Adaptive robust visual servoing/force control for robot manipulator with dead-zone input

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ABSTRACT In this paper, we investigate the adaptive robust visual servoing/force control problem for the uncertain robot manipulator. The robot is considered to work with an unknown constraint surface and the unknown depth of feature point in image plane is assumed to be time-varying. The unknown constraint surface is linearly parameterized, and new adaptive laws are designed to estimate the unknown parameters online. In image based visual servoing control, the unknown time-varying depth plays a special role as it appears nonlinearly in the overall Jacobian matrix and cannot be adapted together with other uncertain kinematic parameters. To handle this problem, a compensatory depth-independent interaction matrix framework and the corresponding adaptive laws are proposed to compensate for the depth in the closed-loop dynamics. Moreover, the dead-zone input constraint is considered and handled by designing the robust compensatory terms and adaptive laws. With the proposed control scheme, it is proved that the image position and force tracking errors converge to zero asymptotically. Finally, numerical simulation and experimental results illustrate the effectiveness of the proposed approach.

INDEX TERMS visual servoing, force control, depth-independent interaction matrix, constraint surface, dead-zone input

I. INTRODUCTION

In various robotic manipulator applications such as spot welding, cutting and packaging, the manipulator always makes an interaction with the surrounding environment. When the manipulator performs operations on a surface, both the position and the interaction force should be well controlled in order to guarantee the successful execution of the tasks. Many force control schemes have been put forward for the robot manipulator and they can be classified into two categories namely impedance control [1] and hybrid position/force control [2]. The impedance control describes the relationship between the interaction force and manipulator position. Hence, the force control can be achieved through the position control of the end effector by adjusting the equivalent system parameters. On the contrary, the hybrid position/force control attempts to realize both position and force control simultaneously. According to the principle of orthogonality, the constrained surface is decomposed into the constrained direction where the force is controlled and the unconstrained direction where the motion is controlled.

For the hybrid position/force control, a major problem is that it is assumed the knowledge of the constraint surface can be obtained exactly. However, the robots usually work in an unidentified environment which means the model of the constrained surface cannot be obtained exactly. Several researches have been proposed to handle the constraint problem which formulates the control problem into joint space or Cartesian space [3], [4]. For the reason that the constraint surface is unknown, we can not get the desired position on the constraint surface neither in Cartesian space nor in joint space. Then, image based visual servoing/force control methods which formulate the desired end effector position from the Cartesian space into image space are proposed [5]–[7]. The position of the end-effector and its desired position on the uncertain constraint surface can be obtained in image space by using cameras. [8] proposes a sensory feedback controller for robot manipulator with uncertainty in kinematics and dynamics. However, the constraint surface is assumed to be known. [9] investigates the visual/force tracking control method for uncertain robots with unknown constraint surface.
In the proposed method, neural networks are adopted to estimate the unknown structure of the constraint surface. Recently, an adaptive visual servoing/force control method is proposed to deal with the uncertain problem [7]. The unknown constraint surface is linearly parameterized and the unknown parameters are estimated by the designed adaptive laws. However, a drawback of these works is that they assume the depth of feature point in the camera coordinate is constant and the image Jacobian matrix can be linearly parameterized directly. It is important to note that the depth of feature points could be constant in very few cases, such as the planar manipulator works in a plane and a fixed camera is placed perpendicular to that plane. In most of time, the depth is time-varying and the image Jacobian cannot be linearly parameterized together with other uncertain kinematic parameters. Hence, the methods proposed by [5]–[7] may be not available for this case. Some researches on image based visual servoing deal the time-varying depth with the depth-independent interaction matrix, which is obtained by eliminating the depth in the traditional image matrix [8]–[14]. However, these works only focus on the free motion control. If the manipulator performs operations on a constraint surface, these proposed methods are no longer available. Thus, it becomes necessary to handle the case when the depth is time-varying and the robot works with an unknown constraint surface. This is the primary focus of this paper.

Nonlinear characteristics such as dead-zone, saturation, backlash, failures are common in actuator and sensors and there are many works dealing with this problem [15]–[22]. Dead-zone input is one of the most important nonsmooth nonlinearities in the practical control system applications and it can cause unsatisfactory control performance. In [13], the dead-zone is decomposed into two parts, respectively a nominal asymmetric dead-zone and an uncertain continuous input function. Then, the dead-input is handled by establishing an adaptive asymmetric dead-zone compensation error. [16] deals with the manipulator control with fuzzy dead-zone. The fuzzy adaptive controller is designed by defuzzifying the fuzzy slop of input dead-zone and the stabilization of closed-loop system is guaranteed. [17] gives the expression of the errors between the dead-zone input torques and the actual torques. Then, neural networks are adopted to estimate the errors online. It is obvious that the dead-zone input also produces a significant effect on the control performance for the visual servoing/force control, and this should not be ignored.

In this paper, we investigate the visual servoing/force control problem for the uncertain robot manipulator with unknown constraint surface, simultaneously taking the dead-zone input into account. The position errors are transformed from the Cartesian space to image space by using a camera. The main contributions of this paper are stated as follows. First, it is the first time to consider both unknown constraint surface and time-varying depth of feature point. By designing a compensatory depth-independent image Jacobian matrix framework and the corresponding adaptive laws, the problem of unknown time-varying depth is handled. Moreover, the knowledge requirements for the lower and upper bounds of unknown parameters are also removed. In order to deal with the unknown constraint surface, robot kinematics, dynamics and camera model, new parameters adaptive laws are developed to update the parameters online. Second, the unknown asymmetric dead-zone input is considered for the manipulator dynamics. The robust compensatory terms and adaptive laws are designed to deal with the dead-zone input. Third, the adaptive robust visual servoing/force control scheme is designed for the manipulator, such that the image position and force tracking errors converge to zero asymptotically.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, we consider the visual servoing system with eye-to-hand configuration. The fixed camera is placed near the manipulator to map the end-effector from Cartesian space to image space, as shown in Fig.1. The camera is supposed to be a pin-hole camera and not calibrated.

