Guided Image Filtering Based Limited-angle CT Reconstruction Algorithm Using Wavelet Frame

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ABSTRACT Computed tomography (CT) has its irreplaceable function in the nondestructive testing and medical diagnosis. In some practical CT imaging applications, the limited-angle scanning is common due to X-ray's potential harm to human and the limitation of the scanning conditions. Under these circumstances, analytic reconstruction algorithms, like filtered backprojection (FBP), will not obtain satisfactory results because of lacking of the projection data. Iterative reconstruction (IR) methods that can incorporate priori knowledge have attracted attention in many fields, and wavelet frame based regularization reconstruction algorithms have proven to be a useful means to reduce slope artifacts and noise for limited-angle CT. However, with the obtained projection data of the scanned object further reduces, the edge structures and the details of the reconstructed image worsen. For the sake of improving the quality of the reconstructed image from the limited-angle projection data, a guided image filtering (GIF) based limited-angle CT reconstruction algorithm using wavelet frame was proposed. In each iteration of the proposed algorithm, the reconstructed result constrained by the wavelet frame was used as the guidance image to transfer the important features it contains to the reconstructed result of SART method by GIF. Furthermore, some simulated experiments and real data tests were conducted to evaluate the feasibility and validity of the proposed algorithm, and the qualitative and quantitative indexes indicated that the proposed algorithm was superior to other iterative reconstruction algorithms in artifacts reduction, noise suppression and structure preservation.

INDEX TERMS Image reconstruction, computer tomography (CT), limited-angle, guided image filtering, wavelet frame.

I. INTRODUCTION
Computed tomography (CT) is a more advanced imaging mode, and it is broadly used in archaeology, non-destructive testing (NDT) and medical diagnosis. It can noninvasively obtain the interior image of the scanned object by using the projection data obtained from the detector, which is a classic inverse problem in mathematics. When the projection data available are sufficient, analytic reconstruction algorithms (such as filtered back projection (FBP) algorithm [1]) can accurately reconstruct images. However, in some practical CT imaging applications, the obtained projection data of the scanned object are usually incomplete. For example, in the field of medicine, the C-arm CT [2], the dental CT [3], and the breast CT [4] want to reduce the radiation dose of X-ray by using shorter time exposure due to X-ray harms the health of human body [5]. The projection data of the scanned object are acquired from different view angles within a limited angular range. In the field of industry, to detect the defects of the long object or check the pipeline in service [6, 7], the projection data of the scanned object are also obtained from different view angles within a limited angular range due to the limitation of scanning conditions. Fig. 1 is the sketch map of the limited-angle CT.
Under these circumstances, image reconstructed by conventional analytic algorithms (such as FBP algorithm) will appear artifacts and edges will be distorted [8], which will make the disease diagnosis and defect detection more difficult. Therefore, many researchers make efforts to improve reconstructed image quality for the limited-angle CT.

Recent years, iterative reconstruction (IR) algorithm have attracted much attention, like expectation maximization (EM) [9], simultaneous algebraic reconstruction technique (SART) [10], and algebraic reconstruction technique (ART) [11], and the results reconstructed by these IR methods were better than FBP method in terms of noise suppression when the projection data are complete. However, these methods cannot obtain satisfactory solution when the projection data are incomplete. Regularized iterative reconstruction methods which can incorporate the prior information of the scanned object can be really helpful toward improving the quality of reconstructed image from incomplete projections with noise. Therefore, many researchers were devoted to design an appropriate transformation to make full use of prior information of the scanned object, and various regularization based CT image reconstruction methods have been proposed [12-14].

