Hybrid chaos communication with Code Index Modulation

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ABSTRACT Hybrid chaos communication (HCC) is promising for covert transmission by delivering confidential information via chaotic signals. To enhance the transmission rate and the energy efficiency while maintaining a low probability of interception, in this paper, we propose a code index modulation aided HCC (CIM-HCC) scheme. Specifically, the input bit stream is split into the chaotic signaling stream, the modulating stream, and the code indexing stream. The chaotic signaling stream is modulated by the hybrid chaos signal and the code indexing stream is mapped onto the Walsh codes. The modulating stream is multiplied to the Walsh codes. Thus, the proposed CIM-HCC achieves higher data rate and lower power consumption than that of the conventional HCC. Another advantage is that, since part of the information bits are conveyed by the indices of the Walsh codes, the CIM-HCC outperforms the conventional HCC and binary phase-shift keying (BPSK) in terms of reliability. In particular, when the number of indexing bits equals to that of the signaling bits, the proposed CIM-HCC doubles the data rate and saves about 50% power consumption. Additionally, numerical results show that, the proposed scheme outperforms the code index modulation differential chaos shift keying in terms of bit error rate over both the additive white Gaussian noise channel and the multipath Rayleigh fading channels.

INDEX TERMS Chaos communication, code index modulation, data rate.

I. INTRODUCTION

The future wireless networks are expected to provide higher spectral efficiency, better reliability and security. Hybrid chaos communication (HCC), which conveys information via wideband chaotic signals, constitutes a potential candidate for reliable and covert transmission. In this paper, we investigate a code index modulation aided HCC (CIM-HCC) scheme, which benefits from high transmission rate, low power consumption, enhanced security and satisfactory error rate.

The HCC is a promising chaos communication scheme based on the hybrid chaos system [1]. The chaos signal used in this scheme is generated by the hybrid chaos system defined by a continuous differential equation and a discrete switching condition with symbolic information encoded in the chaos waveform. Previous works show that it outperforms the correlation-based chaos communication schemes such as differential chaos shift keying (DCSK), coherent chaos shift keying (CSK) in terms of BER performance. Owing to the improved BER performance and simplicity in implementation, this newly-proposed chaos system has been intensively investigated in chaos communication [2]–[7], chaos radar [8], [9], and underwater ranging and navigation [10]. However, most of the communication schemes based on the hybrid chaos system focus on the experimental implementation in circuit and encoding method, and little attention has been given to improve the data rate and the energy efficiency of the conventional HCC.

Another promising modulation technique referred to as index modulation (IM) has been developed to offer a higher data rate, energy and spectral efficiencies yet relatively simple realization [11]–[14]. Extra information can be conveyed via various physical resources such as transmit antennas, subcarriers, spreading code, time slots and so on. To achieve a higher data rate and energy efficiency while keeping the low probability of interception, several schemes combining index modulation and chaos communication are proposed. Among these schemes, carrier index DCSK (CI-DCSK) utilizes the subcarrier frequencies to convey information and achieves higher data rate and energy efficiency compared with the conventional DCSK [15]. Commutation code index DCSK (CCI-DCSK) was proposed to exploit the orthogonality be-
tween the commuted chaotic sequences to convey information [16]. In addition, code index modulation DCSK (CIM-DCSK) was proposed to encode extra bits onto the Walsh code [17], [18]. These DCSK variants with IM techniques outperform the conventional DCSK scheme in BER performance and significantly improve the data rate. Nevertheless, they all have a delayed-correlation-based structure so that they perform worse than the coherent chaos communication schemes in BER.

Based on the hybrid chaos system and the code index modulation technique, this paper proposes a CIM-HCC scheme to improve the reliability and data rate. The bit stream is divided into the chaotic signaling bits, the modulating bit and the code indexing bits. The chaotic signaling bits are transmitted via hybrid chaos signal and the code indexing bits are mapped onto Walsh code. The modulating bit is multiplied by the Walsh code, which is the same with the direct sequence spread spectrum (DSSS). Then a DS-BPSK signal conveying the selected Walsh code and the modulating bit is generated with the carrier frequency as integer multiples of the basic frequency of the chaos signal. After that, the transmitted signal is obtained by adding the DS-BPSK signal and the hybrid chaos signal up. At the receiver, the chaotic signaling bits and the code indexing bits are recovered individually with no interference to each other. The chaotic signaling bits are estimated by the differential-integral-based non-coherent detector. The code indexing bits are retrieved by demapping the estimated code index which is estimated via a correlation-based detector. The modulating bit is estimated as the sign of the correlation output of the estimated code index. Since the hybrid chaos signal is orthogonal to the DS-BPSK signal, the intervention of the chaos signal has no impact on demodulating the code indexing bits and the modulating bit. Therefore, the proposed scheme offers a higher data rate than the conventional HCC. The analytical BER performance over the additive white Gaussian noise (AWGN) channel is derived and then validated by the computer simulations. The error performance of the proposed scheme over multipath Rayleigh fading channels is discussed by simulations.