A. KINEMATICS

Let \( x_b \in \mathbb{R}^{4 \times 1} \) represent the homogenous coordinates of feature point with respect to the robot base frame. Let \( q \in \mathbb{R}^{n \times 1} \) and \( \dot{q} \) represent the joint angle and velocity vector respectively. From the perspective projection model, we have

\[
\begin{pmatrix}
  u \\
  v \\
  1
\end{pmatrix} = \frac{1}{z_c} M x_b
\]

(1)

where \( u \) and \( v \) denote the two axes of projection on the image plane, \( z_c \) denotes the time-varying depth of the feature point, \( M \in \mathbb{R}^{3 \times 4} \) denotes the unknown constant perspective projection matrix.

From (1), we have

\[
z_c = m_{3}^T x_b
\]

(2)

where \( m_3^T \) is the third row of \( M \). The time derivative of \( z_c \) is written as

\[
\dot{z}_c = w \dot{q}
\]

(3)
where \( w = m_3^T \frac{\partial (x_b)}{\partial q} \), \( w \in \mathbb{R}^{1 \times n} \).

Define \( y = [u, v]^T \) as the image coordinate of feature point. Then, the following holds

\[
y(t) = \frac{1}{z_c} \hat{M} x_b
\]

where \( \hat{M} \in \mathbb{R}^{2 \times 4} \) is the first two rows of \( M \).

Differentiating (4) with respect to time, the following relation can be obtained

\[
y'(t) = \frac{\dot{A}}{z_c} \hat{q}
\]

where \( A \) is called the depth-independent image Jacobian matrix and represents the following matrix

\[
A = \begin{bmatrix} \bar{m}_1^T - u \bar{m}_1^T & \theta(x_b) \\ \bar{m}_2 - v \bar{m}_3 & \theta(x_b) \end{bmatrix}
\]

where \( \bar{m}_i \) is the \( i \)th row of \( \hat{M} \).

In order to handle the unknown time-varying depth of feature point, we define a compensatory depth-independent image Jacobian matrix as follows

\[
W = A + \frac{1}{2} \Delta y w
\]

where \( \Delta y = y - y_d \), \( y_d \) is the desired image trajectory. Then, the overall kinematics has the following properties.

**Property 1** The time-varying depth \( z_c \) can be linearly parameterized as follows

\[
z_c = Y_z(q) \theta_z
\]

where \( Y_z(q) \in \mathbb{R}^{1 \times h_1} \) is the regressor matrix, and \( \theta_z \in \mathbb{R}^{h_1 \times 1} \) represents the corresponding unknown parameters. The detail proof can be referred to [8, 12].

**Property 2** The product \( W \rho \) can be linearly parameterized as follows

\[
W \rho = Y_{kc}(y, q, \rho) \theta_{kc}
\]

where \( \rho \in \mathbb{R}^{n \times 1} \) is any known vector, \( Y_{kc}(y, q, \rho) \in \mathbb{R}^{2 \times h_2} \) is the regressor matrix, \( \theta_{kc} \in \mathbb{R}^{h_2 \times 1} \) is the unknown parameter vector which is determined by the products of unknown robot kinematics and camera parameters. The detail proof is given as follows.

**Proof.** By considering the row vectors \( m_i^T \frac{\partial (x_b)}{\partial q} \), it can be derived as follows

\[
m_i^T \frac{\partial (x_b)}{\partial q} = \frac{\partial}{\partial q} (m_{i1} x_1^1 + m_{i2} x_2^2 + m_{i3} x_3^3 + m_{i4})
\]

where \( m_{ij} \) is the \( (i, j) \) element of \( M \), \( x_b^k \) is the \( k \)th element of \( x_b \).

According to the manipulator forward kinematic [23, 24], \( x_b^k \) can be described as

\[
x_b^k = \sum_{h=1}^{n} l_h t_r^{h}(q),
\]

\( l_h \) is the unknown length of \( h \)th DOFs of manipulator, \( t_r^{h}(\cdot) \) represents the corresponding known trigonometric function, \( h = 1, 2, ..., n \).

Thus, (10) can be rewritten as

\[
\frac{\partial (x_b)}{\partial q} = \sum_{h=1}^{n} l_h t_r^{h}(q) = \frac{\partial}{\partial q} (m_{i1} \sum_{h=1}^{n} l_h t_r^{h}(q) + m_{i2} \sum_{h=1}^{n} l_h t_r^{h}(q) + \sum_{h=1}^{n} m_{i3} l_h t_r^{h}(q) + m_{i4})
\]

(11)

From (11), we can easily conclude that \( m_i^T \frac{\partial (x_b)}{\partial q} \) depends linearly on the unknown parameters i.e. \( m_i^T \frac{\partial (x_b)}{\partial q} \rho = f_k c\rho \). Here, \( f_k c\rho \) is the matrix whose elements are \( f_k(c_{\rho})(\delta r_i^{h}(q), \rho) \), \( \rho_i \) is the \( i \)th element of \( \rho \), \( f_k c\rho \) is the unknown vector whose elements are \( f_k(c_{\rho})(m_{i1}, l_h), i = 1, 2, 3, j = 1, 2, 3, 4, k = 1, 2, 3, h = 1, 2, ..., n \). The functions \( f_k(c_{\rho}) \) and \( f_k(c_{\rho}) \) are multiplicative functions whose exact expressions can be obtained if the structure of the manipulator is specified.

