User-Coupling Angle-Domain Adaptive Filtering based Frequency Synchronization for Massive MIMO Multiuser Uplink

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ABSTRACT In this paper, we develop a user-coupling angle-domain adaptive filtering (UC-ADAF) based frequency synchronization method for the uplink of a massive multiple-input multiple-output (MIMO) multiuser network, in the presence of highly overlapping angle-of-arrival (AoA) regions among coexisting users. In order to deal with the extremely severe multiuser interference (MUI) among users with highly overlapping AoA region, we propose to couple them together and perform joint carrier frequency offset (CFO) estimation and data detection. For the coupled users, the UC angle-constraining matrix (UC-ACM) can inherently suppress the interference from non-overlapping users and the UC-ADAF vectors are adaptively optimized to further eliminate the MUI from slightly overlapping users. Moreover, the challenging problem of addressing the severe MUI caused by the highly overlapping users is circumvented through joint estimation and detection. The effectiveness of the proposed UC-ADAF approach and its robustness against overlapping degree among coupled users and total number of mutually overlapping users, as well as its superiority over the existing methods, are demonstrated by numerical results.

INDEX TERMS Frequency synchronization, carrier frequency offset (CFO), user-coupling angle-domain adaptive filtering (UC-ADAF), multiuser interference (MUI) suppression, multiuser massive MIMO.

I. INTRODUCTION

Over the past few years, large-scale multiple-input multiple-output (MIMO) or massive MIMO systems have drawn exploring interests from both academia and industry [1]–[11]. Massive MIMO is considered as a promising technique for the next generation wireless systems due to its high spectral and energy efficiency as well as the ability to substantially relieve the multiuser interference (MUI) with simple linear transceivers. However, these potential gains depend heavily on perfect frequency synchronization, which is quite challenging owing to the existence of carrier frequency offsets (CFOs) [12], [13] among different users.

Various conventional synchronization schemes have been proposed for multiuser MIMO [14]–[19]. In [16] for example, a robust multi-CFO estimation with iterative interference cancellation (RMCE-IIC) scheme has been proposed for a coordinated multi-point (CoMP) orthogonal frequency division multiplexing (OFDM) system. Based on the Zadoff-Chu (ZC) sequences, the optimal set of training sequences is designed to minimize the mutual interference, which is also robust to different CFOs. Besides, a few frequency synchronization methods have also been designed for massive MIMO systems [20]–[25]. A blind frequency synchronization approach has been developed in [23] for multiuser OFDM uplink with a large number of receive antennas. However, many blocks are necessary to obtain the satisfactory CFO estimation performance, which delays the decoding and cannot be directly applied to latency-sensitive scenarios.

More recently, a multiuser frequency synchronization scheme has been proposed in [26] when a large-scale uniform linear array (ULA) is configured at the BS. The CFO estimation and data detection therein are performed individually for
design in this paper a user-coupling ADAF (UC-ADAF) based frequency synchronization approach to cope with the multi-CFO estimation of users with highly overlapping AoA region. Fig. 1 exemplifies a multiuser scenario consisting of users with highly overlapping AoA region\(^1\) and illustrates the similarities and dissimilarities between the previous ADAF \([29], [30]\) and the proposed UC-ADAF. First, among all the coexisting users, those with highly overlapping AoA region will be categorized into one or several couples. Note that there might be several users whose AoA regions contiguously intersect, yet only those sharing most of their AoA regions in common will be coupled. In Fig. 1 for example, Users 1-2 and Users 3-4 should be categorized into two couples, rather than all coupled together. Next, all the MF beamformers falling into the union AoA region of the coupled users form the UC-ACM, which can inherently suppress the interference stemming from non-overlapping IUs. Then, the UC-ADAF vectors are optimized to minimize the MUI on the coupled users caused by slightly overlapping IUs. Finally, joint CFO estimation and subsequent data detection can be carried out for the coupled users. By coupling together users with highly overlapping AoA regions, the troublesome and challenging problem of addressing the extremely severe MUI among them can be circumvented. For each couple of users, UC-ADAF will be exploited to conduct joint CFO estimation and data detection; while the ADAF algorithm will be applied for each of the rest users. Numerical results demonstrate the effectiveness of the proposed UC-ADAF approach and its superiority over existing approaches.

The remainder of this paper is organized as follows. The system model is described in Section II. Section III elaborates the proposed UC-ADAF algorithm. Simulation results are provided in Section IV. Section V concludes the paper.

**Notations:** Superscripts \((\cdot)^{*}, (\cdot)^{T}, (\cdot)^{H}, (\cdot)^{-1}, (\cdot)^{†}\) and \(E(\cdot)\) represent conjugate, transpose, Hermitian, inverse, pseudo-inverse and expectation, respectively; \(j = \sqrt{-1}\) is the imaginary unit; \(\| \cdot \|\) denotes the Frobenius norm operator; \(\text{tr}(\cdot)\) denotes the trace operation. For a vector \(x\), \(\text{diag}(x)\) is a diagonal matrix with \(x\) as the main diagonal; \(\otimes\) denotes the Kronecker product; \(\text{vec}(\cdot)\) is the vectorization operator; \(\mathbb{C}^{m \times n}\) denotes the vector space of all \(m \times n\) complex matrices; \(I_{Q}\) is the \(Q \times Q\) identity matrix.

**II. SYSTEM MODEL**

We consider a multiuser OFDM uplink system that consists of \(K\) distributed single-antenna users and one BS with

\(^1\)It should be pointed out that there exists two points of view when describing the users. On the one hand, when all the users are considered as an integral, we refer to User 1 as “user with highly overlapping AoA region”, since its AoA region highly overlaps with that of User 2 (similarly for Users 2-4). User 5 which occupies an exclusive AoA region is referred to as “user with exclusive AoA region”. On the other hand, when we focus on a specific user and want to perform MUI suppression, the other users will be considered as interfering users (IUs). The designation for IUs depends on their position relative to the interested user. For instance, if the focus is put on User 2, then User 1, Users 3-4 and User 5 are referred to as “highly overlapping IU”, “slightly overlapping IUs” and “non-overlapping IU”, respectively.
of CFO estimation to simplify the receiver design. In fact, we can send one
uplink. The CFO will affect the uplink received OFDM signal i.e.,
where \( m, n \) is adopted to characterize the multiuser OFDM uplink
multi-tap channel matrix \( \mathbf{H}_k \) of \( \mathbf{F}_L \) consisting of its first
blocks, including a whole training block at the begin-
ning of each uplink frame and
\( \sum_{i=1}^{K} x_i^{(k)}(0), x_i^{(k)}(1), \ldots, x_i^{(k)}(N-1) \) as the frequency
domain data symbols transmitted from the \( k \)th user regardless of how severe the MUI might be, our pro-
posed UC-ADAF approach differs greatly from the UG. Actually, the UG directly adopts the simplest MF beam-
formers by introducing UC-ADAF vectors.