The advantages of the proposed scheme are three-fold. First, it utilizes the index of the Walsh code to convey the extra information bits and thus it achieves a higher data rate and energy efficiency than the conventional HCC. Second, the transceiver structure is much simpler than CIM-DCSK which requires a complicated process of frame forming. Third, the hybrid chaos signal is orthogonal to the DS-BPSK signal so that they share a common bandwidth and the spectral efficiency increases correspondingly. Both the theoretical analysis and the simulation results are provided to show the advantages of our scheme.

The rest of this paper is organized as follows. In section II, the structure of the proposed scheme is described. In section III, the BER performance, the energy efficiency and the breakability are analyzed and discussed. In section IV, the BER performance of CIM-HCC and the parameters influence are tested through computer simulations. Finally, section V concludes the paper.

II. SYSTEM MODEL
A. HYBRID CHAOS COMMUNICATION SCHEME

The chaos signal used in HCC is generated from a differential equation defined as

\[ \ddot{u} - 2\beta \dot{u} + (\omega^2 + \beta^2)(u - s) = 0 \]

where \( \omega \) determines the symbol period as \( T = 2\pi/\omega; \beta \in (0, 1/2 \ln 2) \) affects the fluctuating amplitude. The basic frequency of the hybrid chaos system is \( f_0 = \frac{\omega}{2\pi} \). The state \( s \in \{\pm 1\} \) is set as the sign of current \( u(t) \) once the following guard condition meets and it keeps unchanged until the next time when the condition arrives

\[ \dot{u} = 0 \Rightarrow s = \text{sgn}\{u(t)\}. \]

Hence \( s(t) \) is a continuous rectangular waveform with samples at the integer multiples of \( T \) as \( s_m = s(mT) \), where \( m \) is an integer. For the segment of \( mT \leq t < (m + 1)T \), the exact analytic solution of (1) is

\[ u(t) = s_m + (u_m - s_m)e^{\beta(t-mT)}(\cos \omega t - \frac{\beta}{\omega} \sin \omega t) \]

with the iteration relation \( u_{m+1} = e^{\beta T}/\omega u_m - (e^{2\beta T}/\omega - 1)s_m \), where the return points of \( u(t) \) are \( u_m = u(mT) \). Based on the iteration relation, the \( N \)th return point \( u_N \) and the former state samples \( \{s_0, s_1, ..., s_{N-1}\} \) determine the initial value \( u_0 \) of the chaos generator (1) as

\[ u_0 = e^{-N\beta T}u_N + (1 - e^{-\beta T}) \sum_{m=0}^{N-1} s_m e^{-m\beta T} \]

where \( m \) is an integer. From the analytic solution (3) and the iteration relation, \( u(t) \) can also be expressed as a linear convolution of a basis function and a symbol sequence as

\[ u(t) = \sum_{m=0}^{\infty} s_m P(t - mT) \]

where \( m \) is an integer and the basis function \( P(t) \) is given by

\[ P(t) = \begin{cases} (1 - e^{-\beta T})e^{\beta t}(\cos \omega t - \frac{\beta}{\omega} \sin \omega t), & t < 0 \\ 1 - e^{\beta(t-T)}(\cos \omega t - \frac{\beta}{\omega} \sin \omega t), & 0 \leq t < T \\ 0, & T \leq t. \end{cases} \]

The iteration relationship (4) implies that the future symbol sequence determines the initial value when \( N \) is large.
enough. Base on it, an easy encoding method of mapping the information symbols into the initial value is proposed in [4]. The block diagram of the hybrid chaotic communication system based on a non-coherent detector is shown in Fig. 1. The input bits stream \( \{s_{m}\} \) \((m = 0, 1, ..., N - 1)\) determines the initial value \(u_0\), then the chaotic waveform \(u(t)\) is generated by solving the differential equation (1). The received signal over AWGN channel is modeled as

\[
r(t) = u(t) + n(t)
\]

(7)

where the white Gaussian noise term \(n(t)\) has zero mean and power density \(N_0/2\). At the receiver, the difference between the received signal and its time-delayed version is computed as

\[
\tilde{\eta}(t) = u(t + T) - u(t) + n(t + T) - n(t)
\]

\[
= \sum_{m=0}^{\infty} s_m P(t + T - mT) - P(t - mT))
+ n(t + T) - n(t)
\]

