Neural Dynamics for Control of Industrial Agitator Tank with Rapid Convergence and Perturbations Rejection

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Abstract—The industrial agitator tank is a widely used equipment in the chemical industry for the production of the chemical reagents. The high-performance agitator tank controller is critical to increase its productivity. In this paper, we propose an agitator tank controller based on a neural dynamics method with a shorter error-converging time in comparison with the existing methods. In addition, the controller also has a strong capability to reject perturbations. Ultimately, the superiority of the proposed agitator tank controller is theoretically analyzed. Numerical results validate the superior performance of the proposed controller.

Index Terms—Chemical industry, Automatic control, Control design, Neural dynamics method, Rapid convergence, Perturbations rejection.

I. INTRODUCTION

CHEMICAL industry is indispensable to heavy industry, to a large extent [1], [2]. The chemical industry mainly produces chemical products [3], [4], such as chemical reagents, biological reagents and farm chemicals [5], [6]. Industrial agitator tanks are frequently used to the production of these reagents. Therefore, the agitator tank performs a crucial role in the chemical industry [7], [8]. As the step of science and technology development accelerates, the variety and function of chemical reagents are increasing abundantly [9], [10]. As a typical nonlinear system, the agitator tank has received extensive attention from researchers in order to develop a high-performance agitator tank controller [11]–[17].

The rapid development of artificial intelligence has attracted the wide attention of researchers in recent years. As a significant subtopic of machine learning, neural networks are widely used in various spheres [18]–[32]. In the field of control systems for agitator tanks, many control methods have been presented [33], [34]. For example, Cai et al. designed a dry mortar mixing storage control system on the basis of the backward propagation (BP) neural proportional-integral-derivative (PID) controller [33]. The system can solve the control problem of complex dynamic systems by correcting the weight coefficient of the neural network in the control process according to the gradient descent method. Moreover, Zhang et al. first presented a controller of agitator tank based on Taylor finite difference [34]. The controller is an error-based dynamic method, which can make the error function converge to zero. The agitator tank equipped with the controller is able to prepare the reagent of desired concentration quickly. Nevertheless, the controller does not consider that the agitator tank may be disturbed during the actual operation. Note that the agitator tank may be disturbed in a practical manufacturing scenario during operation. The interference could lead to inaccuracy of the reagent concentration, and lead to a waste of materials. Consequently, the existing agitator tank control methods have the above mentioned defects [33]–[44].

To compensate for these defects of the existing agitator tank controllers, a new controller based on the neural dynamics method is proposed in this paper [45]–[47]. The proposed controller enables the concentration of the output solution of the agitator tank to quickly converge to the desired one. Therefore, the controller has perturbations rejection performance. As an evolution beyond existing methods, the proposed controller ensures that the system of the agitator tank can effectively eliminate perturbations. This advantage of perturbations rejection is of great significance for improving the performance of the existing agitator tank. As a result, the agitator tank equipped with the proposed controller continuously outputs the solution of the target concentration.

The remainder of this paper is organized as follows. Section II describes the dynamic equation of the agitator tank system and gives the system parameters. The design steps of the agitator tank controller are presented in Section III. In addition, Section IV demonstrates the rapid convergence performance and perturbations rejection performance of the agitator tank from the theoretical perspective. Simulation results are presented in Section V to show the superior performance of the proposed controller. Section VI concludes this paper. At the end of this section, the main contributions of this paper are summarized as follows.

- In this paper, a new controller of agitator tank is proposed...
III. DESIGN STEPS OF THE AGITATOR TANK CONTROLLER

The agitator tank controller is designed and given in this section. First of all, a neural dynamics method is introduced here. Specifically, the residual error function is defined to start the derivation from the viewpoint of control as follows:

\[ \epsilon(t) = q(t) - q_d(t), \]  

where \( q(t) \) represents the actual value and \( q_d(t) \) denotes the expected value. When \( \epsilon(t) \) converges to zero, the actual value \( q(t) \) is approximately equal to the expected one \( q_d(t) \). Inspired by the neural dynamics design equation \( \dot{\epsilon}(t) = -c\epsilon(t) \) introduced in [34], where \( \dot{\epsilon}(t) \) represents the first derivative of \( \epsilon(t) \), and \( c > 0 \) is the scaling factor related to the convergence rate of the residual error, it is modified in [48]–[53] that

\[ \dot{\epsilon}(t) = -cP_1(\epsilon(t)) - dP_2 \left\{ \epsilon(t) + c \int_0^t P_1(\epsilon(\tau))d\tau \right\}, \]  

where the convergence rate parameters of the residual error are denoted by \( c \) and \( d > 0 \); \( P_1(\cdot) \) and \( P_2(\cdot) \) stand for the monotonically-increasing odd function. Combining (3) and (4) obtains

\[ \dot{\epsilon}(t) = -cP_1(q(t) - q_d(t)) 
- dP_2 \left\{ (q(t) - q_d(t)) + c \int_0^t P_1(q(\tau) - q_d(\tau))d\tau \right\}. \]

