Gravity Aided Positioning based on Real-time ICCP with Optimized Matching Sequence Length

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ABSTRACT The existing real-time iterative closest contour point (ICCP) algorithm uses fixed matching sequence length, which is set according to human experience, and cannot obtain the best positioning accuracy on all tracks. This paper proposes a real-time ICCP algorithm with optimized matching sequence length (OMSL-ICCP) based on the analysis of the shortcomings of the existing real-time ICCP. The optimal matching sequence length of OMSL-ICCP under the current measured gravity anomaly sequence is obtained by the golden section search. The Hausdorff distance is utilized to obtain the search range of the closest contour point, which can effectively shrink the search range of the closest contour point. And the gravitation field algorithm is applied to further improve the positioning accuracy. In simulation tests with different gravity sensor measurement noises, different INS positioning errors and different gravity map resolutions, the difference of positioning performance between the existing real-time ICCP and the OMSL-ICCP is compared. Under the same test conditions, the simulation results show that the positioning accuracy of OMSL-ICCP is higher than the positioning accuracy obtained by the existing real-time ICCP algorithm with the optimal sequence length.

INDEX TERMS Gravity aided positioning, real-time ICCP, golden section search, gravitation field algorithm, Hausdorff distance

I. INTRODUCTION

Inertial navigation system (INS) has the characteristics of well concealment and full autonomy. It can continuously provide the parameters such as position, attitude and speed in all-weather. The INS is the core navigation system of underwater vehicle because of the concealment and reliability. For the inherent characteristics of inertial sensors, the positioning error of the INS is accumulated and diverged over time, which makes it difficult to ensure the long-term accuracy. Other methods are needed to correct systematic positioning errors periodically [1]–[3]. The irregular shape and the inhomogeneous density of the earth result in different gravity anomaly at various points of the earth, which shows a function of spatial location (latitude, longitude, altitude) [4]–[7]. The underwater vehicle can collect the surrounding gravity anomaly through the sensor and matching with the pre-stored data to correct the INS positioning error.

Matching algorithm is the core of the gravity aided navigation and an important factor affecting the positioning accuracy. Gravity aided matching algorithm can be divided into two categories: single point matching method and sequence matching method. Iterative closest contour point (ICCP) is a typical sequence matching method [8]–[11]. It has the characteristics of high accuracy and easy implementation. The ICCP algorithm achieves matching positioning through the optimal transformation between the measured data along the track and their closest contour points on the reference map, and can simultaneously correct the position error and heading error of the INS [12]. Two prerequisites for applying the ICCP algorithm in gravity aided navigation are that the gravity sensor has no measurement noise and the actual position of the underwater vehicle is near the position indicated by the INS [13]. By analyzing the principle of the basic ICCP algorithm, there are two main shortcomings.

(1) As a typical sequence iterative matching algorithm, the basic ICCP can only perform one matching operation after...
it receives a certain number of data. Because the velocity of underwater vehicle is very slow and the gravity anomaly is a slow-changing signal, in order to ensure that the gravity anomaly values of the two adjacent measurement points are quite different, the distance between the two adjacent measurement points needs to be relatively large. Therefore, the real-time performance of the basic ICCP is very poor, and the basic ICCP cannot be used to correct the INS positioning error in time. (2) The basic ICCP algorithm requires that the actual position of the underwater vehicle is near the position indicated by the INS. In order to ensure that the basic ICCP can get the closest contour point, a large search range needs to be set in advance. The method of finding the closest contour point is cumbersome, these operations will consume a lot of time during the operation of the basic ICCP.

In response to the shortcomings of the basic ICCP, many scholars have proposed improved methods. TONG [14], BAI [15] and LIU [16] proposed real-time ICCP algorithm (real-time ICCP), respectively. They use the first-in-first-out queue (FIFO) of the computer data structure to improve the storage mode of INS positions and measured gravity anomaly in basic ICCP. The real-time ICCP can complete one matching operation after it receives a new measured data, which improves the real-time performance of the basic ICCP. Some scholars adopt the "rough and precise" matching strategy to improve the positioning accuracy of the basic ICCP. Firstly, the rough matching algorithm is used to limit the position of the underwater vehicle to a small range, and then the basic ICCP is used for precise matching to get the exact position of underwater vehicle [17]–[19]. Some scholars use the intelligent optimization algorithms to improve the positioning accuracy of the basic ICCP, such as PMF algorithm [20], BP neural network algorithm [21], genetic algorithm [22], constrained particle swarm optimization algorithm [23], particle swarm optimization algorithm [24], [25] and ant colony search algorithm [26], etc. Through the continuous optimization of the fitness function, the intelligent optimization algorithm quickly converges to the optimal solution, which can get higher positioning accuracy than the basic ICCP [12]. Some scholars narrow down the search range of the closest contour point by sliding window, which can reduce the computation of basic ICCP for finding the closest contour point [27]–[29].