Thus, \( A \rho \) can be linearly parameterized i.e. \( A \rho = Y_{kcA}(y, q, \rho) \theta_{kc} \), where \( \theta_{kc} = \left[ \theta_{kc1}^T, \theta_{kc2}^T, \theta_{kc3}^T \right]^T \) and \( Y_{kcA}(y, q, \rho) \) is written as

\[
Y_{kcA}(y, q, \rho) = \begin{bmatrix} \phi_{kc1}(q, \rho) & 0 & -w \phi_{kc3}(q, \rho) \\ 0 & \phi_{kc2}(q, \rho) & -v \phi_{kc3}(q, \rho) \end{bmatrix}
\]

Moreover, we can obtain that \( \frac{1}{2} \Delta y w = Y_{kdz}(y, q, \rho) \theta_{kc} \), \( Y_{kdz}(y, q, \rho) \) can be expressed as

\[
Y_{kdz}(y, q, \rho) = \begin{bmatrix} 0 & 0 & \frac{1}{2} (u - u_d) \phi_{kc3}(q, \rho) \\ 0 & 0 & \frac{1}{2} (v - v_d) \phi_{kc3}(q, \rho) \end{bmatrix}
\]

Then, we can obtain that \( W \rho = Y_{kc}(y, q, \rho) \theta_{kc} \), where \( Y_{kc}(y, q, \rho) = Y_{kcA}(y, q, \rho) + Y_{kdz}(y, q, \rho) \).

**Assumption 1** In this paper, we assume that the desired image trajectory \( y_d \) and its derivatives \( \dot{y}_d \) and \( \ddot{y}_d \) are continuous and bounded.

**B. DYNAMICS**

The dynamics of the constrained robotic manipulator are given as follows [5, 7]

\[
H(q) \ddot{q} + Q(q, \dot{q}) \dot{q} + g(q) = \Gamma(u) + J^T(q) f
\]

(12)

where \( H(q) \in \mathbb{R}^{n \times n} \) represents the inertia matrix, \( Q(q, \dot{q}) \in \mathbb{R}^{n \times n} \) represents the Coriolis and centrifugal forces, \( g(q) \in \mathbb{R}^{n \times 1} \) denotes the gravitational force, \( \Gamma(u) \in \mathbb{R}^{n \times 1} \) is the joint torque with dead-zone input, \( J \in \mathbb{R}^{4 \times n} \) denotes the robot Jacobian matrix, \( f \in \mathbb{R}^{4 \times 1} \) represents the contact force vector.

Define the constraint surface with an algebraic term and shown as follows

\[
\Psi(x_b) = 0
\]

(13)

Assuming \( \Psi(x_b) \) is continuously differentiable, the expression of contact force can be written as

\[
f = \frac{\partial \Psi(x_b)}{\partial x_b} \lambda = d(x_b) \lambda
\]

(14)
where $\lambda$ denotes the magnitude of the contact force, the unit vector $d(x_b) \in R^{4 \times 1}$ represents the direction of the force which is perpendicular to the contact surface.

Hence, \(12\) can be represented as

\[
H(q) \ddot{q} + Q(q, \dot{q}) \dot{q} + g(q) = \Gamma(u) + D^T(q)\lambda
\]

(15)

where $D(q) = \begin{bmatrix} [\partial\Psi(x_b)/\partial x_b]^T / ||\partial\Psi(x_b)/\partial x_b|| \end{bmatrix} J(q)$, $D(q) \in R^{1 \times n}$ is the Jacobian of the constraint function and the following property holds

\[
D(q)\ddot{q} = 0
\]

(16)

The fundamental properties associated with the dynamics are listed as follows $12, 25$.

**Property 3**: The inertia matrix $H(q)$ is positive-definite.

**Property 4**: The matrix $H(q) - 2Q(q, \dot{q})$ is a skew-symmetric matrix.

**Property 5**: The robot dynamics depends linearly on an unknown constant dynamic parameter vector $\theta_d$, which can be described as

\[
H(q) \dot{\dot{q}} + Q(q, \dot{q}) \dot{q} + g(q) = Y_d(q, \dot{q}, \ddot{q})\theta_d
\]

(17)

where $Y_d(q, \dot{q}, \ddot{q}) \in R^{m \times h_s}$ is the dynamic regressor matrix, $\dot{q} \in R^{h_s \times 1}$ is a differentiable vector, and $\ddot{q}$ is the derivative of the vector $\dot{q}$.

**C. DEAD-ZONE INPUT**

This paper concerns the unknown asymmetric dead-zone input $\Gamma(u_i(t))$ as shown in Fig. 2. $m_{il}$ and $m_{ir}$ represent the left and right slopes of the dead-zone respectively, $b_{il}$ and $b_{ir}$ represent the two breakpoints. Then, the characteristics of dead-zone input can be defined as follows

\[
\Gamma(u_i(t)) = \begin{cases} 
  m_{ir}(u_i(t) - b_{ir}), & \text{if } u_i(t) \geq b_{ir}, \\
  0, & \text{if } -b_{il} < u_i(t) < b_{ir}, \\
  m_{il}(u_i(t) + b_{il}), & \text{if } u_i(t) \leq -b_{il}.
\end{cases}
\]

(18)

**Assumption 2**: In this paper, the unknown parameters $m_{il}$, $m_{ir}$, $b_{il}$ and $b_{ir}$ are assumed to be positive constants and have known upper bounds. Moreover, the slopes $m_{il}$ and $m_{ir}$ are assumed to be equal. Then, the dead-zone input can be rewritten as follows $18, 19$

\[
\Gamma(u_i(t)) = m_i(t)u_i(t) + o_i(t)
\]