Total variation (TV), which utilized the prior information that the gradient magnitude image of the reconstructed image is sparse, was proposed for image denoising [15]. Whereafter, a two dimensional regularization function was proposed, and it was implemented by an expectation maximization algorithm in the bayesian reconstruction [16]. Then, Sidky and Pan [17] proposed a CT reconstruction algorithm from under-sampling projection data, data consistent term was treated by the projection onto convex sets (POCS) method, and sparseness constraint was solved by the TV minimization. Later, an iterative reconstruction framework which combined POCS with adaptive steepest descent (ASD-POCS) was developed [5]. In general, TV-based methods can effectively suppress noise and streak artifacts in some applications, nevertheless, blocky artifacts or staircase effect will appear on the reconstructed image because of the assumption that reconstructed image is piecewise constant. In order to solve this problem, Yu [18] built a pseudo-inverse to solve discrete gradient transform, and developed a soft-threshold filtering algorithm (TV-STF). Meanwhile, total difference minimization (TDM) algorithm was developed (TDM-STF), and experimental results showed better performance with fewer parameters to adjust. Chen [19] developed an anisotropic total variation (ATV) method, which utilized the actual scanning range as prior knowledge to improve the quality of the reconstructed image for limited-angle CT. Besides, Chen [20] used the CT image previously scanning as the prior image to propose an image reconstruction algorithm based on prior image. In summary, the TV based methods can improve the performance in reducing artifacts, however, the edges and details information of the scanned object was partly distorted in the limite-angle CT.

In the past several decades, wavelet frame theory was well studied and developed [21-23]. Nowadays, wavelet frame have been introduced into image reconstruction [24-26]. The fundamental theory of the wavelet frame based reconstruction methods is using a properly wavelet frame to sparsely represent the reconstructed image to suppress noise and reduce artifacts. Jia [24] developed an iterative tight-frame (TF) based reconstruction algorithm for cone-beam CT from incomplete projection data with noise. Then, Zhou [25] proposed an adaptive wavelet tight frame method to reconstruct a variety of diversified images. Recently, Zeng [26] developed a wavelet frame based limited-angle CT image reconstruction method which utilized the wavelet owns the property of multiscale, experimental results indicated that the proposed method has advantage in suppressing slope artifacts and noise. However, the slope artifacts will increase and the edge will be distorted when the obtained projection data further decrease.

In the last few years, He [27] proposed an edge-preserving smoothing operator named as guided image filtering (GIF), which showed better performance than other state-of-the-art filters (such as the bilateral filter). GIF improved the image quality by transferring the important features of the guidance image to the filtered image. Motivated by the GIF, Ji [28] used a CT image previously scanned as the prior image and combined the SART method with GIF for few-view CT reconstruction. However, it is difficult to obtain the prior image in many applications. So, Yu and Wang [29] proposed a sparsity-induced dynamic guided filtering approach for sparse-view CT, which took intermediate reconstruction results by TDM-STF method as the guidance images.

Inspired by the above work, we developed a GIF based limited-angle CT reconstruction algorithm using wavelet frame. The proposed algorithm is denoted as SART-WF-GIF. For the sake of improving reconstructed image quality for limited-angle CT, the proposed algorithm incorporated the multiscale properties of wavelet frame, the prior

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**FIGURE 1.** Sketch map of the limited-angle CT. S denotes the X-ray source, T denotes the objective table, b denotes the scanned object, D denotes the detector, o denotes the rotation center of the X-ray source, o1 denotes the rotation angle which is less than 180° plus a fan-angle.
information that the reconstructed image is the sparseness in wavelet domain, and the edge-preserving smoothing operator GIF. In the implementation process of the proposed algorithm, the results of the SART method, which was utilized to enforce the data fidelity, were used as the input image of the GIF, and the reconstructed images constrained by the wavelet frame were regarded as the guidance image of the GIF, finally, the GIF was used to transfer the important features of the results constrained by wavelet frame to the results reconstructed by SART method during the iterative process. The experimental results indicated that the proposed algorithm was superior to other iterative reconstruction algorithms in artifacts reduction, noise suppression and structure preservation.

The overall structure of the rest of this article is arranged as follows. In Section II, first, CT imaging model is introduced, then, the regularized reconstruction model and the GIF are briefly reviewed, finally, the proposed algorithm is presented. In Section III, we present and analyze the simulated and real data experiment results. In Section IV, some discussions and conclusions are given.