The matching sequence lengths of the basic ICCP and the real-time ICCP are all fixed and set according to the experience. Influenced by the characteristics of gravity anomaly near the underwater vehicle track, the matching sequence length required for ICCP to obtain the best positioning accuracy is different. In the region where the gravity anomaly changes obviously, the algorithm can get the best positioning accuracy by using shorter matching sequence length. While in the region where the gravity anomaly changes slowly, the algorithm requires longer matching sequence length to obtain the best positioning accuracy. This paper proposes a real-time ICCP algorithm with optimized matching sequence length (OMSL-ICCP) based on the analysis of the shortcomings of the existing real-time ICCP. The OMSL-ICCP through the golden section search [30], [31] to get the optimal matching sequence length, so that the OMSL-ICCP can obtain the best positioning accuracy under the current measured gravity anomaly sequence. The Hausdorff distance [32] is utilized to narrow down the search range of the closest contour point after each iteration. The gravitation field algorithm is used to further improve the positioning accuracy of the OMSL-ICCP. Compared with the existing real-time ICCP algorithm, the OMSL-ICCP can obtain higher positioning accuracy and can be utilized to correct INS positioning error in time. The rest of this paper is structured as follows. In Section 2, we analyzed the shortcomings of existing real-time ICCP algorithm. The OMSL-ICCP is explained in detail in Section 3. Then the performance of the OMSL-ICCP is verified in Section 4. Finally, Section 5 is the conclusion of the full paper.

II. ANALYSIS OF EXISTING REAL-TIME ICCP

The ICCP algorithm is essentially a method of matching multilateral arcs. The principle of basic ICCP is to take the Euclidean distance between the multilateral arc formed by measured gravity anomaly and the multilateral arc formed by the closest contour points as the fitness function. In the iterative process, the centroids of two multilateral arcs are matched by the rigid transformation (rotation and translation) to minimize the fitness function value.

In the real-time ICCP algorithm, the endpoint of the multilateral arc is used as the output. According to the principle of basic ICCP algorithm, the positioning at the endpoint of the multilateral arc may not be the optimal data. Therefore, the positioning error of the track obtained by the endpoints of the multilateral arcs is larger than that of the basic ICCP.

Using the basic ICCP and the existing real-time ICCP, the matching experiments are performed in the gravity maps shown in Figure 1. The resolution of the gravity maps is 1′ × 1′, and the range is 1° × 1°. Real-time ICCP is implemented by the method in reference [16].

The output period of INS is 1 Hz, and the data record is 12 hours. The main parameters of gyro and accelerometer are given in Table 1. Due to the errors of gyro and accelerometer, the position error when INS works alone increases with time, as showed in Figure 2. GPS and INS tracks are shown in Figure 3. The five-pointed star is the starting position of the underwater vehicle, which is supplied by GPS. The underwater vehicle sailed from the northwest to the southeast, and after about two hours, it turned to the northwest and continued sailing for about 10 hours.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gyro random walk /°/h</td>
<td>0.01</td>
</tr>
<tr>
<td>Accelerometer biases /μg</td>
<td>100</td>
</tr>
</tbody>
</table>

The initial parameters of the basic ICCP and the real-time ICCP are set as follows. The period of gravity anomaly
measurement is 6 minutes, the maximum iteration is 10, and the closest contour point search radius is $7'$. During the simulation, the real-time measured gravity anomaly is simulated by the gravity anomaly at the actual position of the underwater vehicle (provided by GPS). Data that is not on the gravity map grid point is obtained by bilinear interpolation. The basic ICCP and the real-time ICCP are used to perform matching experiments on each gravity map, respectively. Because the INS and GPS tracks are derived from the sea test, and the track parameters cannot be changed. By clockwise rotating the gravity map $90^\circ$, the underwater vehicle tracks starting from different directions are obtained. Two tracks in gravity map (a) are marked as A1 and A2, and two tracks in gravity map (b) are marked as B1 and B2. Each matching algorithm performs 20 matching experiments on each track. The RMSE of longitude and latitude errors is calculated. The formula of RMSE is as follows.

$$
RMSE_t = \left( \frac{1}{20} \sum_{m=1}^{20} \left\| \left( \hat{\varphi}_t, \hat{\lambda}_t \right)_{ICCP, (m)} - (\varphi, \lambda)_{GPS, (m)} \right\|^2 \right)^{1/2} \tag{1}
$$

Where, $(\hat{\varphi}_t, \hat{\lambda}_t)_{ICCP, (m)}$ is the longitude and latitude position obtained by the ICCP at the $t$ time in the $m$th experiment,
TABLE 2. RMSE of the basic ICCP with different matching sequence lengths on each track

<table>
<thead>
<tr>
<th>Tracks</th>
<th>RMSE(m)</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>962.2981</td>
<td>922.3528</td>
<td>600.9502</td>
<td>679.9201</td>
<td>666.2026</td>
<td>805.7608</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>1293.433</td>
<td>1146.192</td>
<td>1247.535</td>
<td>1296.682</td>
<td>1208.882</td>
<td>996.2735</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>1683.386</td>
<td>1496.615</td>
<td>1727.372</td>
<td>1560.39</td>
<td>1094.735</td>
<td>1111.746</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>1429.414</td>
<td>1210.561</td>
<td>1062.091</td>
<td>1367.97</td>
<td>1039.827</td>
<td>913.2455</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 3. RMSE of the real-time ICCP with different sequence lengths on each track

<table>
<thead>
<tr>
<th>Tracks</th>
<th>RMSE(m)</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1030.505</td>
<td>947.2756</td>
<td>808.8503</td>
<td>867.6261</td>
<td>954.4986</td>
<td>1145.053</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>1295.368</td>
<td>1263.89</td>
<td>1284.69</td>
<td>1307.243</td>
<td>1335.215</td>
<td>1411.824</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>1671.721</td>
<td>1618.998</td>
<td>1575.566</td>
<td>1495.09</td>
<td>1411.237</td>
<td>1445.22</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>1635.19</td>
<td>1523.797</td>
<td>1468.365</td>
<td>1430.377</td>
<td>1348.234</td>
<td>1460.383</td>
<td></td>
</tr>
</tbody>
</table>

The matching sequence lengths are set to 6, 8, 10, 12, 15, and 20, respectively. The RMSE results of the basic ICCP and the real-time ICCP on each track are shown in Table 2 and Table 3.

