DEA Evaluation Method Based on Interval Intuitionistic Bayesian Network and Its Application in Enterprise Logistics

Mengdi Xu1, Shousheng Liu1, Zeshui Xu2, (Fellow, IEEE), Wei Zhou3

1 Basic Department, Army Engineering University of PLA, Nanjing, Jiangsu 211101, China
2 Business School, State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu, Sichuan 610064, China
3 School of Finance, Yunnan University of Finance and Economics, Kunming, Yunnan 650221, China

Corresponding author: Zeshui Xu (e-mail: xuzeshui@263.net).

ABSTRACT Data envelopment analysis (DEA) is a model for evaluating the effectiveness of relative effectiveness of decision-making units with multiple input and output data based on non-parametric modeling using mathematical programming (including linear programming, multi-parameter programming, stochastic programming, etc.). Due to the complexity of real life, sometimes it is hard to get the exact value of the input and output data directly. The fuzzy DEA (FDEA) proposed to solve this problem well and is widely used in practice. However, the data of FDEA have certain subjectivity, and in addition, some indicators cannot be quantified intuitively. Owing to the complexity of society, there are often some causal relationships in the indicator system. As a method of combining uncertainty and graph theory for uncertainty reasoning, Bayesian network (BN) can effectively deal with the causal chain problem existing in the index and discover the potential relationship between data. The BN is often used to process accurate numerical information and does not handle uncertain information with ambiguity favorably. In order to solve the above issue, the interval-valued intuitionistic fuzzy number (IVIFN) is introduced into the BN to construct the interval-valued intuitionistic fuzzy BN (IVIFBN). Then based on the index data obtained by the IVIFBN, the crossover efficiency is proposed, and the super efficiency interval FDEA (SEIFDEA) model is constructed. According to the different optimisms of the decision makers, the Hurwitz decision criterion is introduced for sorting. In addition, the model is applied to the performance evaluation system of logistics enterprises. Compared with the traditional DEA model, the validity and superiority of the model in fuzzy environment are verified.

INDEX TERMS Bayesian network, data envelopment analysis, interval-valued intuitionistic fuzzy number, logistics enterprises

I. INTRODUCTION

With the progress of the times and the continuous development of society, logistics has continuously entered our sights and gradually affected our lives. Modern logistics is the product of economic globalization and an important service industry that promotes economic globalization and has become an important support for social and economic development. At present, scholars have made many analyses and researches on the performance evaluation of logistics enterprises. Many researchers believe that data envelopment analysis (DEA) is an effective method to quantitatively analyze the operation of the logistics industry. The DEA is an operational research method that evaluates the efficiency of decision making units through multiple inputs and multiple outputs. Charnes et al. [1] proposed the first classical model, the CCR model to make decisions. However, according to the research findings, there are certain defects in the practical application of the CCR model directly. In this regard, scholars continue to modify and improve the model, and successively propose some new DEA models. The most famous and widely used ones are the super efficiency model (SEDEA) and the cross efficiency model (CEDEA). The SEDEA model evaluates and ranks all decision making units (DMUs) through self-assessments, solving the shortcomings of using traditional DEA methods. The basic idea of the cross-efficiency evaluation model is to use the optimal weight of the DMU to calculate the efficiency values of other DMUs, and obtain the evaluation value of the cross-efficiency. Then a decision support ranking method [2] based on crossover efficiency and analytic hierarchy process is proposed. Sometimes, it is necessary to consider the level of satisfaction with the decision makers. So Dai et al. [3] defined the utility function of satisfaction and maximize...
overall satisfaction to optimize results. However, due to the lack of information in the real world and the limitations of subjective cognition, the information we obtain is often ambiguous. To solve this problem, Triantis and Girod [4] extended the fuzzy set to the DEA model and proposed the fuzzy DEA (FDEA). Therefore, decision makers or experts can more accurately evaluate the information. Based on this advantage, more and more attention has been paid to the FDEA which is widely used in various fields. A data envelopment of fuzzy expectation values [5] is proposed in a fuzzy environment. The DMU is sorted by measuring the optimistic efficiency and pessimistic efficiency of the decision unit by weighting the input and output values. Then Ignatius et al. [6] improved the FDEA. When the fuzzy numbers have asymmetry, a new fuzzy framework is proposed and evaluated at different levels of certainty. Taking twenty-three of the European Union member states about their energy efficiencies as an example, the applicability and effectiveness of the model for asymmetric fuzzy numbers are illustrated. Aiming at the defects of the traditional FDEA model, Agarwal [7] proposed a FDEA model based on α-cut method to deal with the efficiency metric and ordering problem of given fuzzy input and output data. Finally, a numerical example of the fuzzy DEA model is given, which has stronger application value. However, one disadvantage of the α-cut method is that it does not include the uncertainty of all information. So Zerafat Angiz et al. [8] introduced a linear programming model, and poses “local α-level” to obtain multi-objective linear programming measurements for efficiency research under uncertainty. Since the FDEA model cannot evaluate a fuzzy sample DMU (SDMU), then a generalized FDEA model capable of evaluating SDMU [9] was proposed and a new vector-based evaluation method was adopted.

Due to the good performance of the DEA method, it has been widely used in various industries. However, in practical applications, we often have to change and adjust the model according to the actual situation. Therefore, the combination of different models can solve practical problems more effectively. We found that the combination of FDEA and AHP is the most common method. FDEA and AHP combine to deal with plant equipment layout problems [10]. FDEA is combined with fuzzy analytic hierarchy process (FAHP) for the performance evaluation ranking problem of decision-making units in fuzzy environments [11]. On the other hand, due to the rise of artificial intelligence, some scholars have combined FDEA with various networks, which opens up new ideas for traditional FDEA models and provides a more diverse model and solution to problems. FDEA is combined with AHP for evaluation, and then a risk assessment model is created through Bayesian belief network for product development [12]. The shipping company is resource-configured through a centralized network data envelopment method [13]. By constructing a Dynamic Network Data Envelopment Analysis (DNDEA) model to evaluate the relative efficiency of DMUs with network structure over several time periods, better reflecting the uncertainty in actual evaluation problems [14].

The results obtained by the FDEA model can better reflect the preferences of decision makers. However, sometimes the decision makers are hesitant about the membership and non-affiliation of the fuzzy numbers, so we will use the interval to divide them, that is, the form of the IVIFN as the original data, which can more accurately reflect the preferences of decision makers. In addition, the phenomenon that the attribute value is missing or null and the qualitative indicator cannot be quantified often occurs or even is unavoidable in real life. Due to the complexity of society, there are often some causal relationships in the indicator system. As a process analysis and diagnostic reasoning method, BN can effectively deal with the causal chain problem existing in the above indicators. For example, “We obtained experimental data from A to get 1/3 of the probability of getting lung cancer. Both smoking and drinking lead to lung cancer. So, can we get the probability of lung cancer by smoking and drinking?” At this point, the appearance of the BN perfectly solved this problem. The BN combines known historical information (a priori probability) with a causal network to efficiently handle incomplete information and other problems. The good performance of the BN in dealing with uncertainties makes it widely used in environmental detection [15], fault diagnosis [16], and risk assessment [17]. According to actual needs, some events change with time, so the time series is added to the Bayesian network to construct dynamic BN (DBN), such as the risk assessment and grade prediction of traffic accidents [18], the dynamic and comprehensive quantitative risk analysis (DCQRA) of natural gas station accident scenarios and risk modeling [19]. Many data are uncertain due to various factors such as man-made and environment. So John et al. [20] used BN for modeling, and then used FAHP to evaluate the effects between variables. Besides, some scholars directly use the fuzzy Bayesian network (FBN) to conduct the safety risk analysis on the uncertainty of tunnel construction [21], the safety evaluation on oil and gas pipelines [22], and the risk assessment on industrial production [23]. By combining the above practical problems in the DEA model and the excellent characteristics of BN, we propose a super-efficient cross-DEA (SECDEA) model based on the BN in the interval intutionistic fuzzy environment. Firstly, the probability of the interval intutionistic fuzzy event is defined and applied to the BN. Some properties of the interval intutionistic fuzzy BN are obtained. It is found that the interval intutionistic fuzzy event probability is transitive in the BN. Then we transform the incomplete information collected and convert it into interval direct fuzzy probability through BN as the original data of DEA evaluation method. Finally, the super-efficiency interval
DEA model is constructed. Based on the introduction to cross-efficiency, the fuzzy SECDEA (FSECDEA) model is established. According to the different optimism of decision makers, the Hurwitz decision criterion is introduced to evaluate the decision-making unit. The framework of the paper is demonstrated in the Figure 1. According to the model proposed in this paper, the case study of enterprise logistics is carried out. Through the collected imperfect data and the development status of the enterprise at the present stage, the model structure is applied to evaluate the future development trend of the enterprise and make decision analysis for investors.