The time derivatives of \( q(t) \) and \( q_d(t) \) are denoted by \( \dot{q}(t) \) and \( \dot{q}_d(t) \), respectively. Then, according to the parameters of the agitator tank system, the residual error function (3) is expanded as follows:

\[ \epsilon_{\Gamma}(t) = \Gamma(t) - \Gamma_d(t), \]  
\[ \epsilon_{\Upsilon_b}(t) = \Upsilon_b(t) - \Upsilon_{bd}(t), \]

where \( \epsilon_{\Gamma}(t) \) and \( \epsilon_{\Upsilon_b}(t) \) stand for the residual error of \( \Gamma(t) \) and \( \Upsilon_b(t) \), separately; \( \Gamma_d(t) \) represents the desired liquid volume of the agitator tank; \( \Upsilon_{bd}(t) \) denotes the desired concentration of the effluent liquid. Calculating the time derivative of \( \epsilon_{\Gamma}(t) \) and \( \epsilon_{\Upsilon_b}(t) \) obtains

\[ \dot{\epsilon}_{\Gamma}(t) = \dot{\Gamma}(t) - \dot{\Gamma}_d(t) \]
\[ = -cP_1(\Gamma(t) - \Gamma_d(t)) 
- dP_2 \left\{ (\Gamma(t) - \Gamma_d(t)) + c \int_0^t P_1(\Gamma(\tau) - \Gamma_d(\tau))d\tau \right\}, \]

\[ \dot{\epsilon}_{\Upsilon_b}(t) = \dot{\Upsilon}_b(t) - \dot{\Upsilon}_{bd}(t) \]
\[ = -cP_1(\Upsilon_b(t) - \Upsilon_{bd}(t)) 
- dP_2 \left\{ (\Upsilon_b(t) - \Upsilon_{bd}(t)) + c \int_0^t P_1(\Upsilon_b(\tau) - \Upsilon_{bd}(\tau))d\tau \right\}, \]

where \( \dot{\Gamma}_d(t) \) and \( \dot{\Upsilon}_{bd}(t) \) denote the time derivatives of \( \Gamma_d \) and \( \Upsilon_{bd}(t) \), respectively. Combining (7)-(8) and (1)-(2) gets

\[ \dot{v}_1(t) + \dot{v}_2(t) - 0.2\sqrt{\dot{\Gamma}(t) - \dot{\Gamma}_d(t)} = -cP_1(\Gamma(t) - \Gamma_d(t)) 
- dP_2 \left\{ (\Gamma(t) - \Gamma_d(t)) + c \int_0^t P_1(\Gamma(\tau) - \Gamma_d(\tau))d\tau \right\}, \]  

where the concentrations of the influent liquids and effluent liquid, respectively; \( v_1, v_2 \) and \( v_0 \) stand for the flow rates of the influent liquids and effluent liquid, respectively; \( \Gamma \) denotes the liquid volume of the agitator tank. The concentrations of the two influent liquids are unequal for the most part without special claims. Research targets of the agitator tank in this paper are that the concentration of actual solution in the tank can quickly reach the desired value, and that the error of the concentration of solutions can converge to zero with external perturbations.

The system equations of the agitator tank are presented as follows [34]:

\[ \dot{\Gamma}(t) = v_1(t) + v_2(t) - 0.2\sqrt{\Gamma(t)}, \]  

\[ \dot{\Upsilon}_b(t) = (\Upsilon_{b1} - \Upsilon_{b2}(t)) \frac{v_1(t) + (\Upsilon_{b2} - \Upsilon_{b1}) v_2(t)}{\Gamma(t)} 
- \frac{n_1 \Upsilon_{b1}(t)}{(1 + n_2 \Upsilon_b(t))^2}, \]

where \( t \) denotes time; \( \dot{\Upsilon}_b(t) \) is the time derivative of \( \Upsilon_b(t) \); \( \dot{\Gamma}(t) \) is the time derivative of \( \Gamma(t) \); the outflow rate constants in the dynamic process of the system are denoted by \( n_1 \) and \( n_2 \).

The schematic of traditional industrial agitator tank equipment is depicted in Fig. 1 with illustrative parameters. As far as the parameters are concerned, \( \Upsilon_{b1}, \Upsilon_{b2} \) and \( \Upsilon_b \) denote the concentrations of the influent liquids and effluent liquid, respectively; \( v_1, v_2 \) and \( v_0 \) stand for the flow rates of the influent liquids and effluent liquid, respectively; \( \Gamma \) denotes the liquid volume of the agitator tank.
and
\[
(\Psi_{bl} - \Psi_{b}(t)) \frac{v_{1}(t)}{\Gamma(t)} + (\Psi_{b2} - \Psi_{b}(t)) \frac{v_{2}(t)}{\Gamma(t)} - \frac{n_{1} \Psi_{b}(t)}{(1 + n_{2} \Psi_{b}(t))} \tilde{\Psi}_{bd}(t) \\
= -cP_{1}(\Psi_{b}(t) - \Psi_{bd}(t)) \\
- dP_{2} \left\{ (\Psi_{b}(t) - \Psi_{bd}(t)) + c \int_{0}^{t} P_{1}(\Psi_{b}(\tau) - \Psi_{bd}(\tau)) d\tau \right\},
\]
(10)

