A New Geosynchronous SAR Constellation And Its Signal Characteristics

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This work was supported by the National Natural Science Foundation of China under Grant 60972149, Grant 61101200.

ABSTRACT The Geosynchronous Synthetic Aperture Radar (GEO SAR) has a long synthetic aperture time. The temporal decoherence and azimuth variance characteristics of echo are significant, which brings great challenges to signal processing. In order to reduce the synthetic aperture time and thus improve the observation flexibility and efficiency, this paper proposes a SAR system mounted on a special satellite in reverse equatorial geosynchronous orbit, so called the reverse equatorial GEO SAR. The orbit is nearly circular, and the orbital plane coincides with the equatorial plane. The satellite flies against the direction of Earth’s rotation. The relative velocity is accelerated twice and the synthetic aperture time is shorter. The satellite does not drift in latitude direction and drift uniformly in longitude direction. The echo has azimuth invariance property. Meanwhile, the 24-hour continuous observation can be achieved for areas with a certain range of latitude by means of large squint technique and constellation design. As the GEO SAR has high orbit, large delay, large squint angle and wide pulse width, the complex modulation effects are generated by continuous motion between satellite and target during imaging period. In this paper, the uniform acceleration curve motion is used to describe the motion of both satellite and target, and the linear approximation is used to describe the slant range change in the process of transmitting and receiving one pulse. Accordingly, an accurate signal model for reverse equatorial GEO SAR is established, which provides a theoretical model for signal characteristics analysis and imaging algorithms development.

INDEX TERMS Geosynchronous SAR, SAR constellation, SAR signal model

I. INTRODUCTION

The satellite-based Synthetic Aperture Radar (SAR) is known as spaceborne SAR. Currently, the spaceborne SAR satellites are all in low-orbit. After years of development, the technology of low-orbit SAR has been relatively mature. However, due to the restriction of orbital height, the temporal resolution cannot be greatly improved. The geosynchronous (GEO) SAR, mounted on the geosynchronous satellite platform, has higher orbital altitude, larger field of view, shorter orbital period, and better temporal resolution [1], [2]. It is able to quickly revisit the fixed area and to continuously observe the fixed area for a long time [3]. Therefore, the GEO SAR has gradually become a research hotspot in the spaceborne SAR field [4] [5], [6].

The resolution of spaceborne SAR is proportional to slant range and inversely proportional to relative velocity and synthetic aperture time. Therefore, the GEO SAR usually takes hundreds or even thousands of seconds of synthetic aperture time to achieve high resolution [7]. In that case, the variation of both radar system and target scattering characteristics will cause decoherence problems, which will affect the azimuth focusing ability of SAR [8]. In addition, the long synthetic aperture time will cause problems such as modeling of slant range histories and batch processing of azimuth variant signals, which will make the imaging algorithms more difficult [9], [10].

The spaceborne SAR generally adopts side looking geometry, so the observable orbit arc of target area is short. The use of large squint technology can extend the observable orbit arc to a great extent and increase the flexibility and efficiency of observation [11]. However, it leads to further increment of synthetic aperture time and still has imaging
blind areas such as forward-looking, backward-looking and downward-looking areas. Therefore, it is impossible for a single GEO SAR satellite to achieve 24-hour continuous observation of a fixed area. The satellite network is a way to overcome this problem [12]. In [13], NASA and JPL proposed the global earthquake satellite system (GESS) program, which included five groups of satellites. Each group consists of two GEO SAR satellites with the same ground track of nadir point and 180 degrees apart in satellite's latitude angle. Each GEO SAR satellite runs on a circular orbit with an inclination of 60 degrees. The GESS constellation (abbreviated as ConI below) has nearly continuously global observation ability, but is faced to long synthetic aperture time, great difficulty in signal processing and big difference in global imaging performance.

To overcome these problems above, a new GEO SAR constellation (abbreviated as ConII below) is proposed in this paper. Firstly, the satellites are evenly distributed on a circular orbit that coincides with the equatorial plane. The angular velocity of the satellite is the same as that of the Earth, and the direction is opposite. It means that the satellite flies from east to west. In Earth-Centered Earth-Fixed (ECEF) coordinate, the angular velocity of the satellite is doubled and the synthetic aperture time is greatly shortened. A single GEO SAR adopts the large squint technology to observe a specific target area for a long time. When the acceptable squint angle range is exceeded, the latter GEO SAR takes the place of the current one to achieve 24-hour continuous observation. This GEO SAR ConII has exactly the same observation geometry for the surface with the same latitude and different longitude (Without considering the surface topography, the Earth is assumed to be an ellipsoid). The echo has invariant characteristic along azimuth direction, which greatly simplifies the imaging algorithms and maintains imaging performance consistency.