(19)

where $m_i(t)$ and $o_i(t)$ are defined by

\[
m_i(t) = \begin{cases} 
  m_{il}, & \text{if } u_i(t) \leq 0, \\
  m_{ir}, & \text{if } u_i(t) > 0.
\end{cases}
\]

\[
o_i(t) = \begin{cases} 
  -m_{il}b_{ir}, & \text{if } u_i(t) \geq b_{ir}, \\
  -m_{il}u_i(t), & \text{if } -b_{il} < u_i(t) < b_{ir}, \\
  m_{il}b_{il}, & \text{if } u_i(t) \leq -b_{il}.
\end{cases}
\]

Furthermore, we can obtain that $o_i(t)$ is upper bounded with a known constant $\bar{o}_i$ e.g., $|o_i(t)| \leq \bar{o}_i$.

**FIGURE 2**: The dead-zone nonlinearity property of the actuator

Let $m_i = \frac{1}{n}, u_i = \hat{a}_i\hat{u}_i$, $\Delta a_i = \hat{a}_i - a_i$, $i \in (1, n)$. Then, we can rewrite $\Gamma(u)$ as

\[
\Gamma(u) = \bar{u} + m\bar{u}_x + o
\]

(20)

\[
u = \hat{a}_\Lambda \bar{u}
\]

(21)

where $\bar{u} = [\bar{u}_1, ..., \bar{u}_n]^T$ is the virtual controller, $\bar{u}_x = [\Delta a_1 \bar{u}_1, ..., \Delta a_n \bar{u}_n]^T$, $o = [o_1, ..., o_n]^T$. It should be noted that we define $\hat{a} = [\hat{a}_1, ..., \hat{a}_n]^T$, $\hat{a}_\Lambda = \text{diag}\{\hat{a}_1, ..., \hat{a}_n\}$, $m = \text{diag}\{m_1, ..., m_n\}$.

**III. ADAPTIVE ROBUST VISUAL SERVOING/FORCE CONTROLLER DESIGN**

In this section, a visual servoing/force controller is proposed. Because of the unknown constraint surface, the desired trajectory in Cartesian space cannot be obtained exactly. By using cameras, we can get the desired trajectory in image space. To simplify the problem and discussion, we assume that there is only one feature point which is always in the field of view to be traced.

Let us rewrite the contact force as follows

\[
D^T(q)\lambda = J^T(q)d(x_b)\lambda + J^T(q)\ddot{d}(x_b)\lambda
\]

(22)

where $\ddot{d}(x_b)$ is a fixed estimation of $d(x_b)$ which is not updated. Then, the two terms of the right side of (22) can be linearly parameterized as

\[
J^T(q)d(x_b)\lambda = Y_{cs}(q)\hat{\theta}_{cs}\lambda
\]

(23)

\[
J^T(q)\ddot{d}(x_b)\lambda = Y_J(q)\hat{\theta}_J\lambda
\]

(24)

where $Y_{cs}(q) \in R^{n \times h_s}$ and $Y_J(q) \in R^{n \times h_J}$ are the regressor matrices, $\hat{\theta}_{cs} \in R^{h_s \times 1}$ and $\hat{\theta}_J \in R^{h_J \times 1}$ are the corresponding unknown parameter vectors.
where $a_x = \beta \Delta y^T K_y (\dot{JW}^+)^{-1} \Delta F + a_p (\Delta \lambda + \gamma \Delta F)$, $a_p = J (\dot{q} - \dot{\bar{z}}_d \dot{W}^+ (\dot{y}_d - \alpha \Delta y))$.

We can rewrite the position of the end effector $x_b$, as $x_b = [x_{bp}^T, x_{bo}^T]^T$, where $x_{bp}$ denotes the position vector and $x_{bo}$ denotes the orientation vector. Thus, $a_x$ can be partitioned as $a_x = [a_x^T, a_o^T]^T$. $\dot{d}(\bar{x}_b)$ can also be written as $\dot{d}(\bar{x}_b) = \left[ \dot{d}_r^T (\bar{x}_b), \dot{d}_o^T (\bar{x}_b) \right]^T$. Then, we can define the rotation matrix $R(x)$ as

$$R(x) = \begin{bmatrix} R_p(n_p, \phi_p) & 0 \\ 0 & R_o(n_o, \phi_o) \end{bmatrix}$$

(32)

where $n_p$ is a unit vector normal to both the vectors $a_{xp}$ and $d_p(\bar{x}_b)$ as shown in Fig.3, $\phi_p$ is the angle between $d_p(\bar{x}_b)$ and $R_p(n_p, \phi_p)d_p(\bar{x}_b)$, which can be determined from the angle $\phi$ between $d_p(\bar{x}_b)$ and $a_{xp}$, the vector $d_p(\bar{x}_b)$ is rotated by the rotation matrix $R_p(n_p, \phi_p)$ about the axis $n_p$, so that the vector $R_p(n_p, \phi_p)d_p(\bar{x}_b)$ is perpendicular to the vector $a_{xp}$, as illustrated in Fig.3. Here, the symbol $\times$ represents cross product. We can design the rotation matrix $R_o(n_o, \phi_o)$ by the similar way. In fact, the constraint surface is usually independent of $x_{bo}$. Thus, we can set the rotation matrix $R_o(n_o, \phi_o)$ as an identity matrix.