II. METHOD
A. CT imaging model
In brief, the CT image reconstruction is actually resolved into solving the following equation:

\[ Af = g \]  
(1)

where the system matrix \( A = (a_{i,j})_{M \times N} \) has \( M \times N \) elements, the column vector \( g = (g_1, g_2, ..., g_N)^T \) denotes the projection data obtained by the detector, and \( f = (f_1, f_2, ..., f_N)^T \) represents discrete attenuation coefficients of the reconstructed image. The goal of the CT reconstruction is to acquire \( f \) from \( A \) and \( g \). When obtained projection data are sufficient, analytic reconstruction algorithm (such as FBP algorithm) can accurately reconstruct images. The least squares method is usually used to solve the Eq. (1) by minimizing the following objective function:

\[ \arg \min_f \| Af - g \|_0 \]  
(2)

where \( D \) is the weight matrix, and \( \| \cdot \|_0 \) denotes the weighted \( L_0 \) norm with \( D \) positive definite (i.e., \( \| f \|_0 = \sqrt{f^T D f} \) with \( D > 0 \)). The problem (2) also can be solved by the iterative reconstruction methods, such as SART method [30], and the sequence of results obtained by the SART method converge to a weighted least square solution under the condition that coefficients of the linear imaging system are nonnegative [31].

B. Regularized reconstruction model
In some practical CT imaging applications, the limited-angle scanning is common because of some restricted conditions, which causes an ill-posed inverse problem, and the Eq. (1) is underdetermined. Thus, the reconstructed results will present serious artifacts and noise if the SART-type method be applied. Hence, scholars started to research the regularized reconstruction model, which can use far fewer projection data to reconstruct high-quality images, to maintain stability, suppress artifacts, and protect structures. The core concept of the regularized reconstruction model is using the prior information that the reconstructed images or its coefficients under an appropriate transformation are sparse, and the key is find an appropriate regularization term which usually is a sparse transformation.

In recent years, wavelet transform has great contribution in the image reconstruction from incomplete projection data due to the property of multiscale [23, 26, 32, 33]. And we will focus on the wavelet transform in this paper. In the [23], the authors proposed a wavelet frame based \( f_0 \) quasi-norm to perform image restoration, and it can be formulated as

\[ f^* = \arg \min_{f \in F} \left\{ \frac{1}{2} \| Af - g \|^2_0 + \sum_{i} \lambda_i \| (Wf_i) \|_0 \right\} \]  
(3)

where \( F \) is a convex subset of \( R^N \), \( \lambda_i \) denotes the weight sparsity parameters, and \( W \) denotes the multi-level wavelet tight frame transform (i.e., \( W^TW = I \) ), \( W^T \) denotes the wavelet inverse transform, and \( \| (Wf_i) \|_0 \) is the number of nonzero terms \( \#(q(Wf_i) \neq 0) \). The coefficients of the \( L \)-level wavelet decomposition of \( f \) was denoted as

\[ a = \{ W_{ik} f : 0 \leq i \leq L-1, k \in I \} \]

where \( I \) denotes the index set of all framelet bands, and numerical results indicated that the proposed algorithm can improve image quality by preserving sharp features and smoothing the noise. Then, Zeng [26] developed a wavelet frame \( f_0 \) quasi-norm based limited-angle CT reconstruction method, and experimental results showed that the proposed algorithm can suppress slope artifacts and preserve the structure, but it will gets worse when the scanning angle further decreases.

C. Guided image filtering
Guide image filtering [27] can better smooth image and preserve structure, and it makes an hypothesis that the filtered image \( f^\text{out} \) in the local window \( i \) was expressed as a linear transformation of the guidance image \( I_{\text{guide}} \), and the filtered image \( f_i^\text{out} \) can be expressed as follows:

\[ f_i^\text{out} = a_i I_{\text{guide}} + b_i, \forall i \in a_k \]  
(4)

where \( a_i \) and \( b_i \) are the coefficients in the local window \( a_k \) which centered at the pixel \( k \) with the radius of \( r \). The coefficient \( a_i \) and \( b_i \) are obtained by solving the following cost function:

\[ E(a_i, b_i) = \sum_{I_{\text{in}}} [(a_i I_{\text{in}} + b_i - I_i^0)^2 + \lambda(a_i^2)] \]  
(5)

where \( I_i^0 \) is the input image of the filter, and \( \lambda \) is a regularization parameter which can penalize large \( a_i \).
D. The proposed SART-WF-GIF algorithm

It can be seen from the above sections, if the scanning angle further decreases, the results of the SART showed severe noise and slope artifacts, while the information of details and structures were preserved well. And the results constrained by the wavelet frame can better suppress noise and slope artifacts, whereas the boundaries and details were distorted and over-smoothing. In addition, GIF can make the filtered image have similarity with guidance image by transferring the important features of the guidance image to the input image. Inspired by these facts, we used edge-preserving smoothing operator GIF to incorporate the results of the SART with the results constrained by the wavelet frame to make it possible to preserve important features and suppress slope artifacts at the same time. The proposed algorithm was denoted as SART-WF-GIF, and each iteration of the SART-WF-GIF was consisted of three steps.