Due to the large squint angle and large propagation delay of the proposed GEO SAR, the radial velocity becomes larger, and the slant range changes faster. Thus, the assumption of stop-and-go gets invalid, which causes two effects [14-18]. One is the slow-time effect, which makes accurate calculation of two-way slant range difficult. The other is the fast-time effect, which modulates each pulse and makes the signal modeling complicated. [16] and [17] take these two effects into account in airborne SAR geometry, where both the orbit bending and the Earth rotation are ignored. [14], [15], [18], and [22] research in spaceborne SAR geometry. In [14], an “equivalent position” model is proposed to deal with the slow-time effect. In [22], a uniformly linear motion is used instead of stop-and-go to get the analytical expression of two-way slant range. Meanwhile, a simplified compensation for fast-time effect is given in azimuth time domain. In [15], the linear terms in azimuth frequency domain are compensated for fast-time effect, but the complete formation is not established and the effects of higher order terms cannot be revealed. In [18], a complete formation for compensation is carefully established, which is based on a modified hyperbolic rang equation (MHRE). MHRE approximates to the cubic terms of actual slant range precisely, but the quartic error cannot be ignored as the synthetic aperture time is very long in GEO SAR.

This paper establishes an analytical solution for two-way slant range calculation based on curvilinear motion of uniform acceleration of both satellite and target. Then, the linear approximation is carried out to describe the slant range changing over fast time within transmission and reception process of one pulse. Accordingly, the complete formation of GEO SAR echo is derived, which provides a precise model for signal characteristics analysis and imaging algorithms development.

The rest of this paper is organized as follows: Section II describes how to design the new GEO SAR constellation, and shows its advantages in mid-latitude observation by simulation results. Section III constructs the curvilinear motion of constant acceleration for satellite and target, and then derives the accurate slant range expression, the precision of which is validated by contrast with the other slant range expression. Section IV establishes the modified signal model in both time domain and frequency domain, in terms of the linear slant range variation within each pulse process. The simulation results verify that the modified signal model is effective in both side looking and squint looking geometries.

II. GEO SAR CONSTELLATION DESIGNING

The relative movement between the spaceborne SAR and the observed target is comprised of satellite orbital motion and Earth’s rotation. If the included angle between two angular velocity vectors of satellite and Earth is below 90 degrees, the Earth’s rotation cancels out the satellite motion to some extent. Otherwise, the relative motion is enhanced, and the synthetic aperture time is shortened. The imaging geometry changes a great deal over the orbit, that is the reason why the received echo shows azimuth variance property. As the orbital drift in latitude gets larger, the azimuth variety gets more serious, and vice versa. Based on the two points above, a new GEO SAR ConII flying in reverse equatorial plane is introduced in this section. The range and azimuth resolution expressions are deduced out to establish the SAR parameters. The controlling strategy of satellite platform and radar antenna is given to make the sensor footprint illuminate the target for a long time. The constellation is design to realize 24-hour continuous observation for mid-latitude region. At last, the performance of the proposed ConII is analyzed by comparing with ConI.

A. IMAGING TIME OF SINGLE GEO SAR

The single GEO SAR geometry in relation to a given target is shown in Fig. 1. Compared with the GEO SAR given in ConI, this new GEO SAR has the following characteristics: (1) During one orbital period, the satellite orbits the Earth twice and repeats the observation period for 12 hours; (2) The
ground track of nadir point is always limited within the equator, and the relative motion of the satellite and the Earth remains unchanged; (3) The imaging performance for targets at the same latitude is consistent, as long as the satellite system is invariable and only delayed in time.

\[ \nabla f_a = \left[ \frac{\mathbf{V}_s - (\mathbf{V}_s \cdot \mathbf{n}) \mathbf{n}}{|\mathbf{R}_s - \mathbf{R}_T|} \left( \mathbf{R}_s - \mathbf{R}_T \right) \right] \]

As given in [19], the vectors, \( \mathbf{R}_s \), \( \mathbf{R}_T \), and \( \mathbf{n} \), have analytical expressions, so it is more concise to calculate \( \mathbf{VR} \) and \( \nabla f_a \).

For each target area, at \( t = 0 \), the incident angle \( \theta_{IA0} \) is calculated by (3). If \( \theta_{IA0} < \theta_{IAMIN} \) (Acceptable minimum value of incident angle) or \( \theta_{IA0} > \theta_{IAMAX} \) (Acceptable maximum value of incident angle), the target cannot be observed by this GEO SAR ConII. Otherwise, the imaging conditions are analyzed in 12 hours centred on \( t = 0 \).

\[ \theta_{IA0} = \left\{ \mathbf{n}, \mathbf{R}_S - \mathbf{R}_T \right\} \]  

The incident angle \( \theta_{IA} \) is given by (4). At \( t = 0 \), \( \theta_{IA0} \) is minimum. As \( t \) increases or decreases, \( \theta_{IA} \) gets larger to 90° when the line of sight is tangent to Earth’s surface.

\[ \theta_{IA} = \left(\mathbf{n}, \mathbf{R}_S - \mathbf{R}_T\right) \]  

The ground resolution included angle and Doppler gradient directions \( \theta_{GRIA} \) is given by (5). At \( t = 0 \), when the side looking geometry is formed, \( \theta_{GRIA} = 90° \). As \( t \) increases or decreases, \( \theta_{GRIA} \) gets smaller first and then gets larger to 90° again at the time when \( \theta_{IA} = 90° \).

\[ \theta_{GRIA} = \cos^{-1} \left( \frac{\mathbf{VR} \cdot \nabla f_a}{|\mathbf{VR}| |\nabla f_a|} \right) \]

For azimuth resolution \( \rho_{az} \), the required synthetic aperture time \( T_s \) is given by (6), where \( \lambda \) is the wavelength. At \( t = 0 \), \( T_s \) is minimum. As \( t \) increases or decreases, \( T_s \) gets larger.