(11)

Finally, after solving (9) and (10), we can get
\[
v_{1}(t) = \frac{1}{(\Psi_{b1} - \Psi_{b}(t))} \left\{ \frac{n_{1} \Psi_{b}(t) \Gamma(t)}{(1 + n_{2} \Psi_{b}(t))} + \tilde{\Psi}_{bd}(t) \Gamma(t) \\
- cP_{1}(\Psi_{b}(t) - \Psi_{bd}(t)) - (\Psi_{b1} - \Psi_{b}(t)) v_{2}(t) \\
- dP_{2} \left\{ (\Gamma(t) - \Gamma_{a}(t)) + c \int_{0}^{t} P_{1}(\Gamma(\tau) - \Gamma_{a}(\tau)) d\tau \right\},
\]
\[
v_{2}(t) = \frac{1}{(\Psi_{b2} - \Psi_{b}(t))} \left\{ \frac{n_{1} \Psi_{b}(t) \Gamma(t)}{(1 + n_{2} \Psi_{b}(t))} + \tilde{\Psi}_{bd}(t) \Gamma(t) \\
- cP_{1}(\Psi_{b}(t) - \Psi_{bd}(t)) - (\Psi_{b2} - \Psi_{b}(t)) v_{1}(t) \\
- dP_{2} \left\{ (\Psi_{b}(t) - \Psi_{bd}(t)) + c \int_{0}^{t} P_{1}(\Psi_{b}(\tau) - \Psi_{bd}(\tau)) d\tau \right\},
\]
(12)

Simplifying the above two equations obtains the proposed controller:
\[
v_{1}(t) = \left\{ \frac{0.2 \sqrt{\Gamma(t)} + \tilde{\Gamma}_{d}(t) - cP_{1}(\Gamma(t) - \Gamma_{d}(t))}{(\Psi_{b1} - \Psi_{b}(t))} \\
- dP_{2} \left\{ (\Gamma(t) - \Gamma_{d}(t)) + c \int_{0}^{t} P_{1}(\Gamma(\tau) - \Gamma_{d}(\tau)) d\tau \right\} \times (\Psi_{b1} - \Psi_{b}(t)) - \frac{n_{1} \Psi_{b}(t) \Gamma(t)}{(1 + n_{2} \Psi_{b}(t))} \tilde{\Psi}_{bd}(t) \Gamma(t) \\
+ dP_{2} \left\{ (\Psi_{b}(t) - \Psi_{bd}(t)) + c \int_{0}^{t} P_{1}(\Psi_{b}(\tau) - \Psi_{bd}(\tau)) d\tau \right\} \times \frac{1}{(\Psi_{b1} - \Psi_{b}(t))} + cP_{1}(\Psi_{b}(t) - \Psi_{bd}(t)),
\]
(13)

\[
v_{2}(t) = \left\{ \frac{0.2 \sqrt{\Gamma(t)} + \tilde{\Gamma}_{d}(t) - cP_{1}(\Gamma(t) - \Gamma_{d}(t))}{(\Psi_{b2} - \Psi_{b}(t))} \\
- dP_{2} \left\{ (\Gamma(t) - \Gamma_{d}(t)) + c \int_{0}^{t} P_{1}(\Gamma(\tau) - \Gamma_{d}(\tau)) d\tau \right\} \times (\Psi_{b2} - \Psi_{b}(t)) - \frac{n_{1} \Psi_{b}(t) \Gamma(t)}{(1 + n_{2} \Psi_{b}(t))} \tilde{\Psi}_{bd}(t) \Gamma(t) \\
+ dP_{2} \left\{ (\Psi_{b}(t) - \Psi_{bd}(t)) + c \int_{0}^{t} P_{1}(\Psi_{b}(\tau) - \Psi_{bd}(\tau)) d\tau \right\} \times \frac{1}{(\Psi_{b2} - \Psi_{b}(t))} + cP_{1}(\Psi_{b}(t) - \Psi_{bd}(t)),
\]
(14)

with \( \Psi_{T}(t) = -M_{1}(t) - M_{2}(t) \), and \( \Psi_{T_{b}}(t) = -M_{1}(t) - M_{2}(t) \). Then, at the end

\[
\dot{\Psi}_{T}(t) = -cP_{1}(\Psi_{b}(t) - \Psi_{bd}(t)) + cP_{2}(\Psi_{b}(t) - \Psi_{bd}(t)),
\]
(19)

\[
\dot{\Psi}_{T_{b}}(t) = -cP_{1}(\Psi_{b}(t) - \Psi_{bd}(t)) + cP_{2}(\Psi_{b}(t) - \Psi_{bd}(t)),
\]
(20)

