Optimal Path Planning-Based Finite-Time Control for Agile CubeSat Attitude Maneuver

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\textbf{ABSTRACT} A finite-time controller based on optimal path planning was developed for the agile CubeSat attitude maneuver. This achieved a stable attitude large angle maneuver just by using momentum wheels and a magnetorquer, which is unlike other larger satellites that need a control moment gyroscope or thruster. The proposed optimal path planning algorithm was based on a quintic polynomial with three path segments in order to decrease angular velocity in the attitude maneuver process as far as possible to improve system stability under the condition of satisfying CubeSat's limited control capability. And there are also an improved differential evolution (DE) algorithm for the optimization of path planning, in which the variation and crossover coefficients all obey the beta distribution. The finite-time controller was based on the nonsingular terminal sliding mode, and an improved PD type sigmoid function was proposed to replace the sign function. Finally, the physical simulation platform is built by using NJUST-2 as a object, the results revealed that the finite-time controller based on Beta DE path planning can decrease angular velocity in the attitude maneuver process as far as possible to improve system stability under the condition of satisfying CubeSat’s limited control capability, then achieve a stable attitude large angle maneuver just by using momentum wheels and a magnetorquer.

\textbf{INDEX TERMS} CubeSat; Attitude maneuver; Path planning; Quintic polynomial; Differential evolution; Finite-time control

\section{I. INTRODUCTION}

Recently, the study of attitude determination and control system (ADCS) for CubeSat remains mainly focused on three-axis stabilized earth oriented control, which can perform simple missions that does not require good attitude agility, such as platform verification, earth observation and science experiments \cite{1-4}.

However, with the rapid development of CubeSat, more mission requirement have been proposed to improve access to space, such as employing the unique features of CubeSat for science, exploration and space operations at a much lower cost than traditional satellites \cite{5-10}. For example, the CubeSat Proximity Operations Demonstration (CPOD) mission with a pair of 3U CubeSats will demonstrate rendezvous, proximity operations and docking in the future \cite{5,6}. Moreover, staring imaging \cite{8}, deep-space exploration \cite{9} and space debris tracking \cite{10} have also been proposed for CubeSat with game-changing potential. Nevertheless, these missions have only been previously performed by much larger and more sophisticated satellites \cite{5-10}. In the process of performing these missions in the future, the ADCS of CubeSat will play a key role, especially in the aspect of attitude agility, in order to achieve a large angle maneuver \cite{5-10}.

CubeSat’s motion is governed by kinematic and dynamic equations, but most of the references \cite{11} for CubeSat attitude kinematic and dynamic equations have not considered the situation that CubeSat’s angular velocity has influence on wheel control input torque. This would limit the precision of the control input torque model, since the agile CubeSat is always at a high angular velocity in the attitude maneuver process. Furthermore, the path planning algorithm for the CubeSat maneuver has remained unmentioned in most references, while some studies have been conducted in other fields \cite{12,13}. A path planning
algorithm based on quintic polynomial was presented for aircraft wing automatic positioning by Zhu [12], while a flight path planning algorithm based on quintic polynomial was proposed for unmanned aerial vehicles by Luitpold [13]. Moreover, Yao [14] and Albert [15] proposed a quintic polynomial path planning algorithm for the attitude maneuver of large complex satellites. However, there was only one path segment in these references. Hence, it cannot guarantee a smaller angular velocity under the condition of satisfying CubeSat’s constraints in control torque, so a larger angular velocity will result in attitude instability. Then, the present study proposes a path planning algorithm with three path segments (speed-up, constant speed, and speed-down), and makes a time optimization using Beta DE. For DE, some studies have been conducted [16-18]. Borko [16] and Swagatam [17] proposed a self-adaptive approach for control parameters, in which the variation and crossover coefficients were random numbers that followed a uniform distribution. In addition, for the variation and crossover coefficients, normal (gauss) distribution [18] was also proposed. Nevertheless, although these coefficients can produce random numbers to maintain the group’s diversity, these cannot guarantee a suitable value for coefficients in different stages. For the finite-time controller, some references [19-31] have presented different control methods. For instance, Xu [24] proposed an adaptive sliding mode controller, and Junquan [25] presented a nonsingular fast terminal sliding mode controller. In addition, Haichao [29], Binglong [30] and Yunhua [31] proposed different sliding mode controllers for the attitude maneuver of large complex satellites. However, the chattering phenomenon continued to exist in these sliding mode controllers, and the gain coefficient of the sliding surface cannot be adaptive to different attitude maneuver missions in these controllers [29-31]. These problems limit the controllers’ performance.

