Delay-dependent Stability Analysis of Modular Microgrid with Distributed Battery Power and SoC Consensus Tracking

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This work was supported in part by National Natural Science Foundation of China under Grant 61603325, in part by the Science and Technology Planning Project of Guangdong Province under Grant 2017A010102013, in part by the Natural Science Foundation of Guangdong Province under Grant 2018A0303130111, and in part by the Guangzhou Science and Technology Planning Project under Grant 201902020003

ABSTRACT The modular microgrid based on distributed battery storages is a simple and reliable power supply way for the islands. Modules are interconnected to the transmission network through the three-port converter. The three-port converter supplies the local ac bus voltage within the module and realizes the power exchange between the module and the transmission network. Distributed battery power and SoC consensus tracking model considering communication time delay is established on the leader-following multi-agent consensus tracking theory. By using the delay decomposition method, where the delay interval is discretized into two segmentations with an equal width, a modified Lyapunov-Krasovskii functional is constructed. A delay-dependent sufficient condition for the absolute stability of the battery power and SoC consensus tracking is derived in terms of linear matrix inequalities (LMIs) by using the free-weighting matrix method. The maximum delay margin corresponding to different controller parameters is calculated with the LMI toolbox of MATLAB. The case study is carried out based on the DongAo Island microgrid demonstration project. Simulations of battery power and SoC consensus tracking with constant time or time-varying delays verifies the effectiveness of the maximum delay margin determined by the stability criteria. Experiments reveal that the three-port converter can meet the requirements of the modular microgrid networking and operation control. The distributed battery power and SoC consensus tracking of the modular microgrid can be achieved when the communication time delay is less than the maximum delay margin.

INDEX TERMS Modular microgrid, power and SoC consensus tracking, distributed battery storages, time delay, multi agent system, Lyapunov-Krasovskii functional

I. INTRODUCTION

THERE are many islands worldwide. With the growth of coastal city economy, the development of islands is becoming more and more popular. How to meet the increasing energy demand has become an urgent problem. Traditional diesel generator power system has high power cost, poor power quality, which leads to environmental problems such as noise and air pollution. Furthermore, it is too expensive to connect the urban power grid to remote small islands. Islands usually possess rich wind and solar radiation resources. The island power loads are also distributed in different geographical locations. Therefore, it is necessary to develop a clean, efficient, and flexible power supply system for the islands.

Microgrid is an autonomous multidirectional power network that integrates distributed renewable energy, load, and storage systems together. There are already many microgrid demonstration projects constructed on islands [1]-[3]. The configuration of the traditional microgrid mimics large power system by including a common ac bus. Because of the low system inertia and high penetration of renewable energy, it is
challenging to maintain voltage and frequency stability in the stand-alone microgrid. Reference [4], [5] introduce a standard three-layer hierarchical control structure conforming to the ISA-95 standards. The primary control usually based on the droop principle and virtual resistance offers the voltage and frequency immediate regulations of the local device subjects to local disturbances [6], [7]. The secondary control usually based on the consensus theory is the coordination control among devices to regulate the voltage and frequency to the rated values [8], [9]. The tertiary control usually based on the intelligent algorithm realizes multi-objective optimization on long-term time horizons [10], [11]. Implementing such a control scheme is expensive and complicated.

The important feature of microgrid is that there are various kinds of controllable converters, which provide the possibility of different system configurations. A modular microgrid based on distributed battery storages is proposed in [12], [13]. One module is built for a concentrated load zone. Modules are connected through 10kV medium voltage transmission network. There is no common ac bus in the modular microgrid and each module is responsible for the ac voltage quality within the module. So the control of modular microgrid only considers the ac voltage control within the module [14]–[16] and exchange power control between the module and the transmission network [17]. Such control scheme is much simpler than that of the traditional one. In addition, the ac voltage quality within the module remains unchanged when modules exit from or connected to the transmission network. The unified modular structure and control strategy make the system easy to expand and maintain.

The battery is the power and energy balancing component in the module. The renewable generation such as wind power and photovoltaic is random and load power is also time-varying. When the module runs independently, the battery is applied to compensate for the supply-demand imbalance, where the battery works in uncontrollable bad conditions. The space-time complementarity of generation and load exist in different geographical locations. Power and SoC consensus control of distributed batteries among modules makes the battery in a controllable mode that helps to reduce the battery power variation and make full use of the distributed battery capacity. Though local ac voltage control technology within the module is much matured, the energy coordination control among modules still needs to be explored in depth.

Multi-agent system (MAS) consensus theory is a powerful tool for research on the distributed system [18]. Existing consensus theory can be divided into two categories: consensus with a leader and consensus without a leader. A novel distributed cooperative control scheme for hybrid energy storage system based on a leaderless consensus protocol is proposed to realize the power splitting between batteries and super capacitors [19]. A MAS-based distributed optimal strategy has been proposed to minimize the generation cost of the distributed renewable generator/energy storage system and meanwhile maintains the supply-demand equality conditions within the microgrid [20]. A novel hierarchical control strategy combined sliding mode control and leader-following consensus control is proposed for islanded microgrid [21]. Aforementioned MAS-based research doesn’t consider the effect of time delay. Transmission delay exists in almost all sparse communication networks, which has a great impact on the system stability.