Further, given that in the actual operation, the agitator tank may be disturbed by external perturbations. We add the interference amount in equation (1) and equation (2) of the agitator tank system. Thus, it has
\[
v_{1}(t) + M_{1}(t) + v_{2}(t) + M_{2}(t) - 0.2 \sqrt{\Gamma(t)} \tilde{\Gamma}_{d}(t) = \tilde{\Gamma}_{d}(t) - cP_{1}(\Gamma(t) - \Gamma_{d}(t)) \\
- dP_{2} \left\{ (\Gamma(t) - \Gamma_{d}(t)) + c \int_{0}^{t} P_{1}(\Gamma(\tau) - \Gamma_{d}(\tau)) d\tau \right\},
\]
(15)

\[
(\Psi_{bl} - \Psi_{b}(t)) \frac{v_{1}(t)}{\Gamma(t)} + (\Psi_{b2} - \Psi_{b}(t)) \frac{v_{2}(t)}{\Gamma(t)} \\
- \frac{n_{1} \Psi_{b}(t)}{(1 + n_{2} \Psi_{b}(t))} - \tilde{\Psi}_{bd}(t) = -cP_{1}(\Psi_{b}(t) - \Psi_{bd}(t)) \\
- dP_{2} \left\{ (\Psi_{b}(t) - \Psi_{bd}(t)) + c \int_{0}^{t} P_{1}(\Psi_{b}(\tau) - \Psi_{bd}(\tau)) d\tau \right\},
\]
(16)

where \( M_{1} \) and \( M_{2} \) stand for perturbations. Furthermore, we have
\[
v_{1}(t) + v_{2}(t) - 0.2 \sqrt{\Gamma(t)} \tilde{\Gamma}_{d}(t) = -cP_{1}(\Gamma(t) - \Gamma_{d}(t)) \\
- dP_{2} \left\{ (\Gamma(t) - \Gamma_{d}(t)) + c \int_{0}^{t} P_{1}(\Gamma(\tau) - \Gamma_{d}(\tau)) d\tau \right\} \\
- M_{1}(t) - M_{2}(t),
\]
(17)

\[
(\Psi_{bl} - \Psi_{b}(t)) \frac{v_{1}(t)}{\Gamma(t)} + (\Psi_{b2} - \Psi_{b}(t)) \frac{v_{2}(t)}{\Gamma(t)} \\
- \frac{n_{1} \Psi_{b}(t)}{(1 + n_{2} \Psi_{b}(t))} - \tilde{\Psi}_{bd}(t) \\
= -dP_{2} \left\{ (\Psi_{b}(t) - \Psi_{bd}(t)) + c \int_{0}^{t} P_{1}(\Psi_{b}(\tau) - \Psi_{bd}(\tau)) d\tau \right\} \\
- (\Psi_{bl} - \Psi_{b}(t)) \frac{M_{1}(t)}{\Gamma(t)} - (\Psi_{b2} - \Psi_{b}(t)) \frac{M_{2}(t)}{\Gamma(t)} \\
- cP_{1}(\Psi_{b}(t) - \Psi_{bd}(t)).
\]
(18)

Combining formulas (1), (2), (5), (6), (7), and (8), we get
\[
\dot{\Psi}_{T}(t) = -cP_{1}(\Gamma(t) - \Gamma_{d}(t)) + \Psi_{T}(t) \\
- dP_{2} \left\{ (\Gamma(t) - \Gamma_{d}(t)) + c \int_{0}^{t} P_{1}(\Gamma(\tau) - \Gamma_{d}(\tau)) d\tau \right\},
\]
(19)

\[
\dot{\Psi}_{T_{b}}(t) = -cP_{1}(\Psi_{b}(t) - \Psi_{bd}(t)) + \Psi_{T_{b}}(t) \\
- dP_{2} \left\{ (\Psi_{b}(t) - \Psi_{bd}(t)) + c \int_{0}^{t} P_{1}(\Psi_{b}(\tau) - \Psi_{bd}(\tau)) d\tau \right\},
\]
(20)
of this section, the existing controller in [34] as follows is employed to provide a benchmark for comparison.

\[
v_1(t) = \left\{ \begin{array}{c}
0.2\sqrt{\Gamma(t)} + \hat{\Gamma}_d(t) - cP(\Gamma(t) - \Gamma_d(t)) \\
\times (\gamma_{b_1} - \gamma_{b_1}) - \frac{n_1 \gamma_{b_1}(\gamma_{t})(1 + n_2 \gamma_{b_1}(t))^2 - \gamma_{b_1}(t)\gamma_{t})(t) + cP(\gamma_{b_1}(t) - \gamma_{b_1}(t)\gamma_{t})(t) \\
+ cP(\gamma_{b_1}(t) - \gamma_{b_1}(t)\gamma_{t})(t) \times \frac{1}{(\gamma_{b_2} - \gamma_{b_1})},
\end{array} \right.
\]

(21)

\[
v_2(t) = \left\{ \begin{array}{c}
0.2\sqrt{\Gamma(t)} + \hat{\Gamma}_d(t) - cP(\Gamma(t) - \Gamma_d(t)) \\
\times (\gamma_{b_2} - \gamma_{b_2}) - \frac{n_1 \gamma_{b_2}(\gamma_{t})(1 + n_2 \gamma_{b_2}(t))^2 - \gamma_{b_2}(t)\gamma_{t})(t) + cP(\gamma_{b_2}(t) + \gamma_{b_2}(t)\gamma_{t})(t) \\
+ cP(\gamma_{b_2}(t) + \gamma_{b_2}(t)\gamma_{t})(t) \times \frac{1}{(\gamma_{b_2} - \gamma_{b_1})},
\end{array} \right.
\]

(22)

In this paper, (21)-(22) is called the original controller, and the proposed controller (13)-(14) is called the new controller.