\[ T_s = \frac{\lambda}{2 \rho_{az} |\nabla f_a|} \sin \theta_{GRIA} \]

For range resolution \( \rho_{r} \), the required signal bandwidth \( B \) is given by (7), where \( c \) is the speed of light in vacuum. At \( t = 0 \), \( B \) is maximum. As \( t \) increases or decreases, \( B \) gets smaller.

\[ B = \frac{c}{2 \rho_{r} |\nabla f_a|} \sin \theta_{GRIA} \]

According to Equations (4) to (7), we can find four time span L1, L2, L3, and L4 from four imaging conditions. First, L1 is found when \( \theta_{IA} > \theta_{IAMAX} \) and \( \theta_{IA} < \theta_{IAMIN} \). Second, L2 is found when \( \theta_{GRIA} > \theta_{GRIAMAX} \) (Acceptable minimum value of ground resolution included angle). Third, L3 is found when \( T_s < \text{SATMAX} \) (Acceptable maximum value of synthetic aperture time). Fourth, L4 is found when \( B < \text{SBMax} \) (Acceptable maximum value of signal bandwidth). Due to the geometry symmetry, L1, L2, L3, and L4 are centred on \( t = 0 \), the intersection of which gives the imaging time of signal GEO SAR, T_{Single}.
B. DISTRIBUTION AND OPERATION OF GEO SAR CONSTELLATION

In case several duplicated GEO SAR satellites are launched in the same orbit, the given target can be observed continuously by constellation relay. That is, the former satellite comes into the imaging time span first, and then the latter one comes into the imaging time span when the former one cannot satisfy imaging conditions. We assume that the satellites have 1-D roll steering capability and the antennas have 1-D azimuth scanning capability. In order to achieve 24-hour continuous observation, the way to control the rolling angle and the azimuth scanning angle under the guidance of target is established.

To fulfill continuous observation, the numbers of GEO SAR satellites that composes the constellation is given by \( N_{\text{Constellation}} = \left\lfloor \frac{12}{T_{\text{Single}}} \right\rfloor \), where \( \left\lfloor \cdot \right\rfloor \) calculates the nearest integer toward infinity. Then, the imaging time of each GEO SAR in constellation is renewed as follows:

\[
T_{\text{Constellation}} = \frac{12}{N_{\text{Constellation}}} \tag{8}
\]

In an optimized design, the satellite would be evenly distributed in longitude, and the longitude difference can be expressed as

\[
\text{LON}_{\text{Diff}} = \frac{360}{N_{\text{Constellation}}} \tag{9}
\]

To move the beam centre line to the given target, the rolling angle is set by \( (10) \). If the target locates in northern hemisphere, the satellite platform turns right where the rolling angle is defined as positive, and vice versa. At \( t=0 \), \( \theta_{\text{roll}}(t) \) is maximum. As \( t \) increases or decreases, \( \theta_{\text{roll}}(t) \) gets smaller.

\[
\theta_{\text{roll}}(t) = \tan^{-1} \left( \frac{R_s \sin \alpha}{a - R_s \cos \alpha \cos(2\omega t)} \right) \tag{10}
\]

To move the beam centre point to the given target, the azimuth angle is set by \( (11) \). At \( t=0 \), \( \theta_{\text{az}}(t) = 0^\circ \). As \( t \) increases, \( \theta_{\text{az}}(t) \) gets larger to minus direction. As \( t \) decreases, \( \theta_{\text{az}}(t) \) gets larger to plus direction.

\[
\theta_{\text{az}}(t) = -\sin^{-1} \left( \frac{R_s \cos \alpha \sin(2\omega t)}{|R_s - R_t|} \right) \tag{11}
\]

The given GEO SAR constellation running on the equatorial geosynchronous orbit share the same orbit with those geostationary commercial communication satellites, [20] and [21]. The collision cannot be occurred between the satellites in constellation. The collision between the constellation and the other commercial communication satellites can be avoided by increasing or decreasing the orbit height to some extent. By the way, this change has no effect on the overall design of the constellation and the subsequent analysis of the signal characteristics.

C. SIMULATION

In the following, an example of GEO SAR ConII is demonstrated. The range and azimuth resolution in ground plane is set to be 5m. The L-band (wavelength 24cm) is chosen, as that in [13]. The imaging conditions are shown in Table I.

<table>
<thead>
<tr>
<th>TABLE I IMAGING CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SATMax</td>
</tr>
<tr>
<td>300s</td>
</tr>
</tbody>
</table>

The area required for 24-hour continuous observing is from 20\(^\circ\)N to 45\(^\circ\)N. Two representative cities (Harbin and Haikou) are selected at the latitude edges, the locations of which are shown in Table II.

<table>
<thead>
<tr>
<th>TABLE II LOCATION OF TWO REPRESENTATIVE CITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>City</td>
</tr>
<tr>
<td>Central longitude</td>
</tr>
<tr>
<td>Central latitude</td>
</tr>
</tbody>
</table>

According to Equations (4) to (7), the available imaging time of Harbin is 3.134h (1.567h before its side looking time and 1.567h after) and that of Haikou is 2.666h (1.333h before its side looking time and 1.333h after). Therefore, five satellites are required for 24-hour continuous observation. According to Equation (10), the rolling angle required for Harbin is more demanding which is from 6.73deg to 6.88deg. According to Equation (11), the azimuth angle required for Haikou is more demanding, which is from -5.37deg to 5.37deg. The parameters of the constellation are shown in Table III.