Analysis and synthesis of time-delay system is an important research topic in the field of control theory. Lyapunov-Krasovskii functional method, which is developed by Razu-mikhin and Krasovskii, has been widely used to analyze the system stability [22]. Because the delay-dependent stability conditions are sufficient conditions, there exist different analysis methods to reduce the conservativeness of criteria, such as cross term estimation, system model transformation, integral inequality, time delay decomposition and free weight matrix method [23]–[25]. By constructing Lyapunov-Krasovskii functional with three items, sampling effect on secondary control of microgrid with consensus protocol is addressed in [26]. A novel secondary voltage and frequency restoration, the optimal active power sharing and the accurate reactive power sharing protocols are proposed and the Lyapunov-Krasovskii function with two items is used to analyze the stability of the scheme [27]. The sampled-delayed-data-based consensus problem for upper-triangular multi-agent nonlinear systems via output feedback is investigated in [28] and a parametric sequence is obtained and its convergence is proved by properly constructing a Lyapunov-Krasovskii function. The key of delay-dependent stability analysis is the construction of the Lyapunov-Krasovskii functional and determination of bound of the derivative term of the functional. The upper bound of time-varying delays is used instead of the time-varying delays themselves in the Lyapunov Krasovskii functional in the above research on microgrid delay-dependent stability, thus the information of time-varying delays is omitted and the conclusion is inevitably conservative. The application of the Lyapunov-Krasovskii functional method to the stability analysis of modular microgrid has also not been reported yet.

The main contribution of this paper is to analyze the delay-dependent stability of the modular microgrid with distributed battery power and SoC consensus tracking protocol by a modified Lyapunov-Krasovskii function. The hardware topology and compound control strategy of three-port converter is designed to meet the requirements of modular microgrid networking and operation control. The distributed battery power and SoC consensus tracking model considering communication time delay is built based on the leader-following multi-agent consensus tracking theory. By adopting the time delay decomposition and piecewise analysis method, a modified Lyapunov-krasovskii function is constructed by decomposing the time delay into two equal intervals. A delay-dependent sufficient condition for the absolute stability of modular microgrid can be derived in terms of linear matrix inequalities (LMIs) by using the free-weighting matrix and integral inequality method.
maximum delay margin with constant time or time-varying delay corresponding to different controller parameters can be determined by applying LMI toolbox in MATLAB. The case study and analyses are carried out on the DongAo Island modular microgrid demonstration project to verify the validity of the stability criterion. The research in this paper gives theoretical guidance for the controller design of the modular microgrid.

The organization of this paper is as follows: Section II illustrates preliminaries and the leader-following multi-agent consensus tracking problem. Section III introduces the distributed battery power and SoC consensus tracking model of modular microgrid considering communication time delay. Section IV discusses the delay-dependent stability criterion based on the Lyapunov-Krasovskii functional. Section V is the case study and analysis on the DongAo Island modular microgrid demonstration project. Section VI is the conclusion.

II. PRELIMINARIES AND LEADER-FOLLOWING MULTI-AGENT CONSENSUS TRACKING PROBLEM

Figure 1 illustrates the undirected communication topology of the leader-following multi-agent system [29]. Agent 0 is the leader agent and other agents are the following agents. Adjacent matrix $B = \text{diag}(b_1, b_2, ..., b_n)$ describes the relationship between the leader agent and the following agents. If there is no communication, then $b_i = 0$, else $b_i > 0$. Adjacent matrix $A = [a_{ij}]$ describes the relationship among the following agents. The adjacent coefficient between the following agent $i$ and agent $j$ exists $a_{ij} = a_{ji}$ for the undirected communication network. The Laplacian matrix $L = [l_{ij}]$ is another matrix describing the communication network and the element is

$$l_{ij} = \begin{cases} -a_{ij} & i \neq j \\ \sum_{j \neq i} a_{ij} & i = j \end{cases} \quad (1)$$

The first order multi-agent system can be expressed as

$$\frac{dx_i(t)}{dt} = u_i(t), i \in I \quad (2)$$

where $x_i$ is the state variable of the agent $i$ and $u_i$ is the control variable.

To ensure that all the following agents can follow the leader’s reference state, a typical continuous-time consensus tracking protocol was presented [30].

$$u_i(t) = b_i (x_0(t) - x_i(t)) + \sum_{j=1}^{n} a_{ij} (x_j(t) - x_i(t)), i = 1, 2, \ldots, n. \quad (3)$$

where $x_0$ is the state variable of the leader agent, $x_i$ and $x_j$ are the state variables of the following agent $i$ and $j$.

When the communication time delay is considered, the control input can be expressed as

$$u_i(t) = b_i (x_0(t - d(t)) - x_i(t - d(t))) + \sum_{j=1}^{n} a_{ij} (x_j(t - d(t)) - x_i(t - d(t))) \quad (4)$$

where $d(t)$ is the time delay.

The following lemmas will be employed for the analysis.

Lemma 1: Considering an undirected communication network topology of the leader-following multi-agent system, the leader is globally reachable only when the Hermite matrix $H = B + L$ is positive definite [31].

Lemma 2: (Jensen integral inequality [32]) For any constant positive definite matrix $R \in R^{n \times n}$, scalar $h_2 \geq h_1 \geq 0$ and function vector $x(t) : [h_1, h_2] \to R^{n \times n}$, the following inequality exists

$$h_{12} \int_{t-h_2}^{t-h_1} x^T(s) ds \geq \left[ \int_{t-h_2}^{t-h_1} x(s) ds \right]^T R \left[ \int_{t-h_2}^{t-h_1} x(s) ds \right] \quad (5)$$

where $h_{12} = h_2 - h_1$.