The classical one-ring channel propagation model [31], [32] is adopted to characterize the multiuser OFDM uplink
scenario. The channel between the BS and the \( k \)th user consists of \( L \) taps. Denote the average power of the \( l \)th tap as \( \sigma_l^2 \). Then, \( \sigma_l^2, l = 1, 2, \ldots, L \) models the power delay
profile (PDP) of the channel with \( \sum_{l=1}^{L} \sigma_l^2 = 1 \) such that the total average channel gain per received antenna is normalized.
Each tap is composed of \( P \) separable subpaths identifiable by its unique AoA \( \theta^{|S_i|}_l \) uniformly distributed within \( (\theta_k-\theta_{\text{na}}, \theta_k+\theta_{\text{na}}) \) and associated independent and identically distributed (i.i.d.) complex gain \( g_i^{(k)}(l) \sim CN(0, \sigma_l^2/P) \). Here, \( \theta_k \) and \( \theta_{\text{na}} \) represent the average AoA of the \( k \)th user and AS, respectively. Note that accurate average AoAs are assumed available at the BS when addressing the CFO estimation issue in this paper.

\[
\mathbf{a}(\theta_{l,p}) = \begin{bmatrix} 1, e^{-j2\pi \lambda d \sin \theta_{l,p}}, \ldots, e^{-j2\pi \lambda (M-1) \sin \theta_{l,p}} \end{bmatrix},
\]

where \( \chi = \frac{\pi}{\lambda} d \) is the antenna spacing and \( \lambda \) denotes the carrier wavelength. Then, the multi-tap channel matrix between the \( k \)th user and the BS can be expressed as

\[
\mathbf{H}_k = [\mathbf{h}_1^{(k)}, \mathbf{h}_2^{(k)}, \ldots, \mathbf{h}_L^{(k)}] \in \mathbb{C}^{L \times M},
\]

where \( \mathbf{h}_l^{(k)} = \sum_{p=1}^{P} g_i^{(k)}(l) \mathbf{a}(\theta_{l,p}) \) corresponds to the \( l \)th tap channel vector.

Define \( \mathbf{F}_L \) as the \( N \times N \) normalized discrete Fourier transform (DFT) matrix whose \( (m, n) \)-th entry is given by

\[
\frac{1}{\sqrt{N}} e^{-j2\pi \frac{(m-1)(n-1)}{N}}.
\]

Besides, \( \mathbf{F}_L \) stands for the submatrix of \( \mathbf{F} \) consisting of its first \( L \) columns. Each frame consists of \( N_b \) blocks, including a whole training block at the begin-
ning of each uplink frame and \( N_b-1 \) data blocks. Denote \( \mathbf{x}^{(k)} = [x_i^{(k)}(0), x_i^{(k)}(1), \ldots, x_i^{(k)}(N-1)]^T \) as the frequency
domain data symbols transmitted from the \( k \)th user in the \( i \)th block. Then, the received time-domain signal in the \( i \)th block after cyclic-prefix (CP) removal is given by

\[
\mathbf{Y}_i = \sum_{k=1}^{K} \eta_i(\phi_k) \mathbf{E}(\phi_k) \mathbf{B}_i^{(k)} \mathbf{H}_k + \mathbf{N}_i,
\]

where \( \eta_i(\phi_k) = e^{j2\pi \frac{N-M}{N} \phi_k} \) represents the accumu-
lative phase rotation introduced by \( \phi_k \) with \( N_{cp} \) being the length of the CP, \( \mathbf{B}_i^{(k)} = \sqrt{N_{cp}} \mathbf{H} \) diag \( (x_i^{(k)} \mathbf{F}_L) \in \mathbb{C}^{N \times L} \), and \( \mathbf{N}_i \in \mathbb{C}^{N \times M} \) denotes the corresponding additive white Gaussian noise (AWGN) matrix. We assume that each element of \( \mathbf{N}_i \) follows i.i.d. complex Gaussian distribution with variance \( \sigma_n^2 \), i.e., \( \mathbb{E} [\mathbf{N}_i \mathbf{N}_i^H] = M \sigma_n^2 \mathbf{I}_N \). Note that \( i = 1 \) denotes the training block, and \( \mathbf{B}_i^{(k)} \) and \( \mathbf{Y}_1 \) will be simplified as \( \mathbf{B}_k \) and \( \mathbf{Y}_1 \) hereinbelow for notation convenience.

### III. PROPOSED UC-ADAF FREQUENCY SYNCHRONIZATION METHOD

As already explained in Introduction, the ADAF approach proposed in [29], [30] might fail to combat the severe MUI among users with highly overlapping AoA region, which motivates us to develop the UC-ADAF. Unlike the ADAF which performs separate estimation and detection for each user regardless of how severe the MUI might be, our pro-
posed UC-ADAF couples together the users with highly overlapping AoA region, and circumvents the severe mutual interference among the coupled users by performing joint CFO estimation and data detection.

#### A. UC-ACM AND UC-ADAF VECTORS

The UC-ACM originates from the ACM used by ADAF approach. Analogous to the ADAF, the UC-ACM inherently suppresses the MUI from non-overlapping IUs and the UC-ADAF vectors are optimized to minimize the interference on the coupled users caused by slightly overlapping IUs. Let the AoA samples be drawn from \( \mathcal{D}_{\text{FFT}} = \{ \mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_M \} \) with \( \mathcal{D}_l = \sqrt{\cos \left( \frac{\pi(l-1/2)}{M} \right)} \). Then, \( \mathcal{S}_0 = \{ \theta_0^{(q)} : \theta_k < \theta_0 \cdots < \theta_{M} \} \) represents the set of AoA samples falling into the AoA region of the \( k \)th user, and the information of the \( k \)th user should be constrained by the ACM defined as

\[
\mathcal{U}_k = \frac{1}{\sqrt{M}} \left[ \mathbf{a}(\theta_{0}^{(1)}), \mathbf{a}(\theta_{0}^{(2)}), \ldots, \mathbf{a}(\theta_{M}^{(Q_k)}) \right],
\]

where \( Q_k \) is the cardinality of \( \mathcal{S}_0 \), namely \( Q_k = |\mathcal{S}_0| \). Note that there is \( \mathcal{U}_k^H \mathcal{U}_k = \mathbf{I}_{Q_k} \).