By comparing the data in Table 2 and Table 3, it can be seen that the positioning error obtained by the real-time ICCP is larger than that of the basic ICCP in most cases under the same matching sequence length.

By clockwise rotating the gravity map (a), the underwater vehicle tracks starting from different directions are obtained. Four tracks in gravity map (a) are marked as A1, A2, A3 and A4. Using the same method, tracks B1, B2, B3 and B4 are obtained on the gravity map (b). Under the mentioned matching experimental conditions and the real-time ICCP parameters, and the matching sequence length is set to [3, 20], 20 matching experiments are performed on each track, and the RMSE results of the real-time ICCP are obtained. As showed in Figure 4.

(\(\phi, \lambda\))_{GPS,m}^t is the latitude and longitude actual position of the underwater vehicle at the \(t\) time.

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It can be seen from Figure 4 that the positioning error of real-time ICCP decrease with the increase of matching sequence length in the same track, but when the matching sequence length increases to a certain length, the positioning error increases instead of decreases. That is, there is an optimal matching sequence length on each track which can minimize the positioning error of real-time ICCP. The optimal matching sequence lengths are different between different tracks. The optimal matching sequence length on A1 track is 10, the optimal matching sequence length on A2 track is 8, the optimal matching sequence length on A3 track is 14, the optimal matching sequence length of A4 track is 14, the optimal matching sequence length on B1 track is 15, the optimal matching sequence length on B2 track is 15, the optimal matching sequence length on B3 track is 12, and the optimal matching sequence length on B4 track is 19. The different gravity anomaly characteristics along each track lead to different matching sequence lengths for the real-time ICCP to obtain the best positioning accuracy. At present, the existing real-time ICCP adopts fixed matching sequence length, which cannot obtain the best positioning accuracy on all tracks. Therefore, the real-time ICCP can obtain better positioning accuracy under the current measured gravity anomaly sequence by finding the optimal matching sequence length.

III. REAL-TIME ICCP WITH OPTIMIZED MATCHING SEQUENCE LENGTH

Aiming at the shortcomings of existing real-time ICCP, this paper proposes a real-time ICCP with optimized matching sequence length. The OMSL-ICCP through the golden section search to get the optimal matching sequence length, so that the OMSL-ICCP can obtain the best positioning accuracy under the current measured gravity anomaly sequence. The Hausdorff distance is utilized to narrow down the search range of the closest contour point after each iteration. The gravitation field algorithm is used to further improve the positioning accuracy of the OMSL-ICCP.
A. MATCHING SEQUENCE LENGTH OPTIMAL METHOD BASED ON GOLDEN SECTION SEARCH

Golden section search is a search method to find extreme points by narrowing down the scope of the search interval in the given interval through golden ratio division. This method does not care about the setting of the search starting point, and the accuracy of the search result is only related to the setting of the termination condition.

Suppose the objective function is \( f(x) \), then within the interval range \([a, b]\), the inner point recursion formula of the golden section search is as follows [33]–[35].

\[
\begin{align*}
t_{k+1} &= a + 0.382(b - a) \\
t'_{k+1} &= a + 0.618(b - a)
\end{align*}
\]

(2)

Where, \( t_{k+1} \) and \( t'_{k+1} \) are the inner points obtained by the golden section search. If \( f(t_{k+1}) \leq f(t'_{k+1}) \), the optimal value is in \([a, t_{k+1}]\), and the next time continues to search within \([a, t'_{k+1}]\). If \( f(t_{k+1}) > f(t'_{k+1}) \), the optimal value is in \([t_{k+1}, b]\), and the next time continues to search within \([t_{k+1}, b]\) until the termination condition \( \text{eps} \) is satisfied. Because the matching sequence length of the ICCP is an integer, the value of the golden section search inner point is standardized to an integer by rounding. Similarly, the termination condition \( \text{eps} \) is also set to integer.

In this paper, the Euclidean distance between the endpoint of multilateral arc obtained by the ICCP under the current matching sequence length and the its closest contour point is used as the search criterion of the golden section search. The flow chart of getting optimal matching sequence length of OMSL-ICCP based on the golden section search is shown in Figure 5.

B. NARROW DOWN THE SEARCH RANGE OF CLOSEST CONTOUR POINT

The closest contour point calculation of the basic ICCP is cumbersome. If the matching sequence length is too long, it will consume lots of time. The calculation of finding the closest contour point can be reduced by reducing the matching sequence length, reducing the iteration numbers, and reducing the search range of the closest contour point. Without affecting the positioning accuracy of ICCP, the calculation of finding the closest contour point can be effectively reduced by optimizing the search range of the closest contour point.

In this paper, the Hausdorff distance between the positions obtained by the ICCP and its closest contour points is used to narrow down the search range of the closest contour point after each iteration.