The wavelet frame based image reconstruction algorithm can be represented as solving the following optimization problem:

\[
    f = \arg \min \| Af - g \|_D^2 \text{ s.t } \| Wf \| \leq \varepsilon \tag{6}
\]

It is quite difficult to solve this optimization problem since the minimization of objective function involving with \( l_p \)-norm is usually NP-hard. In this work, we used the iterative hard thresholding (IHT) algorithm [36] to address this optimization problem.

1. SART step

The gradient descent method was used to minimize the objective function in the typical IHT, and in this paper, we used the SART method to replace it to address CT image reconstruction problem. The first step of the SART-WF-GIF is the SART step: \( f_{n+1}^{\text{SART}} = SART(f^n) \), which is used to minimize the objective function to emphasize the data consistency, and the iterative formula of the SART method [10] was expressed as:

\[
    f_{i}^{(n+1)} = f_{i}^{(n)} + \beta \sum_{j=1}^{H} \sum_{l=1}^{N} a_{ij} \sum_{i=1}^{N} g_{i} - f_{i}^{n}, \quad 0 \leq \beta \leq 1 \tag{7}
\]

where \( n \) is the number of iterations and \( \beta \) is the relaxing factor.

2. WF step

The second step of the SART-WF-GIF is the WF step which is used hard thresholding operator to address the wavelet frame (WF) constraint. It utilized the fact that the reconstructed image is sparse in the wavelet domain, and the iterative formula is as follows:

\[
    f^{*} = W^T(H_f(Wf_{SART})) \quad i = L \text{ or } H \tag{8}
\]

the hard thresholding operator with threshold \( t \in R^* \) was defined as

\[
    H_f(x) = \begin{cases} 
        0 & \text{if } |x| < t \\
        0 & \text{if } |x| = t \\
        x & \text{if } |x| > t 
    \end{cases} \tag{9}
\]

\( \lambda_H, \lambda_L \) are the threshold of the high frequency and low frequency coefficient of the wavelet. \( W \) denotes the multi-level wavelet tight frame transform which we chose is the piecewise-constant linear B-spline [23], and the associated filters are arranged as follows:

\[
    h_0 = \frac{1}{4} [1,2,1], \quad h_1 = \frac{\sqrt{2}}{4} [1,0,-1], \quad h_2 = \frac{1}{4} [-1,2,-1] 
\]

The two-dimensional mask of the corresponding piecewise-constant linear B-spline wavelet tight frame transform are as follows:

\[
    \begin{align*}
    h_{00} &= \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}, & h_{10} &= \frac{1}{8} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, & h_{20} &= \frac{1}{16} \begin{bmatrix} 2 & 4 & 2 \\ 1 & 0 & -1 \\ -1 & 2 & 1 \end{bmatrix}, \\
    h_{01} &= \frac{1}{8} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, & h_{11} &= \frac{1}{8} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 \end{bmatrix}, & h_{21} &= \frac{1}{16} \begin{bmatrix} -1 & 2 & -1 \\ -1 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}, \\
    h_{02} &= \frac{1}{16} \begin{bmatrix} 2 & 4 & 2 \\ -1 & -2 & -1 \\ -1 & -2 & -1 \end{bmatrix}, & h_{12} &= \frac{1}{8} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 1 \\ 2 & 0 & -2 \end{bmatrix}, & h_{22} &= \frac{1}{16} \begin{bmatrix} -1 & -2 & -1 \\ -1 & -2 & -1 \\ 1 & 2 & 1 \end{bmatrix}
    \end{align*}
\]

