c_{sn} \dot{\theta}_n) = T_l + \dot{L}_c - \sum_{n=1}^{N} r_n (k_{nrx} \dot{x}_n + c_{nrx} \dot{\theta}_n)
\]  
(8)

The differential equations of motion for the first-stage planetary gear can be represented by

\[
\sum_{n=1}^{N} (k_{nrx} \dot{\delta}_{nrx} \cos \theta_{sn} + c_{nrx} \dot{\delta}_{nrx}) - m_s \ddot{\theta}_s = \\
\sum_{n=1}^{N} (k_{nrx} \dot{\theta}_n + c_{nrx} \dot{\theta}_n + k_{nax} \dot{x}_n + c_{nax} \dot{\phi}_n) \cos \phi_{sn} \\
- \sum_{n=1}^{N} (k_{nax} \dot{x}_n + c_{nax} \dot{\phi}_n + k_{hax} \dot{\theta}_n + c_{hax} \dot{\phi}_n) \sin \phi_{sn} \\
- \sum_{n=1}^{N} (k_{hny} \dot{\theta}_n + c_{hny} \dot{\theta}_n + k_{hby} \dot{\phi}_n + c_{hby} \dot{\phi}_n) \cos \phi_{sn}
\]  
(9)

The differential equations of motion for the second-stage planetary gear can be represented by

\[
m_{na} \ddot{x}_{na} + (k_{naa} \dot{x}_{na} + c_{naa} \dot{\phi}_{na}) - (k_{sn} \dot{x}_n + c_{sn} \dot{\theta}_n) \sin \phi_{sn} \\
= (k_{sn} \dot{x}_n + c_{sn} \dot{\theta}_n) \sin \phi_{sn} \\
m_{na} \ddot{y}_{na} + (k_{nay} \dot{\phi}_{na} + c_{nay} \dot{\phi}_{na}) - (k_{sn} \dot{x}_n + c_{sn} \dot{\theta}_n) \cos \phi_{sn} \\
= (k_{sn} \dot{x}_n + c_{sn} \dot{\theta}_n) \cos \phi_{sn}
\]  
(10)

The differential equations of motion for the second-stage planetary gear can be represented by

\[
m_{nb} \ddot{x}_{nb} + (k_{nby} \dot{x}_{nb} + c_{nby} \dot{\phi}_{nb}) - (k_{rn} \dot{x}_n + c_{rn} \dot{\theta}_n) \sin \phi_{sn} \\
= (k_{rn} \dot{x}_n + c_{rn} \dot{\theta}_n) \sin \phi_{sn} \\
m_{nb} \ddot{y}_{nb} + (k_{nby} \dot{\phi}_{nb} + c_{nby} \dot{\phi}_{nb}) - (k_{rn} \dot{x}_n + c_{rn} \dot{\theta}_n) \cos \phi_{sn} \\
= (k_{rn} \dot{x}_n + c_{rn} \dot{\theta}_n) \cos \phi_{sn}
\]  
(11)
The above equations are organized and expressed as a matrix form, and the dynamic equation of the system is obtained as follows

\[ M\ddot{q} + C\dot{q} + Kq = Q \]  

Where M, C, K and Q represent the mass matrix, the damping matrix, the stiffness matrix, and the excitation matrix, respectively. \( q \) is the coordinate matrix, which is given by

\[ q = [x_a, y_a, \theta_a, x_r, y_r, \theta_r, x_c, y_c, \theta_c, x_w, y_w, \theta_w, \theta_{na}] \]  

C. Load Sharing Coefficient

The load sharing coefficient is an important indicator for characterizing the load distribution of planetary gear transmissions. By solving the above system dynamic model using Runge-Kutta numerical integration method, the internal and external mesh forces \( F_{\text{int}} \) and \( F_{\text{ext}} \) of the corresponding planetary gears can be obtained. According to equations (14-17), the load sharing coefficient \( g_{\text{int}} \) and \( g_{\text{ext}} \) corresponding to each planet in the internal and external mesh of the train can be calculated. Also, the first-stage external mesh and the second-stage internal mesh load sharing coefficient \( G_{\text{int}} \) and \( G_{\text{ext}} \) can be obtained.