II. Preliminaries

A. The intuitionistic fuzzy set

In this part, we will first review some basic concepts about IFS, IVIFS and BN related theory which will be used in the next parts.

Zadeh [24] originally introduced the concept of fuzzy sets. Later, Atanassov [25] came up with the intuitionistic fuzzy set (IFS) to consider more about the hesitancy of the decision maker. Then, Xu and Yager [26] described the intuitionistic fuzzy value (IFV) to indicate the fuzzy information. The concept of IFS and IFV can be defined. An IFS A in X is given by $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$, where $\mu_A(x)$ and $\nu_A(x)$ represent the degrees of membership and non-membership of A respectively. For each $x \in X$, $\mu_A(x) \in [0, 1]$, $\nu_A(x) \in [0, 1]$ and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

For convenience, Xu [3] expressed the IFV as $\alpha = (\mu_v, \nu_v)$, where $\mu_v \in [0, 1]$, $\nu_v \in [0, 1]$ and $\mu_v + \nu_v \leq 1$.

However, sometimes, the decision makers are hesitant about the choice of fuzzy number membership and non-affiliation, or the estimation is not accurate. Atanassov [25] introduced interval-valued intuitionistic fuzzy set (IVIFS) in order to express more flexible information. The mathematical expression of an IVIFS is given by $N = \{(x, \mu_N(x), \nu_N(x)) | x \in X\}$, where $\mu_N(x) \in [0, 1]$, $\nu_N(x) \in [0, 1]$ and $\sup \mu_N(x) + \sup \nu_N(x) \leq 1$, $x \in X$.

Ren et al. [27] next transformed the IVIFS to the simplified interval-valued intuitionistic fuzzy set (SIVIFS) as $N = \{(x, \mu^+ x(x), \nu^+ x(x)) | x \in X\}$, where $\alpha$ and $\nu$ represent two IFVs. Besides, they satisfy the condition that $\alpha \cup \nu = \alpha (or \alpha \cap \nu = \nu)$. The relationship between them can be shown in Figure 2 and the operations of SIVIFS can be defined as follows:

$$\begin{align*}
\mu_A(x) &\in [0, 1] \\
\nu_A(x) &\in [0, 1] \\
0 &\leq \mu_A(x) + \nu_A(x) \leq 1
\end{align*}$$
Guided by the definition of the score function and the accuracy function of the IVIFN [28], Ren et al. [27] presented the two functions of SIVIFN as follows:

**Definition 2.2.** [18] Let \( a_i = \{ \alpha_i, \beta_i \} \) and \( a_j = \{ \alpha_j, \beta_j \} \) be two SIVIFSs. Then we call the score of \( a_i (i = 1, 2) \) as
\[
\text{s}(a_i) = \frac{1}{2}[s(\alpha_i) + s(\beta_i)]
\]
and the accuracy function of
\[
a_i (i = 1, 2) \quad \text{as} \quad h(a_i) = \frac{1}{2}[h(\alpha_i) + h(\beta_i)]
\]
respectively, where \( s(\alpha_i) \in [-1, 1], h(\alpha_i) \in [0, 1] \). If \( s(\alpha_i) < s(\alpha_j) \), then \( a_i < a_j \); If \( s(\alpha_i) = s(\alpha_j) \), then \( a_i = a_j \) for \( h(\alpha_i) = h(\alpha_j) \) and \( a_i < a_j \) for \( h(\alpha_i) < h(\alpha_j) \).

B. The concept of Bayesian Network

The Bayesian network (BN) is a tool that helps people apply probabilistic statistics to complex areas for uncertainty reasoning and data analysis. It is also called belief network or directed acyclic graph (DAG) which is a method based on the combination of probability theory and graph theory. The DAG represents the probability dependence of variables, which provides a method of reasoning for exploiting hidden information.

For example, suppose that the node \( E \) directly affects the node \( H \), that is \( E \rightarrow H \), then we establish a directed arc \((E, H)\) from the node \( E \) to the node \( H \) with an arrow pointing from \( E \) to \( H \), and the weight (i.e., the strength of the connection) is represented by the conditional probability \( P(H|E) \) as shown below:

\[
\begin{align*}
\text{Definition 2.3.} & \quad \text{[41]} \quad \text{Let} \quad G = (N, E) \text{ denote a DAG, where} \quad N \text{ denotes a set of all nodes in the graph and} \quad E \text{ denotes a set of all directed connected line segments. Let} \quad X = (X_1, X_2, \ldots, X_n), \quad i \in N \text{ be the random vector represented by a certain node} \quad i \text{ in the DAG. The joint probability of the node} \quad X \text{ can be expressed as:} \\
& \quad P(X_1, \ldots, X_n) = \prod_{i \in N} P(X_i | X_{pa(i)}) \quad (1)
\end{align*}
\]

So \( X \) is the Bayesian network relative to \( G \), where \( pa(i) \) is the “parent” of the node \( i \), or whose “children” is \( i \).

Furthermore, for any random variable, the joint probability can be derived by multiplying the respective local conditional probability distributions.

That is,
\[
P(X_1, \ldots, X_n) = P(X_1)P(X_2 | X_1) \ldots P(X_n | X_1, X_{n-1}) \quad (2)
\]

**Definition 2.4.** [29] If \( A \) and \( B \) are two IVIFS-events, then the conditional probability can be computed as \( P(A|B) \cdot P(B) = P(A \cdot B) \), where “\( \cdot \)” represents the multiplication on closed intervals and “\( \circ \)” represents the operation of IVIFSs respectively. Besides, suppose that \( A \) and \( B \) are independent \( (P(B > 0)) \), then \( P(A|B) = P(A) \).

BN has intuitive expressive ability and powerful reasoning ability. The reasoning process assumes that the conditions are independent which are based on probability statistics. When calculating the probability information, only the variable information related to the variable is considered, which reduces the difficulty of solving the uncertainty information expression and reasoning method, and provides a solution to the uncertainty information reasoning. The conditional probability table of BN nodes mainly comes from expert guidance, statistical literature and prior probability. Due to the conditional independence of BN, in the process of using it for uncertainty information reasoning, only the probability information of the relevant nodes needs to be calculated. Therefore, BN has a good application in the reasoning of uncertainty information. Perkusich et al. [30] constructed a BN model that uses top-down methods and reasoning to define the key metrics needed to build models and their relationships. The BN inference is verified by 10 simulation scenarios, which improves the accuracy of reasoning. Herring et al. [31] proposed a regional risk assessment using the Bayesian Network Relative Risk Model (BN-RRM), providing an adaptive template for the decision makers interested in managing non-native species (NIS) in other coastal areas and large waters. A fuzzy Bayesian network (FBN) method [32] is brought out to model the causal relationship between risk factors that may occur in offshore operations. The flexibility of this approach makes the risk and safety analysis of offshore engineering systems more practical and easy in many assessment environments.