Suppose that the AoA regions of \( G \) users with user indices \( \kappa_1, \kappa_2, \ldots, \kappa_G \) are highly overlapped, while the other users are either non-overlapping or slightly overlapping with them. Then, these \( G \) users will be coupled. Let \( \mathcal{S}_\mathcal{U} = \mathcal{S}_{\kappa_1} \cup \mathcal{S}_{\kappa_2} \cup \ldots \cup \mathcal{S}_{\kappa_G} \) represent the union set of AoA samples falling into the AoA regions of the coupled \( G \) users, and let \( \mathbf{a}(\theta_{\mathcal{U}}^{(q)}) \) denote the \( q \)th element in \( \mathcal{S}_\mathcal{U} \). Then, the information of

\[3\text{The UC [28] also performs the joint estimation and detection for in-
trougress users. However, the proposed UC-ADAF approach differs greatly from the UC. Actually, the UC directly adopts the simplest MF beam-
formers to suppress the MUI. Thus, all the users whose AoA regions are contiguously overlapping must be grouped together to perform joint estima-
tion and detection, which increases the computational complexity and also the likelihood of underdetermination. In contrast, the UC-ADAF employs the adaptively optimized beamformers by introducing UC-ADAF vectors. Thereby, the MUI cancellation is more effective and only the users with highly overlapping AoA region need to be coupled, significantly reducing the computational burden.}
the coupled $G$ users should be constrained by the UC-ACM defined as

$$U_{UC} = \frac{1}{\sqrt{M}} \left[ a(\vartheta_{UC}(1)), a(\vartheta_{UC}(2)), \ldots, a(\vartheta_{UC}(Q_{UC})) \right],$$  \hspace{1cm} (6)$$

where $Q_{UC} = |S_{UC}|$ is the cardinality of $S_{UC}$. Note that there is $U_{UC}^T U_{UC} = I_{Q_{UC}}$.

For the $k$th uncoupled user, we employ the following $M \times 1$ vectors as the beamformers to effectively perform MUI cancellation

$$\omega^{(q)}_k = U_k \gamma^{(q)}_k, \quad q = 1, 2, \ldots, Q_k,$$  \hspace{1cm} (7)$$

where $\gamma^{(q)}_k = \left[ \gamma^{(q)}_k(1), \gamma^{(q)}_k(2), \ldots, \gamma^{(q)}_k(Q_k) \right]^T \in \mathbb{C}^{Q_k \times 1}$, $q = 1, 2, \ldots, Q_k$ are ADAF vectors for the $k$th user. The optimal ADAF vectors can be acquired with the ADAF algorithm [29], [30]. Similarly, for the coupled $G$ users with user indices $k_1, k_2, \ldots, k_G$, we adopt the following $M \times 1$ vectors as the adaptive beamformers to effectively perform MUI cancellation

$$\omega^{(q)}_u = U_{UC} \gamma^{(q)}_{UC}, \quad q = 1, 2, \ldots, Q_{UC},$$  \hspace{1cm} (8)$$

where $\gamma^{(q)}_{UC} = \left[ \gamma^{(q)}_{UC}(1), \gamma^{(q)}_{UC}(2), \ldots, \gamma^{(q)}_{UC}(Q_{UC}) \right]^T \in \mathbb{C}^{Q_{UC} \times 1}$, $q = 1, 2, \ldots, Q_{UC}$ are UC-ADAF vectors for the coupled users. How to obtain the appropriate UC-ADAF vectors will be detailed in Section III-B. Note that the multi-branch beamforming is used to harvest the spatial diversity [30], [33].

**B. JOINT CFO ESTIMATION FOR COUPLED USERS**

Denote $\phi = [\phi_{k_1}, \phi_{k_2}, \ldots, \phi_{k_G}]^T$. The received pilot signal of the couple $G$ users can be expressed as

$$\sum_{G} \mathbf{E} \left( \phi_{k_1} \right) \mathbf{B}_{k_1} \mathbf{H}_{k_1} = \mathbb{B} (\phi) \mathbb{H},$$  \hspace{1cm} (9)$$

where $\mathbb{B} (\phi) = [\mathbf{E} (\phi_{k_1}) \mathbf{B}_{k_1}, \mathbf{E} (\phi_{k_2}) \mathbf{B}_{k_2}, \ldots, \mathbf{E} (\phi_{k_G}) \mathbf{B}_{k_G}]$ and $\mathbb{H} = [\mathbf{H}_{k_1}^T, \mathbf{H}_{k_2}^T, \ldots, \mathbf{H}_{k_G}^T]^T$. Using the trial CFO values $\hat{\phi} = [\hat{\phi}_{k_1}, \hat{\phi}_{k_2}, \ldots, \hat{\phi}_{k_G}]$, to replace the true CFO $\phi$, we can define $\mathbf{P}_B = \mathbb{B} (\hat{\phi}) \mathbb{B}^H \mathbb{B} + \mathbb{P}_B$ and $\mathbf{P}_B = \mathbb{I} - \mathbf{P}_B$, where $\mathbb{B} (\phi)$ is simplified as $\mathbb{B}$ for notation convenience.

Let $\omega^{(q)}_{UC} = U_{UC} \gamma^{(q)}_{UC}$ denote the $q$th trial adaptive beamformer, where $\gamma^{(q)}_{UC}$ is the $q$th trial UC-ADAF vector for the coupled users. Note that $\omega^{(q)}_{UC}$ aims at cancelling the interference caused by users outside the couple and meanwhile maintaining the desired signal of the coupled $G$ users. Then, the equivalent SINR after receiving beamforming with $\omega^{(q)}_{UC}$ can be expressed as

$$\hat{\rho}_{UC}^{(q)} = \frac{\left\| \mathbf{P}_B Y \omega^{(q)}_{UC} \right\|^2}{\left\| \mathbf{P}_B Y \omega^{(q)}_{UC} \right\|^2} = \frac{\left\| Y U_{UC} ^T \gamma^{(q)}_{UC} \right\|^2}{\left\| Y U_{UC} ^T \gamma^{(q)}_{UC} \right\|^2} - 1,$$  \hspace{1cm} (10)$$

where

$$\Theta_{UC} = U_{UC} ^T Y H Y U_{UC} ^*,$$  \hspace{1cm} (11)$$

$$\Xi (\hat{\phi}) = U_{UC} ^T Y H \mathbf{P}_B ^{1/2} Y U_{UC} ^*.$$  \hspace{1cm} (12)$$