Lemma 3: (Schur complement Lemma) Given symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, $S_{11} \in R^{n \times n}$, the following three inequalities are equivalent:

$$S_{11} < 0$$

$$S_{12} < 0$$

$$S_{22} - S_{12} S_{11}^{-1} S_{12}^T < 0 \quad (6)$$

Lemma 4: Assuming that there is $a \leq h(t) \leq b$ and constant matrices $R_i(i = 1, 2, 3)$, then the inequality [33]

$$R_1 + [h(t) - a] R_2 + [b - h(t)] R_3 < 0$$

holds if and only if the following two inequalities hold

$$R_1 + [b - a] R_2 < 0 \quad (7)$$

$$R_1 + [b - a] R_3 < 0 \quad (7)$$

Lemma 5: For any vectors $x, y \in R^n$ and positive definite matrix $P \in R^{n \times n}$, the following inequality holds [34]

$$2x^T y \leq x^T P^{-1} x + y^T P y \quad (8)$$

III. DISTRIBUTED BATTERY POWER AND SOC CONSENSUS TRACKING MODEL OF MODULAR MICROGRID CONSIDERING COMMUNICATION TIME DELAY

The topology of the modular microgrid is illustrated in Figure 2. The essential characteristic of the microgrid lies in a similar modular structure. Each module is responsible for the power supply in an individual zone. Optimized energy flow among modules can be conducted through the 10kV transmission network. The modular microgrid is a power electronics dominated renewable energy integrated system. Photovoltaic arrays are connected to the local ac bus through the DC/AC converter. Wind turbines are connected to the local ac bus via the AC/AC converter. Both the photovoltaic arrays and wind turbines work in the maximum power point tracking mode to make full use of renewable energy. So the conventional commercial AC/AC converter and DC/AC converter can be used directly.
Module 0 is responsible for establishing the voltage of the 10kV transmission line. So the three-port converter in module 0 is simplified to the inverter which directly connected to the transmission network through a transformer. Diesel generator only works intermittently in emergency condition with rated power output. The seawater desalination device starts up to consume excessive power.

The modular microgrid can be described as a multi-agent system as mentioned in section II. Module 0 is agent 0 which is the leader agent and module N is agent N which is the following agent.

A. THREE-PORT CONVERTER AND ITS CONTROL STRATEGY
The three-port converter is the key equipment to complete system networking and operating control [17]. Figure 3 shows the hardware topology of the three-port converter which consists of rectifier and inverter. The three-phase three-wire rectifier working in PQ mode is connected to the transmission network to exchange power between the module and the transmission network. The three-phase four-wire inverter working in V/F mode provides the local ac bus voltage within the module.

Figure 4 reveals the control block diagram of the three-port converter. The controller of the rectifier is designed under the d-q axis synchronous rotating reference frame. The ac variables in the three-phase stationary coordination system are transformed into the DC variables in the synchronous rotating coordination system, which are easy to be filtered and controlled. The phase-locked loop (PLL) keeps the d-axis aligned with the space vector of transmission-line voltage. Then the active power control is converted to the control of d-axis grid current and the reactive power control is converted to the control of q-axis grid current. The PI controller with decoupling compensation realizes the zero steady-state error to the control of q-axis grid current. The PI controller with axis grid current and the reactive power control is converted aligned with the space vector of transmission-line voltage.

The controller of the rectifier is designed under the d-q axis synchronous rotating reference frame. The ac converter. The controller of the rectifier is designed under grid current working in PQ mode is connected to the transmission line. So the three-port converter in module 0 in order to simplify the system networking and operating control [17]. Figure 3 of the three-port converter is the key equipment to complete the three-port converter.

The battery SoC is

\[ C_{\text{Batim}}(t) = \frac{\int_{t} P_{\text{Batim}}(t) dt}{3600} \]  

The unit of the battery SoC is kWh.

Power balance in the module is expressed as

\[ P_{\text{Batim}}(t) = P_{Ei}(t) + P_{PVi}(t) + P_{WTi}(t) - P_{LDi}(t), \]

\[ i = 1, 2, \ldots, n. \]  

where \( P_{\text{Batim}} \) is the battery power, and positive for charging and negative for discharging, \( P_{PVi} \) is the photovoltaic power, \( P_{WTi} \) is the wind power, \( P_{LDi} \) is the load power, and \( P_{Ei} \) is the exchange power. \( P_{PVi} \), \( P_{WTi} \) and \( P_{LDi} \) are uncontrollable random variables. So \( P_{\text{Batim}} \) can only be regulated by the exchange power \( P_{Ei} \) through the three-port converter.

The battery SoC is

\[ C_{\text{Batim}}(t) = \frac{\int_{t} P_{\text{Batim}}(t) dt}{3600} \]  

The unit of the battery SoC is kWh.

Power balance in modules is

\[ P_{\text{Bat0}}(t) = P_{E0}(t) = - \sum_{i=1}^{n} P_{Ei}(t) \]  

So the battery power \( P_{\text{Bat0}} \) in module 0 is determined by the exchange power \( P_{Ei} \) of other modules.