<table>
<thead>
<tr>
<th>TABLE III CONSTELLATION PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>( T_{\text{Constellation}} )</td>
</tr>
<tr>
<td>( N_{\text{Constellation}} )</td>
</tr>
<tr>
<td>( \text{LON}_{\text{Diff}} )</td>
</tr>
<tr>
<td>Maximum rolling angle</td>
</tr>
<tr>
<td>Maximum azimuth angle</td>
</tr>
</tbody>
</table>

The observation capability of GEO SAR ConII based on the parameters in Table III is compared with that of ConI by simulations. The radar parameters and imaging conditions are coherent except that, in ConI, the SATMax is added four times to 1200s and the satellite number is doubled.

The global imaging time of the two constellations are compared in Fig. 2. The simulation results show that the ConI has global imaging capability, whereas the ConII cannot image the low-latitude and high-latitude areas.
The average imaging time across all longitudes with respect to latitude of ConI and ConII are compared in Fig. 3. It can be seen that, in the latitude range from 15 degrees to 50 degrees in both south and north hemisphere, the ConII can realize 24-hour imaging, which is superior to ConI.

The imaging time varying with longitude of ConI and ConII are compared in Fig. 4, for latitudes of Harbin and Haikou. The simulation results show that, for certain latitude, the imaging time of ConII remains unchanged with longitude, but that of ConI fluctuates greatly.

The average synthetic aperture time across all longitudes with respect to latitude of ConI and ConII are compared in Fig. 5. The synthetic aperture time of Harbin and Haikou in one orbital period (24h) are shown in Fig. 6. The simulation results show that: (1) In the mid-latitude area, the average synthetic aperture time of ConII is between 180s and 200s, and that of ConI is between 450s and 550s. (2) The synthetic aperture time in ConII of Harbin is from 150s to 180s, and that of Haikou is from 140s to 280s, meanwhile that in ConI of Harbin is from 300s to 1300s, and that of Haikou is from 350s to 700s. Compared with ConI, the synthetic aperture time of ConII is shortened more than a half, and changes with regularity, which is a notable feature.
III. ACCURATE SLANT RANGE MODELING

A. LINEAR APPROXIMATION OF RANGE HISTORY WITHIN ONE PULSE PROCESS

GEO SAR has a very high orbital altitude, the imaging distance of which exceeds 36,000 km. During the time from transmitting to receiving one pulse signal, the relative motion between satellite and ground causes that the transmitting slant range and the receiving slant range are no longer equal. Moreover, since the given GEO SAR system adopts the squint technology to observe continuously and the large pulse width signal to enhance transmission power (The pulse width is set to be 1ms in [13], [22]), the signal distortion caused by relative motion during pulse transmission period and reception period is not negligible. Therefore, the stop-and-go assumption is no longer valid.

The comparison of the three relative motion models is shown in Fig. 7. As shown in left figure, the range history is kept constant during the period from transmission to reception of one pulse (of hundreds of ms level), which is based on the traditional stop-and-go assumption. As shown in middle figure, the transmitting range history is kept constant during the transmission period and the receiving range history is kept constant during the reception period (of hundreds of μs level), which is based on the inner pulse stop-and-go assumption. As shown in right figure, the range history is changing all the time, which is the foundation of this paper.

In order to describe the effect of continuous satellite and ground motion on range history, this paper establishes the variation of slant range with both the slow and the fast time. The fast time axis is set as $\tau$, and slow time axis is set as $\eta$. When the signal is received at time $\eta + \tau$, it has experienced the transmitting slant range $R_T(\eta + \tau)$ from SAR antenna to target and the receiving slant range $R_R(\eta + \tau)$ from target to SAR antenna. Since the reception duration is very short, it
can be considered that \( R_{\text{Tr}}(\eta + \tau) \) and \( R_{\text{Re}}(\eta + \tau) \) change linearly with fast time \( \tau \) in terms of slope \( V_{\text{Tr}}(\eta) \) and \( V_{\text{Re}}(\eta) \), respectively. The linear approximations are given as follows:

\[
R_{\text{Tr}}(\eta + \tau) = R_{\text{Tr}}(\eta) + V_{\text{Tr}}(\eta) \tau \\
R_{\text{Re}}(\eta + \tau) = R_{\text{Re}}(\eta) + V_{\text{Re}}(\eta) \tau
\] (12)

If the transmitting and receiving range can be accurately calculated, their changing slope can be obtained by taking derivatives.

### B. RANGE CALCULATION BASED ON CURVILINEAR MOTION OF CONSTANT ACCELERATION

In order to accurately calculate the two-way slant range, it is generally necessary to obtain the propagation delay from transmitting to receiving the pulse by iterative method. However, the iterative method will greatly increase the calculation amount and time. In [22], the uniformly linear motion of satellite and Earth is used to get the analytical solution for propagation delay calculation.