III. The decision-making model

A. Interval-valued intuitionistic fuzzy probability

How to relate fuzzy set theory to probability set theory has been a significant problem for a long time. Zadeh [24] was the first one to calculate the probability of each fuzzy event and introduced such basic properties. Atanassov [25] showed the concept of Intuitionistic Fuzzy Event (IF-event). Later, Grzegorzewski [29] generalized the basic properties of probability measures of IF-events.

Similar to the IF-events, we can also integrate the notion of probability into Interval-valued Intuitionistic Fuzzy Event (IVIF-event). Then the probability of an IVIF-event can be defined as follows:

**Definition 3.1.** Let \( X \) be an IVIF-event in \( X \), where \( X \) is a set in \( \mathbb{R}^n \) and \( A = \{ x, [\mu^L_i(x), \mu^U_i(x)], [\nu^L_i(x), \nu^U_i(x)] \} \). Besides, let \( P \) be a probability measure. Then the probability of the IVIF-event \( A \) is well-defined as
\[
P(A) = [\tilde{P}_{\min}(A), \tilde{P}_{\max}(A)]
\]
where \( \tilde{P}_{\min}(A) \) and
\( \bar{p}_{\max}(A) \) represent the closed interval, specifically expressed as follows:

\[
\bar{p}_{\min}(A) = \left[ \int_X \mu^+_{x}(x) \, dP, \int_X \mu^+_{x}(x) \, dP \right]
\]

\[
\bar{p}_{\max}(A) = \left[ 1 - \int_X v^-_{1}(x) \, dP, 1 - \int_X v^-_{1}(x) \, dP \right]
\]

Consequently, the interval \([\bar{p}_{\min}(A), \bar{p}_{\max}(A)]\) can express the probability of the IVIF-event. For every \( A \in \text{IVIF-event in } X \), we will use \( \bar{p}_{\min}(A) \) and \( \bar{p}_{\max}(A) \) to represent the lowest and highest possible probabilities of the IVIF-event \( A \) respectively.

**Theorem 3.1.** Suppose that \( X \) is a set in \( \mathbb{R}^n \). If there are two IVIF-events \( A \) and \( B \) in \( X \), in other words,
\[
A = \left\{ (x, \tilde{\mu}_A(x), \tilde{\nu}_A(x)) \mid x \in X \right\}
\]
and
\[
B = \left\{ (x, \tilde{\mu}_B(x), \tilde{\nu}_B(x)) \mid x \in X \right\}, \quad A \text{ and } B \text{ are independent if and only if } P(A \cdot B) = P(A) \cdot P(B), \quad \text{where “**” denotes the multiplication operation of the IVIFSs and “* ◦ ” denotes the arithmetic multiplication of closed interval.}
\]

**Proof:** First, as we all know from Definition 2 that
\[
A \cdot B = \left\{ (x, \tilde{\mu}_{A \cdot B}(x), \tilde{\nu}_{A \cdot B}(x)) \mid x \in X \right\}
\]
Where
\[
\tilde{\mu}_{A \cdot B}(x) = \left[ \mu^-_{A}(x) \mu^-_{B}(x), \mu^+_{A}(x) \mu^+_{B}(x) \right]
\]
\[
\tilde{\nu}_{A \cdot B}(x) = \left[ v^-_{1}(x) + v^-_{1}(x) - v^-_{1}(x) v^-_{1}(x), v^-_{1}(x) + v^-_{1}(x) - v^-_{1}(x) v^-_{1}(x), v^+_{1}(x) + v^+_{1}(x) - v^+_{1}(x) v^+_{1}(x) \right]
\]
It is well known that the multiplication for closed interval in the set of real numbers can be represented as:
\[
[x, y] \cdot [z, w] = [xz, yw] \quad (if \ x, y, z, w \geq 0)
\]
\[
[x, y] \cdot [z, w] = \left[ \min \{xz, xw, yz, yw\}, \max \{xz, xw, yz, yw\} \right]
\]
besides, according to Definition 3.1, we can suppose that
\[
P(A) = [\bar{p}_{\min}(A), \bar{p}_{\max}(A)], \quad P(B) = [\bar{p}_{\min}(B), \bar{p}_{\max}(B)]
\]
so
\[
P(A \cdot B) = [\bar{p}_{\min}(A \cdot B), \bar{p}_{\max}(A \cdot B)]
\]
Then
\[
P(A \cdot B) = [\bar{p}_{\min}(A \cdot B), \bar{p}_{\max}(A \cdot B)] = \left[ \int_X \mu^-_{x}(x) \mu^-_{x}(x) \, dP, \int_X \mu^+_{x}(x) \mu^+_{x}(x) \, dP \right]
\]
\[
\left[ 1 - \int_X v^-_{1}(x) v^-_{1}(x) \, dP, 1 - \int_X v^-_{1}(x) v^-_{1}(x) \, dP \right]
\]
\[
\left[ \int_X \mu^-_{x}(x) \mu^-_{x}(x) \, dP, \int_X \mu^+_{x}(x) \mu^+_{x}(x) \, dP \right]
\]
\[
= \left[ \int_X \mu^-_{x}(x) \mu^-_{x}(x) \, dP, \int_X \mu^+_{x}(x) \mu^+_{x}(x) \, dP \right]
\]
\[
\left[ 1 - \int_X v^-_{1}(x) v^-_{1}(x) \, dP, 1 - \int_X v^-_{1}(x) v^-_{1}(x) \, dP \right]
\]
\[
= \left[ \int_X \mu^-_{x}(x) \mu^-_{x}(x) \, dP, \int_X \mu^+_{x}(x) \mu^+_{x}(x) \, dP \right]
\]
\[
\left[ 1 - \int_X v^-_{1}(x) v^-_{1}(x) \, dP, 1 - \int_X v^-_{1}(x) v^-_{1}(x) \, dP \right]
\]
\[
= \left[ \int_X \mu^-_{x}(x) \mu^-_{x}(x) \, dP, \int_X \mu^+_{x}(x) \mu^+_{x}(x) \, dP \right]
\]
\[
\left[ 1 - \int_X v^-_{1}(x) v^-_{1}(x) \, dP, 1 - \int_X v^-_{1}(x) v^-_{1}(x) \, dP \right]
\]
\[
= \left[ \int_X \mu^-_{x}(x) \mu^-_{x}(x) \, dP, \int_X \mu^+_{x}(x) \mu^+_{x}(x) \, dP \right]
\]
\[
\left[ 1 - \int_X v^-_{1}(x) v^-_{1}(x) \, dP, 1 - \int_X v^-_{1}(x) v^-_{1}(x) \, dP \right]
\]
\[
= \left[ \int_X \mu^-_{x}(x) \mu^-_{x}(x) \, dP, \int_X \mu^+_{x}(x) \mu^+_{x}(x) \, dP \right]
\]

**Remark 1.** Similar to the IF-events [29], the conditional probability of \( A \) given \( B \) can be denoted as
\[
P(A \mid B) = P(A \cdot B).
\]
In this article, we do not prove the formulas in detail.