Furthermore, the noise correlation among different beamforming branches is given by $\gamma^{(q)H}_{UC} \Xi (\hat{\phi}) \gamma^{(q)}_{UC}$, $q = 1, 2, \ldots, Q_{UC}$ in order to decorrelate the noises among different beamforming branches and to eliminate the scalar ambiguity of the UC-ADAF vectors, we constrain the UC-ADAF vectors $\gamma^{(q)}_{UC}$, $q = 1, 2, \ldots, Q_{UC}$ to satisfy

$$\gamma^{(q)}_{UC} \Xi (\hat{\phi}) \gamma^{(q)}_{UC} = \mathbb{I}_{Q_{UC}},$$  \hspace{1cm} (13)$$

or equivalently,

$$\gamma^{(q)H}_{UC} \Xi (\hat{\phi}) \gamma^{(q)}_{UC} = \Xi^{-1} (\hat{\phi}),$$  \hspace{1cm} (14)$$

where $\gamma^{(q)}_{UC} = [\gamma^{(1)}_{UC}, \gamma^{(2)}_{UC}, \ldots, \gamma^{(Q_{UC})}_{UC}] \in \mathbb{C}^{Q_{UC} \times Q_{UC}}$ is the set of $Q_{UC}$ trial UC-ADAF vectors. Note that the invertibility of $\Xi (\hat{\phi})$ can be guaranteed in most circumstances since there usually hold $Q_{UC} \ll M$ and $Q_{UC} \ll N$. Apparently, (13) ensures that the noises among different beamforming branches are mutually uncorrelated with equalized power. Thus, the overall SINR attained will simply be the summation of the SINR per branch, when the maximum-ratio-combining (MRC) detection among different branches is employed to recover the transmitted symbols.

The maximization of the overall SINR for the coupled $G$ users leads to

$$\hat{\phi} = \arg \max_{\phi, \gamma_{UC}^{(q)}} \sum_{q=1}^{Q_{UC}} \gamma^{(q)H}_{UC} \Theta_{UC} \gamma^{(q)}_{UC} \Xi (\hat{\phi}) \gamma^{(q)}_{UC},$$

s.t. $\gamma^{(q)H}_{UC} \Xi (\hat{\phi}) \gamma^{(q)}_{UC} = \mathbb{I}_{Q_{UC}},$

$$\hat{\phi} = \arg \max_{\phi, \gamma_{UC}^{(q)}} \left[ \Theta_{UC} \Xi^{-1} (\hat{\phi}) \right] \gamma_{UC}^{(q)H} \Xi (\hat{\phi}) \gamma_{UC}^{(q)},$$

s.t. $\gamma_{UC}^{(q)H} \Xi^{-1} (\hat{\phi}) \gamma_{UC}^{(q)} = \mathbb{I}_{Q_{UC}},$ \hspace{1cm} (15)$$

Thus, by setting $G (\hat{\phi}) = \text{tr} \left[ \Theta_{UC} \Xi^{-1} (\hat{\phi}) \right]$, the CFOs of the coupled $G$ users can be estimated by solving

$$\hat{\phi} = \arg \max_{\phi} G (\hat{\phi}).$$  \hspace{1cm} (16)$$

Note that (16) is independent of the optimization of UC-ADAF vectors $\gamma_{UC}^{(q)} = [\gamma^{(1)}_{UC}, \gamma^{(2)}_{UC}, \ldots, \gamma^{(Q_{UC})}_{UC}]$. Besides, the UC-ADAF vectors $\gamma^{(q)}_{UC}$, $q = 1, 2, \ldots, Q_{UC}$ satisfying $\gamma^{(q)H}_{UC} \Xi^{-1} (\hat{\phi}) \gamma^{(q)}_{UC} = \mathbb{I}_{Q_{UC}}$ will be optimal. Actually, the optimal set of UC-ADAF vectors can be taken as the square root of $\Xi^{-1} (\hat{\phi})$, i.e., $\gamma_{UC}^{(q)} = \Xi^{-1/2} (\hat{\phi})$.

Instead of directly solving (16) via $G$-dimensional search about $\hat{\phi}$, we propose to acquire the CFO estimates in the following computationally efficient manner.
Suppose \( \hat{\phi}_c = [\hat{\phi}_{c,1}, \hat{\phi}_{c,2}, \ldots, \hat{\phi}_{c,G}]^T \) being the coarse estimate of \( \phi \). Denote \( \Delta \hat{\phi}_{c,g} = \hat{\phi}_{c,g} - \phi_{c,g} \), \( g = 1, 2, \ldots, G \).

We approximate \( \mathbb{E}(\hat{\phi}_{c,g}) \) with Taylor series expansion as

\[
\mathbb{E}(\hat{\phi}_{c,g}) \approx (I_N + \Delta \hat{\phi}_{c,g} D + \frac{\Delta^2 \hat{\phi}_{c,g}^2}{2} D^2) \mathbb{E}(\hat{\phi}_{c,g}),
\]

(17)

where \( D = \frac{1}{N} \text{diag} ([0, 1, \ldots, N-1]) \). Note that although the ADADF approach [29, 30] is originally designed for users with slightly overlapping AoA region, it indeed can provide valid coarse CFO estimates for the considered circumstances. Define \( \Pi(\hat{\phi}) = \mathbb{B} \mathbb{B}^H = \sum_{g=1}^G \mathbb{E}(\hat{\phi}_{c,g}) B_{c,g} B_{c,g}^H \mathbb{E}(\hat{\phi}_{c,g}) \).

Then, with the approximation (17), \( \Pi(\hat{\phi}) \) can be approximated as

\[
\Pi(\hat{\phi}) \approx \sum_{g=1}^G C_{c,g} C_{c,g}^H + \Delta \Pi,
\]

(18)

where \( C_{c,g} = \mathbb{E}(\hat{\phi}_{c,g}) B_{c,g}, g = 1, 2, \ldots, G \)

\[
\Delta \Pi = \sum_{g=1}^G \left( (2D C_{c,g} C_{c,g}^H D^H + D^2 C_{c,g} C_{c,g}^H + C_{c,g} C_{c,g}^H D^2) \right) \frac{\Delta \hat{\phi}_{c,g}^2}{2},
\]

(19)

Eigendecomposing \( \Pi(\hat{\phi}) \) yields

\[
\Pi(\hat{\phi}) = [U_s, U_s] \Sigma \Pi [U_s, U_s]^H,
\]

(20)

where \( U_s \in \mathbb{C}^{N \times GL} \) and \( U_n \in \mathbb{C}^{N \times (N-GL)} \) belong to the signal and noise subspace, respectively, and \( \Sigma \Pi \) is a diagonal matrix consisting of the eigenvectors of \( \Pi(\hat{\phi}) \) in ascending order. Denote the \( i \)th eigenvalue (in ascending order) of \( \Pi(\hat{\phi}) \) as \( \lambda_i \), \( i = 1, 2, \ldots, N \) and its corresponding eigenvector as \( e_i \). Then, the \( i \)th eigenvector of \( \Pi(\hat{\phi}) \) related to the noise subspace can be expressed as \( \alpha_i + \Delta \alpha_i \). Here, the perturbation term \( \Delta \alpha_i \) is given by [34]

\[
\Delta \alpha_i = \sum_{j \neq i} e_j^H \Delta \Pi e_i = \sum_{j \neq i} e_j^H \Delta \Pi e_i = \sum_{j \neq i} \frac{e_j^H \Delta \Pi e_i}{\lambda_i - \lambda_j} = \sum_{j \neq i} \frac{e_j^H \Delta \Pi e_i}{\lambda_j - \lambda_j} = \sum_{j \neq i} \frac{e_j^H \Delta \Pi e_i}{\lambda_j} = \sum_{j=1}^N \left( \alpha_{g,i} \Delta \hat{\phi}_{c,g} + \beta_{g,i} \Delta \hat{\phi}_{c,g}^2 \right), \quad i = 1, 2, \ldots, N-GL,
\]

