
JIAYU ZHANG\textsuperscript{1}, JIE LI\textsuperscript{1}, YUGANG HUANG\textsuperscript{2}, CHENJUN HU\textsuperscript{3}, KAIQIANG FENG\textsuperscript{1}, and XIAOKAI WEI\textsuperscript{1}

\textsuperscript{1}Key Laboratory of Instrumentation Science & Dynamic Measurement, North University of China, Taiyuan 030051, China
\textsuperscript{2}The 60th Research Institute of General Staff Dept of P.L.A, NanJing 210000, China
\textsuperscript{3}Suzhou Fashion Nano Technology Co.,Ltd., SuZhou 215000, China

Corresponding author: Jie Li (lijie@nuc.edu.cn).

This work was supported in part by the National Natural Science Funds for Distinguished Young Scholars (51225504), the National Natural Science Foundation of China (51575500, 51705477), the Fund for Shanxi ‘1331 project’ Key Subject Construction and the Foundation for Middle-Aged and Young Talents in Higher Education Institutions.

ABSTRACT The Rotation Semi-Stripdown Inertial Navigation System (RSSINS) provides a new solution for the high-precision measurement of high-spinning projectile flight parameters. However, the installation error between the complex semi-strapdown mechanical structures is inevitable. Coupled with the carrier motion, the system installation error seriously affects the navigation accuracy of the system, which should be accurately calibrated and compensated. Aiming at the problem, the calibration and compensation methods are proposed in this study. Firstly, the generation mechanism of the installation errors and the error propagation equation are analyzed and derived. Furthermore, the influence of each installation error angle on attitude is analyzed by simulation. Based on the analysis, taking MEMS device errors into account, a graded calibration method suitable for the system is proposed. And according to the characteristics of the calibration method, the corresponding compensation method is proposed. Finally, experiments have been carried out to validate the performance of the method, which show that the proposed method can compensate the installation errors effectively, and the measurement accuracy of RSSINS has been improved.

INDEX TERMS Rotation semi-SINS, installation error, calibration and compensation, rotation modulation

I. INTRODUCTION

The measurement of projectile navigation parameters such as position, velocity, and attitude is the key to guidance and precision strike [1]-[2]. Flight stability and precision guidance of the projectile highly depend on the high-accuracy measurement of attitude. Inertial navigation system (INS) is a self-contained navigation system in which measurements provided by accelerometers and gyroscopes are used to track the attitude, velocity, and position of a carrier relative to a known starting point, orientation and velocity [3]. In the traditional strapdown inertial navigation system (SINS), the inertial measurement unit (IMU) is rigidly connected to the carrier, which is used to measure the angular rate and specific force, and then according to Newton's theorem, the attitude information of carrier can be obtained at different times [4].

Constrained by special application environment, such as high overload, high spin, and narrow space, inertial measurement system based on MEMS inertial sensors is now considered as the first choice for attitudes measurement of high-spinning projectile [5]. The large dynamic range of MEMS gyro is required on the roll-axis if the navigation parameters are measured by SINS. Due to the limitation of the manufacturing process, however, gyros with large dynamic range cannot meet the requirement of high precision measurement, resulting in the traditional SINS is not suitable for the projectile application. Therefore, it is necessary to solve the challenge of high precision attitude measurement for large dynamic range inertial measurement system.

Different from traditional measurement methods, the concept of semi-strapdown INS (SSINS) is proposed by the Key Laboratory of Instrumentation Science & Dynamic
Measurement[6]-[7]. The structure of SSINS is shown in Figure 1, which is mainly composed of an inner cylinder, an outer cylinder, bearings, and elastic coupling. The gyro with large-range, motor and the control module are installed in the outer cylinder, fixedly connected to the carrier. MEMS-based IMU (MIMU) and an encoder are fixed in the inner cylinder. The connection between the outer cylinder and the inner cylinder is the core of the RSSINS. One end of the inner cylinder is fixed to the motor in the outer cylinder through the flexible coupling, and the other end is fitted with the inner wall of the system through the precision bearing. The control module drives the motor to perform reverse rotation according to the carrier rolling information measured by the gyro with large-range, thereby providing a lower dynamic measurement environment for the inner cylinder. The semi-gyro with large-range, thereby providing a lower dynamic according to the carrier rolling information measured by the gyro with large-range, thereby providing a lower dynamic measurement environment for the inner cylinder. The semi-strapdown measurement method can eliminate the interference of the high-speed rotation on the device’s accuracy, make it possible to measure the projectile attitude using small-range gyro and an encoder.

![Diagram of the semi-strapdown INS structure](image)

**FIGURE 1. The structure of semi-strapdown INS**

MIMU outputs, however, are corrupted with significant sensor errors, such as high-frequency noise, bias, scale factors and installation errors [8]-[10]. As a result, the measurement accuracy of SSINS still can not meet the high-precision guidance requirements of the high-spinning projectile. Fortunately, the emergence of rotation modulation technology has found a breakthrough to improve the measurement accuracy of MEMS sensors [11]-[13]. Therefore, without changing the system structure, the rotation modulation technology is introduced into the SSINS, called RSSINS. Combining with the information of MIMU’s axial gyroscope, the motor drives the MIMU to rotate at the corresponding angular rate to complete the single-axis rotation modulation.

In recent years, rotation modulation has attracted a lot of attention, including single-axis RINS, dual-axis RINS, tri-axial RINS[14]-[16]. Most researches on RINS aim at velocity and position accuracy improvement, while attitude output accuracy receives less attention. High accuracy attitude reference is more crucial in the application of projectile guidance. Any of added errors can unavoidably trigger the deterioration of navigation results. Due to the processing accuracy and the assembly method of the inner and outer cylinder, the installation error is inevitable in the RSSINS. For improving the measurement accuracy of attitude, the analysis and compensation of the installation errors is more significant.

Reference [17] indicates that the rotary table’s errors including installation error, wobble error, and angular error, do not affect the position accuracy, but have an influence on attitude accuracy. The propagation characteristics and the effect on navigation results of rotary table’s errors are analyzed and discussed, which supply a good reference for calibration and compensation of installation errors. Meanwhile, based on the analysis of the installation error propagation mechanism and the influence on the modulation effect, Jia establishes the coupling model of the installation error angle and the rotation angular velocity, and the corresponding error is effectively compensated by performing spectrum analysis on the gyro output [18].