(8)

and the integral is

\[
\eta(t) = \int_0^t \tilde{\eta}(\tau)d\tau = \sum_{m=0}^{\infty} s_m I(t, m) + \xi
\]

(9)

where \(I(t, m) = \int_0^T (P(t + T - mT) - P(t - mT))d\tau\); \(\xi\) is the noise term. The samples of the integral result at \(t = mT\) can be derived as \(\eta(mT) \approx s_m T + \xi\), where \(m\) is an integer. By extracting the signs of the samples, the information sequence \(\{s_m\}\) is recovered as

\[
\hat{s}_m = \text{sgn}\{\eta(mT)\}, m = 0, 1, ..., N - 1.
\]

(10)

Fig. 2 demonstrates a typical waveform in the DI-HCC scheme with basic frequency \(f_0 = 1\) kHz and \(\beta = f_0 \ln 2\). The continuous waveform \(u(t)\) and the rectangular waveform \(s(t)\) are shown in Fig. 2(a). After the differential-integral processing, the integral result \(\tilde{\eta}(t)\) and the samples at \(t = mT\) are shown in Fig. 2(b). From the comparison between the sampling dots and the symbol sequence \(s(t)\), we find that the symbols can be recovered by comparing the samples with the zero threshold, as expressed in (10).

**B. TRANSMITTER DESIGN**

The structure of CIM-HCC transceiver is shown in Fig. 3. Suppose a segment of bit stream has \(N_1 + N_2 + 1\) bits. The information bits are firstly divided into chaotic signaling bits \(\{a_i\}|i = 0, 2, ..., N_1 - 1\) and the samples at \(mT\) and the symbol sequence \(N\) power density \(\sum_{m=0}^{N-1} P(t + T - mT) - P(t - mT))\) and the samples at \(t = mT\) are shown in Fig. 2(b). From the comparison between the sampling dots and the symbol sequence \(s(t)\), we find that the symbols can be recovered by comparing the samples with the zero threshold, as expressed in (10).

![Figure 2](image-url) (a) The typical waveform of the transmitter signal \(u(t)\) and the transmitted symbol sequence \(s(t)\) of DI-HCC. (b) The integral results \(\eta(t)\) and the samples at \(t = mT\).

**FIGURE 2.**

![Figure 3](image-url) The block diagram of the transceiver of CIM-HCC.

**FIGURE 3.**

The block diagram of the transceiver of CIM-HCC.

The information bits are firstly divided into chaotic signaling bits \(\{a_i\}|i = 0, 2, ..., N_1 - 1\) and chaotic signaling bits \(\{b_i\}|i = 0, 2, ..., N_2 - 1\) and a modulating bit \(d\) by a commutation operation to bring disorder into the transmitted bits. As shown in the upper branch, the chaotic signaling bits \(\{a_i\}\) are encoded into the regular hybrid chaotic waveform \(u(t)\) by the hybrid chaos generator. The hybrid chaos signal can be expressed in the convolution form as

\[
u(t) = \sum_{i=0}^{N_1-1} a_i P(t - iT).
\]

(11)

In the bottom branch in Fig. 3, the code indexing bits \(\{b_i\}\) are mapped onto a symbol \(j \in \{1, 2, ..., M\}\), where \(M = 2^{N_2}\). Then the symbol selects the \(j\)th Walsh code \(W_j\) from a predetermined Walsh code set \(\{W_1, W_2, ..., W_M\}\). The \(i\)th Walsh code is \(W_i = \{w_{i1}, w_{i2}, ..., w_{iM}\}\) where \(w_{ik} \in \{\pm 1\}, k = 1, 2, ..., M\). After the selection, a rectangular waveform is generated by the selected Walsh code as

\[
c_j(t) = \sum_{i=1}^{M} w_{ij} P_{r_w}(t - iT_w)
\]

(12)

where \(P_{r_w}(t)\) is a rectangular waveform with a width of \(T_w\). In the middle branch, the modulating bit \(d\) is multiplied to \(c_j(t)\), which is the same with DSSS. Then a DS-BPSK signal is generated, in which carrier frequency is \(f_c = K f_0\) where \(K\) is an integer and \(f_0\) is the basic frequency of the hybrid chaos signal. After that, the transmitted signal is composed by combining the hybrid chaos signal and the as

\[
s(t) = Au(t) + Ao dc_j(t) \sin(2\pi f_c t)
\]