IV. THEORETICAL ANALYSES ON SUPERIOR PERFORMANCE OF THE NEW CONTROLLER

The theoretical analyses of the superiority of the new controller (13)-(14) are presented in this section. Specifically, the global stability of the new controller is demonstrated. In addition, considering that there exist perturbations in the actual production of the agitator tank, theoretical analyses prove the perturbations rejection performance of the new controller.

A. Convergence Analysis

In this section, the global convergence of the new controller is proved theoretically. Therefore, the following theorem is given.

Theorem 1: When \( P_1(\cdot) \) and \( P_2(\cdot) \) are monotonically increasing odd functions, the new controller (13)-(14) is globally stable according to the Lyapunov stability theory.

Proof: The \( i \)-th subsystem of the nonlinear excitation design equation (4) can be written as follows (\( \forall i \in \{1, 2, ..., m\} \)):

\[
\dot{\epsilon}_i(t) = -cP_1(\epsilon_i(t)) - dP_2 \left\{ \epsilon_i(t) + c \int_0^t P_1(\epsilon_i(\tau))d\sigma \right\},
\]

(23)

where \( \epsilon_i(t) \) represents the \( i \)-th element of the error function \( e(t) \). For the \( i \)-th subsystem (23), define an auxiliary variable \( z_i(t) \) as

\[
z_i(t) = \epsilon_i(t) + c \int_0^t P_1(\epsilon_i(\tau))d\sigma.
\]

(24)

Taking the time derivative of both sides of (24) obtains

\[
\dot{z}_i(t) = \dot{\epsilon}_i(t) + cP_1(\epsilon_i(t)).
\]

(25)

Furthermore, combining (23), (24) and (25) gets

\[
\dot{z}_i(t) = -dP_2(z_i(t)).
\]

(26)

Further, we design a Lyapunov function candidate \( \nu_i(t) \) for the \( i \)-th subsystem (23) as follows:

\[
\nu_i(t) = \frac{1}{2} \sigma \epsilon_i^2(t) + \frac{1}{2} \sigma_2^2(t),
\]

(27)

where \( \sigma > 0 \) and \( \nu_0 = \nu_i(0) = \sigma \epsilon_i^2(0) + \sigma_2^2(0)/2 \). Obviously, \( \nu_i(t) \) is positive definite. Note that \( \nu_i(t) > 0 \).
for any $\epsilon_i(t) \neq 0$ or $z_i(t) \neq 0$, and $\nu_i(t) = 0$ only for $\epsilon_i(t) = z_i(t) = 0$. Then the time derivative of $\nu_i(t)$ is derived as follows:

$$\frac{d\nu_i}{dt} = \sigma \epsilon_i(t) \dot{z}_i(t) + z_i(t) \dot{\nu}_i(t) = \sigma \epsilon_i(t) \left[ \dot{z}_i(t) - cP_1(\epsilon_i(t)) \right] - d z_i(t) P_2(z_i(t))$$  \hfill (28)

Then, we are supposed to prove $\dot{\nu}_i(t) \leq 0$ in what follows. Obviously, in this situation, $\nu_i(t) \leq \nu_i(0)$. Based on this, the following conclusions can be gained:

$$\frac{1}{2} \sigma \epsilon_i^2(t) \leq \nu_i(t). \quad \frac{1}{2} \nu_i^2(t) \leq \nu_i(0).$$  \hfill (29)

We can further derive

$$|\epsilon_i(t)| \leq \sqrt{2\nu_i(0)/\sigma}, \quad |z_i(t)| \leq \sqrt{2\nu_i(0)}.$$  \hfill (30)

In addition, we set $X_1$ and $X_2$ to represent $\epsilon_i(t)$ and $z_i(t)$, respectively, and obtain

$$X_1 = \left\{ \epsilon_i(t) \in R_1| |\epsilon_i(t)| \leq \sqrt{2\nu_i(0)/\sigma} \right\},$$

$$X_2 = \left\{ z_i(t) \in R_1| |z_i(t)| \leq \sqrt{2\nu_i(0)} \right\}.$$  \hfill (31)

Applying the median theorem in the bounded region $X_2$, we have

$$P_2(z_i(t)) - P_2(0) = (z_i(t) - 0) \frac{\partial P_2(z_i(t))}{\partial z_i}|_{z_i(t) = z_i(0)},$$  \hfill (32)

where $P_2(0) = 0$ and $\partial P_2(z_i(t))/\partial z_i > 0$. Thus, from (32), the following result can be gained:

$$|P_2(z_i(t))| \leq B_0 |z_i(t)|,$$

where $B_0 = \max \{ |\partial P_2(z_i(t))/\partial z_i|, z_i(t) \in X_2 \} > 0$ is bounded. Thus, one can have

$$|\epsilon_i(t)| P_2(z_i(t)) \leq |\epsilon_i(t)| |z_i(t)| \leq B_0 |\epsilon_i(t)| |z_i(t)|.$$  \hfill (33)