C. DISTRIBUTED BATTERY POWER AND SOC CONSENSUS TRACKING PROTOCOL CONSIDERING COMMUNICATION TIME DELAY
The dynamics of the battery power is

\[ \frac{dP_{\text{Batim}}(t)}{dt} = \frac{dP_{Ei}(t)}{dt} + \frac{dP_{PVi}(t)}{dt} + \frac{dP_{WTi}(t)}{dt} - \frac{dP_{LDi}(t)}{dt} \]  

\[ i = 1, 2, \ldots, n. \]  

The battery power \( P_{\text{Batim}} \) and battery SoC \( C_{\text{Batim}} \) are chosen as the consensus variables. According to Figure 5 and (4,12), the distributed battery power and SoC consensus tracking protocol considering communication time-delay is proposed as follows.

\[ \frac{dP_{Ei}(t)}{dt} = u_{i}(t) = k_{a} \left[ b_{0} (S_{\text{Bat0}}(t - d(t))) - S_{\text{Bat0}}(t - d(t)) \right] + \sum_{j=1}^{n} a_{ij} \left( S_{\text{Batj}}(t - d(t)) - S_{\text{Batj}}(t - d(t)) \right) \]  

\[ + k_{p} \left[ b_{0} (P_{\text{Bat0}}(t - d(t))) - P_{\text{Bat0}}(t - d(t)) \right] + \sum_{j=1}^{n} a_{ij} \left( P_{\text{Batj}}(t - d(t)) - P_{\text{Batj}}(t - d(t)) \right) \]  

where \( k_{a} \) and \( k_{p} \) are the controller gains of battery SoC error and battery power error individually.

Let

\[ \Delta P_{\text{Bat0}} = P_{\text{Bat0}} - P_{\text{Bat0}} \]

\[ \Delta C_{\text{Bat0}} = C_{\text{Bat0}} - C_{\text{Bat0}} \]

\[ P_{X_{1}} = \frac{dP_{PVi}(t)}{dt} + \frac{dP_{WTi}(t)}{dt} - \frac{dP_{LDi}(t)}{dt} - \frac{dP_{Bat0(t)}}{dt} \]

Then

\[ \Delta P_{\text{BAT}} = (\Delta P_{\text{Bat1}}, \Delta P_{\text{Bat2}}, \ldots, \Delta P_{\text{Batn}})^{T} \]

\[ \Delta C_{\text{BAT}} = (\Delta C_{\text{Bat1}}, \Delta C_{\text{Bat2}}, \ldots, \Delta C_{\text{Batn}})^{T} \]

\[ P_{X} = (P_{X_{1}}, P_{X_{2}}, \ldots, P_{X_{n}})^{T} \]

According to (10-13), the dynamics of distributed battery
**FIGURE 2.** Topology of the modular microgrid

**FIGURE 3.** Hardware topology of the three-port converter

**FIGURE 4.** Control block diagram of the three-port converter

**FIGURE 5.** Power flow chart of the modular microgrid
power and SoC consensus tracking error can be obtained as

\[
\frac{d\Delta P_{BAT}}{dt} = \begin{bmatrix} 0 & 0 \\ \frac{1}{3600} & 0 \end{bmatrix} \Delta P_{BAT} + \begin{bmatrix} -k_pH \\ -k_sH \end{bmatrix} \Delta C_{BAT}(t-d(t))
\]

\[
+ \begin{bmatrix} P_s \\ 0 \end{bmatrix}
\]

where \( H = B + L \).

\[Q\]

**IV. DELAY-DEPENDENT STABILITY CRITERION BASED ON THE LYAPUNOV-KRASOVSKII FUNCTIONAL**

The delay-dependent robustly stable criterion for the battery power and SoC consensus tracking of modular microgrid is proposed as follows:

**Lemma 6:** If there exist positive definite matrices \( P, Q_1, Q_2, R_1, R_2 \),

\[
\begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix}
\]

and arbitrary matrices \( M_i, N_i \),

\( i = 1, 2 \) with appropriate dimensions satisfying the following LMIs (15) and the dynamic system (14) is globally asymptotic stable under the condition that \( 0 \leq d(t) \leq h \) and \( d(t) \leq \tau_m \).

\[
\begin{bmatrix} \Pi_1 & \Pi_2 \\ \Pi_2 \end{bmatrix} \begin{bmatrix} \sqrt{\frac{\eta}{2}}(R_1 + R_2) & Y_{13} \\ -(R_1 + R_2) & 0 \end{bmatrix} \leq 0, (i = 1, 2; k = 1, 2) \tag{15}
\]

where

\[
\Pi_1 = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ * & c_{22} & c_{23} & c_{24} \\ * & * & c_{33} & c_{34} \\ * & * & * & c_{44} \end{bmatrix},
\]

\[
\Pi_2 = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ * & d_{22} & d_{23} & d_{24} \\ * & * & d_{33} & d_{34} \\ * & * & * & d_{44} \end{bmatrix}
\]

\[
sym(PA) = PA + A^TP^T
\]

\[
Y_{11}^{11} = \sqrt{\frac{\eta}{2}}N_1, Y_{11}^{12} = \sqrt{\frac{\eta}{2}}M_1, Y_{11}^{21} = \sqrt{\frac{\eta}{2}}M_2,
\]