The present study provides an attitude maneuver control algorithm for the agile CubeSat, which can decrease angular velocity in the attitude maneuver process as far as possible to improve system stability under the condition of satisfying CubeSat’s limited control capability, then achieve a stable attitude large angle maneuver just by using momentum wheels and a magnetorquer.

The main contributions of this algorithm relative to others are as follows:

1. An optimal path planning algorithm based on quintic polynomial with three path segments using Beta DE is proposed, in order to decrease angular velocity in the attitude maneuver process as far as possible to improve system stability under the condition of satisfying CubeSat’s limited control capability

2. Compared with Gauss DE [18], Beta DE improves 50% in the aspect of evolution speed, in which variation and crossover coefficients all obey the beta distribution. This not only guarantees a suitable value for coefficients in different stages, but also produces a random number to maintain the group’s diversity.

3. An improved attitude tracking model was proposed, in which the situation were CubeSat’s angular velocity has influence on the wheel control input torque was considered. Because the angular velocity cannot be ignored in attitude maneuver process.

4. Compared with the adaptive sliding mode controller [24], the sign function is replaced by a sigmoid function, which alleviates chattering phenomenon in the control process.

II. MODELING

The CubeSat was modeled as a rigid body with momentum wheels that produce control input torques of approximately three mutually perpendicular axes, and define a body-fixed frame B. The kinematic and dynamic equations of CubeSat can be expressed as [24]:

\[ \dot{q} = \frac{1}{2} q \otimes \omega \]

\[ I \dot{\omega} = -\omega \times (I \omega + h_o) + u + u_d \]

where \( q = [q_0, q_1, q_2, q_3]^T \) represents the attitude quaternion of CubeSat in body frame B with respect to the inertial frame, with \( q_0 \) being scalar component, \( q_1, q_2, \) and \( q_3 \) being the vector component; \( \omega \in \mathbb{R}^3 \) refers to the angular velocity with respect to an inertial frame and expressed in the body frame; \( I \in \mathbb{R}^{3x3} \) is moment of inertia of CubeSat; \( u \in \mathbb{R}^3 \) represents the control torque; \( u_d \in \mathbb{R}^3 \) is the disturbance torque; \( h_o \in \mathbb{R}^3 \) refers to the angular momentum of the wheels. \( \chi^* \in \mathbb{R}^{3x3} \) is the cross product matrix, and for vector \( x = [x_r, x_i, x_j]^T \) it is given by:

\[
\chi^* = \begin{bmatrix}
0 & -x_i & x_j \\
-x_i & 0 & -x_r \\
x_j & x_r & 0 \\
\end{bmatrix}
\]

The attitude tracking kinematic equation is given by:

\[
\dot{q}_e = \frac{1}{2} q_e \otimes \omega_e
\]

where \( \overline{q}_e = [q_{e_0}, q_{e_1}, q_{e_2}, q_{e_3}]^T = \overline{q}_d^{-1} \otimes q \) refers to the error quaternion with respect to an expected frame D, and is expressed in body frame B, with \( q_{e_0} \) being scalar component, and \( q_{e_1}, q_{e_2}, q_{e_3} \) being vector component, and \( \overline{q}_d = [0, \omega_d]^T \) represents the extended error angular velocity, with \( \omega_d \) being error angular velocity with respect to an expected frame D, and expressed in the
where $F^{-1} = \frac{1}{2}(q_e^* + q_e I_3)$. 

The dynamic model of CubeSat’s attitude tracking can be expressed as:

$$
\frac{d}{dt}(I_3 \omega + \dot{h}_s) = \frac{d}{dt}[I_3 (\omega + A(q_3) \omega_3) + \dot{h}_s] + I_3 \frac{d}{dt}[\omega_3 + A(q_3) \omega_3] + h_s
$$

where $h_s = I_w (C \delta + \omega) = I_w [C \delta + \omega_e + A(q_3) \omega_3]$ and $I_3 = I - C_l C_l$.