(13)

where \(d \in \{\pm 1\}\) is the modulating bit; \(\alpha\) is the amplitude of the DS-BPSK signal; \(A\) is the normalized factor to constraint the average signal power as \(P_s = 1\). Considering the power of hybrid chaos signal \(P_a\) is fixed [1], we have
recovered chaotic signaling bits are estimated as

\[ u_A = \frac{1}{\sqrt{P_\text{s} + \frac{1}{2}k\alpha^2}}. \]

Since the total duration time of the hybrid chaos signal encoding \( N_1 \) chaotic signaling bits equals to the duration time of the DS-BPSK signal, we have \( N_1 = \frac{M T_s}{2} = M \frac{L f_c}{f}, \) where \( L \) is an integer and \( L = T_w f_c. \)

Fig. 4 shows the typical waveform of the hybrid chaos signal \( u(t), \) the DS-BPSK signal \( \alpha(t) \) and the transmitted signal \( s(t) \) with the parameters of \( f_0 = 1 \text{ KHz}, \alpha = 1 \) and \( f_c = 2 \text{ KHz}. \) The number of the chaotic signaling bits \( N_1 \) is 16, and the number of the code indexing bits \( N_2 \) is 5. The modulating bit is set as 1. From the waveform of \( s(t), \) we find that the transmitted signal sees no obvious feature of the conventional DS-BPSK signal. In fact, the interference of the hybrid chaos signal makes it difficult to decode the Walsh code that DS-BPSK signal conveys by the regular BPSK receiver, which benefits the low probability of interception.

C. Receiver Design

The receiver has two branches for demodulating the chaotic signaling bits, the modulating bit, and the code indexing bits, as shown in Fig. 3. The received signal over AWGN channel is

\[ r(t) = s(t) + n(t) \]

where \( n(t) \) is the white Gaussian noise with the power spectrum density of \( N_0. \) The received signal \( r(t) \) is input to the upper and the bottom branches simultaneously to recover the information bits. In the upper branch, the chaotic signaling bits \( \{\hat{b}_i\} \) are estimated by the non-coherent differential-integral-based detector, as shown in Fig. 1. The integral of the difference between the received signal and its delayed version is

\[ \eta_r(t) = \int_0^t (r(\tau + T) - r(\tau))d\tau. \]

By sampling the integral results at a fixed time \( t = i T \), the recovered chaotic signaling bits are estimated as

\[ \hat{a}_i = \text{sgn}\{\eta_r(iT)\}, i = 0, 1, ..., N_1 - 1. \]

In the bottom branch, the received signal is first multiplied by a local carrier with the frequency as \( f_c. \) Then we have the correlation integral of all the \( M \) rectangular waveforms generated by the Walsh codes as

\[ I_k = \int_0^{N_i T} r(t)c_k(t)\sin(2\pi f_c t)dt, k = 1, 2, ..., M. \]  (17)

The selected Walsh code can be detected by comparing the absolute values of all the correlator outputs and selecting the maximum. The recovered symbol \( \hat{j} \) is estimated as

\[ \hat{j} = \arg \max_k \{|I_k|\}, k = 1, 2, ..., M. \]  (18)

Then the code indexing bits \( \{\hat{b}_i\} \) are recovered by demapping the estimated \( \hat{j}. \) Based on the recovered Walsh code index, the modulating bit is determined as

\[ \hat{d} = \text{sgn}\{I_{\hat{j}}\}. \]  (19)

Finally, after the inverse commutation operation on \( \{\hat{a}_i\}, \{\hat{b}_i\} \) and \( \hat{d} \) to recover the normal order of all the bits, the information bits stream is retrieved.

III. Performance Analysis

In this section, we first derive the BER of the CIM-HCC system over AWGN channel. Then we analyze the data rate and energy efficiency compared to the conventional HCC system. After that, the breakability of the proposed scheme is discussed.

A. BER Performance

The total BER \( P_e \) of this system is a weighted sum of the BER of the chaotic signaling bits \( P_{\text{ehcc}}, \) the BER of the code indexing bits \( P_{\text{emap}} \) and the BER of the modulating bit \( P_{\text{emod}}. \) Since \( N_1 \) chaotic signaling bits, \( N_2 \) code indexing bits and one modulating bit are transmitted in duration \( N_1 T, \) the total BER is given by

\[ P_e = \frac{N_1}{N_1 + N_2 + 1} P_{\text{ehcc}} + \frac{N_2}{N_1 + N_2 + 1} P_{\text{emap}} + \frac{1}{N_1 + N_2 + 1} P_{\text{emod}}. \]  (20)