\[
Y_{13}^{22} = \sqrt{\frac{\eta}{2}}N_2, Y_{33}^{13} = -R_1, Y_{33}^{23} = -R_2.
\]

\[
c_{11} = sym(PA + N_{11}) + Q_1 + Q_2 + G_{11}
\]

\[
c_{12} = G_{12} + N_{12}^T - M_{11}
\]

\[
c_{13} = N_{13}
\]

\[
c_{14} = sym(PB) + N_{14}^T - N_{11} + M_{11}
\]

\[
c_{22} = -sym(M_{12} + G_{22} - G_{11} - \frac{2}{h}R_2
\]

\[
c_{23} = -G_{12} + \frac{2}{h}R_2 - M_{13}^T
\]

\[
c_{24} = -N_{12} + M_{12} - M_{23}^T
\]

\[
c_{33} = -G_{22} - Q_2 - \frac{2}{h}R_2
\]

\[
c_{34} = M_{13} - N_{13}
\]

\[
c_{44} = sym(M_{14} - N_{14}) - (1 - \tau_m)Q_1
\]

\[
d_{11} = sym(PA) + Q_1 + Q_2 + G_{11} - \frac{2}{h}R_1
\]

\[
d_{12} = G_{12} + 2R_1 + N_{21}
\]

\[
d_{13} = -M_{21}
\]

\[
d_{14} = sym(PB) + M_{21} - N_{21}
\]

\[
d_{22} = sym(N_{22}) + G_{22} - G_{11} - \frac{2}{h}R_1
\]

\[
d_{23} = -G_{12} + N_{23}^T - M_{22}
\]

\[
d_{24} = M_{22} + N_{24}^T - N_{22}
\]

\[
d_{33} = -sym(M_{23}) - G_{22} - Q_2
\]

\[
d_{34} = -M_{24}^T + M_{23} - N_{23}
\]

\[
d_{44} = sym(M_{24} - N_{24}) - (1 - \tau_m)Q_1
\]

\[
\eta \] denotes the entries implied by symmetry.

**Proof:** Considering the following Lyapunov-Krasovskii functional [35]

\[
V(t) = e^T(t)P \xi(t) + \int_{t-d(t)}^{t} e^T(s)Q_1 e(s)ds
\]

\[
+ \int_{t-h}^{t} \left[ e(s) e^T(s) Q_2 e(s) ds ight] (s) 
\]

\[
+ \int_{t-h}^{t} \left[ e(s) e^T(s) R_1 \xi(s) ds ight] (s) 
\]

\[
+ \int_{t-h}^{t} \left[ e(s) e^T(s) R_2 \xi(s) ds ight] (s)
\]

where

\[
e(t) = (\Delta P_{BAT}^T, \Delta C_{BAT}^T)^T
\]

\[
\xi(t) = \begin{bmatrix} e^T(t) & e^T(t-h) & e^T(t-h) & e^T(t-d(t)) \end{bmatrix}
\]

\[
f_X = \begin{bmatrix} P_X^T \\ 0 \end{bmatrix}
\]

\[
\eta = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & 0 & B_1 \\ 0 & 0 & 0 \end{bmatrix}
\]

\[
A_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},
\]

\[
B_1 = \begin{bmatrix} -k_pH \\ -k_sH \end{bmatrix}
\]

Equation (14) can be written as

\[
\dot{\xi}(t) = \eta \xi(t) + f_X
\]

Differentiating (16) leads to

\[
\dot{V}(t) = 2e^T(t)P \dot{\xi}(t) + e^T(t)(Q_1 + Q_2) e(t)
\]

\[
- e^T(t-d(t)) (1 - d(t)) Q_1 e(t-d(t))
\]

\[
- e^T(t-h) Q_2 e(t-h)
\]

\[
+ \begin{bmatrix} e(t) \\ e(t-h) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-h) \end{bmatrix}
\]

\[
+ \begin{bmatrix} e(t) \\ e(t-h) \end{bmatrix}^T \begin{bmatrix} G_{12} & G_{11} \\ * & G_{22} \end{bmatrix} \begin{bmatrix} e(t-h) \\ e(t) \end{bmatrix}
\]

\[
+ \int_{t-h}^{t} \frac{R_1 + R_2}{2} \dot{\xi}(s) + \int_{t-h}^{t} \frac{R_1 + R_2}{2} \dot{\xi}(s) ds
\]

\[
- \int_{t-h}^{t} \frac{R_1 + R_2}{2} \dot{\xi}(s) ds
\]

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According to Lemma 3 and Lemma 4, for any \( t > 0, d(t) \in \left[0, \frac{h}{2}\right]\) or \( d(t) \in \left(\frac{h}{2}, h\right]\), define \( \Delta_1 = \left\{ t : d(t) \in \left[0, \frac{h}{2}\right] \right\} \) and \( \Delta_2 = \left\{ t : d(t) \in \left(\frac{h}{2}, h\right) \right\} \).