**B. Interval-valued Intuitionistic Fuzzy Bayesian network**

The BN also known as the belief network or the directed acyclic graphical model, is a probabilistic graphical model in which the properties of a set of random variables and their conditional probability distributions (CPDs) are known by means of DAGs. In general, nodes in a directed acyclic graph of a BN represent random variables, which can be observable variables or hidden variables, unknown parameters, and so on. In this paper, the interval-valued intuitionistic numbers consist of the nodes. The arrows connecting the two nodes represent that the two random variables are causal or non-conditional independent. If there is no arrow in the nodes, then the random variables are independent of each other. Suppose that two nodes are connected by a single arrow, indicating one of the nodes is “parents” and the other is “descendants” (or children), the two nodes will generate a conditional probability value.

We have given the independence and conditional theorem of interval number probability forms of interval-valued intuitionistic fuzzy events in Theorem 3.1 and Remark 1. All the nodes represent interval-valued intuitionistic fuzzy information, however, the information obtained through the BN will be an interval number. We sincerely hope that all data results are consistent and continuous, so it is proposed to convert the interval probability to the interval-valued intuitionistic fuzzy number (IVIFN), which makes the data easy to express and more logical [33], an interval number can be converted into a fuzzy number [34], the fuzzy number can be converted into an IVIFN. So we will give the independence formula and the conditional probability formula of the intuitionistic fuzzy probability event in the form of IVIFN.

**Theorem 3.2.** For any two interval-valued intuitionistic fuzzy events (IVIF-events) \( A \) and \( B \), then the IVIF-events can be defined as \( P(A \cdot B) = P(A) \cdot P(B) \), where \( P(A) \) and \( P(B) \) are the forms of IVIFN, and “\( \cdot \)” is the multiplication of IVIFN.

**Proof:** Let \( \tilde{P}(A) = \left[ [a^1, a^2], [a^3, a^4] \right] \) and \( \tilde{P}(B) = \left[ [b^1, b^2], [b^3, b^4] \right] \) be the form of IVIFN, and \( P(A) = \left[ a^1, a^2 \right], \left[ 1 - a^2, 1 - a^1 \right] \) and \( P(B) = \left[ b^1, b^2 \right], \left[ 1 - b^2, 1 - b^1 \right] \). Next
\[
P(A \cdot B) = \tilde{P}(A) \cdot \tilde{P}(B) = \left[ [a^1 b^1, a^1 b^2, a^2 b^1, a^2 b^2], [a^3 b^1, a^3 b^2, a^4 b^1, a^4 b^2] \right]
\]
\[
= \left[ a^1 b^1, a^1 b^2, a^2 b^1, a^2 b^2 \right], \left[ 1 - a^2 b^2, 1 - a^1 b^2 \right]
\]
\[
P(A) \cdot P(B) = \left[ [a^1, a^2], [1 - a^2, 1 - a^1] \right], \left[ [b^1, b^2], [1 - b^2, 1 - b^1] \right]
\]
\[
= \left[ a^1 b^1, a^1 b^2, a^2 b^1, a^2 b^2 \right], \left[ 1 - a^2 b^2, 1 - a^1 b^2 \right]
\]
So \( P(A \cdot B) = P(A) \cdot P(B) \)
Remark 2. Through the conversion method of interval numbers and IVIFNs, the probability calculation method of independent events with interval intuitionistic fuzzy forms has been given above. Similarly, it can be proven that the conditional probability formulas of intuitionistic fuzzy forms are also established. In addition, we will come to the theorem that if the intuitionistic fuzzy probability of all nodes in the Bayesian network is in the form of IVIFNs, then the final calculation result is also the form of IVIFNs.

C. Decision-making model based on data envelopment analysis

1) DATA ENVELOPMENT ANALYSIS

DEA is a new system analysis method founded on the basis of "relative efficiency evaluation" by Charnes et al. [1] in 1978. Subsequently, Li et al. [35] introduced the DEA systematized in 1988. As the research on relevant theories continues to deepen, there are many application papers on DEA methods that have been published one after another, involving production, efficiency, economics, and other related fields. Below we briefly introduce some basic probabilities and theories:

An economic system can be regarded as a unit within a certain range, through the input of certain factors of production to output a certain result of the process, so as to maximize the effectiveness of the results, such units are called Decision Making units (DMU). In general, we are more interested in the same type of DMU, that is, have the same goals, external environment and the input and output indicators. Assuming that there are \( n \) decision units \( DMU_j \) \((j = 1, 2, \ldots, n)\), \( m \) input variables and \( s \) output variables, for a \( DMU_j \), its input and output variables are \( s_{ij} \) \((i = 1, 2, \ldots, m)\) and \( s_{rj} \) \((r = 1, 2, \ldots, s)\). If we want to analyze and evaluate these DMUs, it is very complicated and unreasonable to directly compare these variables. We need to make a certain synthesis of its input and output, as an overall input and output production process. However, there is a certain difference between each input variable and each output variable, and their importance degrees in the overall are not the same. Therefore, we will give appropriate weight to the variable.

2) SUPER DATA ENVELOPMENT ANALYSIS

We know that the traditional DEA methods can only divide decision units into valid and invalid. For the effective units, the efficiency values are all 1, and it is impossible to further distinguish the advantages and disadvantages. Therefore, in order to rank decision units with efficiency values of 1, the super-efficiency data envelopment analysis method (SEDEA) was proposed [36]. The basic principles of DEA and SEDEA are similar, except that the \( k \)-th decision unit is excluded when performing the \( k \)-th decision unit for efficiency evaluation. Then an effective decision-making unit can increase the proportion of input, which is the value of the super-efficiency that may be greater than 1. Since the super-efficiency model does not change the relative efficiency value of invalid \( DMU \), besides, the valid DMU can be distinguished, then by comparing the super efficiency values, we can sort the DMUs. Andersen and Petersen [37] proposed the SEDEA model as follows:

\[
\begin{align*}
\text{max } h_d &= \sum_{j=1}^{m} w^i_j s^i_{dj} \\
\text{s.t.} \sum_{j=1}^{m} w^i_j s^i_{dj} &= 1 \\
\sum_{j=1}^{m} w^i_j s^i_{dj} - \sum_{j=1}^{m} w^j_j s^j_{dj} &\leq 0, j = 1, 2, \ldots, n \quad (j \neq d) \\
w^i_j, w^j_j &\geq 0, i = 1, 2, \ldots, m; r = 1, 2, \ldots, s
\end{align*}
\]

If \( h_d \geq 1 \), then \( DMU_d \) \((d = 1, 2, \ldots, n)\) are said to be valid for DEA. If \( h_d < 1 \), then \( DMU_d \) \((d = 1, 2, \ldots, n)\) are said to be invalid for DEA. In Model 1, the optimal weights of DMU are \( w^i_j^* \) and \( w^j_j^* \) respectively. When we evaluate a DMU and calculate the efficiency value for it, we usually choose the most favorable weight, so there may be a phenomenon in which the weight distribution is unbalanced. So Sexton et al. [38] proposed a cross-efficiency evaluation model (CEDEA), which combined the self-assessment with the decision maker's evaluation to evaluate and sort all DMUs, and solved the defects of using the traditional DEA method. The basic idea of the cross-efficiency evaluation model is to use the optimal weight \( w^i_j^* \) and \( w^j_j^* \) of the DMU to calculate the efficiency values of other DMUs, so as to obtain the evaluation value of cross-efficiency.