Since CubeSat’s angular velocity is always large in the attitude maneuver process, it is necessary to consider the influence of CubeSat’s angular velocity to the wheel control input torque. This would allow the control torque from the momentum wheels to be more precise. Notably, the momentum wheels’ model is given as follows:

$$
\begin{align*}
M_i \dot{q}_i + M_i \dot{q}_i + M_i \dot{q}_i = u + u_m
\end{align*}
$$

where $u_m \in R^3$ represents the magnetorquer control torque; $I_w \in R^4$ is moment of inertia of the wheels; is the speed of the wheels; $M \in R^3$ is the magnetic moment vector; $\beta \in R^3$ is the local magnetic field vector. Furthermore, the disturbance torque is $u_d$, in which $T_d$ is the external torque, and $T_{cd}$ is the disturbance torque.
of the wheels.

For the distribution matrix of wheel $C$ given by Equation (9), there are a number of forms. However, two distribution forms were the most popular, four skew momentum wheels, and three orthogonal momentum wheels with one being skew [4,34]. When the four momentum wheels were all skew, the installing configuration was given in Figure 1, in which the axis of each wheel was tilted 54.74 degrees relative to the roll axis in body frame $B$, and tilted 45 degrees relative to the pitch axis or yaw axis in body frame $B$. In this case, $C$ would be defined as:

$$C = \begin{bmatrix}
\cos \theta_j & \cos \theta_j & \cos \theta_j & \cos \theta_j \\
\sin \theta_j & \sin \theta_j & -\sin \theta_j & -\sin \theta_j \\
\sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} \\
\sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2}
\end{bmatrix}$$

where $\theta_j = 54.74 \pi / 180$.

When the three momentum wheels were orthogonal, and the other one was skew (as shown in Fig. 1b), will be defined as:

$$C = \begin{bmatrix}
1 & 0 & 0 & \sqrt{3} / 3 \\
0 & 1 & 0 & \sqrt{3} / 3 \\
0 & 0 & 1 & \sqrt{3} / 3
\end{bmatrix}$$

![FIGURE 1. Installing configuration of momentum wheels](image)

### III. BETA DE PATH PLANNING

There were some constraints for angular velocity and control torque for the limitation of the CubeSat actuator’s performance. Notably, a larger angular velocity would lead to more heavy vibration and lower system stability during the maneuver process. Therefore, in order to decrease angular velocity as far as possible to improve system stability under the condition of satisfying CubeSat’s limited actuator’s performance, the three path segments were designed. Then the constant speed stage can decrease maximum angular velocity. However, in the meantime, it necessary to satisfy CubeSat’s limited actuator’s performance, so an optimal algorithm should be designed.

A Beta DE path planning algorithm was designed in the present, because the quintic polynomial algorithm can satisfy these limitations through the time optimization of the constant speed period using Beta DE. There were three stages: acceleration, constant speed and deceleration. The stages of acceleration and deceleration can guarantee a smooth and continuous angular acceleration during the maneuver process. Furthermore, the time optimization of the constant speed period using Beta DE can guarantee a lower angular velocity under the condition of satisfying the CubeSat actuator’s constraints.

#### A. PATH PLANNING

The quintic polynomial equation is [12-15]:

$$q_p(t)=a_0+a_1t+a_2t^2+a_3t^3+a_4t^4+a_5t^5$$

(10)

where $t$ represents time, and $a_0, a_1, a_2, a_3, a_4, a_5$ are coefficients of the quintic polynomial.

For the attitude maneuver, Equation (10) can be replaced with:

$$q_0 = a_0, q_f = a_1, \dot{q}_0 = 2a_2, \ddot{q}_0 = a_3$$

$$q_f = a_4 + a_5f + a_6f^2 + a_7f^3 + a_8f^4 + a_9f^5$$

$$\ddot{q}_f = 2a_0 + 6a_1f + 12a_2f^2 + 20a_3f^3$$

(11)

where $q_0$ and $q_f$ represent the initial and final attitude quaternion of the attitude maneuver, and $C$ is the time of attitude maneuver.