Reference [19] employed the north and east velocity errors to identify installation error. Similarly, given that the true attitude is constant in a static condition, Gao et al. [20] used velocity and position errors to calibrate installation errors by utilizing the Kalman filter. In the same manner, Jiang et al. [21] used attitude output to calibrate installation errors with the Kalman filter. Based on the establishment of the modulation average frame, the vector projection and the attitude error are obtained. During the calibration period, the carrier was constantly static and the installation error can be calculated directly by the attitude matrix before and after rotation. However, the specific force, gyro measurement, velocity, and attitude error models could be deduced while considering installation error. However, these errors are difficult to obtain and even unavailable without external reference. Hu et al.[22] proposed a self-calibration method based on the configuration consisting of two RINSs to address the installation errors solely, which constructed the Kalman filter utilizing the attitude difference and attitude error model to estimate installation errors. And observability analysis was conducted to improve estimation accuracy and optimize asynchronous time.

In this work, the IMU is installed with an encoder on a motor and rotates following some specific rotation scheme in RSSINS. The encoder is used to measure the angles with the Kalman filter. Based on the establishment of the Kalman filter utilizing the attitude output to calibrate installation error. Similarly, given that the true attitude is constant in a static condition, Gao et al. [20] used velocity and position errors to calibrate installation errors by utilizing the Kalman filter. In the same manner, Jiang et al. [21] used attitude output to calibrate installation errors with the Kalman filter. Based on the establishment of the modulation average frame, the vector projection and the attitude error are obtained. During the calibration period, the carrier was constantly static and the installation error can be calculated directly by the attitude matrix before and after rotation. However, the specific force, gyro measurement, velocity, and attitude error models could be deduced while considering installation error. However, these errors are difficult to obtain and even unavailable without external reference. Hu et al.[22] proposed a self-calibration method based on the configuration consisting of two RINSs to address the installation errors solely, which constructed the Kalman filter utilizing the attitude difference and attitude error model to estimate installation errors. And observability analysis was conducted to improve estimation accuracy and optimize asynchronous time.

In this work, the IMU is installed with an encoder on a motor and rotates following some specific rotation scheme in RSSINS. The encoder is used to measure the angles with the Kalman filter. Based on the establishment of the modulation average frame, the vector projection and the attitude error are obtained. During the calibration period, the carrier was constantly static and the installation error can be calculated directly by the attitude matrix before and after rotation. However, the specific force, gyro measurement, velocity, and attitude error models could be deduced while considering installation error. However, these errors are difficult to obtain and even unavailable without external reference. Hu et al.[22] proposed a self-calibration method based on the configuration consisting of two RINSs to address the installation errors solely, which constructed the Kalman filter utilizing the attitude difference and attitude error model to estimate installation errors. And observability analysis was conducted to improve estimation accuracy and optimize asynchronous time.

In this work, the IMU is installed with an encoder on a motor and rotates following some specific rotation scheme in RSSINS. The encoder is used to measure the angles with the Kalman filter. Based on the establishment of the modulation average frame, the vector projection and the attitude error are obtained. During the calibration period, the carrier was constantly static and the installation error can be calculated directly by the attitude matrix before and after rotation. However, the specific force, gyro measurement, velocity, and attitude error models could be deduced while considering installation error. However, these errors are difficult to obtain and even unavailable without external reference. Hu et al.[22] proposed a self-calibration method based on the configuration consisting of two RINSs to address the installation errors solely, which constructed the Kalman filter utilizing the attitude difference and attitude error model to estimate installation errors. And observability analysis was conducted to improve estimation accuracy and optimize asynchronous time.

In this work, the IMU is installed with an encoder on a motor and rotates following some specific rotation scheme in RSSINS. The encoder is used to measure the angles with the Kalman filter. Based on the establishment of the modulation average frame, the vector projection and the attitude error are obtained. During the calibration period, the carrier was constantly static and the installation error can be calculated directly by the attitude matrix before and after rotation. However, the specific force, gyro measurement, velocity, and attitude error models could be deduced while considering installation error. However, these errors are difficult to obtain and even unavailable without external reference. Hu et al.[22] proposed a self-calibration method based on the configuration consisting of two RINSs to address the installation errors solely, which constructed the Kalman filter utilizing the attitude difference and attitude error model to estimate installation errors. And observability analysis was conducted to improve estimation accuracy and optimize asynchronous time.

In this work, the IMU is installed with an encoder on a motor and rotates following some specific rotation scheme in RSSINS. The encoder is used to measure the angles with the Kalman filter. Based on the establishment of the modulation average frame, the vector projection and the attitude error are obtained. During the calibration period, the carrier was constantly static and the installation error can be calculated directly by the attitude matrix before and after rotation. However, the specific force, gyro measurement, velocity, and attitude error models could be deduced while considering installation error. However, these errors are difficult to obtain and even unavailable without external reference. Hu et al.[22] proposed a self-calibration method based on the configuration consisting of two RINSs to address the installation errors solely, which constructed the Kalman filter utilizing the attitude difference and attitude error model to estimate installation errors. And observability analysis was conducted to improve estimation accuracy and optimize asynchronous time.

In this work, the IMU is installed with an encoder on a motor and rotates following some specific rotation scheme in RSSINS. The encoder is used to measure the angles with the Kalman filter. Based on the establishment of the modulation average frame, the vector projection and the attitude error are obtained. During the calibration period, the carrier was constantly static and the installation error can be calculated directly by the attitude matrix before and after rotation. However, the specific force, gyro measurement, velocity, and attitude error models could be deduced while considering installation error. However, these errors are difficult to obtain and even unavailable without external reference. Hu et al.[22] proposed a self-calibration method based on the configuration consisting of two RINSs to address the installation errors solely, which constructed the Kalman filter utilizing the attitude difference and attitude error model to estimate installation errors. And observability analysis was conducted to improve estimation accuracy and optimize asynchronous time.

In this work, the IMU is installed with an encoder on a motor and rotates following some specific rotation scheme in RSSINS. The encoder is used to measure the angles with the Kalman filter. Based on the establishment of the modulation average frame, the vector projection and the attitude error are obtained. During the calibration period, the carrier was constantly static and the installation error can be calculated directly by the attitude matrix before and after rotation. However, the specific force, gyro measurement, velocity, and attitude error models could be deduced while considering installation error. However, these errors are difficult to obtain and even unavailable without external reference. Hu et al.[22] proposed a self-calibration method based on the configuration consisting of two RINSs to address the installation errors solely, which constructed the Kalman filter utilizing the attitude difference and attitude error model to estimate installation errors. And observability analysis was conducted to improve estimation accuracy and optimize asynchronous time.