In the real life, the occurrence of something is not static, it is often a dynamic process. Therefore, when an event is evaluated for pre-breaking, the selection of multi-stage data can make the result more accurate. In this paper, through the Bayesian network above, we will obtain the interval intuitionistic fuzzy numbers of each attribute at different stages. By comparing the fractional functions, we can find the most likely stage happened, and use the interval intuitionistic fuzzy numbers of this stage as input and output data for the next stage of data envelopment analysis. At this point, the input and output data are interval number forms. For the consistency and connectivity of the article, we denote the data as the fuzzy numbers. That is to say, for an interval number \( P = [a, b] \), according to the connection between the interval number and the fuzzy number [33], we can have the fuzzy form of \( P = (a, 1-b) \). Combined with the characteristics of this paper, we integrate the fuzzy numbers with the interval SEDEA, and in line with the crossover efficiency of the other DMU according to the optimal weight of each DMU obtained by SEDEA, so a new SEDEA interval efficiency model based on fuzzy numbers is defined below:

Definition 3.2. Assuming there are \( n \) decision units \( DMU_j \) \((j = 1, 2, \ldots, n)\), \( m \) input variables and \( s \) output variables, for a \( DMU_d \), its input and output variables are for the decision-making model.
\[ s_{x_{id}} = (\mu_{x_{id}}, \nu_{x_{id}})(i = 1, 2, \ldots, m) \]
and
\[ s_{y_{rd}} = (\mu_{y_{rd}}, \nu_{y_{rd}})(r = 1, 2, \ldots, s) \]. In order to select the most favorable situation for the DMU, we use \( \mu_{x_{id}} \) and \( 1 - \nu_{y_{rd}} \) as input and output values to obtain the fuzzy super efficiency upper bound model for DMU. Similarly, in order to select the most unfavorable case of DMU, we use \( 1 - \nu_{x_{id}} \) and \( \mu_{y_{rd}} \) as the input and output values to obtain the DMU fuzzy super efficiency lower bound model for DMU. The models constructed are shown below:

\[
\max h^U_d = \sum_{i=1}^{m} w_i \mu_{x_{id}}
\]

s. t.
\[
\sum_{i=1}^{m} w_i (1 - \nu_{x_{id}}) = 1
\]
\[
\sum_{i=1}^{m} w_i (1 - \nu_{x_{id}}) - \sum_{i=1}^{m} w_i \mu_{x_{id}} \leq 0, j = 1, 2, \ldots, n (j \neq d)
\]
\[
w_i, \nu_{x_{id}} \geq 0, i = 1, 2, \ldots, m; r = 1, 2, \ldots, s
\]

\[
\max h^L_d = \sum_{i=1}^{m} w_i \mu_{y_{rd}}
\]

s. t.
\[
\sum_{i=1}^{m} w_i (1 - \nu_{x_{id}}) = 1
\]
\[
\sum_{i=1}^{m} w_i (1 - \nu_{x_{id}}) - \sum_{i=1}^{m} w_i \mu_{x_{id}} \leq 0, j = 1, 2, \ldots, n (j \neq d)
\]
\[
w_i, \nu_{x_{id}} \geq 0, i = 1, 2, \ldots, m; r = 1, 2, \ldots, s
\]

Then, \( h^U_d \) and \( h^L_d \) are the upper and lower limits obtained by (7) and (8), so the interval number \([h^L_d, h^U_d]\) is the interval super efficiency value of DMU.

DMU can be classified according to the number of intervals obtained. When \( h^U_d \geq 1 \), DMU \((d = 1, 2, \ldots, n)\) are said to be valid for interval DEA; when \( h^L_d < 1 \) and \( h^U_d \geq 1 \), DMU \(j\) is said to be valid for partial interval DEA; when \( h^U_d < 1 \), it is said that DMU is invalid for the interval DEA. According to the cross efficiency [38], for the DMU, with its the optimal weight \( w_i^* \) and \( w_r^* \), we will calculate the cross efficiency value as shown in Table 1.

\[
E_{dj} = \sum_{i=1}^{m} w_i^* y_{ij}, \quad d, j = 1, 2, \ldots, n; d \neq j
\]

**TABLE 1**

<table>
<thead>
<tr>
<th>CROSS EFFICIENCY MATRIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluated decision unit(d)</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

Among them, \( x_{ij} \) and \( y_{ij} \) are just a kind of mark, which represents input and output data. Let \( E \) denote the cross efficiency matrix obtained in Table 1. Depending on the input and output data, we will get two different cross-efficiency matrices \( \tilde{E} \) and \( \hat{E} \). Let \( \hat{E} \) be the dominant matrix and \( \tilde{E} \) be the inferior matrix. The specific form of expression is as follows:

\[
E_i = \begin{cases} \tilde{E}, & \text{when } x_{ji} = \mu_{y_{ij}}, y_{ij} = 1 - \nu_{x_{ij}}, \quad i, j = 1, 2, \ldots, n \end{cases}
\]

Suppose that we choose the matrix \( \hat{E} \), where \( \hat{E}_{ij} \) is the element of the matrix. Then for any given \( j \ (j = 1, 2, \ldots, n) \), let

\[
E_{j}^L = \min \{E_{ij}, E_{j2}, \ldots, E_{jn}, i = 1, 2, \ldots, n\}
\]
\[
E_{j}^U = \max \{E_{ij}, E_{j2}, \ldots, E_{jn}, i = 1, 2, \ldots, n\}
\]

Then, for the dominant matrix \( \hat{E} \), each DMU \( j \) can be represented by an interval \([\hat{E}_{j}^L, \hat{E}_{j}^U]\), where the interval is called “dominant interval”. We will use \( \tilde{E}_{j} = [\tilde{E}_{j}^L, \tilde{E}_{j}^U] \) to represent it. In the same way, for the inferior matrix \( \tilde{E} \), each DMU \( j \) can also be represented as such. That is to say, “inferior interval” is \( E_{j} = [E_{j}^L, E_{j}^U] \). By this way, we can get \( n \) sets of dominant and inferior intervals respectively. In other words, each DMU will have a dominant interval and an inferior interval. First of all, we will select \( n \) sets of dominant intervals. For the number of intervals, we use the method of probability matrix to deal with in this paper, this means that for any two intervals \( a = [a^L, a^U] \), \( b = [b^L, b^U] \), they can be sorted by the following formula [39]:

\[
P(a > b) = \max \left\{ 1 - \max \left\{ \frac{b^L - a^L}{a^U - a^L + b^U - b^L}, 0 \right\}, 0 \right\}
\]

In this way, a dominant probability matrix \( \hat{P} \) will be obtained. In the same way, for any two intervals \( a = [a^L, a^U] \) and \( b = [b^L, b^U] \) in sets of inferior intervals, we can also use the formula above to get an inferior probability matrix \( \tilde{P} \).

D. IVIF Bayesian network multi-attribute decision making method

According to the research results of fuzzy set DEA in recent years, it is found that the subjective tendencies of different decision makers lead to different efficiency results, which have certain one-sidedness and limitations. Here, we first use the IVIFBN to infer the obtained information, and based on...
the fuzzy numbers of the obtained attributes, we use the interval DEA model to effectively evaluate the DMU.

This article is mainly divided into two phases: The first phase is the interval intuitionistic fuzzy Bayesian network inference. According to the interval fuzzy number of given data, the probability of occurrence of each attribute is obtained through Bayesian network transmission. The greater the probability value, the more likely the attribute will occur. We obtain the fuzzy numbers as the input and output data of the DEA method. The second phase is the analysis and evaluation of the DEA model. According to the fuzzy numbers of the first phase, the maximum and minimum weights of each attribute are obtained by the super-efficient interval DEA model, and then $\tilde{E}$ the dominant matrix $\tilde{E}$ and the inferior matrix $\tilde{E}$ are obtained by the cross efficiency. Then the scheme is evaluated by the probability matrix.