**A. CASE A:**

When \( t \in \Delta_1 \), according to Lemma 2,
\[
- \int_{t - \frac{h}{2}}^{t} e^{\xi(t)}(s) R_2 \dot{e}(s) ds \\
\leq \frac{2}{h} \begin{bmatrix} e(t) \\ e(t - \frac{h}{2}) \end{bmatrix}^T \begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2 \end{bmatrix} \begin{bmatrix} e(t) \\ e(t - \frac{h}{2}) \end{bmatrix} \tag{19}
\]

According to the Leibniz-Newton formula, we can get
\[
e(t) - e(t - d(t)) - \int_{t - d(t)}^{t} \dot{e}(s) ds = 0 \tag{20}
\]

Then exists the following equations
\[
2\xi^T(t) N_1 \begin{bmatrix} e(t) - e(t - d(t)) - \int_{t - d(t)}^{t} \dot{e}(s) ds \end{bmatrix} = 0 \tag{21}
\]
\[
2\xi^T(t) M_1 \begin{bmatrix} e(t) - e(t - d(t)) - \int_{t - d(t)}^{t} \dot{e}(s) ds \end{bmatrix} = 0 \tag{22}
\]

where \( N_1^T = [N_1^T, \ldots, N_1^{T_2}], M_1^T = [M_1^T, \ldots, M_1^{T_4}] \).

According to Lemma 2 and Lemma 5, there exist the following inequalities
\[
-2\xi^T(t) N_1 \int_{t - d(t)}^{t} \dot{e}(s) ds \\
\leq \int_{t - d(t)}^{t} \dot{e}(s) ds \\
+ \int_{t - d(t)}^{t} \dot{e}(s) ds \\
\leq \frac{h}{2} - d(t) \xi^T(t) M_1 R_2^{-1} N_1^T \xi(t) \tag{24}
\]

According to (19-24), we can get
\[
\dot{V}(t) \leq \xi^T(t) \Pi_1 \xi(t) \\
+ \xi^T(t) \begin{bmatrix} \eta \frac{h}{2} (R_1 + R_2) \eta + d(t) N_1 R_2^{-1} N_1^T \\ \frac{h}{2} - d(t) M_1 R_2^{-1} M_1^T \end{bmatrix} \xi(t) \tag{25}
\]

According to Lemma 3 and Lemma 4, \( \dot{V}(t) \leq 0 \) holds when the following LMIs (26) and (27) hold.
\[
\Pi_1 \begin{bmatrix} \eta \frac{h}{2} (R_1 + R_2) & \sqrt{\frac{h}{2}} N_1 \\ * & -(R_1 + R_2) \end{bmatrix} \begin{bmatrix} \frac{h}{2} N_1 \\ 0 \end{bmatrix} \leq 0 \tag{26}
\]
\[
\Pi_1 \begin{bmatrix} \eta \frac{h}{2} (R_1 + R_2) & \sqrt{\frac{h}{2}} M_1 \\ * & -(R_1 + R_2) \end{bmatrix} \begin{bmatrix} \frac{h}{2} M_1 \\ 0 \end{bmatrix} \leq 0 \tag{27}
\]

**B. CASE B:**

When \( t \in \Delta_2 \), according to Lemma 2, there exists
\[
- \int_{t - \frac{h}{2}}^{t} e^{\xi(t)}(s) R_2 \dot{e}(s) ds \\
\leq \frac{2}{h} \begin{bmatrix} e(t) \\ e(t - \frac{h}{2}) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} e(t) \\ e(t - \frac{h}{2}) \end{bmatrix} \tag{28}
\]

According to the Leibniz-Newton formula, we can get
\[
e(t) - e(t - d(t)) - \int_{t - d(t)}^{t} \dot{e}(s) ds = 0 \tag{29}
\]

Then exists the following equations
\[
2\xi^T(t) N_2 \begin{bmatrix} e(t) - e(t - d(t)) - \int_{t - d(t)}^{t} \dot{e}(s) ds \end{bmatrix} = 0 \tag{30}
\]
\[
2\xi^T(t) M_2 \begin{bmatrix} e(t) - e(t - d(t)) - \int_{t - d(t)}^{t} \dot{e}(s) ds \end{bmatrix} = 0 \tag{31}
\]

where \( N_2^T = [N_2^T, \ldots, N_2^{T_2}], M_2^T = [M_2^T, \ldots, M_2^{T_4}] \).

According to the Lemma 2 and Lemma 5, there exist the following inequalities
\[
-2\xi^T(t) N_2 \int_{t - d(t)}^{t} \dot{e}(s) ds \\
\leq \int_{t - d(t)}^{t} \dot{e}(s) ds \\
+ \int_{t - d(t)}^{t} \dot{e}(s) ds \\
\leq \frac{h}{2} - d(t) \xi^T(t) M_2 R_2^{-1} N_2^T \xi(t) \tag{32}
\]

According to (28-33), we can get
\[
\dot{V}(t) \leq \xi^T(t) \Pi_2 \xi(t) \\
+ \xi^T(t) \begin{bmatrix} \eta \frac{h}{2} (R_1 + R_2) \eta + d(t) N_2 R_2^{-1} N_2^T \\ \frac{h}{2} - d(t) M_2 R_2^{-1} M_2^T \end{bmatrix} \xi(t) \tag{34}
\]

According to Lemma 3 and Lemma 4, \( \dot{V}(t) \leq 0 \) holds when the following LMIs (35) and (36) hold.
\[
\Pi_2 \begin{bmatrix} \sqrt{\frac{h}{2}} R_2 \frac{h}{2} (R_1 + R_2) & \sqrt{\frac{h}{2}} N_2 \\ * & -(R_1 + R_2) \end{bmatrix} \begin{bmatrix} \frac{h}{2} N_2 \\ 0 \end{bmatrix} \leq 0 \tag{35}
\]
\[
\Pi_2 \begin{bmatrix} \sqrt{\frac{h}{2}} R_2 \frac{h}{2} (R_1 + R_2) & \sqrt{\frac{h}{2}} M_2 \\ * & -(R_1 + R_2) \end{bmatrix} \begin{bmatrix} \frac{h}{2} M_2 \\ 0 \end{bmatrix} \leq 0 \tag{36}
\]
V. CASE STUDY AND ANALYSIS