(21)

where \( \alpha_{g,i}, \beta_{g,i}, g = 1, 2, \ldots, G \) are defined as

\[
\alpha_{g,i} = \sum_{j=N-GL+1}^N e_j^H C_{c,g} C_{c,g}^H D^H e_i = -U_s A_s^{-1} U_s^H C_{c,g} C_{c,g}^H D^H e_i;
\]

\[
\beta_{g,i} = \sum_{j=N-GL+1}^N e_j \left( 2D C_{c,g} C_{c,g}^H D^H + C_{c,g} C_{c,g}^H D^2 \right) e_i = -U_s A_s^{-1} U_s^H \left( C_{c,g} C_{c,g}^H D^2 + 2D C_{c,g} C_{c,g}^H D^H \right) e_i.
\]

Note that \( \Lambda_s \) is defined as \( \Lambda_s = \text{diag}(\lambda_{N-GL+1}, \lambda_{N-GL+2}, \ldots, \lambda_N) \). For \( g = 1, 2, \ldots, G \), we further denote

\[
C_{c,g} = \begin{cases} \alpha_{c,g,1}, \alpha_{c,g,2}, \ldots, \alpha_{c,g,N-GL} \\ U_s A_s^{-1} U_s^H C_{c,g} C_{c,g}^H D^H U_n, \end{cases}
\]

\[
D_{c,g} = \begin{cases} \beta_{c,g,1}, \beta_{c,g,2}, \ldots, \beta_{c,g,N-GL} \\ -U_s A_s^{-1} U_s^H \left( C_{c,g} C_{c,g}^H D^2 + 2D C_{c,g} C_{c,g}^H D^H \right) U_n. \end{cases}
\]

Then, \( P_{\hat{\phi}} = I - \mathbb{B} \mathbb{B}^H \mathbb{B}^{-1} \mathbb{B}^H \mathbb{B} \) which represents the subspace orthogonal to that spanned by \( \mathbb{B} \) can be approximated as

\[
P_{\hat{\phi}} = \sum_{i=1}^{N-GL} \left( \alpha_i + \Delta \alpha_i \right) \left( \alpha_i + \Delta \alpha_i \right)^H \approx U_n U_n^H + \Delta \Pi_{\hat{\phi}},
\]

(23)

where

\[
\Delta \Pi_{\hat{\phi}}^+ = \sum_{g=1}^G \sum_{i=1}^{N-GL} \left( \alpha_{g,i} e_i^H e_i + \beta_{g,i} \Delta \hat{\phi}_{c,g} \Delta \hat{\phi}_{c,g}^2 \right) = \sum_{g=1}^G \sum_{i=1}^{N-GL} \Delta \hat{\phi}_{c,g} \Delta \hat{\phi}_{c,g}^2,
\]

(24)

Combining (12) and (23), we arrive at

\[
\Xi(\hat{\phi}) \approx \frac{U_{UC}^T Y^H U_n U_n^H Y U_{UC}^T + U_{UC}^T Y^H \Delta \Pi_{\hat{\phi}}^+ Y U_{UC}^T}{A}.
\]

(25)

Moreover, there holds \((P + Q)^{-1} \approx P^{-1} - P^{-1} Q P^{-1}\) when the entries of arbitrary invertible matrix \(P\) is much larger than those of \(Q\). Thus, \(\Xi^{-1}(\hat{\phi})\) can be approximated as

\[
\Xi^{-1}(\hat{\phi}) \approx (A + B)^{-1} \approx A^{-1} - A^{-1} B A^{-1}.
\]

(26)

Then, the joint CFO estimation problem (16) can be equivalently rewritten as

\[
\hat{\phi} \approx \arg \max_{\phi} \left\{ \text{tr}[\Theta_{UC} A^{-1}] - \text{tr}[\Theta_{UC} A^{-1} B A^{-1}] \right\},
\]

\[
= \min_{\hat{\phi}} \text{tr}[\Theta_{UC} A^{-1} U_{UC}^T Y^H \Delta \Pi_{\hat{\phi}}^+ Y U_{UC}^T A^{-1}] - \text{tr}[\Theta_{UC} A^{-1} U_{UC}^T Y^H \Delta \Pi_{\hat{\phi}}^+ Y U_{UC}^T A^{-1}].
\]

(27)
Denote $\Delta \hat{\phi} = [\Delta \hat{\phi}_{\kappa_1}, \Delta \hat{\phi}_{\kappa_2}, \ldots, \Delta \hat{\phi}_{\kappa_G}]^T$. Substituting (24) into (27) leads to

$$\Delta \hat{\phi} \approx \arg \min_{\Delta \tilde{\phi}} \left\{ \Delta \tilde{\phi}^T A \Delta \tilde{\phi} + b^T \Delta \phi \right\},$$

where the diagonal and off-diagonal elements of the real symmetric matrix $A \in \mathbb{C}^{G \times G}$ are given by

$$a_{g,g} = \frac{1}{2} \mathrm{tr} \left[ \Theta_{UC} A^{-1} U_{UC}^H Y H^H D_{\kappa_g} U_n H^H + U_n D_{\kappa_g} \right], \quad g = 1, 2, \ldots, G,$$

and the $g$th element of vector $b \in \mathbb{C}^{G \times 1}$ is determined as

$$b_g = \mathrm{tr} \left[ \Theta_{UC} A^{-1} U_{UC}^H Y H (C_{\kappa_g} U_n H^H + U_n C_{\kappa_g}) Y U_{UC}^H A^{-1} \right].$$

By solving (28), the optimal residual CFO estimates can be obtained as follows

$$\Delta \hat{\phi} = [\Delta \hat{\phi}_{\kappa_1}, \Delta \hat{\phi}_{\kappa_2}, \ldots, \Delta \hat{\phi}_{\kappa_G}]^T,$$

$$= - (A + A^T)^{-1} b - \frac{1}{2} A^{-1} b.$$  \hspace{1cm} (29)

The final CFO estimates for the coupled $G$ users are given by

$$\hat{\phi} = \hat{\phi}_c + \Delta \hat{\phi}.$$  \hspace{1cm} (30)

Remark 1: Acquiring the optimal UC-ADAF vectors requires the inverse of matrix $\Xi(\hat{\phi}) \in \mathbb{C}^{Q_{UC} \times Q_{UC}}$. However, when $Q_{UC}$ is too large, the invertibility of $\Xi(\hat{\phi})$ cannot be guaranteed. Also, a large $Q_{UC}$ may hinder the parameter estimation performance. In order to circumvent the invertibility problem, we propose the following UC-ACM partitioning technique. For illustration, we partition the UC-ACM problem, we propose the following UC-ACM partitioning.