IV. Case study

In this part, we select six different scale logistics companies for case analysis, and evaluate the performance of the company and forecast its future development through the evaluation model proposed above. At present, the new direction of China's logistics industry development is: green, efficient and intelligent. In recent years, the volume of express delivery has increased year by year, and the over-packaging of goods has deepened the degree of environmental pollution and resource shortage, which has caused a heavy burden on the environment, thus restricting the sustainable development of the economy and society. Then, our first keyword for new logistics is “green logistics”. Secondly, due to the needs of society and the increasing market competitiveness, an important indicator for an enterprise to survive is its high efficiency. Improving logistics efficiency is a new requirement of market competition. So we put forward the second keyword “efficient logistics” for new logistics. Finally, with the application of intelligent information technology, various high-tech products are continuously generated and data is increasingly deepened into logistics platforms such as express delivery, which makes the operation of logistics will be more and more inclined to intelligence, and applications such as drones will become a new competitive point. Thence, our third keyword for new logistics is “smart logistics”. Therefore, these three elements occupy an important influence factor in the evaluation system of logistics enterprises. On the other hand, indicators such as customer satisfaction, packaging cost, resource recovery capacity, and order completion rate have certain impact on the performance evaluation of logistics enterprises. For logistics enterprises, the green environmental protection ability directly affects resource recovery capability of the enterprise; and the level of transportation management capability directly affects the indicators such as order completion rate and order completion time, which represents the efficiency level of the enterprise. In addition, whether there is certain intelligent equipment directly affects the future development of the enterprise. According to the above analysis, a BN relationship diagram for the performance evaluation of logistics enterprises can be constructed as follows:

![BN model of logistics](image)

**TABLE II**

<table>
<thead>
<tr>
<th>Logistics performance</th>
<th>Green environmental protection ability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (I)</td>
<td>High (II)</td>
</tr>
<tr>
<td>Bad (I)</td>
<td>$([0.52,0.55],[0.36,0.42])$</td>
</tr>
<tr>
<td>Good (II)</td>
<td>$([0.32,0.40],[0.30,0.48])$</td>
</tr>
</tbody>
</table>

**TABLE III**

<table>
<thead>
<tr>
<th>Logistics performance</th>
<th>Transportation management capability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (I)</td>
<td>High (II)</td>
</tr>
<tr>
<td>$([0.68,0.72],[0.20,0.25])$</td>
<td>$([0.48,0.55],[0.42,0.45])$</td>
</tr>
<tr>
<td>High (II)</td>
<td>$([0.36,0.42],[0.45,0.50])$</td>
</tr>
</tbody>
</table>

**TABLE IV**

<table>
<thead>
<tr>
<th>Logistics performance</th>
<th>Smart device</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (I)</td>
<td>High (II)</td>
</tr>
<tr>
<td>$([0.42,0.50],[0.25,0.32])$</td>
<td>$([0.33,0.40],[0.48,0.55])$</td>
</tr>
<tr>
<td>High (II)</td>
<td>$([0.30,0.45],[0.36,0.52])$</td>
</tr>
</tbody>
</table>

**TABLE V**

<table>
<thead>
<tr>
<th>Package cost</th>
<th>Green environmental protection ability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (I)</td>
<td>High (II)</td>
</tr>
</tbody>
</table>

This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see https://creativecommons.org/licenses/by/4.0/
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2929201, IEEE Access

<table>
<thead>
<tr>
<th>Low (I)</th>
<th>High (II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.30,0.35]</td>
<td>[0.56,0.62]</td>
</tr>
<tr>
<td>[0.63,0.70]</td>
<td>[0.23,0.28]</td>
</tr>
</tbody>
</table>

**TABLE VI**

The interval intuitionistic fuzzy conditional probability of “resource recovery capability”

<table>
<thead>
<tr>
<th>Resource recovery capability</th>
<th>Green environmental protection ability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (I)</td>
<td>High (II)</td>
</tr>
<tr>
<td>[0.60,0.68]</td>
<td>[0.25,0.30]</td>
</tr>
<tr>
<td>[0.36,0.46]</td>
<td>[0.43,0.50]</td>
</tr>
</tbody>
</table>

**TABLE VII**

The interval intuitionistic fuzzy conditional probability of “customer satisfaction”

<table>
<thead>
<tr>
<th>Customer satisfaction</th>
<th>Transportation management capability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (I)</td>
<td>High (II)</td>
</tr>
<tr>
<td>[0.60,0.66]</td>
<td>[0.25,0.30]</td>
</tr>
<tr>
<td>[0.40,0.42]</td>
<td>[0.50,0.54]</td>
</tr>
</tbody>
</table>

**TABLE VIII**

The interval intuitionistic fuzzy conditional probability of “order completion rate”

<table>
<thead>
<tr>
<th>Order completion rate</th>
<th>Transportation management capability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (I)</td>
<td>High (II)</td>
</tr>
<tr>
<td>[0.52,0.55]</td>
<td>[0.28,0.32]</td>
</tr>
<tr>
<td>[0.30,0.35]</td>
<td>[0.60,0.62]</td>
</tr>
</tbody>
</table>

**TABLE IX**

The interval intuitionistic fuzzy conditional probability of “loss rate”

<table>
<thead>
<tr>
<th>Loss rate</th>
<th>Transportation management capability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (I)</td>
<td>High (II)</td>
</tr>
<tr>
<td>[0.40,0.50]</td>
<td>[0.36,0.42]</td>
</tr>
<tr>
<td>[0.68,0.72]</td>
<td>[0.20,0.25]</td>
</tr>
<tr>
<td>High (II)</td>
<td></td>
</tr>
<tr>
<td>[0.60,0.70]</td>
<td>[0.24,0.30]</td>
</tr>
<tr>
<td>[0.42,0.44]</td>
<td>[0.40,0.50]</td>
</tr>
</tbody>
</table>

**TABLE X**

The interval intuitionistic fuzzy conditional probability of “time lapse”

<table>
<thead>
<tr>
<th>Time lapse</th>
<th>Transportation management capability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (I)</td>
<td>High (II)</td>
</tr>
<tr>
<td>[0.45,0.50]</td>
<td>[0.42,0.45]</td>
</tr>
<tr>
<td>[0.65,0.70]</td>
<td>[0.22,0.23]</td>
</tr>
<tr>
<td>High (II)</td>
<td></td>
</tr>
<tr>
<td>[0.62,0.68]</td>
<td>[0.24,0.30]</td>
</tr>
<tr>
<td>[0.20,0.30]</td>
<td>[0.58,0.60]</td>
</tr>
</tbody>
</table>

**TABLE XI**

The interval intuitionistic fuzzy conditional probability of “smart device”

<table>
<thead>
<tr>
<th>Logistics performance</th>
<th>Smart device</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (I)</td>
<td>High (II)</td>
</tr>
<tr>
<td>[0.60,0.65]</td>
<td>[0.20,0.37]</td>
</tr>
<tr>
<td>[0.10,0.20]</td>
<td>[0.40,0.46]</td>
</tr>
</tbody>
</table>

In this paper, we have selected six logistics companies as DMUs. Based on historical experience and relevant data, we get the interval intuitionistic fuzzy prior probability of each enterprise that is given in Table XII at two different stages.