The Hausdorff distance is a minimax distance used to measure the resemblance degree of two point sets. Suppose two finite point sets \( A = \{a_1, a_2, \ldots, a_m\} \) and \( B = \{b_1, b_2, \ldots, b_n\} \), then the Hausdorff distance between sets \( A \) and \( B \) is as follow [36]–[38].

\[
H(A, B) = \max(h(A, B), h(B, A))
\]

(3)

Where,

\[
h(A, B) = \max_{a \in A} \min_{b \in B} \|a - b\|
\]

(4)

\[
h(B, A) = \max_{b \in B} \min_{a \in A} \|b - a\|
\]

(5)

In this paper, \( \|\| \) is the Euclidean distance between set \( A \) and set \( B \). By calculating the Hausdorff distance between the positions obtained by ICCP and its closest contour points, the search radius of the closest contour point used in the next iteration is obtained. The search radius is calculated as follows.

\[
ccspr = \text{round}\left(\frac{H(A, B)}{\text{grid}}\right) + 1
\]

(6)

Where, \( H(A, B) \) is the Hausdorff distance between the positions obtained by the ICCP and its closest contour points; \( \text{grid} \) is the gravity map grid spacing; \( ccspr \) is the search radius of the closest contour point in the next iteration. The search range of the closest contour point is a rectangular area, which is formed by centering on the current position and extending \( ccspr \) grid lines to all sides.

C. POSITIONING ACCURACY OPTIMIZATION METHOD BASED ON GRAVITATION FIELD ALGORITHM

Real-time ICCP algorithm uses the endpoint of the multilateral arc as the final output. It can be known from the basic ICCP principle that the positioning data may not be the optimal data, and the positioning accuracy can be further improved by the optimization algorithm. In this paper, the gravitation field algorithm is used to search the closest contour point which is corresponding to the position obtained by the ICCP in the search range of closest contour point. The

FIGURE 5. Flow chart of getting the optimal matching sequence length base on golden section search.
optimization method consists of gravitation field algorithm and the construction of fitness function.

1) Gravitation Field Algorithm
The gravitation field algorithm (GFA) is a heuristic search algorithm which simulates the solar system nebula model. The Nebula model describes the formation of the solar system: while the dust and nebulae in the universe form stars under the force of gravitation, some of the dust is thrown away by stars, and the ejected dust keeps constantly gathering under the force of gravitation, eventually forming all the planets in the solar system [39]–[41]. In order to avoid excessively dense distribution of optimized objects, the Nebula model is optimized to stipulate that all dust generates a rotation repulsive force against surrounding dust within a certain range. The GFA mainly includes three stages: calculation the attractive force from the central dust, calculation the rotation repulsive force and position update. The virtual force model used in the algorithm does not have a fixed form. The algorithm is described as follows.

The attractive force of central dust:
In order to move the dust to the high-like area, it is prescribed that all the dust is attracted by the central dust \( x_{\text{best}} \) at the \( k \) time, and the rest dust do not generate attractive force. The virtual mechanical model is as follows.

\[
P = K_a d_i (x_{\text{best}} - x_i) \tag{7}
\]

Where, \( P \) represents the virtual attractive force of \( x_{\text{best}} \) on \( x_i \); \( K_a \) is the attractive force coefficient used to adjust the attractive force strength, \( d_i \) is the Euclidean distance between \( x_i \) and \( x_{\text{best}} \).

The rotation repulsive force:
While all the dust is attracted by the central dust, all the dust generates a rotation repulsive force to the dust within the perceived range, so that the optimized objects are sparse and avoid excessive concentration. The virtual mechanical model is as follows [42].

\[
F_{ij} = K_r \left( \frac{1}{d_{ij}^2} - \frac{1}{d_{\text{th}}^2} \right) \frac{(x_j - x_i)}{d_{ij}} \tag{8}
\]

Where, \( F_{ij} \) is the virtual rotation repulsive force of \( x_j \) on \( x_i \), \( d_{ij} \) is the perceived radius, \( K_r \) is the rotation repulsive force coefficient used to adjust the repulsive force strength, and \( d_{ij} \) is the Euclidean distance between \( x_i \) and \( x_j \).

Position update:
The resultant force of the individual \( x_i \) is defined as the vector sum of all the virtual forces.

\[
F_i = \sum_{j=1}^{n} F_{ij} + P \quad j = 1, 2, \ldots, n, j \neq i \tag{9}
\]

Where, \( n \) is the amount of dust. \( x_i \) complete one iteration and update position by (10).

\[
x_i' = x_i + F_i \tag{10}
\]

Where, \( x_i' \) is the updated position. Its value is limited by the minimum displacement and maximum displacement, namely \( x_i' \in [L_{\text{min}}, L_{\text{min}}] \). The GFA terminates after the optimization condition or maximum iterations is satisfied. The Figure 6 is the sketch map of GFA. The subgraph on the left shows the dust distribution that randomly generated within the search range. The subgraph on the right shows the dust moving to the central dust (optimized target) in the iterative process under the resultant force of virtual attractive force and the rotation repulsive force.

2) Construction of Fitness Function
The fitness function of the gravitation field algorithm consists of two parts. The difference between the gravity anomaly of the predicted position and the measured gravity anomaly, and the Euclidean distance between the predicted position and the position obtained by ICCP. By calculating the two parameters, the fitness function value of the predicted position is obtained.

\[
\max \text{Fitness} = \eta_1 \text{fitness}_1 + \eta_2 \text{fitness}_2 \tag{11}
\]

Where,

\[
\text{fitness}_1 = \exp \left( -\frac{1}{2R} (g_{\text{pre}} - g_m)^2 \right) \tag{12}
\]
fitness_2 = \exp \left[ -\frac{1}{2R} \left( (x_{\text{pre}} - x_{\text{ICCP}})^2 + (y_{\text{pre}} - y_{\text{ICCP}})^2 \right)^{1/2} \right] 

(13)

fitness_1 is the fitness function value obtained by gravity anomaly, \( g_m \) is the measured gravity anomaly, and \( g_{\text{pre}} \) is the gravity anomaly at predicted position. \( R \) is the observed noise variance. fitness_2 is the fitness function value obtained by the Euclidean distance between the predicted position and the position obtained by the ICCP. \( x_{\text{ICCP}} \) and \( y_{\text{ICCP}} \) are the latitude and longitude of the position obtained by the ICCP, \( x_{\text{pre}} \) and \( y_{\text{pre}} \) are the latitude and longitude of predicted position. \( \eta_1 \) and \( \eta_2 \) are the adjustable factors used to adjust the proportion of fitness_1 and fitness_2 in fitness function.