**Remark 1** $a_x$ and $d(\bar{x}_b)$ are all known vectors which can be obtained by using the estimated parameters $J$, $W$, $\dot{z}_c$ and the measurement parameters $\dot{q}$, $\dot{y}_d$, $\dot{y}_d$, $\dot{\lambda}_d$, $\dot{\lambda}_d$ and $\Delta F$. Therefore, there must be a matrix which can rotate a known vector to perpendicular to another known vector. For the vectors in Cartesian space, the method to design rotation matrix $R(x)$ can be referred to Rodrigues’ rotation formula [20].

Substituting (20), (21), (22), (23), (24), (30) into the dynamics (15) and then subtracting (29) yields

$$H(q) \dot{s} = -Q(q, \dot{q}, s) - K_s s + (o - \bar{\sigma} \bar{s}(s))$$

$$-\dot{W}^T K_y \Delta y - \dot{J}^T R(x) \dot{d}(\bar{x}_b) (\Delta \lambda + \gamma \Delta F) + m \ddot{u}_e + Y_d(q, \dot{q}, \dot{\dot{q}}, \dot{\dot{q}}_r) \Delta \theta_d - Y_{cs}(q, s) \Delta \theta_{cs}$$

$$- Y_{jj}(q, s) \Delta \theta_j$$

(33)

where $\Delta \theta_d = \dot{d}_d - \dot{d}_d, \Delta \theta_{cs}$ and $\Delta \theta_j$ are similarly defined. The parameters $\dot{d}_d, \dot{\theta_d}, \dot{\theta_{cs}}, \dot{\theta_{kc}}$ and $\dot{\theta_z}$ are estimated by the adaptive laws

$$\dot{\theta_d} = -\Gamma_{-d}^{-1} Y_{d}^T(q, \dot{q}, \dot{\dot{q}}, \dot{\dot{q}}_r) s$$

$$\dot{\theta_j} = -\Gamma_{-j}^{-1} Y_{j}^T(q, s) s$$

$$\dot{\theta_{cs}} = -\Gamma_{-cs}^{-1} Y_{cs}^T(q, s) s$$

$$\dot{\theta_{kc}} = -\Gamma_{-kc}^{-1} Y_{kc}^T(q, s) K_y \Delta y$$

$$\dot{\theta_z} = -\Gamma_{-z}^{-1} Y_{z}^T(q, s) K_y \Delta y$$

(34) - (39)

where $\bar{u}_\Lambda$ is a diagonal matrix whose diagonal element is $\bar{u}_i$, the matrices $\Gamma_z, \Gamma_{kc}, \Gamma_{cs}$, $\Gamma_j$ and $\Gamma_d$ are positive definite.
Choose the Lyapunov function candidate as follows:

\[ V = \frac{1}{2} s^T H(q) s + \frac{1}{2} z_c \Delta y^T K_y \Delta y + \frac{1}{2} \beta \Delta F^2 + \frac{1}{2} \Delta \theta_z^T \Gamma_z \Delta \theta_z + \frac{1}{2} \Delta \theta_{kc}^T \Gamma_{kc} \Delta \theta_{kc} + \frac{1}{2} \Delta \theta_{cs}^T \Gamma_{cs} \Delta \theta_{cs} + \frac{1}{2} \Delta \theta_{kc}^T \Gamma_{kc} \Delta \theta_{kc} + \frac{1}{2} \Delta \theta_{d}^T \Gamma_{d} \Delta \theta_{d} + \frac{1}{2} tr(m \Delta a \Delta a^T) \]

(40)

where \( \Delta \theta_z, \Delta \theta_{kc}, \) and \( \Delta \alpha \) are similarly defined with \( \Delta \theta_d. \)

Differentiating (40) with respect to time, then applying (33) and the adaptive laws (34), (35), (36), (37) results in

\[ \dot{V} \leq -s^T K_s s + z_c \Delta y^T K_y \Delta y + \frac{1}{2} \Delta \theta_z^T \Gamma_z \Delta \theta_z + \frac{1}{2} \Delta \theta_{kc}^T \Gamma_{kc} \Delta \theta_{kc} + \frac{1}{2} \Delta \theta_{cs}^T \Gamma_{cs} \Delta \theta_{cs} + \frac{1}{2} \Delta \theta_{kc}^T \Gamma_{kc} \Delta \theta_{kc} \]

(41)

Using \( J \) and \( W \) to multiply both sides of (27) respectively, we have

\[ J \dot{s} = \dot{J}(\dot{q} - \dot{z}_c \dot{W}^+(\ddot{y}_d - \alpha \Delta y)) - \beta R(x) \dot{\bar{d}}(\bar{x}_b) \Delta F \]

and

\[ W \dot{s} = \dot{W} (\dot{q} - \dot{z}_c (\dot{y}_d - \alpha \Delta y)) - \beta (\dot{J} \dot{W}^+)^{-1} R(x) \dot{\bar{d}}(\bar{x}_b) \Delta F \]

(42)

(43)

According to (28), (29), property 1 and property 2, we have

\[ \dot{W} = z_c \Delta \dot{y} + \frac{1}{2} \Delta \theta_{kc}^T \Gamma_{kc} \Delta \theta_{kc} + \frac{1}{2} \Delta \theta_{cs}^T \Gamma_{cs} \Delta \theta_{cs} + Y_{kc}(y,q) \dot{\Delta \theta}_{kc} - Y_{cs}(y,q) \dot{\Delta \theta}_{cs} \]

(44)

Substituting (42), (43) and the adaptive laws (38), (39) into (41), and with the fact that \( R^T(x) R(x) = I, \) \( \dot{d}^T(\bar{x}_b) \dot{d}(\bar{x}_b) = \dot{I}, \) we have

\[ \dot{V} \leq a_p^T(\Delta \lambda + \gamma \Delta F) R(x) \dot{d}(\bar{x}_b) + (\Delta \theta_{kc}^T \Gamma_{kc} (J \dot{W}^+)^{-1} \Delta F) R(x) \dot{\bar{d}}(\bar{x}_b) \]

(45)

\[ -s^T K_s s - \alpha z_c \Delta y^T K_y \Delta y - \beta \gamma \Delta F^2 \]

(46)

(47)

Using (31) in the previous equation, we finally have

\[ \dot{V} \leq -s^T K_s s - \alpha z_c \Delta y^T K_y \Delta y - \beta \gamma \Delta F^2 \]

(48)

Now, it is the time to give the main result of this paper.