The above UC-ACM partitioning technique can improve the parameter estimation performance and ensure the invertibility of $\Omega_j(\hat{\phi}), \ k = 1, 2$ by reducing the dimension of UC-ADAF vectors. Nevertheless, the CFOs of the coupled two users are still jointly estimated by (32).

C. JOINT DATA DETECTION FOR COUPLED USERS

After joint CFO estimation, the equivalent channel response vector for the $q$th beamforming branch can be obtained as

$$\hat{h}_q(\kappa_q) = [\hat{h}_{k_1}(\kappa_q), \hat{h}_{k_2}(\kappa_q)^T, \ldots, \hat{h}_{k_G}(\kappa_q)^T]$$

$$= \mathrm{E}^T(\hat{\phi}) Y U_{UC}^H \hat{x}_i(q), \quad q = 1, 2, \ldots, Q_{UC},$$

where $\hat{h}_{k_1}(\kappa_q)$ is the estimated channel response for the $k_q$th user. Performing beamforming with $\hat{\Omega}_{q}(\kappa_q) = U_{q}^H \hat{\Omega}_{q}(\kappa_q)$ on the received signal of the $q$th block $Y_i(q)$, we can approximate $y_i(q) = Y_i(q)^T \hat{\Omega}_q(y_i(q))$ as

$$y_i(q) \approx \sum_{g=1}^{G} \eta_q(k_q) E(\hat{\phi}_{k_q}) F H^H \mathrm{diag}(x_i^{(k_q)}) \sqrt{N} F_j \hat{h}_q^{(k_q)} + n_i(q),$$

$$= \sum_{g=1}^{G} \eta_q(k_q) E(\hat{\phi}_{k_q}) \hat{H}_q^{(k_q)} F H^H x_i^{(k_q)} + n_i(q).$$

where $\hat{H}_q^{(k_q)} = \sqrt{N} F H^H \mathrm{diag}(F_j \hat{h}_q^{(k_q)}) F$ and $q = 1, 2, \ldots, Q_{UC}$ stands for the circular convolution matrix corresponding to $\hat{h}_q^{(k_q)}$, and $n_i(q) = (\sum_{k \neq k_1, k_2, \ldots, k_G} \eta_q(k) \times E(\hat{\phi}_k) B_k^H H_k + N_i) \hat{\Omega}_q^{(k_q)}$.

Define $Y_i = [y_i^{(1)}, y_i^{(2)}, \ldots, y_i^{(Q_{UC})}]$. Then, there holds

$$\mathrm{vec}(Y_i) \approx \sum_{g=1}^{G} \eta_q(k_q) \left( I_{Q_{UC}} \otimes E(\hat{\phi}_{k_q}) \hat{H}_q^{(k_q)} F H^H x_i^{(k_q)} + N_i \right),$$

$$= \hat{H}_q \left( \eta_i \otimes F H^H \right) x_i + N_i,$$  \hspace{1cm} (35)

where $x_i = [x_i^{(k_1)} T, x_i^{(k_2)} T, \ldots, x_i^{(k_G)} T]^T$ is the transmitted symbols in the $q$th block of the coupled $G$ users, and $\eta_i = \mathrm{diag}(\eta_i(k_1), \eta_i(k_2), \ldots, \eta_i(k_G))$.  \hspace{1cm} (36)
The uniform channel PDP is assumed for simulation, i.e., coupled $G$ is taken as to evaluate the impact of various factors on it and analyze the effectiveness of the proposed approach. In this section, we demonstrate the effectiveness of the proposed UC-ADAF, ADAF, JSFA and UG. Unlike the UC-ADAF, the joint CFO estimation and data detection is performed with multidimensional search as in (16) and in the computationally efficient manner as in (28), with corresponding lower MSE bound.

**IV. SIMULATION RESULTS**

In this section, we demonstrate the effectiveness of the proposed UC-ADAF based frequency synchronization scheme, evaluate the impact of various factors on it and analyze the computational complexities. The total number of subcarriers is taken as $N = 64$ and the channel length is set as $L = 10$. The uniform channel PDP is assumed for simulation, i.e., $\sigma^2 = \frac{1}{L}$, $l = 1,2, \ldots , L$. The training symbols are randomly drawn from QPSK constellations, while data transmission employs 16-QAM. The average power of either pilot or data symbols is normalized. The normalized CFO is randomly generated from $-0.2$ to $0.2$. The AS is fixed as $\theta_{\text{AS}} = 10^\circ$, and we assume that both average AoAs and AS are perfectly known at BS. Unless otherwise stated, the number of receive antennas is set as $M = 64$. The mean squared error (MSE) of the normalized CFO and symbol error rate (SER) are adopted as the performance metrics. For comparison, we also include the existing ADAF approach [29], [30] designed for users with slightly overlapping AoA region, JSFA scheme [26] which is only applicable to mutually non-overlapping users, and UG scheme [28] which groups all the users with contiguously intersecting AoA regions to perform joint CFO estimation and data detection.

First, we consider $K = 9$ users whose average AoAs are configured as $\{38^\circ, 50^\circ, 70^\circ, 75^\circ, 95^\circ, 97^\circ, 117^\circ, 122^\circ, 142^\circ\}$. In accordance with the relative proximity of AoA regions, the first 8 users can be paired into 4 couples, and the coupled users for joint estimation and detection are underlined. Note that the partitioned UC-ACM (30) is used for the UC-ADAF. The joint CFO estimation and data detection is performed for each couple of two users with the UC-ADAF, while the isolated 9th user employs the ADAF to estimate its own CFO and recover the transmitted symbols.

**FIGURE 2:** CFO estimation MSE performance comparison of UC-ADAF, when the joint CFO estimation is performed with multidimensional search as in (16) and in the computationally efficient manner as in (28), with corresponding lower MSE bound.
floor, which is within expectation, since it cannot combat the severe MUI caused by highly or slightly overlapping IUs.