**TABLE XII**

The interval intuitionistic fuzzy prior probability for the enterprise

We convert the data collected by each enterprise into the interval intuitionistic fuzzy probability. According to the conditional probability between each node and the known probability of the bottom node, the bottom-up reasoning method is adopted through the BN to obtain the interval intuitionistic fuzzy posterior probability of the root node, and then the root node is corrected. According to the posterior probability of the root node and the conditional probability between each node, the BN adopts the top-down reasoning method to obtain the interval direct fuzzy probability that each unknown attribute node is most likely to be obtained. The specific calculation method is as follows:

For the enterprise $u_1$, assume that the “Package cost” is known, we will use the BN to get the fuzzy information of the other six nodes. According to the prior probability of the root node and the conditional probability between the nodes. We can get the posterior probability of the root node under the “Package cost” known.

$$P_n(Package|\text{Logistics1}) = \frac{P_n(Logistics1\cap Package)}{P_n(Package)}$$

where $P_n(Package|\text{Logistics1})$ represents the attribute “Logistics performance” that is the low degree, $P_n(Package|\text{Logistics2})$ represents the attribute “Logistics performance” that is the high degree, $P_n(Package)$ represents the attribute “Package cost” that is the low degree, $P_n(Package)$ represents the attribute “Package cost” that is the high degree. Besides, the conditional probability is given by the table.

$P_n(Package) = P_n(Package|\text{Logistics1}) \cdot P_n(Package|\text{Logistics2})$ and $P_n(Package|\text{Logistics1})$ and $P_n(Package|\text{Logistics2})$ can be calculated by

$$P_n(Package|\text{Logistics1}) = \frac{P_n(Package\cap \text{Logistics1})}{P_n(\text{Logistics1})}$$

$$P_n(Package|\text{Logistics2}) = \frac{P_n(Package\cap \text{Logistics2})}{P_n(\text{Logistics2})}$$

This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see https://creativecommons.org/licenses/by/4.0/.
where Green1 represents the attribute “Green environmental protection ability” that is the low degree, Green2 represents the attribute “Green environmental protection ability” that is the high degree.

According to the division of the operation of the interval intuitionistic fuzzy value proposed by [40], we can get the four posterior probability of the root node to correct the prior information. For the obtained $P_n (\text{Logistics1} | \text{Package1})$ and $P_n (\text{Logistics1} | \text{Package2})$, we select the median value as the posterior probability $P_n (\text{Logistics1})$ of the root node in the low degree and $P_n (\text{Logistics2})$ in the low degree. Then, for the attribute nodes that cannot quantize the data, the interval intuitionistic fuzzy probability can be obtained by the downward transfer of the Bayesian network. Suppose that the node “Resource recovery capability” is unknown, we can get the information in the same way as follows:

$$P_n (\text{Resource}) = P(\text{Resource} | \text{Green1}) \otimes P(\text{Green1}) \otimes P(\text{Resource} | \text{Green2}) \otimes P(\text{Green2})$$

Similarly, the possible interval intuitionistic fuzzy probabilities for each node can be calculated. For each node, there are two interval intuitionistic fuzzy probabilities $a_1 = (\alpha_1, \beta_1)$ and $a_2 = (\alpha_2, \beta_2)$. Their score functions are $s(a_1)$ and $s(a_2)$ respectively. We get the magnitude of $a_1$ and $a_2$ by comparing the size of the score function. The larger the interval intuitionistic fuzzy probability, the more likely it is to occur. We choose a larger interval intuitionistic fuzzy probability. For the convenience of subsequent calculations, the median value of the interval intuitionistic fuzzy probability is reduced to the intuitionistic fuzzy probability. Similarly, for each $u_i (i = 1, \ldots, 6)$, there are seven intuitionistic fuzzy probabilities, the specific values are visible in Table X. III.

<table>
<thead>
<tr>
<th>Indicator node Enterprise type</th>
<th>Package cost</th>
<th>Resource recovery capability</th>
<th>Customer satisfaction</th>
<th>Order completion rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>(0.407,0.3646)</td>
<td>(0.494,0.2922)</td>
<td>(0.5957,0.2136)</td>
<td>(0.5556,0.2256)</td>
</tr>
<tr>
<td>$u_2$</td>
<td>(0.3893,0.4107)</td>
<td>(0.4752,0.3398)</td>
<td>(0.6181,0.2416)</td>
<td>(0.5562,0.2685)</td>
</tr>
<tr>
<td>$u_3$</td>
<td>(0.3872,0.3856)</td>
<td>(0.4730,0.3153)</td>
<td>(0.6194,0.2188)</td>
<td>(0.5563,0.2434)</td>
</tr>
<tr>
<td>$u_4$</td>
<td>(0.4077,0.3306)</td>
<td>(0.4984,0.3805)</td>
<td>(0.6071,0.2249)</td>
<td>(0.5601,0.2418)</td>
</tr>
<tr>
<td>$u_5$</td>
<td>(0.4641,0.3410)</td>
<td>(0.5592,0.2678)</td>
<td>(0.6471,0.1955)</td>
<td>(0.6233,0.1961)</td>
</tr>
<tr>
<td>$u_6$</td>
<td>(0.3860,0.3824)</td>
<td>(0.4679,0.3589)</td>
<td>(0.5563,0.3396)</td>
<td>(0.5241,0.3330)</td>
</tr>
</tbody>
</table>

The six enterprise logistics to be evaluated are taken as one decision-making unit $DMU_j (j = 1, 2, \ldots, 6)$, and the six attributes such as packaging cost are taken as indicators. The index that is smaller and the better is used as the input indicator, and the bigger the better is used as the output indicator to construct the input and output system. The specific indicators are divided as shown in Table X. IV.

<table>
<thead>
<tr>
<th>input indicator</th>
<th>Package cost, Loss rate, Time lapse</th>
</tr>
</thead>
<tbody>
<tr>
<td>output indicator</td>
<td>Resource recovery capability, Customer satisfaction, Order completion rate, Smart device</td>
</tr>
</tbody>
</table>

According to the SEDEA model proposed in Definition 3.2 and the input and output indicators defined above, we can use Matlab to solve the model to obtain the interval efficiency value and the two optimal weight by Model (2) and Model (3).

<table>
<thead>
<tr>
<th>Indicator node</th>
<th>Interval efficiency value</th>
<th>optimal weight by (7)</th>
<th>optimal weight by (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>[0.5836, 2.1040]</td>
<td>(2.4558, 2.600, 0.0, 0.0, 0.3667)</td>
<td>(0.0, 0.0, 0.9797, 0.0, 0.3870)</td>
</tr>
<tr>
<td>$u_2$</td>
<td>[0.6218, 2.1402]</td>
<td>(2.5867, 2.4523, 0.0, 0.0, 0.7167)</td>
<td>(0.0, 0.0, 0.9797, 0.0, 0.3870)</td>
</tr>
<tr>
<td>$u_3$</td>
<td>[0.0680, 2.1678]</td>
<td>(2.5826, 2.7343, 0.0, 0.0, 0.3586)</td>
<td>(0.0, 0.0, 0.9797, 0.0, 0.3870)</td>
</tr>
<tr>
<td>$u_4$</td>
<td>[0.5833, 2.0586]</td>
<td>(2.4528, 2.5968, 0.0, 0.0, 0.3662)</td>
<td>(0.0, 0.0, 0.9797, 0.0, 0.3870)</td>
</tr>
<tr>
<td>$u_5$</td>
<td>[0.5958, 1.9565]</td>
<td>(2.5970, 2.7428, 0.0, 0.0, 0.3868)</td>
<td>(0.0, 0.0, 0.9797, 0.0, 0.3870)</td>
</tr>
<tr>
<td>$u_6$</td>
<td>[0.5952, 2.0184]</td>
<td>(2.5970, 2.7428, 0.0, 0.0, 0.3868)</td>
<td>(0.0, 0.0, 0.9797, 0.0, 0.3870)</td>
</tr>
</tbody>
</table>

For any $DMU_j (d = 1, 2, \ldots, 6)$, we can get two efficiency values $h_d^U$ and $h_d^L$ through Models (7) and (8), then form the interval efficiency value $[h_d^L, h_d^U]$. At the same time, the optimal weights $w^1$ and $w^2$ are also obtained respectively. According to (9) and (10), we obtain the dominant cross
value and the inferior cross value, as shown in Tables X VI-X VII.