According to the principle of ICCP, the matching algorithm takes the closest contour point as the optimization target. By setting \( \eta_1 \) and \( \eta_2 \) reasonably, adjusting the fitness function value, so that the dust of the GFA can be gathered around the closest contour point in the search range of the closest contour point. The position of the central dust is taken as the optimal position to improve the positioning accuracy of the OMSL-ICCP.

According to the above description, the flow chart of the positioning optimization method based on the gravitation field algorithm is shown in Figure 7.

**D. THE STEPS OF OMSL-ICCP**

Through the above methods, the calculation process of OMSL-ICCP can be obtained. Because the golden section search requires the objective function \( f(x) \) to satisfy the single peak condition within a certain range, the golden section search is applied to each segment by the segmentation method, and the optimal matching sequence length is obtained by comparing the results of the segments. In this paper, the matching sequence length search space is divided into two subspaces, and the golden section search is applied in each subspace. The flow chart of the OMSL-ICCP is shown in Figure 8.

**IV. EXPERIMENTS AND RESULTS ANALYSIS**

**A. EXPERIMENTAL METHODS AND PARAMETERS SETTING**

Simulation experiments are performed by using the tracks in Section 2 and the gravity maps shown in Figure 1. By clockwise rotating the gravity maps (a) and (b), the tracks entering the gravity map from different directions are obtained. The tracks are marked A1 and A2 on gravity map (a), and B1 and B2 on gravity map (b). Using the real-time ICCP algorithm in reference [16] and OMSL-ICCP, 20 matching experiments are performed on each track respectively, and the RMSE results of each track are calculated by (1). The initial parameters of the real-time ICCP adopt the parameters in Section 2. Taking the optimal matching sequence

**FIGURE 7.** Flow chart of positioning optimization method base on gravitation field algorithm.

**FIGURE 8.** Flow chart of OMSL-ICCP.
length and its positioning data as reference data, which are obtained by experiments in the matching sequence lengths search range of \([3, 20]\). The initial parameters of OMSL-ICCP are set as follows: the gravity measurement period is 6 minutes. Maximum iteration number is 10, and the closest contour point search radius is 7′. The search range of golden section search is \([6, 20]\). The initial parameters of GFA are set as follows: the perceptual radius \(d_{th} = 10\)m, the attractive force coefficient \(K_a = 20\), the rotation repulsive force coefficient \(K_r = 1\), the maximum displacement \(L_{max} = 60\)m, and the minimum displacement \(L_{min} = -L_{max}\). The population is 100 and the maximum iteration number is 30. The adjustable factors \(\eta_1\) and \(\eta_2\) are set to 0.1 and 0.8, respectively. During the simulation, the real-time gravity anomaly is simulated by the gravity anomaly at the actual position of the underwater vehicle (provided by GPS). Data that are not on the grid are obtained by bilinear interpolation. Because the search range maximum value of the golden section search is set to 20. In order to prevent the phenomenon that the OMSL-ICCP has no output before it receives 20 sets of INS data, it is stipulated that the OMSL-ICCP begin to work after it receives the 10th set of INS data, and take the received INS data length as the search range maximum value. When the OMSL-ICCP receives the INS data over 20, the golden section search searches for the optimal matching sequence length in the range of \([6, 20]\).

The application of the ICCP in gravity aided navigation requires that the gravity sensor has no measurement noise and the actual position of the underwater vehicle is near the position indicated by the INS. Obviously, no measurement noise is difficult to achieve in actual uses. It can be seen from the principle of the basic ICCP algorithm that the gravity sensor measurement noise, the INS measurement noise, and the gravity map resolution have a great influence on the positioning accuracy. This paper will compare and analyze the performance of the real-time ICCP and OMSL-ICCP under the influence of the above factors.

### B. PERFORMANCE ANALYSIS WITH NO GRAVITY SENSOR MEASUREMENT NOISE

Under the test condition of no gravity sensor measurement noise, the real-time ICCP and OMSL-ICCP are applied to the matching experiments on A1, A2, B1 and B2 tracks, respectively. After the OMSL-ICCP works, the RMSE results and position error curve obtained by the two methods are shown in Table 4 and Figure 9, respectively. The matching sequence length of the real-time ICCP that obtains the best positioning accuracy is 10 on A1 track, the optimal matching sequence length is 8 on A2 track, the optimal matching sequence length is 15 on B1 track, and the optimal matching sequence length is 15 on B2 track.

<table>
<thead>
<tr>
<th>Tracks</th>
<th>RMSE(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real-time ICCP</td>
</tr>
<tr>
<td>A1</td>
<td>808.8503</td>
</tr>
<tr>
<td>A2</td>
<td>1277.613</td>
</tr>
<tr>
<td>B1</td>
<td>1382.356</td>
</tr>
<tr>
<td>B2</td>
<td>1322.502</td>
</tr>
</tbody>
</table>

In Figure 9, subgraph (a) is the positioning errors of the two methods on A1 track. subgraph (b) is the positioning errors of the two methods on A2 track. subgraph (c) is the...
positioning errors of the two methods on B1 track. subgraph (d) is the positioning errors of the two methods on B2 track.