In view of the satellite orbital motion and the Earth’s rotation, the uniform acceleration curve motion is used instead in the following. During the period from transmitting to receiving one pulse, it is assumed that the acceleration of satellite and target remains constant and the velocity keeps uniformly changing, which is illustrated in Fig. 8.

![FIGURE 8. The uniform acceleration curve motion of the satellite and the target](image)

In Fig. 8, at the transmitting moment (marked by circle), the satellite position vector is denoted by \( R_{\text{Tr}} \). After transmission delay \( \tau_{\text{Tr}} \) (marked by triangle), the target position vector is denoted by \( R' \). After reception delay \( \tau_{\text{Re}} \) (marked by square), the position and velocity vectors of satellite are denoted by \( R_{\text{Re}} \) and \( V_{\text{Re}} \), and those of target are denoted by \( R'_{\text{Re}} \) and \( V'_{\text{Re}} \). The acceleration vector of satellite and target are constant within one pulse period, which are denoted by \( A' \) and \( A'^{\prime} \).

According to the uniform acceleration curve motion of target, we have \( R = R_{\text{Re}} - V_{\text{Re}} \tau_{\text{Re}} - \frac{1}{2} A'^{\prime} \tau_{\text{Re}}^2 \). The receiving delay \( \tau_{\text{Re}} \) meets the equation as follows:

\[
\| R' - R_{\text{Re}} \| = c \tau_{\text{Re}}
\] (14)

Rearranging the equation (14), we can get the quartic equation of \( \tau_{\text{Re}} \) as follows:

\[
a_4 \tau_{\text{Re}}^4 + a_3 \tau_{\text{Re}}^3 + a_2 \tau_{\text{Re}}^2 + a_1 \tau_{\text{Re}} + a_0 = 0
\] (15)

In equation (15), the coefficients of the quartic equation are given by:

\[
\begin{align*}
a_4 & = \frac{1}{2} \| A' \|^2 \\
a_3 &= \left( V_{\text{Re}} + A'^{\prime} \right) \\
a_2 &= \left( V_{\text{Re}} + A'^{\prime} \right) \\
a_1 &= 2 \left( R_{\text{Re}} - R_{\text{Re}}^\prime, V_{\text{Re}} \right) \\
a_0 &= \left( R_{\text{Re}} - R_{\text{Re}}^\prime \right)^2
\end{align*}
\] (16)

Solving this quartic equation, four solutions are obtained, including two negative numbers and two positive numbers. The smaller positive number is the effective result for \( \tau_{\text{Re}} \).

Similarly, according to the uniform acceleration curve motion of satellite, we have \( R_{\text{Tr}} = R_{\text{Re}} - V_{\text{Re}} \tau_{\text{Re}} - \frac{1}{2} A' \left( \tau_{\text{Re}} + \tau_{\text{Tr}} \right)^2 \).

The transmitting delay \( \tau_{\text{Tr}} \) satisfies:

\[
\| R'_{\text{Tr}} - R_{\text{Re}} \| = c \tau_{\text{Tr}}
\] (17)

Rearranging the equation (17), we can get the quartic equation of \( \tau_{\text{Tr}} \) as follows:

\[
b_4 \tau_{\text{Tr}}^4 + b_3 \tau_{\text{Tr}}^3 + b_2 \tau_{\text{Tr}}^2 + b_1 \tau_{\text{Tr}} + b_0 = 0
\] (18)

In equation (18), the coefficients of the quartic equation are given by:

\[
\begin{align*}
b_4 & = \frac{1}{2} \| A' \|^2 \\
b_3 &= \left( V_{\text{Re}} + A'^{\prime} \right) \\
b_2 &= \left( V_{\text{Re}} + A'^{\prime} \right) \\
b_1 &= 2 \left( R_{\text{Re}} - R_{\text{Re}}^\prime, V_{\text{Re}} \right) \\
b_0 &= \left( R_{\text{Re}} - R_{\text{Re}}^\prime \right)^2
\end{align*}
\] (19)

Solving this equation, the smaller positive number is the solution for \( \tau_{\text{Tr}} \).

Through the above steps, the two-way slant range \( R_{\text{Tr}}(\eta) + R_{\text{Re}}(\eta) \) at any given time can be acquired, and the variation slope \( V_{\text{Tr}}(\eta) + V_{\text{Re}}(\eta) \) can be obtained by taking derivatives.
C. SIMULATION
The simulation results given below reveal the range model error caused by the four motion assumptions, including the stop-and-go assumption, the uniform velocity linear motion, the uniform acceleration curve motion, and the MHRE model. The parameters of satellite and SAR systems follow those in Section II. The iterative method is used to calculate the accurate two-way slant range.