### TABLE X VI
**DOMINANT CROSS EFFICIENCY VALUE**

<table>
<thead>
<tr>
<th>Evaluation decision unit(d)</th>
<th>Evaluated decision unit(j)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>2.1040</td>
<td>2.0744</td>
<td>2.1678</td>
<td>2.0584</td>
<td>1.9140</td>
<td>2.0187</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2.1525</td>
<td>2.1402</td>
<td>2.2359</td>
<td>2.1102</td>
<td>1.9613</td>
<td>2.0717</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>2.1042</td>
<td>2.0745</td>
<td>2.1678</td>
<td>2.0584</td>
<td>1.9140</td>
<td>2.0187</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>2.1041</td>
<td>2.0744</td>
<td>2.1677</td>
<td>2.0586</td>
<td>1.9139</td>
<td>2.0186</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>2.1558</td>
<td>2.1032</td>
<td>2.1987</td>
<td>2.1031</td>
<td>1.9565</td>
<td>2.0600</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>2.1042</td>
<td>2.0744</td>
<td>2.1678</td>
<td>2.0583</td>
<td>1.9140</td>
<td>2.0184</td>
</tr>
</tbody>
</table>

### TABLE X VII
**INFERIOR CROSS EFFICIENCY VALUE**

<table>
<thead>
<tr>
<th>Evaluation decision unit(d)</th>
<th>Evaluated decision unit(j)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0.5837</td>
<td>0.6323</td>
<td>0.6132</td>
<td>0.6051</td>
<td>0.6180</td>
<td>0.6175</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.4888</td>
<td>0.6218</td>
<td>0.6080</td>
<td>0.5387</td>
<td>0.4584</td>
<td>0.4729</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0.4898</td>
<td>0.6228</td>
<td>0.6090</td>
<td>0.5397</td>
<td>0.4594</td>
<td>0.4739</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0.5627</td>
<td>0.6095</td>
<td>0.5911</td>
<td>0.5833</td>
<td>0.5957</td>
<td>0.5953</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0.5627</td>
<td>0.6095</td>
<td>0.5911</td>
<td>0.5833</td>
<td>0.5957</td>
<td>0.5953</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>0.5627</td>
<td>0.6096</td>
<td>0.5911</td>
<td>0.5833</td>
<td>0.5958</td>
<td>0.5953</td>
</tr>
</tbody>
</table>

According to Formula (11), the “dominant interval” \( \tilde{E}_j = [\tilde{E}_j^L, \tilde{E}_j^U] \) and the “inferior interval” \( E_j = [E_j^L, E_j^U] \) can be obtained respectively, where the interval is called “dominant interval”. We will use \( \tilde{E}_j = [\tilde{E}_j^L, \tilde{E}_j^U] \) to represent it. In the same way, for the inferior matrix \( E_j \), each \( DMU_j \) can also be represented as such. That is to say, “inferior interval” is \( E_j = [E_j^L, E_j^U] \).

### TABLE X VIII
**TWO INTERVAL EFFICIENCY VALUES**

<table>
<thead>
<tr>
<th>dominant matrix ( \tilde{E} )</th>
<th>inferior matrix ( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_1 )</td>
<td>( [2.1040, 1.558] )</td>
</tr>
<tr>
<td>( E_2 )</td>
<td>( [2.0744, 2.1402] )</td>
</tr>
<tr>
<td>( E_3 )</td>
<td>( [2.1677, 2.235] )</td>
</tr>
<tr>
<td>( E_4 )</td>
<td>( [2.0583, 2.1102] )</td>
</tr>
<tr>
<td>( E_5 )</td>
<td>( [1.9139, 1.9613] )</td>
</tr>
<tr>
<td>( E_6 )</td>
<td>( [2.0184, 2.0717] )</td>
</tr>
</tbody>
</table>

According to the ordering method of the likelihood matrix by Formula (12), we sort the two matrices dominant matrix \( \tilde{E} \) and inferior matrix \( E \) and the following two possible degree matrices are obtained: the dominant probability matrix and the inferior probability matrix.

Then we sort the likelihood matrix separately according to Formula (13). For the convenience, we use \( r_i = \sum_{j=1}^{n} p_{ij}, i = 1, 2, \ldots, n \) to sort the scheme:

\[
\frac{1}{n(n-1)} \sum_{i=1}^{n} p_{ij}^n j = 1, 2, \ldots, n
\]

According to the optimisms of the decision makers, we introduce the Hurwitz decision criterion, that is, the optimistic index \( \alpha \) to sort each scheme.

\[
SE_j = \alpha E_j + (1-\alpha) \tilde{E}_j, 0 \leq \alpha \leq 1, j \in n
\]

### FIGURE 4
Scheme values in different situations
From Figure 4, we can know that when the dominant matrix is $\alpha=0$, $r_5 < r_6 < r_4 < r_2 < r_1 < r_3$. When the inferior matrix is $\alpha=1$, $r_1 < r_2 < r_3 < r_4 < r_5 < r_6$. When $\alpha=0.5$, $r_5 < r_6 < r_2 < r_3 < r_4 < r_1$. We found that no matter how $\alpha$ changes, the companies with the best returns always produce in $u_i$, that is, companies with good initial performance and declining performance in the short term can represent large enterprises, which reflects the process of capital accumulation. Despite that the short-term benefits of the company have declined, the overall benefits are still high. The companies with the worst returns are generated in $u_e$, that is, companies with poor initial performance and rapid performance in the short term can represent small businesses. Although the short-term benefits are high, the accumulated capital is still small. From this we can conclude that although the short-term performance of large enterprises has declined, the overall strength still exists, and the overall income is still good. That means a lean camel is bigger than a horse. This is also consistent with the actual situation. In reality, in order to stabilize, people will still choose large enterprises, which will be more secure. It also shows that the model proposed in this paper is an objective and effective, the authentic method of performance evaluation. Therefore, when the investors want to pursue higher returns in the short term, they can choose small businesses, and at the same time, the risk will be greater. When the investors want to pursue long-term and stable returns, they can choose large companies with less risk.

V. Conclusions

With the development of social economy, online shopping has become inseparable from our life, and the logistics industry that has followed has become more and more popular. Logistics enterprises have gradually become the pillar of social emergency development, so the evaluation of corporate logistics performance is crucial to the development of enterprises. This paper proposes a super-efficient cross-DEA model in a fuzzy environment to evaluate the performance of logistics enterprises. In view of the new requirements of today’s logistics system and the inability to quantify some of the attributes of indicators, we have constructed a range of intuitionistic fuzzy Bayesian networks under logistics enterprises. First, the BN has been transmitted based on the data given, and then the fuzzy data of the indicator have been obtained. Then, based on the above fuzzy data, we have selected the SEDEA model for evaluation, and then have introduced the cross-efficiency. By combining the self-evaluation and his-evaluation, the optimal and worst cross-matrix has been used to rank the decision-making units (enterprise performance). Finally, we have evaluated and predicted the future development trend of the company by introducing the Hurwitz decision criterion according to the different optimisms of the decision makers.

This paper has proposed a method to evaluate performance by combining BN with DEA. It has certain advantages. The qualitative or missing data can be refined to improve the accuracy of the data. However, there are still some limitations, such as strong subjectivity and insufficient comprehensive selection of indicators. In the future, we will conduct thorough research, continuously optimize the DEA model, and apply the model to solve more practical problems in the real life.

REFERENCES