![FIGURE 2. The transform results of image under the one level piecewise-constant linear B-spline framelets transform.](image-url)
regard the results of step 1 as the input image \( f^\circ \), and regard the results of step 2 as the guidance image \( f^g \). The optimal values of \( a_i \) and \( b_i \) for (4) can be computed as:

\[
a_i = \frac{1}{|\omega|} \sum_{m=0}^{N} f^* f_m - f^* f_m^r \quad \sigma_i^2 + \lambda
\]

\[
b_i = f^* - a_i f^r
\]

In the local window \( \omega_i \), \(|\omega|\) denotes the number of the pixels. \( f^\circ_i \) is the mean of \( f^* \), \( f^\circ_i \) and \( \sigma_i^2 \) are the mean and the variance of \( f^\circ \). To consider the total local windows of the image, filtered image \( f^m \) can be expressed as:

\[
f^m = a_i f^\circ_i + b_i
\]

where \( a_i = \frac{1}{|\omega|} \sum_i a_i \) and \( b_i = \frac{1}{|\omega|} \sum_i b_i \).

Table I is the main steps of SART-WF-GIF algorithm, and \( N \) is the maximum iterations.

<table>
<thead>
<tr>
<th>Implementation steps of the SART-WF-GIF algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initialization</strong> ( f^\circ = 0 ), ( n = 1 ), ( \lambda_i ), ( \lambda_H ), ( N ), ( r ), ( \lambda )</td>
</tr>
<tr>
<td>While ( n \leq N ) do</td>
</tr>
<tr>
<td>SART step: ( f^s = \text{SART}(f^{n-1}) ) via Eq. (7)</td>
</tr>
<tr>
<td>WF step: Computing ( f^w ) via Eq. (8)-Eq. (9)</td>
</tr>
<tr>
<td>GIF step: Using guide image ( f^g ) and the input image ( f^s ) to computing the output image ( f^m ) via Eqs. (10)-(12)</td>
</tr>
<tr>
<td>( f^n = f^m )</td>
</tr>
<tr>
<td>( n = n + 1 )</td>
</tr>
<tr>
<td>end while</td>
</tr>
<tr>
<td>return ( f^m )</td>
</tr>
</tbody>
</table>

III. EXPERIMENTAL RESULTS

In this section, we used some simulation experiments and real experiments to test the validity of the proposed algorithm for limited-angle CT image reconstruction by some qualitative and quantitative studies. We utilized the fan-beam scanning to implement all the experiments. In addition, some other methods such as SART and wavelet frame based reconstruction algorithm (SART-WF) are compared with the proposed algorithm. The computer platform was configured as follows: CPU is Inter(R) Core(TM) i5-6500 3.20GHz, GPU was NVIDIA GTX 1080 with 8GB memory, system environment was Windows 7 64bit, and the Matlab version was R2016b combined with Visual Studio C++ 2015. In this study, three classic metrics were utilized to appraise the reconstructed image quality, including the root mean square error (RMSE), the structural similarity index measure (SSIM), and the peak signal to noise ratio (PSNR).

A. Parameters and iteration number selections

In the CT image reconstruction, it is a difficult task to select parameters and iteration number due to some factors (such as the reconstructed object, model and the available projection data), and the selection of parameters and iteration number are important for obtaining a better result.

In this paper, the most important parameters of the proposed algorithm are \( \lambda_i \), \( \lambda_H \), \( r \) and \( \lambda \). The parameters \( \lambda_i \) and \( \lambda_H \) control the sparsity of reconstructed image, \( r \) control the radius of the local window of the GIF, \( \lambda \) control the similarity of the output image and the guide image. First, we referred to the parameter selection in the previous papers [26, 29]. Then, we used the single variable method, which is adjusted one parameter and left the other parameters unchanged, to fine tuning these parameters to get a better result by considering the values of the evaluation index and visual inspection for eye-appealing results together with the full-scan image for comparison.

In every experiment, we chose the minimum iteration number, which make the result reach the optimal evaluation index and visual inspection under some parameters, as the optimal iteration number.