To derive the exact expression of \( P_e, \) we first define \( E_{b0}, \) i.e., the bit signal-to-noise ratio (SNR) as

\[ \frac{E_b}{N_0} = \frac{E_s}{N_1 + N_2 + 1} \frac{E_s}{N_0} = \frac{N_i T}{2(N_1 + N_2 + 1)T_s} SNR \]  (21)

where \( E_s \) is the symbol energy; \( SNR = \frac{P_{\text{emod}}}{P_N} \) and \( P_N = \sigma^2 \) is the average power of noise; \( T_s \) is the sampling period which determines the sampling frequency of the system and the discrete transmitted signal can be expressed as \( s_i = s(iT_s), \) where \( i = 0, 1, ..., \frac{N_i T}{T_s}. \)
1) Error probability for detecting the code indexing bits

The correlator output for the $k$th Walsh code is expressed as

$$I_k = \int_0^{N_1T} A u(t)c_k(t)\sin(2\pi f_c t)dt + \int_0^{N_1T} A e\delta c_j(t)\sin(2\pi f_c t)\sin(2\pi f t)dt + \int_0^{N_1T} n(t)c_k(t)\sin(2\pi f_c t)dt.$$  

(22)

One of the properties of hybrid chaos signal is the orthogonality between itself and the sinusoid waveforms with frequencies of integer multiples of the basic frequency of the chaos signal. Consider the cross-correlation of the hybrid chaos signal $u(t)$ and the sine waveform $\sin(2\pi n f_0 t)$ as

$$c(\tau) = \int_{-\infty}^{\infty} u(t)\sin(2\pi nf_0(t-\tau))dt = \sum_{m=0}^{\infty} s_m \int_{-\infty}^{\infty} P(t-mT)\sin(2\pi nf_0(t-\tau))dt$$  

(23)

where $n$ is an integer and $f_0$ is the basic frequency of the hybrid chaos system. The integral term $\int_{-\infty}^{\infty} P(t-mT)\sin(2\pi nf_0(t-\tau))dt$ is proven to be zero [1]. Hence, $c(\tau)$ keeps zero. Then the hybrid chaos signal $u(t)$ is orthogonal to the DS-BPSK signal approximately for large integration period $N_1T$ or Walsh code length, and the mean of the first term in (22) is zero approximately. Considering the ideal orthogonality among the different Walsh codes, we have the correlator output as

$$I_k = \left\{ \begin{array}{ll} \frac{N_1 A_\alpha}{2} + \chi, & k = j \\ \chi, & k \neq j \end{array} \right.$$  

(24)

where $\chi = \int_0^{N_1T} n(t)c_k(t)\sin(2\pi f_c t)dt$ is the noise term with zero mean and variance $\sigma_\chi^2 = \frac{N_1 T_\delta}{2} \sigma_n^2$. As proven in (18), the code indexing bits are retrieved by demapping the estimated symbol $\hat{j}$ from determining the maximum absolute value of the correlator output $I_k$. The error probability of detecting the selected Walsh code is

$$P_{ec} = \Pr\{|I_j| < \max\{|I_k|\}, 1 \leq k \leq M\}.$$  

(25)

Let $Y = |I_j|$. Then $Y$ follows folded normal distribution with the probability density function as

$$f_Y(y) = \frac{1}{\sqrt{2\pi} \sigma_Y} \left( \exp\left( -\frac{(y - \mu_Y)^2}{2\sigma_Y^2} \right) + \exp\left( -\frac{(y + \mu_Y)^2}{2\sigma_Y^2} \right) \right)$$  

(26)

where $\mu_Y$ and $\sigma_Y^2$ are mean and variance of $Y$, given by

$$\mu_Y = \sqrt{\frac{2\sigma_Y^2}{\pi}} \exp\left( -\frac{\mu_Y^2}{2\sigma_Y^2} \right),$$  

(27)

$$\sigma_Y^2 = \mu_Y^2 + \sigma_\chi^2 - \mu_Y^2$$  

(28)

where $\mu_I = \frac{N_1 A_\alpha}{2}$. Let $X = \max\{|I_k|\}, 1 \leq k \neq j \leq M$. Since all the correlator outputs are independent, we have the error probability of detecting the symbol $\hat{j}$ as

$$P_{ec} = \Pr(Y < X) = \int_0^{\infty} \Pr(Y < X)f_Y(y)dy = \int_0^{\infty} (1 - \Pr(X \leq Y))f_Y(y)dy$$

$$= \int_0^{\infty} \left( 1 - \prod_{n=1}^{M-1} \Pr(|I_n| \leq y) \right)f_Y(y)dy$$

$$= \frac{1}{\sqrt{2\pi} \sigma_Y} \int_0^{\infty} \left( 1 - \left( \frac{1 - \text{erf}\left( \frac{y}{2\sigma_Y} \right)\sqrt{M-1}}{\sqrt{M-1}} \right) \right)$$

$$\left( \exp\left( -\frac{(y - \mu_Y)^2}{2\sigma_Y^2} \right) + \exp\left( -\frac{(y + \mu_Y)^2}{2\sigma_Y^2} \right) \right)dy$$  