In this work, the IMU is installed with an encoder on a motor and rotates following some specific rotation scheme in RSSINS. The encoder is used to measure the angles with the Kalman filter. Based on the establishment of the modulation average frame, the vector projection and the attitude error are obtained. During the calibration period, the carrier was constantly static and the installation error can be calculated directly by the attitude matrix before and after rotation. However, the specific force, gyro measurement, velocity, and attitude error models could be deduced while considering installation error. However, these errors are difficult to obtain and even unavailable without external reference. Hu et al.[22] proposed a self-calibration method based on the configuration consisting of two RINSs to address the installation errors solely, which constructed the Kalman filter utilizing the attitude difference and attitude error model to estimate installation errors. And observability analysis was conducted to improve estimation accuracy and optimize asynchronous time.
ment of the system error model, the working principle of rotation modulation technology is briefly introduced, and the system installation error is modeled. In Section III, according to the system working principle, the output characteristics of gyros are analyzed, from which the coupling form between system installation error and carrier motion is complicated, and the influence on the system accuracy cannot be neglected. Meanwhile, the influence of each installation error angle on attitude errors can be obtained by analyzing the propagation characteristics of installation error, which can provide a reference for error compensation. Then, the methods to calibrate and compensate the errors are put forward in Section IV. Section V provides the experiment results to prove the effectiveness of the proposed method. Finally, conclusions are drawn in Section VI.

II. ERROR MODELING

A. ERROR MODE OF RSSINS

In RSSINS, the actual output of the MIMU is:

\[
\dot{\omega}_s^n = (\mathbf{I} + \delta A_s) \left[ \mathbf{I} + \delta K_s \right] \omega_s^n + \epsilon
\]

(1)

\[
\dot{f}_s^n = (\mathbf{I} + \delta A_s) \left[ \mathbf{I} + \delta K_s \right] f_s^n + V^n
\]

(2)

Where \(\omega_s^n\), \(f_s^n\) are ideal angular rate and acceleration in s-frame, and s-frame is the inertial sensor frame, whose axes are aligned with the sensing axis of inertial sensors. \(\dot{\omega}_s^n\), \(\dot{f}_s^n\) are actual angular rate and acceleration along s-frame. \(\epsilon = \left[ \epsilon_x \quad \epsilon_y \quad \epsilon_z \right]^T\), \(V^n = \left[ V_x \quad V_y \quad V_z \right]^T\) are gyro constant drifts and accelerometer biases along s-frame. \(\delta K_s = \begin{bmatrix} \delta k_{gx} & \delta k_{gy} & \delta k_{gz} \\ \delta k_{gy} & \delta k_{gy} & \delta k_{gxy} \\ \delta k_{gz} & \delta k_{gxy} & \delta k_{gzz} \end{bmatrix}\), \(\delta k_s = \begin{bmatrix} \delta k_{ax} & \delta k_{ay} & \delta k_{az} \end{bmatrix}\) are scale factor error matrix of gyros and accelerometers respectively. \(\delta A_s\) are the installation error matrix of gyros and accelerometers respectively, which could be expressed as follow:

\[
\delta A_s = \begin{bmatrix} 0 & \delta A_{gy} & \delta A_{gz} \\ \delta A_{gy} & 0 & \delta A_{gxy} \\ -\delta A_{gz} & \delta A_{gxy} & 0 \end{bmatrix}, \delta A_s = \begin{bmatrix} 0 & \delta A_{ay} & \delta A_{az} \\ \delta A_{ay} & 0 & \delta A_{azy} \\ -\delta A_{az} & \delta A_{azy} & 0 \end{bmatrix}
\]

Different from SINS, MIMU and carrier are no longer fixed in the RSSINS. Thus, the error equations of RSSINS are expressed as:

\[
\dot{\phi} = \mathbf{f} \times \omega^n + \delta \omega^n = \mathbf{C}_s^n \left[ \mathbf{C}_s^n \right] (\delta K_s + \delta A_s) \left[ (\delta K_s + \delta A_s) \right] (\omega^n + \epsilon)
\]

(3)

\[
\dot{\mathbf{r}}^s = \mathbf{f} + \mathbf{C}_s^n \left[ \mathbf{C}_s^n \right] \left[ \delta K_s + \delta A_s \right] f_s^n + V^n + \mathbf{C}_s^n \left[ \mathbf{C}_s^n \right] \left[ 2 \delta \omega^n \times \delta \omega^n \right]
\]

(4)

Where \(\mathbf{f}\) represents the misalignment angle. \(\omega^n\) is the rotation angular rate of the navigation system relative to the inertial system, which is \(\omega_s^n = \omega_e^n + \omega_{ie} + \omega_{iee}\). \(\omega_{ie}\) is the relative rotation between the s-frame and b-frame. \(\delta \omega^n\) is the navigation calculation error. \(\delta \mathbf{r}\) and \(V^n\) represent the velocity error and velocity in n-frame. \(\mathbf{C}_s^n\) represents the direction cosine matrix from A-frame to B-frame. The other variables are defined as above.

It can be seen from the above error equations that the error of the inertial sensors does not change by the rotation, but the direction cosine matrix \(\mathbf{C}_s^n\) is changed by the periodic rotation, so that the error term is modulated into periodic signal, which is eliminated in the navigation solution process, and no longer diverges with time[23], thus improving the navigation accuracy.

B. MODELLING OF SYSTEM INSTALLATION ERRORS

Due to machining and assembly accuracy, installation errors are inevitable in the inner and outer cylinders as shown in Figure 2. The installation error angles not only cause the carrier motion to couple with each other, resulting in new errors in navigation solutions, but if them exceed a certain range, the stability of the structure will be greatly affected.

FIGURE 2. Schematic diagram of installation errors

As shown in Figure 2, the carrier coordinate system (b-frame) of the RSSINS is defined as follows: X-axis coincides with the outer cylinder axis, the Y-axis points to the top of the body and the Z-axis is defined according to the right-hand rule. And the coordinate system of motor is defined as \(\theta_0\)-frame in the initial time, which coincides with carrier coordinate system in the ideal situation. However, there are installation error angles \(\beta_1, \beta_2, \beta_3\) between the b-frame and \(\theta_0\)-frame as shown in Figure 3, which is called outer cylinder installation error.