After the OMSL-ICCP works, the average value curve of the optimal matching sequence length obtained at each moment is shown in Figure 10.

It can be seen from Table 4 that the positioning accuracy of the OMSL-ICCP is higher than that of the real-time ICCP with the optimal matching sequence length. It can be seen from Figure 9 that the latitude error and the longitude error of the underwater vehicle obtained by OMSL-ICCP are superior to or close to that of the real-time ICCP in most cases. But sometimes it is worse than the real-time ICCP. Taking the subgraph (a) in Figure 9 as an example to analyze the reasons for these cases.

The OMSL-ICCP uses the golden section search to get the optimal matching sequence length. The Euclidean distance between the endpoint of the multilateral arc and its closest contour point is used as the search criterion of golden section search. On this basis, the GFA is utilized for precise matching. The GFA guides dust to gather to the closest contour point of the current position through fitness function, thus optimizing the positioning accuracy of the OMSL-ICCP.

Figure 11 shows the position error curve of the real-time ICCP, ICCP optimized by golden section search (GSS, OMSL-ICCP without precisely matched by GFA), and ICCP precisely matched by GFA (OMSL-ICCP). Figure 12 indicates the Euclidean distance between the actual position (GPS) and the matching position. The matching positions are derived from the real-time ICCP, the ICCP optimized by golden section search (GSS, OMSL-ICCP without precisely matched by GFA), and the ICCP precisely matched by the GFA (OMSL-ICCP).

As can be seen from Figure 11 and Figure 12, when the longitude error or the latitude error of the GSS is larger than that of the real-time ICCP, the Euclidean distance between the position obtained by GSS and the actual position is smaller than that of the real-time ICCP. This phenomenon is caused by the search criterion of the matching sequence length search method. The longitude error, the latitude error or the Euclidean distance obtained by OMSL-ICCP can usually be better than that of GSS. However, the position error at some moments is larger than the GSS, even larger than the real-time ICCP. This is because the GFA falls into local optimum, which can be improved by optimizing the algorithm or optimizing the fitness function. The search criterion of the optimized matching sequence length search method, optimizing the precise matching algorithm and its fitness function can be focus as future research contents.

In Figure 13, the search range of the closest contour point is obtained by the Hausdorff distance, and the five-pointed star is the matching point position. The OMSL-ICCP searches the closest contour point in the initially search range of $14' \times 14'$ while it receives the INS data. After the first matching operation of OMSL-ICCP, the closest contour point search range obtained by the method in Section 3.2 is $3' \times 3'$, as showed in the red box. The new search range is much
FIGURE 13. Use Hausdorff distance to get the closest contour point search range.

FIGURE 14. Position error curves of the two methods on each track (Gravity anomaly superimposing Gaussian white noise with mean of 0mGal and standard deviation of 1mGal).

smaller than the initial search range.

C. INFLUENCE OF DIFFERENT GRAVITY SENSOR MEASUREMENT NOISES ON POSITIONING PERFORMANCE

Gravity sensor without measurement noise is difficult to realize in reality. By superimposing Gaussian white noise on the gravity anomaly at the actual position of the underwater vehicle (provided by GPS), the influence of gravity sensor measurement noise on the algorithm performance is simulated.

By superimposing Gaussian white noise with mean of 0m-Gal and standard deviation of 1mGal on the measured gravity anomaly. Using the experimental methods and parameters in Section 4.1, the real-time ICCP and OMSL-ICCP are used to perform matching experiments on the A1, A2, B1 and B2 tracks, respectively. After the OMSL-ICCP works, the RMSE of the two methods are shown in Table 5, and the position error curve of the two methods on each track is shown in Figure 14. The best positioning data of the real-time ICCP on each track are obtained by matching experiments, and these data are used as reference data. The matching sequence length of the real-time ICCP that obtains the best positioning accuracy is 10 on A1 track, the optimal matching sequence
length is 10 on A2 track, the optimal matching sequence length is 20 on B1 track, and the optimal matching sequence length is 16 on B2 track.

**TABLE 5.** RMSE results of the two methods on each track (Gravity anomaly superimposing Gaussian white noise with mean of 0mGal and standard deviation of 1mGal).

<table>
<thead>
<tr>
<th>Tracks</th>
<th>Real-time ICCP</th>
<th>OMSL-ICCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1024.148</td>
<td>854.159</td>
</tr>
<tr>
<td>A2</td>
<td>1299.481</td>
<td>1015.658</td>
</tr>
<tr>
<td>B1</td>
<td>1592.376</td>
<td>1479.824</td>
</tr>
<tr>
<td>B2</td>
<td>1812.942</td>
<td>1487.383</td>
</tr>
</tbody>
</table>

After the OMSL-ICCP works, the average value curve of the optimal matching sequence length obtained at each moment is shown in Figure 15.

**FIGURE 15.** The average matching sequence length curve obtained by OMSL-ICCP at each moment (Gravity anomaly superimposing Gaussian white noise with mean of 0mGal and standard deviation of 1mGal).

Table 5 shows that the positioning accuracy of both methods decreases due to the influence of gravity sensor measurement noise. The positioning accuracy of OMSL-ICCP is higher than that of the real-time ICCP on each track.