(29)

where $\text{erf}(\cdot)$ denotes the error function. The error in the symbol detection causes the corresponding errors in the demapping process of the code indexing bits. The expectation of the number of errors in the code indexing bits is given as

$$Q = \frac{1}{M - 1} \sum_{i=1}^{N_2} \binom{N_2}{i}$$  

(30)

where $\binom{n}{m} = \frac{n!}{m!(n-m)!}$. Then the error probability of the code indexing bits is

$$P_{emap} = \frac{Q}{N_2} P_{ec}.$$  

(31)

2) Error probability for detecting the modulating bit

As shown in Fig. 3, detecting the modulating bit $d$ depends on the correct estimation of the code index $j$. Therefore, the error of detecting the modulating bit occurs in two different cases. The first case is when the code index is recovered correctly, but the modulating bit $d$ is detected with an error. The second case is when there is an error in the code index detection and thus the modulating bit has an error probability of $\frac{1}{2}$ under this condition. Hence, the error probability of detecting the modulating bit is a sum of the error probability in two cases, which is expressed as

$$P_{mod} = \frac{1}{2} P_{ec} + P_{SS}(1 - P_{ec}).$$  

(32)

where $P_{SS} = 0.5\text{erfc}\left( \frac{\sqrt{E_b}}{N_0} \right)$ is the BER for the conventional DS-BPSK and $\text{erfc}(\cdot)$ denotes the complementary error function.
The integral of the differential $\eta(t) = r(t + T) - r(t)$ is composed of three terms as

$$
\eta(t) = A \sum_{i=0}^{N_1-1} a_i (P(t + T - iT) - P(t - iT)) + A\alpha d \int_0^t c_j (\tau + T) \sin(2\pi f_c (\tau + T)) d\tau - A\alpha d \int_0^t c_j (\tau) \sin(2\pi f_c \tau) d\tau + \xi
$$

where the noise term $\xi = - \int_{T-T}^T n(\tau) d\tau$ is normally distributed with zero mean and variance $\frac{N_0}{2} T$. At the sampling time $t = i T$, the integrals of the sine waves are zero. Hence, the samples of the integral are

$$
\eta(iT) \approx A a_i T + \xi,
$$

which is the same as (9). Obviously, the added BPSK signal has no impact on the demodulation process for the chaotic signaling bits encoded in the hybrid chaos signal. The error probability of detecting the chaotic signaling bits is

$$
P_{\text{errc}} = P(\eta(iT) < 0|a_i = 1) P(a_i = 1) + P(\eta(iT) > 0|a_i = -1) P(a_i = -1)
$$

where $P(a_i = 1) = P(a_i = -1) = 0.5$. Then we have

$$
P_{\text{errc}} = \frac{1}{2} \text{erfc} \left( \frac{\mathbb{E}(\eta(iT))}{\sqrt{2D(\eta(iT))}} \right)
$$

where $\mathbb{E}(\cdot)$ denotes the mean; $D(\cdot)$ denotes the variance. The average signal power of the hybrid chaos signal is

$$
P_s = 1 + \frac{1 - e^{-\beta T}}{2\beta T} \left( \omega^2 - \frac{2e^{2\beta}}{\beta^2} \right).
$$

Finally, the total BER of CIM-HCC over AWGN channel is obtained by substituting the (36), (32) and (31) into average BER expression (20).

### B. DATA RATE AND ENERGY EFFICIENCY

In the chaotic hybrid system, each information bit is transmitted in duration $T$, thus the data rate is 1 bit/T. In the proposed CIM-HCC scheme, $N_1$ modulated bits are transmitted through the regular HCC and $N_2$ code indexing bit are mapped to the selection of the Walsh code. Additionally, one modulating bit is multiplied by the Walsh code. Hence, this process enhances the data rate to $\frac{N_1 + N_2 + 1}{N_1}$ bit/T. Apparently, the data rate $R_b$ increases as the ratio of $N_2$ to $N_1$ grows, as shown in Table 1.