\[ g_{\text{int}} = \frac{N \times F_{\text{int}}}{\Sigma_{n=1}^{N} F_{\text{int}}} \]  

\[ g_{\text{ext}} = \frac{N \times F_{\text{ext}}}{\Sigma_{n=1}^{N} F_{\text{ext}}} \]  

\[ G_{\text{int}} = \frac{N \times \text{max}(f_{\text{int}})}{\Sigma_{n=1}^{N} \times \text{max}(f_{\text{int}})} \]  

\[ G_{\text{ext}} = \frac{N \times \text{max}(f_{\text{ext}})}{\Sigma_{n=1}^{N} \times \text{max}(f_{\text{ext}})} \]  

IV. Dynamic Characteristic Analysis

A. Dynamic Mesh Force

The dynamic mesh force of the high and low speed stages under the rated load in time and frequency domain are shown in Figures 6 and 7. It shows that the waveforms of the dynamic mesh forces in time domain for two planetary stages are basically similar, but the magnitudes of the dynamic mesh forces are inconsistent. The spectral components of the dynamic mesh force are highly consistent for two planetary stages. The main frequency components of the dynamic mesh forces are \( f_0 \) and \( f_6 \), which are mesh frequencies for the first external stage and the second internal stage, respectively. In terms of amplitude, the low-speed stage \( f_6 \) is roughly only 1/10 of the high-speed stage \( f_0 \).

Since the power is directly transmitted from the motor to the tire through the planetary gear transmission, the torque load has a wide range. Therefore, it is necessary to study the impacts of torque load on the dynamic characteristics. The speed sweeping
up analysis for dynamic mesh force under different torque load levels are shown in Figures 9 and 10. From the speed sweeping up analysis, the dynamic mesh forces increase both for the low-speed and high-speed stages with the increase of the torque load. Also, the main magnitudes of the main first peak response in 485 rpm and second peak response in about 1770 rpm is increased. However, for the third main peak response, both the magnitude and location are increased due to the increase of the torque load. For the low-speed stage, the location of the maximum dynamic mesh force is changed from higher rotation speed about 4300 rpm to relative lower rotation speed 1770 rpm.

![Figure 9](image1.png)

**FIGURE 9.** Sweep up analysis of the first stage external mesh force under variable torque loads. (a) The planetary gear 1a. (b) The planetary gear 2a. (c) The planetary gear 3a.

![Figure 10](image2.png)

**FIGURE 10.** Sweep up analysis of the second stage internal mesh force under variable torque loads. (a) The planetary gear 1b. (b) The planetary gear 2b. (c) The planetary gear 3b

### B. Center Trajectory and Offset Analysis for Planetary Gear

The dual planetary gears are mounted on the same planet gear shaft, but mesh with the sun gear and inner ring gear, respectively. This will cause a shearing and bending effects for the two gear stages on the planetary shaft as a result of biased gear loaded condition. Therefore, it is very necessary to quantify the amount of offset between the centers of the two planetary gears. The vibration displacement response of the planetary gear center relative to the planet carrier is shown in Figure 11. The planetary gear has a driving action along tangential direction of the planet carrier, so that the range of the vibration displacement distribution in this direction is larger. For radial direction, the mesh force directions of sun gear and ring gear are opposite, so the displacements are distributed on both positive and negative sides. Setting the origin located at the center of low-speed stage planetary gear system, the offset distribution between the planet wheels as shown in Figure 12 can be identified by the relative position between the two stages of planet wheels. It shows that the offsets of the three planets are distributed in almost overlapped flat ellipses. The mean and fluctuation range of the offsets in the radial direction are both larger than the tangential direction.

![Figure 11](image3.png)

**FIGURE 11.** Center track diagram of two planetary gear stages
The spectrum analysis of offsets under rated load is shown in Figure 13. The locations of main components for the offsets in frequency domain for both directions are consistent with the dynamic mesh force, which are the mesh frequencies of the two planetary stages. And the high-speed stage makes a major contribution. However, the magnitude of the main frequency component for radial offset is much larger than the tangential direction, as opposed to the case where the planetary gears have a larger range of tangential fluctuations than radial direction.

Although the fluctuating range of the planet trajectory increases as the load increases, the offset between the coaxial planet gears decreases significantly. The spectrum analysis of offset under peak torque load is shown in Figure 16. It shows that the frequency components of the inter-planetary offsets along two directions do not change, but the amplitudes of the main frequency components decrease with the increase of load.