In summary, when there are only one parameter in the model, L-curve method [37] can be used to select the optimal parameter. When the number of parameters in the model are greater than or equal to 2, most of them are selected by trial and error [38]. So, a more detailed study of the parameters selection strategy is needed, and several methods can help us to select parameters, such as adaptive selection of parameters and deep learning, which are our future research directions.

B. Simulation experiment study

First, we used some simulation experiments to validate the proposed algorithm, which utilized a digital NURBS based caradec-torso phantom (NCAT), and the size of the resolution is \( 256 \times 256 \). Table II illustrated the geometrical scanning structure of the simulated CT system, and three different scanning ranges \( \{ [0, 90], [0, 120] \} \) and \( \{ [0, 150] \} \) were chose to present the algorithm performance. For a fair comparison, the parameters of these competing methods were optimized to obtain the best results in terms of the values of the evaluation index, and \( \beta \) equals 1 in all these experiments. The iterative steps of the SART algorithm were 150, 170 and 230 for the scanning ranges \( \{ [0, 90], [0, 120] \} \) and \( [0, 150] \). For the SART-WF method, the
The distance from source to rotation axis (mm) & 500 \\
Sampling interval between two adjacent projection views (deg) & 1 \\
The detector bin numbers & 372 \\
Size of each detector element (mm) & 1 \\
Voxel size of the object (mm) & 1 \\

Fig. 3 showed the reconstructed results by the SART, SART-WF and the proposed algorithms from different scanning angle. The images in the first column is the reference image, and the subsequent columns are the results reconstructed using the SART, SART-WF, and the proposed algorithm. The images from top to bottom are reconstructed results from $[0^\circ, 90^\circ]$, $[0^\circ, 120^\circ]$ and $[0^\circ, 150^\circ]$ scanning angle. The location of red arrows present some obvious artifacts. In addition, we used the same display window to display the reconstructed results of these three methods to make their comparison fairer, due to different images have different dynamic range of the discrete attenuation coefficients. And the display window is $[0 1]$ in this experiments. Fig. 4 is the error maps for the reconstructed images in Fig. 3 by different algorithm, with different scanning angle. Horizontal profile (123th row) of the reconstructed images shown in Fig. 3 with different scanning angle are given in Fig. 5, which can further illustrate the difference between different algorithms. From the Fig. 3, Fig. 4 and Fig. 5, it can be see that the reconstructed images quality increases as the scanning angle increases. And the slope artifacts presented in the SART images and SART-WF images have been suppressed by the proposed algorithm, in addition, the proposed algorithm can better preserve the edge structure and the small detail, which are seriously distorted in the results of other algorithms, with the same scanning angle.

Table III is the quantitative characterization of the reconstruction quality. As seen from Table III, the reconstructed images from the proposed algorithm were closest to the reference image according to the maximum PSNR, SSIM and the minimum RMSE compared to the other two algorithms.
FIGURE 5. Horizontal profile (123th row) of the reconstructed images shown in Fig. 3 by different algorithm, with different scanning angle.

TABLE III. Comparison of the PSNR, SSIM and RMSE of the reconstructed results by different reconstruction algorithms with different projection angles

<table>
<thead>
<tr>
<th>Scanning ranges</th>
<th>Algorithm</th>
<th>PSNR</th>
<th>SSIM</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 90°]</td>
<td>SART</td>
<td>22.4961</td>
<td>0.8650</td>
<td>0.0750</td>
</tr>
<tr>
<td></td>
<td>SART-WF</td>
<td>23.5952</td>
<td>0.9145</td>
<td>0.0661</td>
</tr>
<tr>
<td></td>
<td>SART-WF-GIF</td>
<td>28.0419</td>
<td>0.9499</td>
<td>0.0396</td>
</tr>
<tr>
<td>[0, 120°]</td>
<td>SART-WF</td>
<td>29.9675</td>
<td>0.9653</td>
<td>0.0317</td>
</tr>
<tr>
<td></td>
<td>SART-WF-GIF</td>
<td>34.4656</td>
<td>0.9853</td>
<td>0.0189</td>
</tr>
<tr>
<td>[0, 150°]</td>
<td>SART</td>
<td>33.5293</td>
<td>0.9289</td>
<td>0.0399</td>
</tr>
<tr>
<td></td>
<td>SART-WF</td>
<td>33.9843</td>
<td>0.9838</td>
<td>0.0200</td>
</tr>
<tr>
<td></td>
<td>SART-WF-GIF</td>
<td>39.7552</td>
<td>0.9946</td>
<td>0.0103</td>
</tr>
</tbody>
</table>