Fig. 9 shows the range model error of the point target located in Haikou at side looking and squint looking geometry, where the squint angles are 0deg and 5.37deg respectively. The error caused by stop-and-go assumption is composed of constant term and linear term. The constant term becomes negligible in side looking geometry and reaches 70m in squint looking geometry. The linear term maintains at about 10m in two geometries. The error caused by uniform velocity linear motion is also composed of constant term and linear term. The constant term reaches 0.0139m in squint looking geometry, which will cause constant phase error of 41.7° and further cause deviation to some applications like interferometry. The Doppler center frequency deviation caused by the linear term error is less than 1Hz, which can be ignored. What’s more, the error based on uniform velocity linear motion is 10⁻²m orders of magnitude, whereas the error based on uniform acceleration curve motion is only 10⁻⁸m orders of magnitude. It means that, the range model precision is enhanced by 6 orders of
magnitude. According to the trend of the range error, the error based on uniform velocity linear motion increases linearly as the synthetic aperture time extends. Meanwhile, the error based on uniform acceleration curve motion is kept at 10⁻⁴m orders of magnitude. The error based on MRHE includes quadratic and quartic terms, the maximum value is 0.03m, which will cause phase error of 90°. The precision of MRHE is not appropriate for the proposed GEO SAR. Therefore, we can get better precision by the proposed range model, which can be applied to conditions with higher requirements of precision. Similarly, through the simulation results of Harbin, we can come to the same conclusion.

IV. SIGNAL MODEL AND CHARACTERISTICS

A. SIGNAL MODEL IN 2-D TIME DOMAIN

The signal model of GEO SAR is established according to the accurate slant range given above. The quadrature demodulated baseband signal is written by:

\[
 s(\tau, \eta) = \text{rect} \left\{ \frac{1}{T} \left[ \tau - \frac{2}{c} R(\eta + \tau) \right] \right\} \times \exp \left\{ j\pi k_s \left[ \tau - \frac{2}{c} R(\eta + \tau) \right]^2 \right\} \times w_\eta(\eta) \text{exp} \left\{ -j \frac{4\pi f_c}{c} R(\eta + \tau) \right\}
\]

(20)

where, \text{rect}( ) is the unit rectangular window, \( k_s \) and \( T \) are the chirp rate and pulse width of the transmitted chirp pulse, \( w_\eta(\eta) \) is the antenna beam pattern in azimuth direction, \( f_c \) is the radar carrier frequency, \( R(\eta) \) is the average of one-way range history.

In terms of the linear approximation given in equation (12) and (13), we have:

\[
 R(\eta + \tau) = R(\eta) + V(\eta) \tau \quad \text{(21)}
\]

where, \( R(\eta) = \frac{1}{2} R_c(\eta) + \frac{1}{2} R_a(\eta) \), \( V(\eta) = \frac{1}{2} V_c(\eta) + \frac{1}{2} V_a(\eta) \).

By substituting equation (21) into equation (20), we can get

\[
 s(\tau, \eta) = \text{rect} \left\{ \frac{1 - \frac{2}{c} V(\eta)}{T} \left[ \tau - \frac{2}{c} \left( R(\eta) + \frac{1}{2} V(\eta) \right) \right] \right\} \times \exp \left\{ j\pi k_s \left( 1 - \frac{2}{c} V(\eta) \right) \right\}^{2} \times w_\eta(\eta) \text{exp} \left\{ -j \frac{4\pi f_c}{c} R(\eta + \tau) \right\}
\]

(22)

Rearranging the equation (22), we can get

\[
 s(\tau, \eta) = \text{rect} \left\{ \frac{1}{T + \Delta T} \left[ \tau - \frac{2}{c} (R(\eta) + \Delta R) \right] \right\} \times \exp \left\{ j\pi (k_s + \Delta k_s) \left[ \tau - \frac{2}{c} (R(\eta) + \Delta R) \right]^2 \right\} w_\eta(\eta) \times \exp \left\{ -j \frac{4\pi f_c}{c} (f_c + \Delta f_c) (R(\eta) + \Delta R) + j2\pi \Delta f_c \tau \right\}
\]

(23)

where \( \Delta f_c \), \( \Delta k_s \), \( \Delta T \), \( \Delta R \) respectively represent the amount of change in carrier frequency, chirp rate, pulse width and slant range, all of which depend on the azimuth time. It reveals the modulation effects induced by the slant range variation within one pulse process, by contrast with the traditional signal model.

Let the instantaneous Doppler frequency history is \( f_d(\eta) \), where \( f_d(\eta) = -2V(\eta) / \lambda \), we have:

\[
 \frac{\Delta f_c}{k_s} = \left[ 1 + \frac{f_d(\eta)}{f_c} \right]^2 - 1 \quad \text{(24)}
\]

\[
 \frac{\Delta T}{T} = - \frac{f_d(\eta)}{f_c + f_d(\eta)} \quad \text{(25)}
\]

\[
 \frac{\Delta R}{R(\eta)} = - \frac{f_d(\eta)}{f_c + f_d(\eta)} \quad \text{(26)}
\]

It shows that the modulation degree has positive correlation with the Doppler frequency and negative correlation with the carrier frequency. Thus, the higher the resolution and the larger the squint angle, the more serious is the modulation effects. The lower the radar carrier frequency, the more serious is the effects.