The vibration displacement response distribution of the center of each planet relative to the planet carrier under the peak torque load is shown in Figure 14. Each of the planet gear trajectories exhibits a large flattened elliptical shape along the tangential direction. As the load increases, the fluctuating range of the planet trajectory becomes larger. And tangential direction is more obvious than the radial direction. The position distribution of the offset of each planetary gear under the peak torque load is shown in Figure 15.

Table V

| Different Load Conditions for NW Planetary Gear Transmission |
|-------------|-------------|
| Torque (Nm) | Velocity (rpm) |
| Case 1      | 250         | 2600        |
| Case 2      | 750         | 870         |
| Case 3      | 1250        | 520         |
| Case 4      | 1750        | 380         |
| Case 5      | 2250        | 320         |
| Case 6      | 2750        | 220         |

Figure 17 shows the mean offsets between two stages of planetary gears under different loads. The load levels are listed in Table V. From Figure 17(a), the mean value of the radial offset increases and the magnitude decreases significantly as the load increases. In Figure 17(b), as the load increases, the mean value of the tangential offset decreases significantly, but
the magnitude increases first and then decrease. Figure 18 shows the variance and peak-to-peak value of the average offset in radial and tangential direction. As the load increases, the variation and the peak-to-peak values of in radial offset drops. However, the variance and peak-peak values of tangential offset shows a tendency to increase first and then decrease as the load increases.

**FIGURE 17.** Mean offset under variable loads. (a) Radial direction. (b) Tangential direction.

**FIGURE 18.** Fluctuations in the average offset under different loads. (a) Variance of average offset. (b) Peak-to-peak value of average offset.

### C. Dynamic load sharing coefficient

Through the calculation in Section 3.3, the dynamic load sharing coefficients of the two stages can be obtained. Figure 19 shows the load sharing coefficients under rated load and maximum torque load conditions.

**FIGURE 19.** Load sharing coefficient for rated and maximum load conditions. (a) Rated torque load. (b) Maximum torque load.

It can be seen that dynamic load sharing coefficients are at a good level, 1.008 for rated torque load and about 1.003 under peak torque load. For rated torque load condition, the dynamic load sharing coefficients are almost the same both for the high and low speed planetary gear pairs. But for maximum torque load level, the dynamic load sharing coefficients are better for the low speed stage than the high speed stage.

**FIGURE 20.** Load sharing coefficients under variable loads. (a) High speed stage. (b) Low speed stage.

**FIGURE 21.** The maximum load sharing coefficients under variable loads

Figure 20 shows the change of the load sharing coefficients of the planetary gear transmission under different load conditions, which is listed in Table V. It shows that the load sharing performance of the two-stage planetary transmission is at a very good level, even in the worst case, the maximum load factor of 1.035. The load sharing characteristics of the high speed stage with lower torque load are slightly worse than the low speed level with relative higher torque load. A closer look at the load-carrying characteristic curve reveals that as the torque load increases, the two-stage load-carrying characteristic curve tends to be completely uniform.

Figure 21 shows the variation of the maximum load sharing coefficients under different load conditions, which more intuitively reflects the effect of load on the load sharing coefficient. The figure shows that the load sharing performance
of a two-stage planetary transmission get better as the torque load increases. The difference in load sharing between the high-speed and low-speed stages also decreases with the increase of the load. Corresponding to the previous load-sharing characteristic curve, when the load increases, the two-stage load-carrying characteristics tend to be consistent.

V. Conclusions

(1) The dynamic mesh force waveform and spectral characteristics of each planetary gear at high and low speeds under the rated and maximum torque load are almost identical. Higher torque load tends to increase the magnitudes of the peak responses. Compared to the peak response at speed 485 rpm and 1770 rpm, both the magnitude and location of the third response at relative high speed about 4300 rpm are increased due to the increase of the torque load.

(2) The mean and fluctuation range of the offsets in the radial direction are both larger than the tangential direction. As the load increases, although the fluctuating range of the planet trajectory increases, the offset between the coaxial planet gears decreases significantly. The mean value of the radial offset increases and the magnitude decreases significantly. The mean value of the tangential offset decreases significantly, but the magnitude increases first and then decreases.

(3) The load sharing characteristics of the low-speed stage of the NW planetary gear is better than the first stage. With the increase of the load, the load sharing performance of a two-stage planetary transmission gets better. The difference in load sharing between the high-speed and low-speed stages also decreases with the increase of the load. In addition, it is suggested that the stiffness of planet carrier should be increased and the offset between planet gears should be decreased to ensure the high performance of the wheel hub.

REFERENCES

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