In some practical applications, the available projected data are mixed with noise. Hence, we evaluated the robustness of these three algorithm by adding the Gaussian noise \( \mu, \sigma^2 \) [31] to the projection data. In this experiment, the average value \( \mu \) is set to zero, and the variance \( \sigma^2 \) is set to 5. The key parameters of these three algorithms are respectively set as: The iterative steps of the SART algorithm were 20, 10 and 10 for the scanning ranges [0°, 90°], [0°, 120°] and [0°, 150°]. For the SART-WF method, the \( \lambda_s = 0.001 \), the \( \lambda_w = 0.0004 \) and the iterative steps were 900, 600 and 600 for the scanning ranges [0°, 90°], [0°, 120°] and [0°, 150°]. For the proposed algorithm, the \( \lambda_s = 0.001 \), the \( \lambda_w = 0.0004 \), \( \nu = 1 \) and \( \beta = 0.00015 \). The iterative steps were 3000, 3000 and 1200 for the scanning ranges [0°, 90°], [0°, 120°] and [0°, 150°].

TABLE IV. Comparison of the PSNR, SSIM and RMSE of the reconstructed results by different reconstruction algorithms with different projection angles with Gaussian noise

<table>
<thead>
<tr>
<th>Scanning ranges</th>
<th>Algorithm</th>
<th>PSNR</th>
<th>SSIM</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 90°]</td>
<td>SART</td>
<td>21.1332</td>
<td>0.7750</td>
<td>0.0878</td>
</tr>
<tr>
<td></td>
<td>SART-WF</td>
<td>23.5352</td>
<td>0.9055</td>
<td>0.0666</td>
</tr>
<tr>
<td></td>
<td>SART-WF-GIF</td>
<td>25.8581</td>
<td>0.9101</td>
<td>0.0509</td>
</tr>
<tr>
<td>[0, 120°]</td>
<td>SART-WF</td>
<td>29.7654</td>
<td>0.9548</td>
<td>0.0352</td>
</tr>
<tr>
<td></td>
<td>SART-WF-GIF</td>
<td>33.2350</td>
<td>0.9633</td>
<td>0.0218</td>
</tr>
<tr>
<td>[0, 150°]</td>
<td>SART</td>
<td>34.3265</td>
<td>0.9739</td>
<td>0.0195</td>
</tr>
<tr>
<td></td>
<td>SART-WF</td>
<td>37.1939</td>
<td>0.9772</td>
<td>0.0138</td>
</tr>
</tbody>
</table>

FIGURE 6. Horizontal profile (123th row) of the reconstructed images shown in Fig. 3 by different algorithm, with different scanning angle.

FIGURE 6. Reconstructed results of the NCAT image with Gaussian noise. The images in the first column is the reference image. The subsequent columns are the results reconstructed using the SART, SART-WF, and the proposed algorithm. The images from top to bottom are reconstructed results from [0°, 90°], [0°, 120°] and [0°, 150°] scanning angle. The location of red arrows present some obvious artifacts. The display window is [0 1].

FIGURE 7. Error maps for the reconstructed images shown in Fig. 6 by different algorithm, with different scanning angle.
Fig. 6 is the reconstructed results of the NCAT image with Gaussian noise, and the location of red arrows present some obvious artifacts. Fig. 7 is the Error maps for the reconstructed images shown in Fig. 6. Fig. 8 is the horizontal profile (123th row) of the reconstructed images shown in Fig. 6. From the Fig. 6, Fig. 7 and Fig. 8, it can be seen that the image quality degradation because the adding of Gaussian noise, and the proposed algorithm better protected the edge structure and anatomical detail than other algorithms. The results of SART present serious slope artifacts and noise which decreases the reconstructed images quality. The results of SART-WF show that the edges are distorted and blurred. Table IV showed the PSNR, SSIM and RMSE values of the noise-added NCAT phantom by different reconstruction algorithms with different projection angles. From this table, it can be seen that the proposed algorithm was superior to SART and SART-WF, in terms of slope artifacts suppression and edge preservation.