B. SIGNAL MODEL IN 2-D FREQUENCY DOMAIN

The range Fourier transform is applied to the signal in 2-D time domain, and the expression of the signal in range frequency and azimuth time domain is obtained as follows:

\[
 s(f, \eta) = \text{rect} \left\{ \frac{f - \Delta f_c}{(k_s + \Delta k_s)(T + \Delta T)} \right\} w_\eta(\eta)
\]

\[
 \times \exp \left\{ j\psi_0(f, \eta) + \psi_1(f, \eta) + \psi_2(f, \eta) \right\}
\]

(28)

\[
 \psi_0(f, \eta) = \frac{\pi \Delta f_c^2}{k_s + \Delta k_s} + \pi \frac{f^2}{k_s} \quad \text{(29)}
\]

\[
 \psi_1(f, \eta) = -\frac{4\pi}{c} \left( f_c + f \right) \Delta R \quad \text{(30)}
\]
Performing azimuth Fourier transform on the signal in range frequency and azimuth time domain. Since the magnitude of \( \psi_1(f, \eta) \) and \( \psi_2(f, \eta) \) is much less than that of \( \psi_0(f, \eta) \), by ignoring the effect of these two terms on the solution of stationary phase expression, we can get
\[
\xi = \left[ 1 + \frac{f}{f_c} \right] f_a(\eta) \tag{31}
\]
where \( \xi \) is the azimuth frequency axis. By substituting equation (31) into equation (28), the signal in 2-D frequency domain can be expressed as:
\[
s(f, \xi) = \text{rect} \left[ \frac{f - \xi/(f/f_c + 1)}{k_T} \left[ 1 + \xi/(f + f_c) \right] \right] w_\eta(\xi) \times \exp \left\{ i \left[ \psi_0(f, \xi) + \psi_1(f, \xi) + \psi_2(f, \xi) \right] \right\} \tag{32}
\]
The phase expression in 2-D frequency domain includes three terms, where \( \psi_0(f, \xi) \) also has the same expression with that of the traditional signal model , \( \psi_1(f, \xi) \) and \( \psi_2(f, \xi) \) are given by:
\[
\psi_1(f, \xi) = -\pi \left[ \frac{f - \xi/(f/f_c + 1)}{k_c} \right]^2 + \frac{\pi f^2}{k_c}, \tag{33}
\]
\[
\psi_2(f, \xi) = \frac{4\pi}{c} \left[ \frac{\xi}{1 + \xi/(f + f_c)} \right] R(\eta(\xi)). \tag{34}
\]
It shows that \( \psi_1(f, \xi) \) is independent of target position but dependent on range frequency and azimuth frequency. Therefore, batch processing in 2-D frequency domain is fit for \( \psi_1(f, \xi) \) compensation. Meanwhile, \( \psi_2(f, \xi) \) is dependent on range frequency, azimuth frequency, and target position, which needs block processing for compensation.

In practical applications, the spaceborne SAR parameters are generally satisfied: \( f/f_c \approx 1, \xi/(f + f_c) \ll 1 \). Therefore, Equation (33) can be approximated as:
\[
\psi_1(f, \xi) \approx 2\pi f \xi/k_c. \tag{35}
\]
It is degraded to the linear term to compensate the stop-and-go assumption in the 2D frequency domain in [15].

### C. SIMULATION

In order to verify the signal model in this paper, we generate the echo of the point target, conduct matched filtering in 2-D frequency domain, and then test the imaging qualities. Two matched filter functions are compared. The first one is based on the traditional signal model by calculating the slant range according to the uniform velocity linear motion [22] and by compensating \( \psi_0(f, \xi) \) and \( 2\pi f \xi/k_c \) [15]. The second one is based on the modified signal model given in this paper by calculating the slant range according to the uniform acceleration curve motion and by compensating \( \psi_0(f, \xi) \), \( \psi_1(f, \xi) \) and \( \psi_2(f, \xi) \).

The point target is located in the central area of Haikou. The side looking geometry and squint looking geometry are both shown. In squint case, the edge of a single satellite observable orbit arc is used. The main simulation parameters are shown in Table IV. As the squint angle increases, the imaging parameters \( f_c, B_r \) and \( T \) decrease correspondingly. According to equations (6) and (7), the theoretical values of both range and azimuth resolution in ground plane are approximately 2m and 5m respectively.

| TABLE IV SIMULATION PARAMETERS |
|-----------------|--------|--------|--------|--------|--------|
| Resolution      | 5m     | 2m     | 5m     | 2m     |
| Radar parameters|        |        |        |        |
| Value1(Side)    | 89.8   | 72.8   | 224.6  | 182.0  |
| Value2(Squint)  | 74.9   | 60.7   | 187.1  | 151.7  |
| Value3(Side)    | 116.9  | 94.7   | 292.2  | 236.8  |
| Value4(Squint)  | 125.7  | 1.25   | 1.25   | 1.25   |
| Unit            | MHz    | MHz    | s      | s      |
| Pulse repetition frequency | PRF | 300    | 800    | 700    | 1500   |
| Ground resolution included angle | \( \theta_{GRIA} \) | 90     | 42.6   | 90     | 42.6   |