Substituting (33) into (28), we have

$$\frac{d\epsilon_i}{dt} = -\sigma \epsilon_i(t) P_2(z_i(t)) - \sigma c \epsilon_i(t) P_1(\epsilon_i(t)) - d z_i(t) P_2(z_i(t)) \leq -\sigma B_0 |\epsilon_i(t)| |z_i(t)| - \sigma c B_1 \epsilon_i^2(t) - d B_2 z_i^2(t) = -\sigma - d \frac{B_2}{\sigma} \frac{d B_0}{z_i(t)} |\epsilon_i(t)|,$$

where $B_1 > 0$ and $B_2 > 0$. In addition, in a similar way, $B_1 = \min \{ |\partial P_1(\epsilon_i(t))/\partial \epsilon_i|, \epsilon_i(t) \in X_1 \}$ and $B_2 = \min \{ |\partial P_2(z_i(t))/\partial z_i|, \epsilon_i(t) \in X_2 \}$ are obtained by applying the median theorem. From the above discussions, we can get

$$\dot{\nu}_i(t) \leq 0 \text{ provided that } 0 < \sigma \leq \frac{4 c B_1 B_2}{d B_0^2}. \hfill (34)$$

The above results can guarantee the negative definite of $\nu_i(t)$. Therefore, according to Lyapunov stability theory, it can be concluded that the $i$th subsystem (23) is globally stable. In this sense, the error function $e(t)$ generated by the model (4) globally converges to zero. This proof is completed.

B. Robustness Analysis

Conventional agitator tank controllers do not take the effects of disturbances into account, which may have a significant impact on the accuracy of preparing reagents. In this section, we add the unknown constant perturbation to the agitator system and prove that the perturbation injected model (19)-(20) is anti-disturbing.

**Theorem 2:** When polluted by the unknown constant perturbation, the state output of the perturbation injected model (19)-(20) globally converges to the optimal solution to equation (1)-(2) of the agitator tank system from any randomly-generated initial value.

**Proof:** Substitute (5)-(6) into (19)-(20), and simplify to get

$$\dot{e}(t) = -cP_1(e(t)) - dP_2 \left\{ e(t) + c \int_0^t P_1(e(\tau)) d\tau \right\} + \Psi(t). \hfill (35)$$

Further, the $i$th subsystem of the above equation can be written as

$$\dot{\epsilon}_i(t) = -cP_1(\epsilon_i(t))$$

$$- dP_2 \left\{ \epsilon_i(t) + c \int_0^t P_1(\epsilon_i(\tau)) d\tau \right\} + \Psi(t). \hfill (36)$$

We introduce the same auxiliary variable $z_i(t)$ as (24). Thus, the time derivative of $z_i(t)$ is $\dot{z}_i(t) = \dot{\epsilon}_i(t) + cP_1(\epsilon_i(t))$. Then substituting $z_i(t)$ and $\dot{z}_i(t)$ into (36), we have

$$\dot{z}_i(t) = -dP_2(z_i(t)) + \Psi(t), \hfill (37)$$

According to the above equation, the $i$th perturbation interference subsystem (36) selects the Lyapunov function candidate as

$$\kappa_i(t) = \left( dP_2(z_i(t)) - \Psi(t) \right)^2/2,$$

which ensures the positive definiteness of $\kappa_i$. Then, the solution of $\kappa_i$ can be written as

$$\frac{d\kappa_i}{dt} = \left( dP_2(z_i(t)) - \Psi(t) \right) \frac{\partial dP_2(z_i(t))}{\partial z_i(t)} \dot{z}_i(t)$$

$$= -d \frac{\partial dP_2(z_i(t))}{\partial z_i(t)} \left( dP_2(z_i(t)) - \Psi(t) \right)^2. \hfill (38)$$

Since $P_2(\cdot)$ is a monotonically increasing odd activation function, it makes $\partial dP_2(z_i(t))/\partial z_i > 0$. Therefore, we conclude that $\dot{\epsilon}_i(t) = z_i(t) - P_2^2(\Psi(t)/d)$. Namely, $z_i(t)$ converges to $-dP_2^2(\Psi(t)/d)$ and $\lim_{t \to \infty} z_i(t) = -dP_2^2(\Psi(t)/d)$. Considering that $\dot{z_i}(t) = \dot{\epsilon}_i(t) + cP_1(\epsilon_i(t))$ and $\lim_{t \to \infty} \dot{z_i}(t) = 0$, we can derive that $\dot{\epsilon}_i(t) = z_i(t) - cP_1(\epsilon_i(t))$. When $t \to \infty$, $\dot{\epsilon}_i(t) = z_i(t) - cP_1(\epsilon_i(t))$ reduces to $\dot{\epsilon}_i(t) = -cP_1(\epsilon_i(t))$.  