C. Real data study

Next, we used a real CT projection data to demonstrate the ability of these three algorithms in practical applications for limited-angle CT reconstruction. The projection data of a walnut [35] was used in this study, and it consists of various complex internal structures, which presenting some challenging tasks for limited-angle CT reconstruction. The projection data were obtained by utilizing the tube current 200 μA and the acceleration voltage of X-ray tube was 80 kV, its resolution was 2304 x 2296, and the angular step is 0.3 degrees evenly distributed over 360°. In addition, we used the middle of the two-dimensional projected images to form a fan-beam sinogram, which is equivalent to the central horizontal cross-section of the walnut. The geometry structure of the real CT system was the same as Ref. [35], and two types of limited-angle projection data were utilized: (1) [0, 120], 400 views with the angle increment is 0.3. (2) [0, 150], 500 projection views with the angle increment is 0.3. The resolution of the reconstructed image is 256 x 256. The key parameters of these three algorithms are respectively set as: The iterative steps of the SART algorithm were 7 and 7 for the scanning ranges [0,120] and [0,150]. For the SART-WF method, the $λ_e = 0.000001$, the $λ_H = 0.0000004$, the iterative steps were 300 and 200 for the scanning ranges [0,120] and [0,150]. For the proposed algorithm, the $λ_e = 0.000001$, the $λ_H = 0.0000004$, $r = 2$ and $λ = 0.000000143$. The iterative steps were 2000 and 1000 for the scanning ranges [0,120] and [0,150].

Fig. 9 presents the results reconstructed of the walnut image from the scan ranges [0,120] and [0,150]. As can be seen from the images above, the proposed algorithm is better than the other two algorithms on the visual quality. Under the circumstance of [0,150], the overall structures and details are better reconstructed by these three algorithms, nevertheless, the proposed algorithm has better performance than other algorithms in boundary protection, as indicated by the red arrow. In terms of noise reduction, the proposed algorithm is better than the other two
algorithms. When the scanning angle changed to $[0, 120]$, the quality of the reconstructed images is reduced, as indicated in Fig. 9. In summary, it is showed that the denoising ability of the proposed algorithm is better than that of SART and SART-WF, and the proposed algorithm preserved the detailed structures better than the other two algorithms.

IV. DISCUSSIONS AND CONCLUSIONS

In this paper, we developed a GIF based limited-angle CT reconstruction algorithm using wavelet frame. The proposed algorithm uses edge-preserving smoothing operator GIF to incorporate the results of the SART with the results constrained by the wavelet frame. Experimental results show that the proposed algorithm was superior to other iterative reconstruction algorithms in artifacts reduction, noise suppression and structure preservation.

In three representative experimental studies, the proposed algorithm has a noticeable effect over the SART and SART-WF method in slope artifacts reduction, noise suppression, and edge preservation for limited-angle CT reconstruction. The proposed algorithm also obtained the maximum PSNR, SSIM and the minimum RMSE compared to the other two algorithms. In addition, the horizontal profile and error maps clearly show that the proposed algorithm has advantages in structure preservation and artifacts reduction.

The proposed algorithm has many parameters need to be chose, and they were all adjusted by manually and empirically. A more detailed study of the parameters selection strategy is needed, and we will find an adaptive adjustment method in future work. In addition, the parameter setting will affect the convergence procedure of the proposed method. Because the step 2 of the proposed algorithm is a non-smooth and non-convex regularization method, and our method can only obtain a local optimal value rather than a global optimal value (i.e. our method does not really solve the NP-hard problem). Nevertheless, our method can work effectively in practical application by some experiments.

The proposed algorithm has a limitation that it need more iteration numbers than other iterative reconstruction algorithms. Because of the proposed algorithm need to use GIF to incorporate the results of the SART with the results constrained by the wavelet frame, and it will slows down the convergence rate of the algorithm. However, this limitation can be solved by the GPU acceleration technique.

In the future, we can extend the proposed algorithm to other topics, such as sparse sampling CT or low dose CT, or combining the proposed algorithm with the deep learning, or considering the weighted guided image filtering [39].

REFERENCES