By superimposing Gaussian white noise with mean of 0mGal and standard deviation of 5mGal on the measured gravity anomaly, the above experiments are repeated. After the OMSL-ICCP works, the RMSE of the two methods are shown in Table 6, and the position error curve of the two methods on each track is shown in Figure 16. The best positioning data of the real-time ICCP on each track are obtained by matching experiments, and these data are used as reference data. The matching sequence length of the real-time ICCP that obtains the best positioning accuracy is 12 on A1 track, the optimal matching sequence length is 16 on A2 track, the optimal matching sequence length is 19 on B1 track, and the optimal matching sequence length is 20 on B2 track.

After the OMSL-ICCP works, the average value curve of the optimal matching sequence length obtained at each moment is shown in Figure 17.

**FIGURE 16.** The average matching sequence length curve obtained by OMSL-ICCP at each moment (Gravity anomaly superimposing Gaussian white noise with mean of 0mGal and standard deviation of 5mGal).

It can be seen from Table 6 that as the gravity sensor measurement noise increases, the positioning accuracy of both methods is further reduced. The positioning accuracy of OMSL-ICCP is higher than that of the real-time ICCP on each track.

**D. INFLUENCE OF DIFFERENT INS POSITION ERRORS ON POSITIONING PERFORMANCE**

The ICCP requires that the actual position of the underwater vehicle is near the position indicated by the INS. Therefore, the performance of the INS will affect the positioning accuracy of ICCP. Because the INS and GPS tracks are derived from the sea test, and the track parameters cannot be changed. The INS position errors can be increased by superimposing Gaussian white noise on the INS position data.

By superimposing Gaussian white noise with mean of 0 and standard deviation of 1 on the latitude position and longitude position of the INS. Using the experimental methods and parameters in Section 4.1, the real-time ICCP and OMSL-ICCP are used to perform matching experiments on the A1, A2, B1 and B2 tracks, respectively. After the OMSL-ICCP works, RMSE of the two methods are shown in Table 7, and the position error curve of the two methods on each track is shown in Figure 18. The best positioning data of the real-time ICCP on each track are obtained by matching experiments, and these data are used as reference data. The matching sequence length of the real-time ICCP that obtains
FIGURE 16. Position error curve of the two methods on each track (Gravity anomaly superimposing Gaussian white noise with mean of 0mGal and standard deviation of 5mGal).

FIGURE 18. Position error curve of the two methods on each track (Latitude and longitude of the INS superimposing Gaussian white noise with mean of 0′ and standard deviation of 1′).
the best positioning accuracy is 15 on A1 track, the optimal matching sequence length is 17 on A2 track, the optimal matching sequence length is 20 on B1 track, and the optimal matching sequence length is 15 on B2 track.

**TABLE 7.** RMSE of the two methods on each track (Latitude and longitude of the INS superimposing Gaussian white noise with mean of $0'$ and standard deviation of $1'$).

<table>
<thead>
<tr>
<th>Tracks</th>
<th>Real-time ICCP</th>
<th>OMSL-ICCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>2973.347</td>
<td>2410.236</td>
</tr>
<tr>
<td>A2</td>
<td>2992.743</td>
<td>2471.887</td>
</tr>
<tr>
<td>B1</td>
<td>3061.094</td>
<td>2593.594</td>
</tr>
<tr>
<td>B2</td>
<td>3181.001</td>
<td>2672.424</td>
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</tbody>
</table>

After the OMSL-ICCP works, the average value curve of the optimal matching sequence length obtained at each moment is shown in Figure 19.

**FIGURE 19.** The average matching sequence length curve obtained by OMSL-ICCP at each moment (Latitude and longitude of the INS superimposing Gaussian white noise with mean of $0'$ and standard deviation of $1'$).

It can be seen from Table 7 that the performance of the INS has a great influence on the positioning accuracy of the ICCP. After Gaussian white noise is superimposed on the latitude and longitude of the INS, the positioning accuracy of both methods decreases. The positioning accuracy of OMSL-ICCP is higher than that of the real-time ICCP on each track.

By superimposing Gaussian white noise with mean of $0'$ and standard deviation of $2'$ on the latitude position and longitude position of the INS, the above experiments are repeated. After the OMSL-ICCP works, the RMSE of the two methods are shown in Table 8, and the position error curve of the two methods on each track is shown in Figure 20. The best positioning data of the real-time ICCP on each track are obtained by matching experiments, and these data are used as reference data. The matching sequence length of the real-time ICCP that obtains the best positioning accuracy is 15 on A1 track, the optimal matching sequence length is 15 on A2 track, the optimal matching sequence length is 15 on B1 track, and the optimal matching sequence length is 16 on B2 track.

**TABLE 8.** RMSE of the two methods on each track (Latitude and longitude of the INS superimposing Gaussian white noise with mean of $0'$ and standard deviation of $2'$).

<table>
<thead>
<tr>
<th>Tracks</th>
<th>Real-time ICCP</th>
<th>OMSL-ICCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>5303.258</td>
<td>5114.394</td>
</tr>
<tr>
<td>A2</td>
<td>5569.936</td>
<td>4919.48</td>
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<tr>
<td>B1</td>
<td>5752.627</td>
<td>5163.221</td>
</tr>
<tr>
<td>B2</td>
<td>5720.813</td>
<td>5309.41</td>
</tr>
</tbody>
</table>

After the OMSL-ICCP works, the average value curve of the optimal matching sequence length obtained at each moment is shown in Figure 21.

**FIGURE 21.** The average matching sequence length curve obtained by OMSL-ICCP at each moment (Latitude and longitude of the INS superimposing Gaussian white noise with mean of $0'$ and standard deviation of $2'$).