3) The ADAF suffers from obvious MSE and SER performance floor at $M = 64$. In fact, the interference incurred by highly overlapping IUs is too strong such that even the carefully designed ADAF vectors are incapable of attenuating the MUI. Hence, the approximate single-user transmission model cannot be obtained for users with highly overlapping AoA regions, resulting in significant performance deterioration.

4) The proposed UC-ADAF approach is noticeably superior over the other existing methods in the considered scenario. By coupling together the users with highly overlapping AoA regions, the UC-ADAF can not only effectively eliminate the MUI from non-overlapping and slightly overlapping IUs, but also circumvent the extremely severe interference caused by highly overlapping IUs. As a result, it outperforms the ADAF by 3dB in terms of SER performance at $M = 128$, and avoids the SER floor which the ADAF exhibits at $M = 64$.

Next, we will investigate the impact of number of coupled users $G$, overlapping degree $\theta_{OL}$ and total number of mutually overlapping users $K$ on the MSE and SER performance of our proposed UC-ADAF algorithm. Note that the overlapping degree $\theta_{OL}$ refers to the degree of the overlapped AoA region of two adjacent users.

For assessing the impact of number of coupled users $G$, we consider $G = K = 2, 3, 4$ with average AoAs distributed as $\{85°, 95°\}$, $\{80°, 90°, 100°\}$ and $\{75°, 85°, 95°, 105°\}$, respectively. Note that the overlapping degree is $\theta_{OL} = 10°$ for all the coupled users. Fig. 4 depicts the MSE and SER performance of UC-ADAF with different numbers of coupled users. The results of ADAF are also included for comparison. Clearly, the performance of ADAF deteriorates sharply with the increasing number of coupled users, while our UC-ADAF can still work even when $G = 4$ users are coupled together for joint CFO estimation and data detection. Moreover, compared to $G = 2$, the performance degradation of UC-ADAF for $G = 4$ is expected and within acceptable range.

In order to evaluate the robustness of the proposed UC-ADAF algorithm and other methods against overlapping degree $\theta_{OL}$, we consider $K = 5$ users with average AoAs distributed as $\{50° + \theta_{OL}, 70°, 90°, 110° - \theta_{OL}, 130° - 2\theta_{OL}\}$. We gauge the MSE and SER performance of UC-ADAF, ADAF, JSFA and UG under different overlapping degrees $\theta_{OL} = \{4°, 6°, 8°, 10°, 12°, 14°, 16°\}$ in Fig. 5. Note that
the MSEs and SERs are obtained at SNR = 14 dB and SNR = 8 dB, respectively.

FIGURE 5: (a) MSE and (b) SER performance comparison of UC-ADAF, ADAF, JSFA and UG against overlapping degrees $\theta_{OL} = \{4^\circ, 6^\circ, 8^\circ, 10^\circ, 12^\circ, 14^\circ, 16^\circ\}$, with $K = 5$ users whose average AoAs are distributed as $\{50^\circ + \theta_{OL}, 70^\circ, 90^\circ, 110^\circ - \theta_{OL}, 130^\circ - 2\theta_{OL}\}$.

The following observations can be drawn from Fig. 5:
1) The proposed UC-ADAF scheme is quite insensitive to the overlapping degrees, which confirms its effectiveness.
2) The ADAF can achieve comparable performance for slight overlapping among users, e.g., $\theta_{OL} = 4^\circ$. However, its performance degrades drastically with the increasing overlapping degree, whether in terms of MSE or SER.
3) The JSFA scheme cannot be applied to scenarios with AoA overlapping among users. In fact, it only depends on the quasi orthogonality among steering vectors to eliminate MUI and is not applicable unless there is sufficient angular separation among users.
4) The UG also relies on the quasi orthogonality among steering vectors to eliminate the interference from intra-group users. Thus, it must group all the users with contiguously overlapping AoA regions to perform joint CFO estimation and data detection. In the case of $K = 5$ users, $K(L+1) = 55$ parameters must be jointly estimated based on $N = 64$ pilot symbols, resulting in its performance exacerbation.

Then, the impact of total number of mutually overlapping users $K$ on the performance of different methods will be assessed. To this end, we consider the distributions of average AoAs shown in Table 1. The MSE and SER performances against total numbers of mutually overlapping users $K = 2, 3, 4, 5, 6, 7$ are given in Fig. 6. Similar to Fig. 5, the MSEs and SERs are obtained at SNR = 14 dB and SNR = 8 dB, respectively.

FIGURE 6: (a) MSE and (b) SER performance of UC-ADAF, ADAF, JSFA and UG against numbers of mutually overlapping users $K = 2, 3, 4, 5, 6, 7$, with $\theta_{OL} = 10^\circ$ for users within each couple and $M = 64$.

From Fig. 6, it can be observed that:
1) The proposed UC-ADAF scheme is quite insensitive to the total number of mutually overlapping users, except for $K = 2$. In fact, this accords with the results in Fig. 4.

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TABLE 1: Distributions of average AoAs for different numbers of users $K = 2, 3, 4, 5, 6, 7$

<table>
<thead>
<tr>
<th>Number of users</th>
<th>Distribution of average AoAs ($\theta_{OL} = 10^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K = 2$</td>
<td>${85^\circ, 95^\circ}$</td>
</tr>
<tr>
<td>$K = 3$</td>
<td>${80^\circ, 90^\circ, 100^\circ}$</td>
</tr>
<tr>
<td>$K = 4$</td>
<td>${70^\circ, 90^\circ, 100^\circ, 110^\circ}$</td>
</tr>
<tr>
<td>$K = 5$</td>
<td>${60^\circ, 70^\circ, 90^\circ, 100^\circ, 110^\circ}$</td>
</tr>
<tr>
<td>$K = 6$</td>
<td>${50^\circ, 60^\circ, 80^\circ, 90^\circ, 100^\circ, 120^\circ, 130^\circ}$</td>
</tr>
<tr>
<td>$K = 7$</td>
<td>${50^\circ, 60^\circ, 80^\circ, 90^\circ, 100^\circ, 120^\circ, 130^\circ}$</td>
</tr>
</tbody>
</table>

2) The ADAF scheme is quite insensitive to the total number of mutually overlapping users, except for $K = 2$.
3) The JSFA scheme is not applicable to scenarios with AoA overlapping among users.
4) The UG scheme is also quite insensitive to the total number of mutually overlapping users, except for $K = 2$.
There is only two mutually overlapping users for \( K = 2 \) yet a couple of three users with severer interference must be coped with for \( K > 2 \), which accounts for the fact that the SER performance of \( K = 2 \) users is superior over that acquired with \( K > 2 \) users.

2) As expected, the ADAF fails to work properly with large overlapping degree \( \theta_{OL} = 10^\circ \) among the coupled users. This is consistent with the results in Fig. 5, which reveals that the ADAF tolerates at most an overlapping degree of \( \theta_{OL} = 4^\circ \).