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For this dynamic system, based on the results of the previous discussion, it is easy to prove that $\lim_{t \to \infty} \epsilon_i(t) = 0$.

Based on the above analyses, when $P_1$ and $P_2$ are monotonically increasing odd activation functions, the error function $\epsilon(t)$ generated by the constant perturbation injected model (19)-(20) converges globally to zero. That is to say, starting from arbitrary initial value, the system state output of the perturbation injected model can globally converge to the optimal solution to (1)-(2), in the presence of unknown constant perturbations. The verification of the anti-perturbation ability of the proposed agitator tank controller (13)-(14) is completed.

In summary, the above two theorems about global convergence and robustness to constant perturbations are demonstrated in detail. The superiority of the model is theoretically explained.

V. ILLUSTRATIVE EXAMPLE

In this section, we perform computer simulations on the process of preparing reagents in an agitator tank and present simulation results. Specifically, the parameters of the agitator tank are set as follows: At the initial time, the concentrations of the two input liquids are set as 24.9 mg/L and 0.1 mg/L, respectively; the inflow rates $v_1$ and $v_2$ of both liquids are set as 5 L/s; the coefficient associated with the outflow rate $v_1$ and $v_2$ are set as 1; parameters $c$ and $d$ are both set as 1.5; the desired solution volume $\Gamma_d$ in the agitator tank is 15 L; the desired solution concentration $\Upsilon_{bd}$ in the agitator tank is 10 mg/L; the monotonically increasing odd functions $P_1(\cdot) = P_2(\cdot) = (\cdot)^3$; the computer simulation time is designed as 50 s. Simulation experiments synthesized by the original controller (21)-(22) and the new controller (13)-(14) for the agitator tank preparing reagents are conducted to demonstrate the high performance of the new controller. For convenient reading, the parameters used in simulations are arranged in Table I.

A. Simulations Without Perturbation

In the absence of disturbances, simulation results of the agitator tank preparing reagents with the aid of the original controller (21)-(22) and the new controller (13)-(14) are given in Fig. 2. For comparison, the simulation results on the agitator tank preparing reagents with the aid of the new controller (13)-(14) are depicted in Fig. 3. Observing Fig. 2(a) with Fig. 3(a), it can be seen that the original controller and the new controller can effectively

![Fig. 4. Simulation results on the production of reagents in the agitator tank equipped with the original controller with constant perturbation. (a) Flow rates of the two input liquids in the agitator tank. (b) The concentration of the output reagent of the agitator tank. (c) The volume of the reagent in the agitator tank. (d) The output reagent concentration error (mg/L) and the liquid volume error (L) of the agitator tank.](image1)

![Fig. 5. Simulation results on the production of reagents in the agitator tank equipped with the original controller with constant perturbation. (a) Flow rates of the two input liquids in the agitator tank. (b) The concentration of the output reagent of the agitator tank. (c) The volume of the reagent in the agitator tank. (d) The output reagent concentration error (mg/L) and the liquid volume error (L) of the agitator tank.](image2)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Upsilon_1$, $\Upsilon_2$</td>
<td>Influent liquid concentration</td>
<td>24.9, 0.1</td>
</tr>
<tr>
<td>$\Upsilon_b$</td>
<td>Effluent liquid concentration</td>
<td>$\text{No}$</td>
</tr>
<tr>
<td>$\Upsilon_{bd}$</td>
<td>Desired effluent liquid concentration</td>
<td>10</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Actual solution volume</td>
<td>$\text{No}$</td>
</tr>
<tr>
<td>$\Gamma_d$</td>
<td>Desired solution volume</td>
<td>15</td>
</tr>
<tr>
<td>$v_1$, $v_2$</td>
<td>Inflow rate</td>
<td>5, 5</td>
</tr>
<tr>
<td>$c$, $d$</td>
<td>Convergence rate</td>
<td>1.5, 1.5</td>
</tr>
<tr>
<td>$n_1$, $n_2$, $\alpha$</td>
<td>Coefficient</td>
<td>1, 1</td>
</tr>
<tr>
<td>$P_1(\cdot)$, $P_2(\cdot)$</td>
<td>Monotonically increasing odd function</td>
<td>$(\cdot)^3$, $(\cdot)^3$</td>
</tr>
</tbody>
</table>

No stands for no fixed value.
control the flow rates $v_1$ and $v_2$ of the input liquid of the agitating tank. As Fig. 2(b) demonstrates, the concentration of the output reagent of the agitator tank reaches the desired one at approximate 4 s. For comparison, in Fig. 3(b), the concentration of the output reagent from the agitator tank reaches the desired one at approximate 1 s. Comparing Fig. 2(c) and Fig. 3(c), the volume of the solution in the agitator tank reaches the desired value at 15 s for the two controllers. In addition, the output reagent concentration error and the volume error of the agitator tank converge to zero at about 4 s as shown in Fig. 2(d). However, Fig. 3(d) shows that the output reagent concentration error and volume error of the agitator tank converge to zero at approximate 1 s. Particularly, in the absence of perturbations, the error of the reagent produced by the agitator tank equipped with the new controller converges to zero for 3 s less than that of the original controller, which is a threefold increase in productivity. The above analyses verify that the new controller proposed in this paper has better convergence performance compared with the original controller.