FIGURE 3. The schematic diagram of outer cylinder installation error

The relationship between the b-frame and \(\theta_0\)-frame can be determined by three basic rotations, which is better described by introducing three matrices. Assume that the first basic rotation is performed around the y-axis and a Euler rotation matrix along y-axis can be used to represent \(\beta_1\), as shown in (5).
Similarly, other two Euler rotation matrices could be imported to represent $\beta_2$ and $\beta_3$, as shown in (6) and (7) respectively.

Therefore, the relationship between the b-frame and r0-frame can be denoted by the matrix $C_b^r$.

$$C_b^r = C(b_o)C(z_o)C(y_o) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$  (8)

Where:

$$a_{11} = \cos \beta_y \cos \beta_z$$
$$a_{12} = \sin \beta_y$$
$$a_{13} = -\cos \beta_z \sin \beta_y$$
$$a_{21} = \cos \beta_x \sin \beta_z$$
$$a_{22} = \cos \beta_x \cos \beta_z$$
$$a_{23} = \sin \beta_x$$
$$a_{31} = -\sin \beta_y \sin \beta_z$$
$$a_{32} = \cos \beta_y \cos \beta_z$$
$$a_{33} = \cos \beta_x$$

Assuming $\beta_x, \beta_y, \beta_z$ are small quantities, then (8) can be approximated as:

$$C_b^r = \begin{bmatrix} 1 & \beta_x & -\beta_y \\ -\beta_x & 1 & \beta_z \\ \beta_y & -\beta_z & 1 \end{bmatrix} = I + \delta C_o$$  (9)

The inner cylinder is connected with the motor through an elastic coupling, which is not an integrated structure with the outer cylinder. Due to factors such as processing technology and assembly error, the installation errors are inevitable. Assume that the MIMU coordinate system is denoted as the s-frame, which should coincide with the r-frame in the ideal condition.

Similar to the outer cylinder installation error, the inner cylinder installation error is described by three basic rotations. The installation error angles are symbolized $\eta_x, \eta_y, \eta_z$. Therefore, the relationship between the s-frame and r0-frame can be denoted by the matrix $C_s^{b_0}$.

$$C_s^{b_0} = C(y_s)C(z_s)C(x_s) = \begin{bmatrix} 1 & \eta_x & -\eta_y \\ -\eta_x & 1 & \eta_z \\ \eta_y & -\eta_z & 1 \end{bmatrix} = I + \delta C_i$$  (10)

We clarify that $\beta_x, \beta_y, \beta_z, \eta_x, \eta_y, \eta_z$ and $\eta_x, \eta_y, \eta_z$ are defined as virtual installation error angles to be estimated and calibrated in this paper.

### III. EFFECT OF INSTALLATION ERRORS ON SYSTEM ACCURACY

For the MEMS-IMU, the change of curvature radius and gravity field of the earth's surface can be neglected during the short time flight [24]. Since the selected MEMS sensors in this system can not sense the rotation angular velocity of the earth, the coupling term generated by the rotation angular velocity of the earth can also be neglected. Meanwhile, the scale factor error and installation error of MIMU have been calibrated and compensated before the system integration. In this paper, only the influence of system installation errors on measurement accuracy is analyzed. Considering the system installation errors, hence, the actual gyro output model is as follows.

$$\omega^{s}_{ho} = C_s^r(C_s^0 \omega^b_{ho} + \omega^s_{ho}) + \epsilon^s$$  (11)

Where $\omega^{s}_{ho}$ is the measurement value of the gyros, $\epsilon^s$ is the gyro constant drift and denoted as $\epsilon^s = [\epsilon_x, \epsilon_y, \epsilon_z]^T$, $\omega^s_{ho}$ is the rotational angular rate of the motor, $r_0$-frame is the motor coordinate system in the initial time. $C_s^r$ is a transformation matrix constructed by the relative rotation angle between the motor and the carrier.

Therefore, the angular rate of the carrier is:

$$\tilde{\omega}^b_{ho} = C_s^r(\omega^s_{ho} + \omega^s_{ho})$$
$$= C_s^rC_s^0C_s^b_0 \omega^b_{ho} + C_s^rC_s^0 \omega^s_{ho} + C_s^r \omega^s_{ho} + C_s^r \epsilon^s$$  (12)

Where $\omega^s_{ho}$ represents the relative rotation between the s-frame and b-frame, which is measured by photoelectric encoder. The accuracy of the photoelectric encoder can reach arc-second degree, and the measurement error is negligible. Therefore $\omega^s_{ho}$ is equivalent to $\omega^s_{ho}$, and the rotation matrix constructed by it is equivalent to $C_s^r$.

As a result of analysis, in addition to sensing the actual carrier motion, the gyro can also sense the error caused by the coupling of the installation error with carrier motion and rotation mechanism motion in the actual output, and then the carrier angular rate measurement error can be deduced as:

$$\delta \omega^b_{ho} = \tilde{\omega}^b_{ho} - \omega^s_{ho}$$
$$= (C_s^r \delta C_s \delta C_s + \delta C_s \delta C_s + \delta C_s) \omega^b_{ho} + C_s^r \delta C_s \omega^s_{ho} + C_s^r \epsilon^s$$  (13)

From the foregoing, the coupling form between the system installation errors and the carrier motion is complicated, and the coupling term between the inner cylinder installation error and the rotation rate of the motor causes the equivalent error of the Y-axis and Z-axis gyroscopes. In high dynamic environment, the magnitude of angular rate is considerable, whether in the axial roll motion of carrier or in the modulation motion of motor, which leads to the equivalent error after coupling even greater than the constant drift of the device. Therefore, it is necessary to calibrate and compensate the system installation errors.