It can be seen from Table 8 that the positioning accuracy of both methods is further reduced as the Gaussian white noise superimposed on the latitude and longitude of the INS increases. The positioning accuracy of OMSL-ICCP is higher than that of the real-time ICCP on each track.

**E. INFLUENCE OF DIFFERENT GRAVITY MAP RESOLUTIONS ON POSITIONING PERFORMANCE**

The gravity map resolution has a great influence on the positioning accuracy of the ICCP. Gravity maps with $2' \times 2'$ and $4' \times 4'$ resolutions are obtained by interval point extraction method. Using the experimental methods and parameters in Section 4.1, the real-time ICCP and OMSL-ICCP are used to perform matching experiments on the A1, A2, B1 and B2 tracks on the gravity maps with resolution $2' \times 2'$, respectively. After the OMSL-ICCP works, the RMSE of the two methods are shown in Table 9, and the position error curve of the two methods on each track is shown in Figure 22. The best positioning data of the real-time ICCP on each track are obtained by matching experiments, and these data...
FIGURE 20. Position error curve of the two methods on each track (Latitude and longitude of the INS superimposing Gaussian white noise with mean of 0′ and standard deviation of 2′).

FIGURE 22. Position error curve of the two methods on each track (Gravity maps with resolution 2′ × 2′).
are used as reference data. The matching sequence length of the real-time ICCP that obtains the best positioning accuracy is 11 on A1 track, the optimal matching sequence length is 8 on A2 track, the optimal matching sequence length is 18 on B1 track, and the optimal matching sequence length is 17 on B2 track.

**TABLE 9.** RMSE of the two methods on each track (Gravity maps with resolution $2' \times 2'$).

<table>
<thead>
<tr>
<th>Tracks</th>
<th>Real-time ICCP</th>
<th>OMSL-ICCP</th>
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</thead>
<tbody>
<tr>
<td>A1</td>
<td>1019.086</td>
<td>882.2689</td>
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<tr>
<td>A2</td>
<td>1328.509</td>
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<td>B1</td>
<td>1463.909</td>
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<tr>
<td>B2</td>
<td>1268.089</td>
<td>1076.072</td>
</tr>
</tbody>
</table>

After the OMSL-ICCP works, the average value curve of the optimal matching sequence length obtained at each moment is shown in Figure 23.

![Figure 23](image-url)

**FIGURE 23.** The average matching sequence length curve obtained by OMSL-ICCP at each moment (Gravity maps with resolution $2' \times 2'$).

It can be seen from Table 9 that the positioning accuracy of both methods decreases when the gravity map resolution is decreased to $2' \times 2'$. The positioning accuracy of OMSL-ICCP is higher than that of the real-time ICCP on each track.

The above experiments are repeated on the gravity maps with resolution $4' \times 4'$. After the OMSL-ICCP works, the RMSE of the two methods are shown in Table 10, and the position error curve of the two methods on each track is shown in Figure 24. The best positioning data of the real-time ICCP on each track are obtained by matching experiments, and these data are used as reference data. The matching sequence length of the real-time ICCP that obtains the best positioning accuracy is 7 on A1 track, the optimal matching sequence length is 8 on A2 track, the optimal matching sequence length is 14 on B1 track, and the optimal matching sequence length is 17 on B2 track.

After the OMSL-ICCP works, the average value curve of the optimal matching sequence length obtained at each moment is shown in Figure 25.

![Figure 24](image-url)

**FIGURE 24.** The position error curve of the two methods on each track (Gravity maps with resolution $2' \times 2'$).

**TABLE 10.** RMSE of the two methods on each track (Gravity maps with resolution $4' \times 4'$).

<table>
<thead>
<tr>
<th>Tracks</th>
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<tbody>
<tr>
<td>A1</td>
<td>1135.375</td>
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</tr>
<tr>
<td>A2</td>
<td>1682.246</td>
<td>1451.737</td>
</tr>
<tr>
<td>B1</td>
<td>1351.262</td>
<td>1140.168</td>
</tr>
<tr>
<td>B2</td>
<td>1050.963</td>
<td>874.7289</td>
</tr>
</tbody>
</table>

![Figure 25](image-url)

**FIGURE 25.** The average matching sequence length curve obtained by OMSL-ICCP at each moment (Gravity maps with resolution $4' \times 4'$).

It can be seen from Table 10 that as the gravity maps resolution decreases, the positioning accuracy of both methods is further reduced. The positioning accuracy of OMSL-ICCP is higher than that of the real-time ICCP on each track.

**V. CONCLUSION**

This paper proposes a real-time ICCP with optimized matching sequence length based on the analysis of the shortcomings of the existing real-time ICCP. The OMSL-ICCP through the golden section search to get the optimal matching sequence length, so that the OMSL-ICCP can obtain the best positioning accuracy under the current measured gravity anomaly sequence. The Hausdorff distance is utilized to narrow down the search range of the closest contour point. The gravitation field algorithm is utilized to further improve the positioning accuracy of the OMSL-ICCP.

This paper compares and analyses the positioning performance of real-time ICCP and OMSL-ICCP under the test conditions of different gravity sensor measurement noise, different INS positioning errors and different gravity map resolutions. The best positioning data of the real-time ICCP on each track are obtained by matching experiments, and these data are used as reference data. The experiments show that the positioning accuracy of the OMSL-ICCP on each track is higher than the positioning accuracy of the real-time ICCP.

Better matching sequence length search method, optimizing the criteria of the search method, and optimizing the
fitness function of the GFA can be focus in the future.

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REFERENCES


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