To evaluate the energy efficiency of the proposed scheme, the bit energy gain to the conventional HCC scheme is computed. The average power of the hybrid signal in HCC $P_{sH}$ and the signal power in CIM-HCC $P_{sC}$ are equivalent to 1. According to the definition, we have the bit energy of the hybrid chaos signal as

$$
E_b H = P_{sH} T.
$$

And the bit energy of the proposed CIM-HCC scheme is

$$
E_b C1H = \frac{N_1}{N_1 + N_2 + 1} P_{sC} T.
$$

Then the relative bit energy gain $\Delta E_b$ is computed as

$$
\Delta E_b = \frac{E_b C1H - E_b H}{E_b H} = -\frac{N_2 + 1}{N_1 + N_2 + 1} \times 100\%.
$$

Obviously, the relative bit energy gain $\Delta E_b$ keeps negative, meaning the CIM-HCC scheme is more energy efficient than the conventional HCC. For the fixed $N_1$, the scheme saves more bit energy when $N_2$ is larger. Table 1 lists different values of $\Delta E_b$ with various $N_1$ and $N_2$. Obviously, the proposed scheme is more energy efficient when $\frac{N_1}{N_2}$ is larger.

### C. BREAKABILITY

The proposed scheme combines the hybrid chaos signal and the DS-BPSK signal together to convey information bits in a common bandwidth. As derived above, the interference of the DS-BPSK signal has no impact on recovering the chaotic signaling bits. However, the hybrid chaos signal makes it difficult to demodulate the Walsh code carried by the DS-BPSK signal via a typical BPSK receiver. At the conventional BPSK demodulator, the received signal is first multiplied by a local carrier with a frequency of $\frac{f_c}{2}$ and the result is

$$
\hat{z}(t) = A\alpha d c_j(t) \sin(2\pi f_c t) \sin(2\pi f_0 t) + A u(t) \sin(2\pi f_c t) + n(t) \sin(2\pi f_0 t).
$$

As (3) suggests, the analytical expression for the segment of $mT < t < (m + 1)T$ can be rewritten as

$$
u(t) = s_m + (u_m - s_m) e^{\beta(t - mT)} B \cos(2\pi f_0 t + \varphi)
$$

where $B = (1 + \frac{\beta^2}{2\pi^2})^{-1}$; $\varphi = \arctan\left(\frac{\beta}{2\pi}\right)$. Hence, the segmented $z(t)$ is

$$
z(t) = A\alpha d c_j(t) \left( \frac{1}{2} - \frac{\cos(2\pi f_0 t)}{2} \right) + A s_m \sin(2\pi f_c t) + A (u_m - s_m) e^{\beta(t - mT)} B \cos(2\pi f_0 t + \varphi) \sin(2\pi f_c t) + n(t) \sin(2\pi f_0 t).
$$

Based on the structure of BPSK receiver, $z(t)$ is then input

### Table 1. The data rate of CIM-HCC and the relative difference of bit energy between CIM-HCC and HCC.

<table>
<thead>
<tr>
<th>$N_1$</th>
<th>$R_b$ (bit/T)</th>
<th>$\Delta E_b$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>1.55</td>
<td>1.78</td>
</tr>
<tr>
<td>16</td>
<td>1.31</td>
<td>1.44</td>
</tr>
<tr>
<td>32</td>
<td>1.16</td>
<td>1.22</td>
</tr>
</tbody>
</table>
to a low-pass filter. However, the third term in (43) is hard to be filtered for the existence of low frequency components. Hence, the filter result includes not only the rectangular part $\alpha_c(t)$ and the noise part but also the interference part caused by the hybrid chaos signal. Consequently, it is difficult for the potential attackers to retrieve the Walsh code conveyed by the DS-BPSK signal via a typical BPSK receiver.

IV. RESULTS AND DISCUSSION

In this section, computer simulations are conducted to evaluate the BER performance and the energy efficiency of CIM-HCC. Fig. 5 shows the theoretical BERs given in (20) and the BERs of BPSK, DI-HCC and CIM-HCC over AWGN channel. Suppose the basic frequency of the hybrid chaos signal $f_0$ is 1 kHz. The amplitude and the carrier frequency of the BPSK signal are fixed as $\alpha = 1$ and $f_c = 2f_0$. The number of chaotic signaling bits and the code indexing bits are set as $N_1 \in \{8, 16, 32\}$ and $N_2 \in \{3, 4, 5\}$ correspondingly. Since CIM-DCSK and CIM-HCC are conceptually similar index modulation schemes that utilize code as the new dimension to enhance the data rate, the theoretical performance of CIM-DCSK is demonstrated. In the simulation of CIM-DCSK, the repeated number of the chaotic sequence is set as 4 and the spreading factor equals to 256. Compared with CIM-DCSK, the proposed CIM-HCC shows significant improvement in BER performance because the hybrid chaos communication outperforms the delayed-correlation-based chaos shift keying scheme. Compared to DI-HCC, the proposed scheme shows different performance with various system parameters; when $N_1$ equals to 8, the BER shows around 0.5 dB gain. When $N_1$ equals to 16, the BER sees no significant change; when $N_1$ increases to 32, the BER deteriorates around 0.5 dB. Hence, it is necessary to analyze the parameters influence on the BER performance.