The modular microgrid demonstration project has been established on DongAo Island which is a tourism island. Four modules are configured to supply power for four separated concentrated load zones. They are a comprehensive building zone, a cultural center zone, a power plant zone and a wharf zone as shown in Figure 6. Each module is composed of 50kW photovoltaic arrays, 20kW wind turbines, 300kWh battery storages, and necessary converters. Module 0 only consisting of 300kWh battery storages is configured to provide the 10kV transmission-line ac voltage. The uniform method is used to set the adjacent coefficients and the undirected communication network of modular microgrid is established as Figure 7. Red line, blue line, green line, magenta line, and cyan line represent variables of module 0, 1, 2, 3, 4 respectively in the following figures.

According to Figure 7, We can easily get

$$\begin{bmatrix}
\frac{1}{2} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$L = \begin{bmatrix}
\frac{2}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\
-\frac{1}{3} & 1 & -\frac{1}{3} & \frac{1}{3} \\
0 & \frac{1}{2} & \frac{1}{2} & 0 \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 1
\end{bmatrix}$$

(37)

$$\begin{bmatrix}
\frac{7}{6} & \frac{1}{3} & 0 & \frac{1}{3} \\
\frac{1}{3} & \frac{3}{2} & -\frac{1}{3} & \frac{3}{2} \\
0 & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 1
\end{bmatrix}$$

$$H = \begin{bmatrix}
\frac{1}{2} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

Eigenvalues of $H$ are $\lambda_1(H) = 0.1847$, $\lambda_2(H) = 0.9053$, $\lambda_3(H) = 1.4585$ and $\lambda_4(H) = 1.7849$. According to Lemma 1, eigenvalues are positive and the leader agent is globally reachable.

The dynamics of distributed battery power and SoC consensus tracking error is described by (14). The delay-dependent robustly stable criterion is Lemma 6. The delay margin can be obtained through solving the LMIs (15) with the function feasp() in the MATLAB. Table 1 shows the maximum delay margin $h$ according to different controller parameters.

![Figure 6. Modular microgrid in DongAo Island](image)

![Figure 7. Undirected communication network topology of the modular microgrid](image)

A. SIMULATIONS OF DISTRIBUTED BATTERY POWER CONSENSUS TRACKING WITH CONSTANT TIME DELAY

When only battery power consensus is considered, then $k_s = 0$. Load power is 0kW, 5kW, 10kW, 15kW, and 20kW in module 0, 1, 2, 3 and 4 respectively. When the constant time delay is considered, $d(t) = 0$ and $\tau_m = 0$, the delay margin is mainly determined by $k_p$. When $k_p = 1$, the calculated delay margin is $h = 0.7948s$. Figure 8 illustrates the power consensus tracking with different time delays. Figure 8(a) and 8(b) show the battery power and exchange power individually when $d = 0.7948s$. The battery power converges to -10kW asymptotically in spite of oscillation. Accordingly, module 0 and 1 send 10kW and 5kW to the transmission network individually. Module 3 and 4 borrow 5kW and 10kW from the transmission network individually. There is no power exchange between module 2 and the transmission network. Figure 8(c) and 8(d) reveal the power consensus tracking oscillates and diverges when $d = 0.9s$ respectively. It turns out that if the constant time delay is within the delay margin, the system is absolutely stable. The stability criterion is effective for constant time delay.

B. SIMULATIONS OF DISTRIBUTED BATTERY POWER CONSENSUS TRACKING WITH TIME-VARYING DELAY

When the time-varying delay is considered, $0 < d(t) < \tau_m$, the term related to $\tau_m$ in $\Pi_1$, $\Pi_2$ is $-(1 - \tau_m)Q_1$. When $\tau_m < 1$ the term is negative definite and $\tau_m$ has an influence on maximum delay margin. When $\tau_m \geq 1$, the term is positive semidefinite and it is difficult to obtain the feasible solution. Table 1 shows that the delay maximum margin remains unchanged when $\tau_m \geq 1$.When $k_p = 1, k_s = 0$, $\tau_m = 3.418$ and $h = 0.6837s$, Figure 9(a) and 9(b) describe the actual battery power and the delayed sampling battery power for control when $k_p = 1$, $k_s = 0$, and $d = 0.3418(1 + \sin 10t)s$ separately. Although the delayed sampling battery power for control is time-varying, the actual battery power still converges to the consensus power -10kW. It turns out that if the time-varying delay is within the maximum delay margin, the system is absolutely stable. The stability criterion is effective for the time-varying delay.
### TABLE 1. Delay margin with different controller parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_p$</td>
<td>0.5</td>
</tr>
<tr>
<td>$k_s$</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_m$</td>
<td>0</td>
</tr>
<tr>
<td>$h(s)$</td>
<td>1.5890</td>
</tr>
</tbody>
</table>

(a) Battery power ($k_p = 1$, $k_s = 0$, $d = 0.7948 s$)

(b) Delayed sampling battery power ($k_p = 1$, $k_s = 0$, $d = 0.7948 s$)

(c) Battery power ($k_p = 1$, $k_s = 0$, $d = 0.9 s$)

(d) Exchange power ($k_p = 1$, $k_s = 0$, $d = 0.9 s$)

### FIGURE 8. Battery power consensus tracking with constant time delay

### FIGURE 9. Battery power consensus tracking with time-varying delay
FIGURE 10. Battery SoC consensus tracking with $k_p = 1$, $d = 0.5s$ and different $k_s$.