In order to reduce the influence of relative angular measurement error on the system, the attitude matrix is
updated between $s$-frame and $n$-frame to calculate the navigation parameters of MIMU [25]. Then the carrier navigation parameters can be obtained by combining the relative rotation angle information. However, the system installation error directly changes the attitude matrix from $s$-frame to $n$-frame, resulting in reduced navigation accuracy. Assuming that the attitude of the carrier is denoted as follows: the pitch angle is $\theta$, the roll angle is $\gamma$ and the yaw angle is $\psi$, the transformation matrix $C_s^n$ can be expressed as:

$$C_s^n = C_b^n C_b^s = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

(14)

Where

$$C_{11} = \cos \theta \cos \varphi$$
$$C_{12} = \cos \theta \sin \varphi \sin \omega - \sin \theta \cos \omega$$
$$C_{13} = \sin \theta \cos \omega$$
$$C_{21} = \cos \theta \sin \varphi \cos \omega + \sin \theta \sin \omega$$
$$C_{22} = \cos \theta \cos \varphi$$
$$C_{23} = -\cos \theta \sin \varphi \sin \omega - \sin \theta \cos \omega$$
$$C_{31} = -\sin \theta$$
$$C_{32} = \cos \theta \sin \omega$$
$$C_{33} = \cos \theta \cos \omega$$

The attitude angles can be derived from attitude matrix:

$$\gamma_s = \arctan \left( \frac{C_{32}}{C_{33}} \right)$$
$$\theta_s = \arcsin \left( -C_{31} \right)$$
$$\phi_s = \arctan \left( \frac{C_{21}}{C_{11}} \right)$$

(15)

However, considering the system installation error, the actual direction cosine matrix is as shown in (16). Ignoring the second-order small quantity, the actual attitude matrix $\tilde{C}_s^n$ can be derived as:

$$\tilde{C}_s^n = C_b^n C_b^s C_s^n = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix}$$

(16)

Where:

$$D_{11} = \cos \varphi \cos \theta + (-\eta, \cos \omega - \beta, \cos \gamma + \eta, \sin \omega \gamma + \beta, \sin \omega \gamma) \sin \theta$$
$$D_{12} = -\cos \varphi \cos \theta \eta, \cos \omega - \beta, \cos \gamma + \eta, \sin \omega \gamma + \beta, \sin \omega \gamma \sin \varphi$$
$$D_{13} = -\cos \varphi \cos \theta \eta, \cos \omega - \beta, \cos \gamma + \eta, \sin \omega \gamma + \beta, \sin \omega \gamma \sin \varphi$$
$$D_{21} = \cos \theta \cos \omega \varphi \cos \theta + \eta, \cos \omega + \beta, \cos \gamma - \eta, \sin \omega \gamma - \beta, \sin \omega \gamma \sin \theta$$
$$D_{22} = \cos \theta \cos \omega \varphi \cos \theta + \eta, \cos \omega + \beta, \cos \gamma - \eta, \sin \omega \gamma - \beta, \sin \omega \gamma \sin \varphi$$
$$D_{23} = -\cos \theta \cos \omega \varphi \cos \theta + \eta, \cos \omega + \beta, \cos \gamma - \eta, \sin \omega \gamma - \beta, \sin \omega \gamma \sin \varphi$$

$$D_{31} = \cos \eta, \cos \omega + \beta, \sin \gamma + \eta, \sin \omega \gamma + \beta, \sin \omega \gamma \sin \theta$$
$$D_{32} = \cos \eta, \cos \omega + \beta, \sin \gamma + \eta, \sin \omega \gamma + \beta, \sin \omega \gamma \sin \varphi$$
$$D_{33} = \cos \eta, \cos \omega + \beta, \sin \gamma + \eta, \sin \omega \gamma + \beta, \sin \omega \gamma \sin \varphi$$

According to the relationship between $C_s^n$ and attitude angle, the following equations can be derived:

$$\tan (\gamma_s + \Delta \gamma_s) = \frac{D_{32}}{D_{33}}$$
$$\sin (\theta_s + \Delta \theta_s) = \frac{D_{31}}{D_{33}}$$
$$\tan (\phi_s + \Delta \phi_s) = \frac{D_{21}}{D_{22}}$$

(17)

Where $\Delta \gamma_s$, $\Delta \theta_s$, $\Delta \phi_s$ stand for the pitch angle error, the yaw angle error and the roll angle error respectively.

Since there is no relative motion between pitch and yaw between the carrier and IMU, the pitch and yaw angles calculated in the $s$-frame are the pitch and yaw motion of the carrier. The roll angle of the carrier is obtained by combining the relative angle measurement information with the IMU measurement information. Here, only the influence of system installation error on the carrier attitude is analyzed, so that the measurement error of relative rotation angle is not considered. Therefore, the IMU roll angle error is the carrier roll angle error. Ignoring the second-order small quantity, the attitude errors can be deduced from (17), such that:

$$\Delta \gamma_b = \frac{\cos \theta \left( (\beta + \eta) \cos (\omega - \eta) + \sin (\omega - \eta) \right) + \left( \beta \cos (\omega - 2\eta) + \eta, \cos \gamma - \beta, \sin (\omega - 2\eta) + \eta, \sin \gamma \sin \theta \right)}{\cos \theta \left( \cos (\omega - \eta) - (\beta + \eta) \sin (\omega - \eta) \right) - \left( \beta \cos (\omega - 2\eta) + \eta, \cos \gamma + \beta, \sin (\omega - 2\eta) - \eta, \sin \gamma \sin \theta \right)}$$
$$\Delta \theta_b = \eta, \cos (\omega) + \beta, \cos \gamma - \eta, \sin (\omega) - \beta, \sin \gamma + \tan \varphi$$
$$\Delta \phi_b = \frac{\eta, \cos (\omega) + \beta, \cos \gamma + \eta, \sin (\omega) + \beta, \sin \gamma}{\cos \theta + (-\eta, \cos (\omega) - \beta, \cos \gamma + \eta, \sin (\omega) + \beta, \sin \gamma) \sin \theta}$$

(18)

From the roll error equation, it can be seen that the coupling of installation error angles and the actual motion of the carrier has an impact on the measurement of the roll angle. The errors of yaw and pitch are mainly caused by the coupling term of $\eta, \beta, \beta, \beta$, carrier roll motion and modulation motion, but also $\eta, \beta, \beta$ have little influence on them. In order to analyze the influence of the system installation errors on attitude errors, simulations of external ballistic flight trajectory are carried out.

Firstly, the projectile trajectory is generated by setting the ballistic parameters. The ballistic parameters are as follows: the diameter of the projectile is 0.125 m, the polar moment of inertia is 0.8 kg*m², the length of the projectile is 0.5 m, the mass of the projectile is 30 kg, the muzzle
velocity is 400 m/s, the fire angle is 50°, the cross wind speed is 50 m/s, the vertical wind speed is 10 m/s, the muzzle speed is 12 r/s, the sampling rate is 5 kHz. The simulated ballistic trajectory and attitude are as follows.