Fig. 10 and Fig. 11 compare the compressed targets after matched filtering based on the traditional signal model and modified signal model. In the side looking geometry, as shown in Fig. 10, at 5m resolution, the focused quality of both models is high; at 2m resolution, the focused quality in azimuth direction based on traditional signal model deteriorates a little. In the squint looking geometry, as shown in Fig. 11, at 5m resolution, the side lobes of azimuth direction based on traditional signal model begin to rise; at 2m resolution, the mismatch of the matched filter based on traditional signal model is significant, and the point target is totally defocused. Meanwhile, the matched filter constructed based on modified signal model can focus the target very well. The improvement in focusing quality is brought by modified signal model. The larger the squint angle is, the more obvious the improvement is.
FIGURE 10. In side looking geometry, contour plots of compressed point target by 16 times interpolation. (a) 5m resolution, based on traditional signal model. (b) 5m resolution, based on modified signal model. (c) 2m resolution, based on traditional signal model. (d) 2m resolution, based on modified signal model.
The imaging qualities in range and azimuth directions at resolution of 5m and 2m are shown in the Table V and VI respectively. In squint looking geometry at 5m resolution, the azimuth PSLR and ISLR have deteriorated slightly based on the traditional signal model, as given in Fig. 12(a); at 2m resolution, the azimuth broadening, PSLR, and ISLR reach 2.04, -0.02dB, and 16.78dB by traditional signal model, that is the consequence of azimuth envelope of two main lobes, as given in Fig. 12(b). Those imaging qualities improve to 0.99, -13.36dB, and -10.72dB by modified signal model, which are close to the ideal values in terms of the rectangular window, respectively.

### Table V: Comparison of Imaging Qualities at Resolution of 5m

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Model</th>
<th>Direction</th>
<th>Broadening</th>
<th>Resolution (m)</th>
<th>PSLR(dB)</th>
<th>ISLR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side looking</td>
<td>Traditional</td>
<td>Range</td>
<td>1.06</td>
<td>1.877</td>
<td>-13.286</td>
<td>-10.029</td>
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<tr>
<td></td>
<td></td>
<td>Azimuth</td>
<td>0.98</td>
<td>4.690</td>
<td>-13.268</td>
<td>-10.298</td>
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<tr>
<td></td>
<td>Modified</td>
<td>Range</td>
<td>1.06</td>
<td>1.877</td>
<td>-13.282</td>
<td>-10.023</td>
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<tr>
<td></td>
<td></td>
<td>Azimuth</td>
<td>0.99</td>
<td>4.690</td>
<td>-13.314</td>
<td>-10.310</td>
</tr>
<tr>
<td>Squint looking</td>
<td>Traditional</td>
<td>Range</td>
<td>1.00</td>
<td>2.188</td>
<td>-13.281</td>
<td>-10.046</td>
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<tr>
<td></td>
<td></td>
<td>Azimuth</td>
<td>1.01</td>
<td>3.384</td>
<td>-11.972</td>
<td>-9.378</td>
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<tr>
<td></td>
<td>Modified</td>
<td>Range</td>
<td>1.06</td>
<td>2.316</td>
<td>-13.279</td>
<td>-10.026</td>
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<tr>
<td></td>
<td></td>
<td>Azimuth</td>
<td>1.00</td>
<td>3.342</td>
<td>-13.277</td>
<td>-10.746</td>
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</tbody>
</table>

### Table VI: Comparison of Imaging Qualities at Resolution of 2m

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Model</th>
<th>Direction</th>
<th>Broadening</th>
<th>Resolution (m)</th>
<th>PSLR(dB)</th>
<th>ISLR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side looking</td>
<td>Traditional</td>
<td>Range</td>
<td>1.06</td>
<td>0.751</td>
<td>-13.303</td>
<td>-10.039</td>
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<td>Azimuth</td>
<td>0.99</td>
<td>1.884</td>
<td>-13.176</td>
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</tr>
<tr>
<td></td>
<td>Modified</td>
<td>Range</td>
<td>1.06</td>
<td>0.751</td>
<td>-13.279</td>
<td>-10.013</td>
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<tr>
<td></td>
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<td>Azimuth</td>
<td>0.99</td>
<td>1.882</td>
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<td>-10.615</td>
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<tr>
<td>Squint looking</td>
<td>Traditional</td>
<td>Range</td>
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<td>0.875</td>
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<td></td>
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<td>Azimuth</td>
<td>2.04</td>
<td>2.726</td>
<td>-0.019</td>
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<tr>
<td></td>
<td>Modified</td>
<td>Range</td>
<td>1.06</td>
<td>0.927</td>
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<td>-10.018</td>
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<tr>
<td></td>
<td></td>
<td>Azimuth</td>
<td>0.99</td>
<td>1.332</td>
<td>-13.363</td>
<td>-10.715</td>
</tr>
</tbody>
</table>
V. CONCLUSION

This paper proposes a new GEO SAR constellation and establishes a modified signal model for the following signal characteristics analysis and imaging algorithms development. The proposed GEO SAR in reverse equatorial geosynchronous orbit can solve the problem of long synthetic aperture time accompanying with the traditional GEO SAR. The proposed method for GEO SAR constellation design can realize 24-hour availability of the mid-latitude area. The simulation results show that, the given constellation has relatively shorter synthetic aperture time, azimuth invariance property of received echo, and better consistency of imaging performance. That makes benefits for observation flexibility and efficiency. Moreover, by assuming curvilinear motion with constant acceleration for both satellite and target, the instantaneous two-way slant range can be calculated precisely. By approximating linear slant range variation within transmission and reception process of one pulse, the modulation effects of continuous motion on signal can be achieved analytically. By contrast with the traditional signal model, it has additional two phase terms, which need to be compensated to enhance imaging qualities. The simulation results show that, the modified signal model is more suitable for GEO SAR with large squint angle and large time delay.

REFERENCES

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