As the analytical expression implies, the theoretical BER is affected by the numbers of the chaotic signaling bits and code indexing bits $N_1$, $N_2$ and the amplitude of the BPSK signal $\alpha$. We compare the simulated BER curves with various parameter settings. First, we focus on the performance difference with various combinations of $N_1$ and $N_2$ with a fixed amplitude parameter $\alpha = 1$. The results are depicted in Fig. 6. For the same number of the code indexing bits $N_2$, the BER performance shows significant improvement when $N_1$ is smaller. When the number of chaotic signaling bits $N_1$ is fixed, the performance improves as $N_2$ increases. Note that the performance for the parameter combinations of $N_1 = 16$
and \( N_2 = 8 \) is almost the same with that of \( N_1 = 8 \) and \( N_2 = 4 \). Apparently, the BER performance is related to the ratio \( \frac{N_1}{N_2} \). As the ratio \( \frac{N_1}{N_2} \) increases, the performance shows a gradual enhancement. When \( \frac{N_1}{N_2} \) increases to \( \frac{1}{2} \), the proposed scheme performs better than the BPSK scheme when \( \frac{E_b}{N_0} \) is larger than 4 dB. However, it is worth note that the BER performance Fig. 7 shows the BER comparison for various amplitude parameter \( \alpha \) when \( N_1 \) and \( N_2 \) are set as 16 and 8 respectively. When \( \alpha \) is larger than 1, the BERs sees a dramatic deterioration as \( \alpha \) increases. When \( \alpha \) equals to 0.8, the BERs sees a slight improvement.

Fig. 8 compares the BER performances of CIM-HCC and the optimize CIM-DCSK over multipath Rayleigh fading channels with 2 and 3 paths. All paths have an equal average power gain. The delays are assumed to be far less than the symbol period. In the simulation of CIM-HCC, the maximum delay spread is limited as \( \tau_m = 0.17T \) where \( T \) is the symbol period. The parameters are \( N_1 = 16, N_2 = 8, \) and \( \alpha = 1 \). In the simulation of the optimize CIM-DCSK, the maximum delay is limited as \( \tau_m = 0.1SF T_c \), where \( SF = 256 \) is the spreading factor and \( T_c \) is the sampling period of the chaotic signal. The number of code index bit \( m_c \) is set as 4 in the optimized CIM-DCSK. The BER curves show that the proposed CIM-HCC provides a considerable performance gain over the optimize CIM-DCSK over multipath Rayleigh fading channels.

To evaluate the breakability of the proposed scheme, we use the coherent BPSK detector to demodulate the Walsh code conveyed in the received signal. The system parameter is set as \( N_1 = 16, N_2 = 4 \) and \( \alpha = 1 \). The BER curve is illustrated in Fig. 9. Obviously, it is difficult to retrieve any bits conveyed by the BPSK signal from the received CIM-HCC waveform, as proved in (43). Hence, the proposed scheme achieves a lower probability of the interception.

**V. CONCLUSION**

This paper proposed a highly efficient chaos communication scheme based on the amalgamation of hybrid chaos system and code index modulation. The chaotic signaling bits are encoded in the hybrid chaotic waveform and the code indexing bits are mapped onto the Walsh code. The modulating bit is multiplied to the Walsh code and generates the DS-BPSK signal. Hence, the proposed scheme achieves higher data rate and better energy efficiency than the conventional HCC. At the receiver, the chaotic signaling bits are retrieved by a difference-integral-based detector while the code indexing bits are recovered by estimating the active code conveyed by the DS-BPSK signal. The BER performance over AWGN channel is derived and analyzed. The performance over multipath Rayleigh fading channels is then discussed. The results suggest that CIM-HCC outperforms CIM-DCSK with 5 to 7 dB in BER performance over AWGN channel, and CIM-HCC offers a considerable performance gain over CIM-DCSK over multipath Rayleigh fading channels. Meanwhile, the Walsh code conveyed in the DS-BPSK signal is difficult to be detected by a typical BPSK demodulator which is due to the interference of the chaos signal. Hence, the proposed scheme improves the security and covertness compared to conventional DS-BPSK.

**REFERENCES**


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