![Ballistic trajectory](image)

**FIGURE 4.** Simulation of external ballistic trajectory

Due to the short-endurance and high-dynamic application, the modulation angular rate is set to 90°/s. And all initial installation angles are set as 20°. In order to analyse the relationship between equivalent attitude errors and system installation errors, we changed every angle at intervals of 20° while keeping the others 20°. The simulation is designed and carried out with the following conditions.

1) SIMULATION OF OUTER CYLINDER INSTALLATION ERROR

At the initial moment, each installation error angle is set to 20°, then the installation error angle of outer cylinder is changed in turn, and each installation error angle is increased from 20° to 60°. The simulation results are shown in Figure 6-Figure 8.

![Attitude errors](image)

**FIGURE 6.** The simulation of attitude errors caused by changing $\beta_x$, and other installation error angles are set to 20°.

![Pitch errors](image)

**FIGURE 7.** The simulation of attitude errors caused by changing $\beta_y$, and other installation error angles are set to 20°.

![Roll errors](image)

**FIGURE 8.** The simulation of attitude errors caused by changing $\beta_z$, and other installation error angles are set to 20°.
It can be seen from Figure 6 that the increase of $\beta_x$ has little effect on the attitude accuracy of carrier, and even the influence on the pitch and roll angle can be neglected, which is consistent with the theoretical analysis. From Figure 7 and Figure 8, it can be seen that $\beta_y, \beta_z$ have great influence on the pitch and yaw angle, and the attitude errors increases with installation error angles. From the analysis of (18), it can be known that the coupling between $\beta_y, \beta_z$ and carrier rolling motion component is the main reason for the rapid accumulation of attitude errors.

2) SIMULATION OF INNER CYLINDER INSTALLATION ERROR

At the initial moment, each installation error angle is set to 20', then the installation error angle of inner cylinder is changed in turn, and each installation error angle is increased from 20' to 60'. The simulation results are shown in Figure 9-Figure 11.

![Figure 9](image1)

**FIGURE 9.** The simulation of attitude errors caused by changing $\eta_x$, and other installation error angles are set to 20'.

![Figure 10](image2)

**FIGURE 10.** The simulation of attitude errors caused by changing $\eta_y$, and other installation error angles are set to 20'.

![Figure 11](image3)

**FIGURE 11.** The simulation of attitude errors caused by changing $\eta_z$, and other installation error angles are set to 20'.

Similarly, the increase of $\eta_z$ has little effect on the attitude accuracy of the carrier, and even the influence on the pitch and roll angle can be neglected, which is consistent with the analysis of (18). It can be seen from Figure 9 that the attitude errors increase with $\eta_x, \eta_z$, but compared with the above simulation results, the growth rate of attitude error is much smaller than that caused by the outer cylinder installation error. The reason can be obtained by analyzing (18). Through the yaw and pitch error propagation modes in (18), we can see that the installation error angle of outer cylinder is coupled with the sine or cosine of carrier rolling motion, and the installation error angle of inner cylinder is coupled with the sine or cosine of modulation motion. Obviously, the carrier roll angular rate is much larger than modulation angular rate. Therefore, the outer cylinder installation errors have significant influence on the carrier attitude error. In summary, the outer cylinder installation error is the main reason of causing attitude error, which must be calibrated and compensated.

IV. CALIBRATION AND COMPENSATION METHOD

In order to minimize installation errors, the system has adopted an integrated structure. If the system installation errors are calibrated as a whole using conventional methods, the outer and inner cylinder installation errors are coupled to each other, which is difficult to compensate them. Since the system adopts the single-axis rotation, there is only relative movement between inner cylinder and motor in the rolling direction. Insufficient degree of freedom results in that the inner cylinder installation error can not be observed completely, and the outer cylinder installation error can not be calibrated independently. Thus, a graded calibration method suitable for the system is proposed.

The MEMS-IMU used in this system can not sense the rotation angular velocity of the earth, so the cross-coupling term generated by it is ignored during the calibration process. Different from high-precision gyros such as optical
fiber gyros and laser gyros, however, the constant drift of MEMS devices is the main error. Assuming that the gyro constant drift is $15^\circ/h$, the error angle accumulated in 60s is $15^\circ$. Thus, the constant drift of the MEMS sensor must be considered in order to calibrate the installation error more accurately. Inspired by the rotation modulation method, the system installation errors are calibrated by using the forward and backward rotation method with the help of three-axis high precision turntable.

1) CALIBRATION OF INNER CYLINDER INSTALLATION ERROR

Inner cylinder installation error is calibrated by direct calibration method, which uses a certain given input of the IMU to observe the output of it. Fixed the system on the test equipment to ensure that there is no shaking during the calibration process, and the system remains stationary. The motor is controlled to drive the inner cylinder to rotate forward and backward at the angular rate $\omega_s$, and the IMU sensitive value is recorded at the same time. In the forward rotation process, the IMU output model is shown as (19).

$$
\begin{bmatrix}
\omega_{bs}^+ \\
\omega_{by}^+ \\
\omega_{bz}^+
\end{bmatrix} = C^s_x \begin{bmatrix}
\omega_s \\
x_s \\
y_s
\end{bmatrix} + \begin{bmatrix}
\cos \eta_x \cos \eta_y \omega_1 + \epsilon_x \\
\sin \eta_x \omega_2 + \epsilon_y \\
-\cos \eta_y \sin \eta_x \omega_1 + \epsilon_z
\end{bmatrix}
$$

Similarly, during the reverse rotation process, the IMU output model is shown as (20).

$$
\begin{bmatrix}
\omega_{bs}^- \\
\omega_{by}^- \\
\omega_{bz}^-
\end{bmatrix} = C^s_x \begin{bmatrix}
-\omega_s \\
x_s \\
y_s
\end{bmatrix} + \begin{bmatrix}
-\cos \eta_x \cos \eta_y \omega_1 + \epsilon_x \\
-\sin \eta_x \omega_2 + \epsilon_y \\
\cos \eta_y \sin \eta_x \omega_1 + \epsilon_z
\end{bmatrix}
$$

The constant drifts of MEMS sensor can be offset by the difference between (19) and (20). $\eta_x$ and $\eta_y$ can be solved as:

$$
\eta_x = \arctan \left( \frac{\omega_{bs}^+ - \omega_{bs}^-}{\omega_{by}^+ - \omega_{by}^-} \right)
$$

$$
\eta_y = \arcsin \left( \frac{\omega_{bs}^+ - \omega_{bs}^-}{\omega_{bz}^+ - \omega_{bz}^-} \right)
$$