FIGURE 11. Operation of the three-port converter with given exchange power.
**FIGURE 12.** Battery power and SoC consensus tracking of the modular microgrid
C. SIMULATIONS OF DISTRIBUTED BATTERY SOC CONSENSUS TRACKING

Figure 10 shows the battery SoC consensus tracking with \( k_p = 1, d = 0.5s \) and different \( k_s \). The initial battery SoC is 140kWh, 160kWh, 180kWh, 200kWh and 220kWh in module 1, 2, 3, 4 and 0 respectively. Figure 10(a) illustrates the battery power. From 0s to 10s, \( k_s = 0.1 \) and the battery power in modules 1, 2, 3, 4 and 0 converges to -6kW, -8kW, -10kW, -12kW, -14kW respectively. The larger the battery SoC is, the larger the battery discharge power is, thus reducing the difference of the battery SoC. From 10s to 20s, \( k_s = 0.2 \) and the battery power in modules 1, 2, 3, 4 and 0 converges to -2kW, -6kW, -10kW, -14kW, -18kW respectively. Figure 10(b) shows the change of the battery SoC relative to its initial SoC. It reveals that the larger the \( k_s \), the faster the battery SoC of module 0 decreases, thus accelerating the convergence rate of battery SoC consensus tracking. The time to achieve the battery SoC consensus tracking is very long and usually takes several hours.

D. EXPERIMENTS OF DISTRIBUTED BATTERY POWER AND SOC CONSENSUS TRACKING OF MODULAR MICROGRID

1) Operation of the three-port converter

Effective operation of the three-port converter is the basis for battery power and SoC consensus tracking of modular microgrid. The output power of three-port converter equals the load power minus renewable generation power according to Figure 5. Figure 11(a) shows that the rectifier of the three-port converter can realize the desired exchange power control. From 4:30 to 8:00 and from 16:00 to 2:25, the renewable generation power is less than load power, the output power of three-port converter is positive. The exchange power is set to about 40kW to provide extra energy to avoid the overdischarge of batteries. From 8:00 to 16:30, the renewable generation power is greater than load power, the battery is in charge. There is no need for external power replenishment and so exchange power is set to zero. Figure 11(b) illustrates that the real-time SoC of battery is within the rational limits. Improper exchange power will cause the battery SoC to exceed the limits, which requires a larger battery capacity.

Figure 11(c) shows that the inverter of the three-port converter can realize three-phase balanced ac bus voltage output with low distortion at the output terminal even under nonlinear or unbalanced loads. The compound control strategy of the inverter ensures the power quality of local ac bus voltage within the module. Figure 11(d) reveals RMS values of the three-phase ac voltage at a load point far away from the three-port converter. It turns out that the voltage is unbalanced through the transmission line and voltage compensation device is also needed in the modular microgrid.

2) Distributed battery power and SoC consensus tracking

Figure 12 shows the battery power and SoC consensus tracking of modular microgrid when \( k_p = 1 \) and \( k_s = 0.1 \). Figure 12(a) is the photovoltaic power which has similar distribution characteristics. Wind power in Figure 12(b) is different because of the different geographical locations. Load power in Figure 12(c) also varies greatly because of different types of electric equipment. Figure 12(d) illustrates the exchange power of modules. The exchange power changes dramatically to reduce the fluctuation of battery power. Figure 12(e) shows the battery power. The battery power is much smoother than other power, which is beneficial to battery performance. When battery SoC is not consistent, the battery power is also not consistent. Once the battery SoC consensus tracking is realized, the battery power becomes consistent. Figure 12(f) describes the battery SoC of modules. It turns out that the battery SoC consensus is achieved through the regulation of exchange power despite differences in initial battery SoC. The stability of battery power and SoC consensus tracking is assured when delay time is within the delay margin according to Lemma 6.

VI. CONCLUSION

In this paper, the modular microgrid based on distributed battery storages is introduced first. Then the hardware topology and compound control strategy of three-port converter is designed to meet the requirement of the modular microgrid networking and operation control. The distributed battery power and SoC consensus tracking model of modular microgrid considering communication time delay have been established based on the leader-following multi-agent consensus tracking theory. By constructing an improved Lyapunov-Krasovskii functional, the delay-dependent sufficient condition for the target problem has been derived with LMIs by employing the free-weighting matrix and integral inequality method. The maximum delay margin of the constant time delay or the time-varying delay to different controller parameters has been calculated by the LMI toolbox of MATLAB. A case study has been carried out based on the modular microgrid demonstration project on DongAo Island. Simulation results turn out the validity of the stability criterion. The experiments verify the function of the three-port converter. The distributed battery power and SoC consensus tracking of modular microgrid is achieved with the consensus tracking protocol when the time delay is within the stable region.

REFERENCES


12

VOLUME 4, 2016


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