B. Simulations Under Constant Perturbation

In this subsection, we add a constant perturbation to the agitator tank system to verify the robustness of the new controller (13)-(14). For further explanation, the constant perturbation in the numerical simulations is set as 0.5. The corresponding simulations are conducted with simulation results presented in Fig. 4 and Fig. 5. Simulation results of the agitator tank preparing reagents equipped with the original controller (21)-(22) are shown in Fig. 4. As a contrast, simulation results on the agitator tank preparing reagents equipped with the new controller proposed in this paper are provided in Fig. 5. As is vividly depicted in Fig. 4(a), the simulation result shows that the original controller cannot effectively control the flow rates $v_1$ and $v_2$ of the input agitator tank. In addition, Fig. 4(b) shows that there exists a large error between the actual output reagent concentration and the desired output. It is noteworthy that the volume of the solution in Fig. 4(c) in the agitator tank also deviates from the desired value. Fig. 4(d) demonstrates that the error between the concentration of the output reagent and the desired concentration, and the error between the volume of the solution in the agitator tank and the desired volume, which are not convergent to zero. In brief, the original controller has poor performance when polluted by perturbations. Moreover, it is readily discovered that the new controller can effectively control the flow rate of the input liquid to the agitator tank in Fig. 5(a). Additionally, Fig. 5(b) and (c) indicate that both the output reagent concentration of the agitator tank equipped with the new controller and the volume of the solution in the tank can reach the desired values. As can be observed in Fig. 5(d), when the agitator tank system is disturbed by constant perturbations, the output reagent concentration error and the volume error of the agitator tank can converge to zero within about 5 s. Through all the above simulation experiments, we come to the conclusion that the new controller has the perturbation rejection performance compared with the original controller under the constant perturbation.
C. Simulations Under Non-constant Perturbation

The agitator tank may be subjected to non-constant perturbations in actual operation. In response to this situation, the numerical simulations of the agitator tank with the non-constant perturbation are conducted. Specifically, the simulation results of the agitator tank equipped with the original controller (21)-(22) and the new controller (13)-(14) are shown in Fig. 6 and Fig. 7, respectively. It is worth noting that time-varying perturbations are common in industrial production and can increase over time. Therefore, in the simulations, the time-varying perturbation 0.1t + 10 is added to the agitator tank system. It can be seen from Fig. 6(a) that the original agitator tank cannot stably control the flow rate of the input liquid. Furthermore, Fig. 6(b) illustrates that the concentration of the solution produced by the agitator tank equipped with the original controller deviates from the desired value, and the deviation increases as time evolves. Moreover, it is observed in Fig. 6(c) that the actual solution volume in the agitator tank exceeds the desired value. Finally, Fig. 6(d) illustrates that the concentration error and volume error of the solution prepared in the agitator tank cannot converge to zero. Thereafter, it can be viewed from Fig. 7(a) that the agitator tank equipped with the new controller can normally control the flow rate of the input liquid. In addition, by observing Fig. 7(b) and (c), it is found that the concentration and volume of the reagent prepared in the agitator tank are almost the same as the desired values after 2.5 s. Lastly, Fig. 7(d) describes the error convergence effect of the prepared reagent parameters. Concretely, the concentration error and volume error of the solution both converge to near zero. In summary, the simulation experiments verify that the proposed controller has perturbations rejection performance under the non-constant perturbation compared to the original controller.

VI. CONCLUSION

In this paper, a new neural dynamic agitator tank controller has been presented. Compared with the original controller theoretical analysis shows that the new controller has fast convergence performance and strong robustness. In addition, in order to further verify these performances of the new controller, we have executed the computer simulations on the agitator tank formulation reagents. Simulation results have indicated that the agitator tank synthesized by the proposed controller has an excellent performance compared with the original controller. Specifically, the reagent concentration and solution volume of the agitator tank equipped with the proposed controller are able to quickly converge to the desired values. In addition, when polluted by noises, the agitator tank equipped with the new controller can still prepare the target reagents with high performance. These superior properties can reduce the waste of industrial materials and greatly increase production efficiency.

REFERENCES


TABLE II

Comparisons among Different Controllers for the Industrial Agitator Tank

<table>
<thead>
<tr>
<th></th>
<th>Convergent time</th>
<th>Convergent time</th>
<th>Constant perturbation</th>
<th>Non-constant perturbation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>without perturbation</td>
<td>with perturbations</td>
<td>rejection</td>
<td>rejection</td>
</tr>
<tr>
<td>New controller</td>
<td>1 s</td>
<td>2.5 s</td>
<td>No*</td>
<td>Yes*</td>
</tr>
<tr>
<td>Original controller</td>
<td>4 s</td>
<td>+∞</td>
<td>No*</td>
<td>No*</td>
</tr>
</tbody>
</table>

* represents that the controller can reject perturbations with convergence theoretically guaranteed.

* stands for that the controller does not specifically consider perturbations rejection.


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