In solving Equation (11), the following results can be obtained:

$$a_0 = q_0, a_1 = q_f, a_2 = \dot{q}_0/2$$

$$a_3 = [20q_0 - 20q_0 - (8\ddot{q}_f + 12q_f)/(3\ddot{q}_f - \ddot{q}_f)]/2t_f$$

$$a_4 = [30q_0 - 30q_0 + (14q_f + 16q_0)/(3q_0 - 2\ddot{q}_f)]/2t_f$$

$$a_5 = [12q_f - 12q_f - (6\ddot{q}_f + 6q_0)/(\ddot{q}_f - \ddot{q}_f)]/2t_f$$

(12)

The time nodes vector of the attitude maneuver is given as:

$$t = [t_a, t_a + t_c, t_f]^T$$

where $t_a = (1/2)(1 - \rho)t_f$ is the time of acceleration or deceleration period; $t_c = \rho t_f$ is time of constant speed period; $\rho$ is the proportion of constant speed period, which satisfies $0 < \rho < 1$.

In view of Equations (10), (11) and (12), the coefficients of the quintic polynomial in the first stage are solved to be:

$$a_0 = q_0, a_1 = \dot{q}_0, a_2 = \ddot{q}_0/2$$

$$a_3 = 2(\dddot{q}_0 - \dddot{q}_0)/t_a - \ddot{q}_0/t_a$$

$$a_4 = 2(\dddot{q}_0 - \dddot{q}_0)/t_a + 3\dddot{q}_0/4t_a^2$$

$$a_5 = 3(\dddot{q}_0 - \dddot{q}_0)/5t_a^2 - \dddot{q}_0/5t_a^2$$

(13)

In view of Equations (10), (11) and (12), the coefficients of the quintic polynomial in the second stage are solved to be:
\[ a_0 = q_1, a_1 = \dot{q}_1, \]
\[ a_2 = a_3 = a_4 = a_5 = 0 \]  \quad (14)

In view of Equations (10), (11) and (12), the coefficients of the quintic polynomial in the third stage are solved to be:
\[
\begin{align*}
    a_0 &= q_1, a_1 = \dot{q}_1, a_2 = 0, a_3 = 0 \\
    a_4 &= (q_j - \dot{q}_j) / t_j^5 - \ddot{q}_j / 4t_j^3 \\
    a_5 &= 3(q_j - \dot{q}_j) / 5t_j^4 + \dddot{q}_j / 5t_j^2
\end{align*}
\]  \quad (15)

where \( q_1 = q(t_0) = q_0 + (0.4\dot{q}_0 + 0.6\ddot{q}_0)t_0 + 0.05\dddot{q}_0t_0^2 \) represents the initial attitude quaternion of constant speed period, and its time derivative is \( \dot{q}_1 = \dot{q}(t_0) = [20(q_j - \dot{q}_j) - 8t_j(\dot{q}_j + \ddot{q}_j) + t_j^2(\dddot{q}_j - \dddot{q}_0)] / (24t_j^2 + 20t_0) \).

Furthermore, \( q_2 = q_1 + \dot{q}_1 t_0 \) represents the initial attitude angle of the deceleration period, and its time derivative is \( \dot{q}_2 = \dot{q}(t_0) \).

Using the coefficients from Equations (13), (14) and (15), the ultimate expected attitude is given by:
\[
\begin{align*}
    q_j &= a_i t_j^5 + a_i t_j^4 + a_i t_j^3 + a_i t_j^2 + a_i t_j + a_0 \\
    \ddot{q}_j &= 5a_i t_j^4 + 4a_i t_j^3 + 3a_i t_j^2 + 2a_i t_j + a_0 \\
    \dddot{q}_j &= 20a_i t_j^3 + 12a_i t_j^2 + 6a_i t_j + 2a_i
\end{align*}
\]  \quad (16)

Substituting Equation (16) into Equations (3) and (4), the error quaternion \( q_\epsilon \) and the error angular velocity \( \omega_\epsilon \) can be obtained.

**B. BETA DE ALGORITHM**

For the DE algorithm, the optimized object is the proportion of the constant speed period \( \rho \), and the fitness function is:
\[
f_i = w_1 \max(|\omega(t)|) + w_2(u_{\text{max}} - \max(|\omega(t)|)) + w_3 \sum_{t=0}^{t_f} |q(t)| \]  \quad (17)

where \( w_1, w_2 \) and \( w_3 \) are the fitness function gain coefficients, and \( \max(\cdot) \) represents the maximum in the range of \( 0 \leq t \leq t_f \), then
\[
|\omega| \leq \omega_{\text{max}} \\
|\omega| \leq u_{\text{max}}
\]

The initial group can be given by [17]:
\[
x_k(0) = \text{rand}(0,1)[(\text{max}(x_k) - \text{min}(x_k))] + \text{min}(x_k) \]  \quad (18)

where \( x_k(0) \) represents the \( k \)-th individual of the initial group, the individual \( x_k \) is the proportion of the constant speed period \( \rho \); \( \text{rand}(0,1) \) is a random number within the limit of \([0,1] \); \( \text{max}(x_k) \) is the maximum of \( x_k \); \( \text{min}(x_k) \) is the minimum of \( x_k \).