Theorem. By using the control laws (21), (30) and the adaptation laws (34), (39), with the Assumption 1, the image position and contact force tracking errors for the robotic system (3) and (15) converge to zero asymptotically i.e., under the control of the proposed scheme, we have \( g(t) - y_d(t) \to 0 \) and \( \lambda(t) - \lambda_d(t) \to 0 \) as \( t \to \infty. \)

Proof. From (48), it is easy to conclude that \( \dot{V} \leq 0 \) which directly leads to the boundness of \( V. \) Therefore, we have that \( s, \Delta y, \Delta F, \Delta \theta_z, \Delta \theta_{kc}, \Delta \theta_{cs}, \Delta \theta_{jc}, \Delta \theta_{d} \) and \( \Delta \alpha \) are all bounded. Then, \( y \) and the estimated parameters \( \theta_z, \theta_{kc}, \theta_{cs}, \theta_{jc}, \theta_{d} \) and \( \hat{a} \) are also bounded. Furthermore, we can obtain the estimates \( \hat{z}_c, \hat{z}_c, J \) and \( W \) are all bounded. Considering the boundness of \( \dot{y}_d, \) we can obtain that \( \dot{q}_r \) is bounded according to (25). Thus, \( \dot{q} \) is bounded since \( s \) is bounded. Then, we can obtain \( \dot{y}, \dot{x}_b, \dot{z}, \dot{z}_c \) are bounded since they are the functions of bounded signals \( \dot{q}, \dot{d}(x_b), \dot{x}_b \) is bounded because \( x_b \) and \( \dot{x}_b \) are bounded. From (38), we can obtain \( \hat{\theta}_{kc} \) is bounded. Then, the boundness of \( W \) can be guaranteed. From (26), we can see that all the terms on the right-hand side of the equation are bounded. Therefore, we can obtain that \( \ddot{q}_r - \beta \dot{W}^+(\dot{J} \dot{W}^+)^{-1} R(x) \dot{d}(\bar{x}_b) \Delta \lambda \) is bounded.

Differentiating (16) with respect to time and substituting the equation (33) into it yields

\[ -\dot{D} \dot{q} = D \dot{q} = D H(q)^{-1}(-Q(q, \dot{q}) s - K_s s + (o - \bar{\alpha}sign_y(s))) \]

(49)

\[ = \dot{W}^T K_y \Delta y + J^T R(x) \dot{d}(\bar{x}_b) (\Delta \lambda + \gamma \Delta F) \]

\[ + m \dot{u}_x + L_\dot{y}(y, q, \dot{q}) \dot{\Delta \theta}_{kc} - Y_{cs}(y, q) \dot{\Delta \theta}_{cs} \leq Y_{jc}(y, q) \dot{\Delta \theta}_{jc} + \beta H(q) \dot{W}^+(\dot{J} \dot{W}^+)^{-1} R(x) \dot{d}(\bar{x}_b) \Delta \lambda + H(q)(\dot{q}_r - \beta \dot{W}^+(\dot{J} \dot{W}^+)^{-1} R(x) \dot{d}(\bar{x}_b) \Delta \lambda) \]

According to (43) and the boundness of \( m \dot{u}_x \), the term \( m \dot{u}_x \) can be rewritten as \( m \dot{u}_x = L + N \Delta \lambda \), where \( L \) and \( N \) are bounded. Thus, we can rewrite (49) as

\[ p(t) = r(t) \Delta \lambda \]

(50)

where

\[ p(t) = -D \dot{q} - D H(q)^{-1}(-Q (q, \dot{q}) s - K_s s - \dot{W}^T K_y \Delta y - \hat{\alpha}sign_y(s)) + \gamma \dot{J}^T R(x) \dot{\bar{d}}(\bar{x}_b) \Delta F \]

(48)

\[ + L + Y_d(q, \dot{q}) \dot{\Delta \theta}_{kc} + Y_{cs}(y, q) \dot{\Delta \theta}_{cs} \lambda_d - Y_{jc}(y, q) \dot{\Delta \theta}_{jc} \lambda_d - H(q)(\dot{q}_r - \beta \dot{W}^+(\dot{J} \dot{W}^+)^{-1} R(x) \dot{d}(\bar{x}_b) \Delta \lambda) \]

(51)

From the previous equations, we can see that \( p(t) \) and \( r(t) \) are bounded which leads to the boundness of \( \Delta \lambda. \) Furthermore, according to (26), we can obtain that \( \dot{q}_r \) is bounded. Based on the closed-loop dynamics (33), the boundness of \( \tilde{s} \) can be concluded.