2) CALIBRATION OF OUTER CYLINDER INSTALLATION ERROR

When calibrating the outer cylinder installation error, the system as a whole is calibrated with the help of tri-axis high-precision turntable. Hence, the IMU output model are shown as (22)-(23) during the positive and negative rotation.

$$
\begin{bmatrix}
\omega_{bs}^+ \\
\omega_{by}^+ \\
\omega_{bz}^+
\end{bmatrix} = C^s_x C^0_y C^b_y \begin{bmatrix}
\omega_s \\
x_s \\
y_s
\end{bmatrix} + \begin{bmatrix}
\epsilon_x \\
\epsilon_y \\
\epsilon_z
\end{bmatrix}
$$

$$
\begin{bmatrix}
\omega_{bs}^- \\
\omega_{by}^- \\
\omega_{bz}^-
\end{bmatrix} = C^s_x C^0_y C^b_y \begin{bmatrix}
-\omega_s \\
x_s \\
y_s
\end{bmatrix} + \begin{bmatrix}
\epsilon_x \\
\epsilon_y \\
\epsilon_z
\end{bmatrix}
$$

In this case, the installation error matrix produced by the coupling of inner and outer cylinder installation error is $C^s_x C^0_y C^b_y$, which can be expressed as:

$$
C^s_x C^0_y C^b_y = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
$$

Similar to the calibration of inner cylinder installation error, the forward and reverse rotational motions are performed at the fixed angular rates within sensor range, and the elements in the matrix $C^s_x C^0_y C^b_y$ are solved by observing the gyro output. Take z-axis as an example, the IMU output model are shown as (25)-(26).

$$
\begin{bmatrix}
\omega_{zs}^+ \\
\omega_{yz}^+ \\
\omega_{zs}^-
\end{bmatrix} = C^s_x C^0_y C^b_y \begin{bmatrix}
0 \\
\epsilon_y \\
-\omega_z
\end{bmatrix} + \begin{bmatrix}
a_{15} \omega_z + \epsilon_x \\
a_{25} \omega_z + \epsilon_y \\
-a_{35} \omega_z + \epsilon_z
\end{bmatrix}
$$

$$
\begin{bmatrix}
\omega_{zs}^+ \\
\omega_{yz}^+ \\
\omega_{zs}^-
\end{bmatrix} = C^s_x C^0_y C^b_y \begin{bmatrix}
0 \\
\epsilon_y \\
-\omega_z
\end{bmatrix} - \begin{bmatrix}
a_{13} \omega_z + \epsilon_x \\
a_{23} \omega_z + \epsilon_y \\
-a_{33} \omega_z + \epsilon_z
\end{bmatrix}
$$

The constant drifts of MEMS sensor can be offset by the difference between (25) and (26). $a_{13}$, $a_{23}$ and $a_{33}$ can be solved as:

$$
\begin{bmatrix}
a_{13} = \left( \omega_{zs}^+ - \omega_{zs}^- \right) / 2 \omega_z \\
+ a_{23} = \left( \omega_{yz}^+ - \omega_{yz}^- \right) / 2 \omega_z \\
+ a_{33} = \left( \omega_{zs}^+ - \omega_{zs}^- \right) / 2 \omega_z
\end{bmatrix}
$$

All elements in the installation error matrix can be observed by rotating around each axis in turn. Meanwhile, according to the model of system installation errors, (24) can be derived as:

$$
C^s_x C^0_y C^b_y = C(y_i) C(z_j) C(x_k) C(x_o) C(z_o) C(y_o)
$$

(28)

The matrix $C(y_i), C(z_j)$ can be obtained combining with the calibration results of inner cylinder installation error angles, which are substituted into (28) to obtain the following equation.

$$
C(y_i)^{-1} C(y_j)^{-1} C^s_x C^0_y C^b_y = C(x_k) C(x_o) C(z_o) C(y_o) = \begin{bmatrix}
b_{11} & b_{12} & b_{13} \\
b_{21} & b_{22} & b_{23} \\
b_{31} & b_{32} & b_{33}
\end{bmatrix}
$$

where:

$$
b_{11} = \cos \beta_x \cos \beta_y \cos \beta_z,
$$

$$
b_{12} = -\sin \beta_x \cos \beta_y \cos \beta_z,
$$

$$
b_{13} = \sin \beta_x \cos \beta_y \cos \beta_z,
$$

$$
b_{21} = \sin \beta_x \sin \beta_y \cos \beta_z,
$$

$$
b_{22} = -\cos \beta_x \sin \beta_y \cos \beta_z,
$$

$$
b_{23} = -\sin \beta_x \sin \beta_y \cos \beta_z,
$$

$$
b_{31} = -\sin \beta_x \cos \beta_y \sin \beta_z,
$$

$$
b_{32} = -\cos \beta_x \cos \beta_y \sin \beta_z,
$$

$$
b_{33} = \cos \beta_x \cos \beta_y \sin \beta_z.
$$

Therefore, $\beta_x, \beta_y, \beta_z$ are solved:

$$
\begin{bmatrix}
\beta_x + \eta_y \\
\beta_y + \eta_y \\
\beta_z + \eta_y
\end{bmatrix} = \begin{bmatrix}
\beta_x \\
\beta_y \\
\beta_z
\end{bmatrix} + \begin{bmatrix}
\eta_x \\
\eta_y \\
\eta_z
\end{bmatrix}.
$$

(29)

From the above analysis, all the installation error angles can be calibrated. However, $\beta$, $\eta_x$, and $\eta_y$ are coupled with each other which cannot be calibrated separately, which
makes it difficult to compensate the sensors output of (12) directly. Hence, the compensation method can be found from the direction cosine matrix. It can be seen from the above analysis that due to special structure of system , \( \beta_i \) and \( \eta_i \) cannot be separately calibrated, it is impossible to construct \( C_i^b \) and \( C_i^e \) to directly compensate the installation error according to (16). Here, according to the system installation error model, (16) can be expanded as follows:

\[
C_i^t = C_i^e C_i^b C_i^o C_i^y
\]

From above analysis, these matrixes \( C_i^e \), \( C_i^b \), \( C_i^o \) and \( C_i^y \) can be constructed respectively by combining the calibration results of \( \beta_i \), \( \beta_z \), \( \eta_y \) and \( \eta_z \). At the same time, \( C_i^e \), \( C_i^b \) and \( C_i^y \) are coupled with each other. Fortunately, \( C_i^e \), \( C_i^b \) and \( C_i^y \) satisfy the following property due to the single-axis rotation of the system:

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos A & -\sin A \\
0 & \sin A & \cos A
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos B & -\sin B \\
0 & \sin B & \cos B
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(A+B) & -\sin(A+B) \\
0 & \sin(A+B) & \cos(A+B)
\end{bmatrix}
\]

Therefore, the \( C_i^e \) \( C_i^b \) \( C_i^o \) \( C_i^y \) can be constructed by combining the relative rotation information and the calibration result of \( \beta_i \) \( \eta_i \), and the compensation of the system installation error is completed according to (16).