3) The JSFA scheme is unsuitable to deal with the mutually overlapping users, since it requires sufficient angular separation among users for MUI cancellation.

4) The UG slightly outperforms our proposed UC-ADAF scheme in the case of \( K = 2, 3, 4 \) users. However, increasing the total number of mutually overlapping users will degrade its performance. For UG, all the users without enough angular separation must be grouped together to perform joint CFO estimation and data detection. As a result, the number of parameters to be jointly estimated linearly increases with the number of mutually overlapping users \( K \) in the considered scenario, which will gradually deteriorate the performance of UG. Moreover, underdetermination problem occurs once the estimated parameters of UG exceeds the number of pilot symbols, which explains its failure to achieve CFO estimation and data detection at \( K \geq 5 \).

Finally, we evaluate the computational complexities of UC-ADAF, ADAF and UG, in terms of complex multiplications. Both CFO estimation and data detection processes are considered. The detailed calculations of the complexities will be omitted here and the complexities of different methods are summarized in Table 2. Note that the given complexity of UC-ADAF is only for a couple of \( G \) users. There may be several couples, each with different numbers of coupled users. Thus, the overall complexity for the total \( K \) users should be the summation of all couples. Similarly, the given complexity of UG is for a group of \( G_{UG} \) users, which share \( \bar{Q}_{UG} \) beamforming branches. Besides, \( \eta \) is the number of iterations for ADAF.

### TABLE 2: Computational complexities of CFO estimation and data detection for UC-ADAF, ADAF and UG

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>CFO estimation</th>
<th>Data detection</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC-ADAF</td>
<td>( O \left( \frac{N Q_{UC} (2 M + N - G L + 4 Q_{UC}) + N^3 + G N^2 (6 N + Q_{UC} + 2 L) + G (N - L) Q_{UC}^2 + G^2 N^2 (N + Q_{UC} - 4 L - G L)}{N Q_{UC} + 2 G L M (N + Q_{UC}) + 2 G^2 N^2 Q_{UC} + G^3 N^3} \right) )</td>
<td>( O \left( \frac{Q_{UC} (M + 2 Q_{UC}) + G L M (N + Q_{UC}) + G N^2 N Q_{UC} + 2 G^2 N^3 Q_{UC} + G^3 N^3}{Q_{UC} (N + N + 2 G L)} \right) )</td>
</tr>
<tr>
<td>ADAF</td>
<td>( O \left( \frac{K^2 L (N + L)^2 + N^2 (M + 3 N + 3 G L) + N^2 (M + 3 N^3) + Q_{OL} (K + N + 2 G L)}{K^2 Q_{OL} (N + N + 2 G L)} \right) )</td>
<td>( O \left( \frac{K N Q_{OL} N_{OL} (G L N + 1)}{K Q_{OL} (N + N + 2 G L)} \right) )</td>
</tr>
<tr>
<td>UG</td>
<td>( O \left( \frac{(G_{UG} L)^2 (2 N + G_{UG} L) + 2 G_{UG} N^2 N Q_{UC} + G_{UG} N^2 (Q_{UC} + 2 G_{UG} Q_{UG} + G_{UG}^2)}{(G_{UG} L)^2 (N + G_{UG} L) + (G_{UG} L)^3 + N Q_{UG} (M + N) + G_{UG} N^2 (4 N - 2 G_{UG} L + 3 L) + N^3} \right) )</td>
<td>( O \left( \frac{(G_{UG} L)^2 (2 N + G_{UG} L) + 2 G_{UG} N^2 N Q_{UC} + G_{UG} N^2 (Q_{UC} + 2 G_{UG} Q_{UG} + G_{UG}^2)}{(G_{UG} L)^2 (N + G_{UG} L) + (G_{UG} L)^3 + N Q_{UG} (M + N) + G_{UG} N^2 (4 N - 2 G_{UG} L + 3 L) + N^3} \right) )</td>
</tr>
</tbody>
</table>

### TABLE 3: Comparison of computational complexities between UC-ADAF, ADAF and UG

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>CFO estimation</th>
<th>Data detection</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC-ADAF</td>
<td>( 1.52 \times 10^7 )</td>
<td>( 8.26 \times 10^7 )</td>
<td>( 9.78 \times 10^7 )</td>
</tr>
<tr>
<td>ADAF</td>
<td>( 3.84 \times 10^6 )</td>
<td>( 5.94 \times 10^4 )</td>
<td>( 3.90 \times 10^4 )</td>
</tr>
<tr>
<td>UG</td>
<td>( 6.85 \times 10^6 )</td>
<td>( 3.94 \times 10^4 )</td>
<td>( 4.01 \times 10^4 )</td>
</tr>
</tbody>
</table>

For further acquiring an intuition comparison of the complexities, we consider the scenario of \( N = 64, M = 64, K = 5, L = 10, N_{OL} = 5, \eta = 5, \theta_{OL} = 10^\circ \). The average AoAs of \( K = 5 \) users are distributed as \( \{60^\circ, 70^\circ, 90^\circ, 100^\circ, 110^\circ\} \), which corresponds to the scenario considered in Fig. 5 by taking \( \theta_{OL} = 10^\circ \). For ADAF, the average dimension of ADAF vectors \( Q_{OL} \) can be approximated by its expectation \( \bar{Q}_{OL} = \frac{2 \theta_{OL}}{\pi} M \approx 7 \), while \( Q_{UG} \) for UC-ADAF and \( Q_{UG} \) for UG depend on both the average AoA distributions and user-grouping strategies. The required complexities of UC-ADAF, ADAF and UG are provided in Table 3. It can be observed that ADAF is the most efficient method and UG suffers from the highest computational burden, while the complexity of the proposed UC-ADAF scheme falls in between, which provides a tradeoff between complexity and performance.

### V. CONCLUSIONS

In this paper, we developed a UC-ADAF based frequency synchronization scheme for multiuser OFDM uplink in the presence of extremely severe MUI among users whose AoA regions highly overlap. The users with highly overlapping AoA region are coupled together. For the coupled users, we introduce UC-ACM and UC-ADAF vectors to obtain the adaptively optimized beamformers which can effectively suppress the interference from non-overlapping IUs and slightly overlapping IUs, and circumvent the problem of attenuating the extremely severe MUI by performing joint CFO estimation and data detection. As for each of the uncoupled users, the previous ADAF scheme can be applied. Numerical results corroborated the effectiveness of the proposed UC-ADAF, as well as its superiority over existing competitors ADAF, JSFA and UG. The impact of various factors on the performance of UC-ADAF was also investigated, which reveals its robustness against overlapping degree and total number of mutually overlapping users.

### REFERENCES


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