The time derivative of (48) can be written as

\[ \dot{V} \leq -2s^T K_s \tilde{s} - 2z_c \Delta y^T K_y \Delta y - 2\alpha z_c \Delta y^T K_y \Delta y - 2\gamma \beta \Delta F \Delta \gamma \]

Because \( V \) is bounded, and each of \( \dot{s}, \dot{z}_c, \dot{z}, \Delta y, \dot{\Delta \theta}_{kc}, \Delta \theta_{cs}, \Delta \theta_{jc}, \Delta \theta_{d}, \Delta \alpha \) are bounded, we have \( \dot{s}, \dot{z}_c, \dot{z}, \Delta y, \dot{\Delta \theta}_{kc}, \Delta \theta_{cs}, \Delta \theta_{jc}, \Delta \theta_{d}, \Delta \alpha \) are bounded. Thus, \( \dot{\Delta \lambda} \) is bounded.
with the assumption that \( \ddot{y}_d \) is bounded. Since \( \Delta \dot{y}, \dot{q}, \dot{\dot{q}} \) and \( \ddot{q} \) are all bounded, we can obtain that the derivative of \( J, \dot{W}^+, \dot{J} \) and \( \dot{W}^+ \) is bounded. According to (26), we can obtain that the derivative of \( \dot{y}_r = \beta \dot{W}^+ (J \dot{W}^+)^{-1} B(x) \) and \( d(\dot{x}_b) \Delta \lambda \) is bounded. From (50), it can be concluded that \( \Delta \lambda \) is bounded. For the fact that \( \Delta F \to 0 \) as \( t \to \infty \) and the boundness of \( \Delta \lambda \), and using Barbalat’s lemma, we can conclude that \( \Delta \lambda \to 0 \) as \( t \to \infty \). This completes the proof.

**Remark 2** In this paper, the robot is assumed to be operating in a region such that the estimated parameters \( \hat{\theta}_{bc} \) and \( \hat{\theta}_J \) will not lead to the singularity of \( W^+ \) and \( J \dot{W}^+ \) which are used only in the definition of control variable \( \dot{q}_r \). Therefore, we can control this by using a singularity robust inverse of the approximate Jacobian matrix in [7], [28] or adopting the strategy of potential force in [9].

**Remark 3** In this paper, we have shown that the visual and force tracking problem for the manipulator can be handled by the proposed control scheme. Compared with the results in [6, 7], we consider the time-varying depth of feature points instead of constant depth. The novel points of our results lie in the design of the compensatory depth-independent image Jacobian matrix framework (7), the definition of the joint reference velocity (25) and the image space adaptation laws (55), (59). By using the designed framework, we do not need to know the lower and upper bounds of the unknown parameters. A new proof method for the parametric linearization of the compensatory depth-independent image Jacobian matrix (7) is given, see Property 2. Compared with the proof in [11], the proof method proposed in this paper do not need to use exact knowledge of the position of the end-effector. Compared with [8], [9], [11], [14] which only consider the free motion control, we consider the constraint motion control with an unknown surface. The novel points of our results lie in the unknown constraint surface adaptive laws (15), (66) and the associated stability analysis. Furthermore, dead-zone input is considered in this paper. We design a robust compensatory terms in the virtual controller (30) and the adaptation law (37) to handle the problem of dead-zone input.

**Remark 4** Unlike the reference [5]–[7], the controller cannot be directly designed when the dead-zone input is taken into account. In this paper, the dead-zone input is transformed into the form of equation (20). Then, the actual controller can be obtained by using the estimated parameters and the virtual controller which can be seen from equation (21). In order to guarantee the asymptotical convergence, the tracking errors must satisfy the requirement of the Barbalat’s lemma. Thus, the dead-zone input should be compensated in the close system completely. In this paper, we design the robust compensatory term \( -\alpha \dot{q}_r(s) \) and the adaptive laws (37) to handle this problem. Moreover, the dead-zone input brings the challenges to the discussion of the boundness of the parameters. According to the Proof, we must guarantee the boundness of \( \Delta \lambda \) in order to guarantee the convergence of force tracking error. Then, the terms of \( L \) and \( N \) which comes from the effect of dead-zone input must be bounded. Thus, the boundness of the parameters in the terms \( L \) and \( N \) is discussed which can be obtained by the result that \( V \leq 0 \).

**IV. SIMULATION AND EXPERIMENT RESULTS**

In this section, a series of simulation and experimental results are presented to show the effectiveness of the proposed control method. First, the simulation results are presented using two link planar manipulator. Then, the experiments are made on a 3-DOF PHANToM manipulator.

**A. SIMULATION RESULTS AND DISCUSSION**

To verify the theoretical results in this paper, a two link planar manipulator is considered and its end-effector is required to move on a constraint surface. A fixed camera is placed distance away from the robot. The manipulator dynamics and kinematics can be referred to [23], [24] and the mathematical model of the perspective projection matrix \( M \) can be referred to [8], [9]. The true values of the robot joints length are set as \( l_1 = l_2 = 1m \), joints mass are set as \( m_1 = m_2 = 0.5kg \). The camera scaling factors are \( \alpha_u = \alpha_v = 100 \), the angle between the two image axes is \( \pi/2 \), the principal point of the camera is \( (u_0, v_0) = (0, 0) \). The dead-zone input slopes are \( m_\psi = m_\phi = [0.5 0.5]^T \), \( b_\psi = b_\phi = [0.02 0.02]^T \).

In the simulation, the unknown constraint surface is described in Cartesian coordinates as

\[
\Psi(x_b) = x_{b_x} + \theta_{cs} x_{b_y} + \gamma_{cs}
\]

where \( x_b = [x_{b_x}, x_{b_y}]^T, x_{b_x} = l_1 \cos(q_1) + l_2 \cos(q_1 + q_2), x_{b_y} = l_1 \sin(q_1) + l_2 \sin(q_1 + q_2) \), the parameters of the constraint surface are set as \( \theta_{cs} = 0 \) and \( \gamma_{cs} = -0.5 \).