Calibration experiments are performed five times to estimate system installation error angles and the average value is shown in Table I.

<table>
<thead>
<tr>
<th>Installation error angles</th>
<th>( \beta_i + \eta_i )</th>
<th>( \beta_z )</th>
<th>( \eta_y )</th>
<th>( \eta_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (°)</td>
<td>13.83</td>
<td>10.48</td>
<td>-7.39</td>
<td>-9.54</td>
</tr>
</tbody>
</table>

V. EXPERIMENTS AND RESULTS

In order to further prove the practicability and effectiveness of the proposed method in actual application, experiments are designed and implemented with the help of tri-axial flight simulator. Table II summarize the technical parameters of tri-axial flight simulator. Meanwhile, the characteristics of the MIMU in the system is shown in Table III.

<table>
<thead>
<tr>
<th>Angular Rate Resolution</th>
<th>Rotation Rate Accuracy</th>
<th>Rotation Rate(°/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inner frame</td>
<td>Middle frame</td>
</tr>
</tbody>
</table>

In this experiment, tri-axial flight simulator is used to provide high-spin environment, and the installation method of the system is shown in Figure 12. At the initial time, the s-frame coincides with the b-frame, and the x-axis of MIMU is along the inner frame axis of turntable, the y-axis is along the outer frame axis of the turntable, and the z-axis is along the middle frame axis of the turntable.

![Figure 12. RSSINS installation on tri-axial flight simulator](image)

In the experiment, the tri-axial flight simulator is used to simulate the carrier attitude movement, as shown in Figure 13, the modulation angular rate is 90°/s, and the positive and negative rotation scheme is employed. Comparing the navigation information before and after compensation measured by RSSINS with the standard information provided by the flight simulator, the navigation parameter measurement errors of the system can be obtained to analyze the error compensation performance.

![Figure 13. Carrier attitude change process simulated by tri-axial flight simulator](image)
Though navigation solution, the attitude information of IMU is calculated as shown in Figure 14. Then, the carrier attitude information is obtained by combining the relative angle measurement information as shown in Figure 15.

Through the analysis of experiment results, we can see that the pitch error of carrier reduces from -1.515° to -0.769°, the yaw error reduces from 1.237° to 0.5435°, and the roll error reduces from -2.386° to -1.778°. Among them, the roll axis is chosen as the modulation axis, of which the constant drift is not modulated, so roll error is larger than that of other two axes. Therefore, the proposed method can compensate the system installation error effectively.

**VI. CONCLUSION**

This paper investigates the impact of system installation errors on navigation accuracy. Firstly, on the basis of introducing the system structure and working principle, the system installation error model and sensor output model are established. Secondly, by analyzing the relationship between installation errors and attitude errors, we can know that the installation error of outer cylinder is the main cause of attitude error, hence, the installation error of outer cylinder should be minimized during system integration. Then, aiming at the problem of insufficient degree of freedom of inner cylinder, the corresponding calibration scheme is proposed. On this basis, we propose the compensation method according to the system structure and the function characteristic. Finally, the validity of the calibration and compensation methods are proved by experiments. After compensation, experimental results show that the yaw and pitch errors of RSSINS are less than 1°, and the roll error is less than 2°. In many task systems, high accuracy attitude information is more crucial than velocity or position accuracy. The method proposed in this paper can effectively improve the navigation accuracy of the system.

**REFERENCES**


Jie Li was born in Shanxi Province, China, in 1976. He received the Ph.D. degree in navigation, guidance and control from the Department of Automatic Control, Beijing Institute of Technology, Beijing, China, in 2005. From April 2006 to June 2009, he was a Postdoctoral Research Associate with North University of China, Taiyuan, China, and worked on navigation, guidance, and control. Since 2007, he has been with the School of Instrument and Electronics, North University of China, first as an Associate Professor, and then, since 2013, as a Professor. His current research interests include strapdown inertial navigation, integrated navigation, and intelligent information processing.

Yugang Huang was born in Hebei Province, China, in 1990. He received the master of engineering in electronic science and technology from the School of Instrument and Electronics, North University of China, Taiyuan, China, in 2017. He is currently working in The 60th Research Institute of General Staff Dept of P.L.A. His current research interests include inertial navigation and signal processing.

Chenjun Hu was born in Henan Province, China, in 1990. He received the master of engineering in precision instrument and machinery from the School of Instrument and Electronics, North University of China, Taiyuan, China, in 2015. He is currently working in Suzhou Fashion Nano Technology Co., Ltd. His current research interests include inertial navigation, GNSS navigation, and integrated navigation.

Kaiqiang Feng was born in the Shanxi Province, China, in 1989. He received the Ph.D. degree from the School of Instrument and Electronics, North University of China, Taiyuan, China, in 2019. He is currently working in North University of China. His current research interests include inertial navigation, inertial-based integrated navigation systems, and state estimation theory.

Xiaokai Wei was born in the Nei Monggol Autonomous Region, China, in 1992. He received the B.E. degree in weapon system and launch engineering from the College of Mechatronics Engineering, North University of China, Taiyuan, China, in 2015, where he is currently working toward the Ph.D. degree in instrument science and technology. His current research interests include inertial navigation, integrated navigation and adaptive control.

Jiayu Zhang was born in Shanxi Province, China, in 1994. She received the B.E. degree in electronic science and technology from the School of Instrument and Electronics, North University of China, Taiyuan, China, in 2016, where she is currently pursuing the Ph.D. degree in instrument science and technology. Her current research interests include inertial navigation and rotation modulation.