We assume that the robot is initially in contact with the constraint surface and the desired point on the surface is get by the fixed camera. The robot end effector is required to move from the initial point \([0.5 1.8]^T\) to the desired point \([0.5 2.3]^T\) and the image feature point is required to move from the initial point \([-11.9 -17.6]^T\) to the desired point \([-4.6 -6.8]^T\) pixels. The desired contact force is set as \(11N\). We set the initial estimated parameters as \( \hat{l}_1 = 1.1, \hat{l}_2 = 1.2, \hat{m}_1 = 0.55, \hat{m}_2 = 0.55, \hat{\alpha}_u = 110, \hat{\alpha}_v = 115 \) and \( \hat{\theta}_{cs} = 0.1 \) respectively. The control gains are set as \( K_s = 400I, K_y = 490I, \alpha = 30, \beta = 0.01, \gamma = 1, \Gamma_d = 1/20I, \Gamma_f = 1/300I, \Gamma_{cs} = 1/300I, \Gamma_{kc} = 1/50I, \Gamma_z = 1/50I \).

The simulation results are shown in Fig.4-7. Fig.4 shows the path of feature point in the image plane. We can see that the feature point always in the field of view of the camera. This means that the feature point will not be lost throughout the whole tracking process. This characteristic is critical for the reason that if the feature point escape the field of view, the control scheme will be failed. Fig.5 shows the image position tracking errors. We can see that the image position tracking errors converge to zero asymptotically. Fig.6 gives the force tracking error and we can also see that the asymptotical convergence of force tracking error is guaranteed. Fig.7 shows the control inputs of the manipulator, it demonstrates...
that the system will be stable after the convergence of the tracking errors. Thus, we can conclude that the convergence of both the image position and force tracking errors are guaranteed in spite of the uncertainties in the kinematics, dynamics, constraint surface, time-varying depth and dead-zone input. Moreover, according to the image based visual servoing theory, the convergence of feature points on the image plane indicates the convergence of manipulator’s end-effector. Thus, we can say that the manipulator’s end-effector can move along with the constraint surface. Furthermore, based on Fig.6, it can be obtained that the manipulator can move along with the unknown constraint surface with a pre-defined contact force value. This confirms the effectiveness of the proposed control method.

B. EXPERIMENT RESULTS AND DISCUSSION

To verify the effectiveness of the proposed control method from the perspective of practical implementation, experimental studies are performed in this section. The experimental setup is composed of a 3-DOF PHANToM manipulator, custom PC, CCD camera, and force sensor. The 3-DOF PHANToM manipulator is provided by SensAble Technologies, and equipped with absolute encoder to measure the joints and end-effector position. Its dynamics and kinematics can be referred to [29]. The CCD camera is mounted on a fixed place to drive the end-effector of the manipulator from the square to the triangle on the paper. The force sensor is provided by ATI and its type is Nano17. The contact force and torque are measured by the ATI Nano17 and transferred to the custom PC through a shielded high-flex cable. Both the image processing algorithm and the proposed controller are implemented on the custom PC. Fig.8 shows the view of the adopted manipulator setup. It should be noted that we use classical image processing algorithm to get the feature point in the image plane which will not mentioned in this paper.

The manipulator’s end-effector is initially in contact with the square and required to move to the center of the triangle on the paper. We set the desired contact force is 4N. Initial values $l_1 = 0.2, l_2 = 0.15, l_3 = 0.03, \hat{m}_1 = 0.4, \hat{m}_2 = 0.2, \hat{m}_3 = 0.15, \hat{\alpha}_u = 370, \hat{\alpha}_v = 360, \hat{u}_0 = 150$ and $\hat{v}_0 = 110$ are used in the experiments. The control gains are set to $K_x = 210I, K_y = 135I, \alpha = 5, \beta = 0.15, \gamma = 10, \Gamma_d = 1/45I, \Gamma_f = 1/75I, \Gamma_{cx} = 1/160I, \Gamma_{kc} = 15I$ and $\Gamma_z = 20I$.

Fig.9-10 show the path of feature point and the position tracking errors in the image plane. We can see that the image position control can be stabilized asymptotically. It also means that the end-effector of the manipulator can be drove from the square to the triangle effectively. The force tracking error is shown in Fig.11. We can see that the force tracking error can also converge to zero asymptotically. Thus, based on the above experimental results, we can conclude that both position and force tracking can be guaranteed for the manipulator by using the proposed control scheme.
In this section, the simulation and experiment results are conducted with 2-3 DOFs manipulator. From the simulation results, we can see that both position and force tracking errors can converge to zero asymptotically. Furthermore, the proposed controller has been applied on the practical experimental setup and it also shows an satisfactory performance. Thus, we can conclude that the proposed control scheme is feasible and effective in the practice applications.

![Image of experimental setup](image1)

**FIGURE 8:** visual servoing/force control – view of the experimental setup

![Path of feature point in image space](image2)

**FIGURE 9:** The path of feature point in image space – experimental results

**V. CONCLUSION**

In this paper, we have introduced a new result for the visual servoing and force tracking control for robot manipulator. Compared with the existing researches which consider either uncertain constraint surface or unknown time-varying depth, the proposed control scheme is a general approach that can handle both of the two problems. Moreover, the new control scheme can also deal with the dead-zone input. The stability analysis of the closed-loop control systems has been presented by using a Lyapunov function. It has been shown that the image position and force tracking errors converge to zero asymptotically with the proposed controller and adaptive laws. Simulation and experimental results have been presented to show the effectiveness of the proposed control scheme.

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