The variation proceeding can be expressed as:
\[
h_k(t_g + 1) = x_k(t_g) + F[x_k(t_g) - x_i(t_g)] \]  \quad (19)

where \( h_k(t_g + 1) \) represents the \( k \)-th individual with a variation, \( t_g \) is the evolution generation times. \( F \) is the variation coefficient, and it is given by:
\[
F = \begin{cases}
0.5 + 0.4^* \text{beta}(40, g) & t_g < \frac{g}{2} \\
0.5 + 0.4^* \text{beta}(40, 80(g - t_g)) & t_g > \frac{g}{2}
\end{cases} \]  \quad (20)

where \( \text{beta} \) represents the random number following the beta distribution; \( g \) is total generation time; \( t_g \) is the real generation time. At the early searching stage of the algorithm, the value of the variation coefficient should be larger, in order to enlarge the search space and maintain the group’s diversity. At this point, the beta distribution belongs to the left-skewed distribution through the coefficient adjustment. On the contrary, at the later searching stage of the algorithm, the value of the variation coefficient should be smaller, in order to make the search space smaller and improve searching precision. At this point, the beta distribution becomes a right-skewed distribution through the coefficient adjustment. Therefore, it can guarantee a suitable value for the variation coefficient in different stages, and produce a random number to maintain the group’s diversity.

The crossover proceeding is given by:
\[
v_i(t_g + 1) = \begin{cases}
    h_i(t_g + 1) & \text{rand}_l < cr \\
    x_i(t_g) & \text{rand}_l > cr
\end{cases} \]  \quad (21)

where \( v_i(t_g + 1) \) represents the \( k \)-th individual with the crossover at the \((t_g+1)_h\) generation time; \( \text{rand}_l \) represents a random number within a range of \([0,1]\); \( cr \) represents the crossover coefficient, which is given by:
\[
\begin{align*}
    cr &= \begin{cases}
        0.5 + 0.4^* \text{beta}(40, 0, g) & t_g < \frac{g}{2} \\
        0.5 + 0.4^* \text{beta}(40, 80(g - t_g)) & t_g > \frac{g}{2}
    \end{cases}
\end{align*} \]  \quad (22)

Similar with the variation coefficient, the crossover coefficient also obeys the beta distribution. However, at the early searching stage of the algorithm, the value of the crossover coefficient should be smaller, in order to prevent the premature convergence problem. At this point, the beta distribution belongs to the right-skewed distribution through the coefficient adjustment. Conversely, at the later searching stage of the algorithm, the value of the crossover coefficient should be larger to improve the convergence rate. At this point, the beta distribution becomes a left-skewed distribution through the coefficient adjustment. Hence, this not only guarantees the appropriate value for the crossover coefficient in different stages, but also produces a random number to maintain the group’s diversity.

The selecting proceeding can be obtained, as follows:
\[
x_i(t_g + 1) = \begin{cases}
    v_i(t_g + 1) & f_i(v_i(t_g + 1)) < f(x_i(t_g)) \\
    x_i(t_g) & f_i(v_i(t_g + 1)) \geq f(x_i(t_g))
\end{cases} \]  \quad (23)
The differences between Beta DE and Gauss DE are presented in Figure 2, which are in the aspect of the proportion of the constant speed period $\rho$, fitness function $f_i$, variation coefficient $F$, and crossover coefficient $cr$, respectively. The plots show that the convergence of $\rho$ and $f_i$ occurred at the 4th generations for Gauss DE, whereas convergence of $\rho$ and $f_i$ occurred at the 2nd generations for Beta DE. Compared with Gauss DE, Beta DE improves 50% of the evolution speed.

![FIGURE 2. The difference between Beta DE and Gauss DE](image)

**IV. FINITE-TIME CONTROLLER**

The attitude maneuver time for CubeSat should be optimal. Therefore, it requires a better convergence performance, when compared to other control missions. It has been verified that the terminal sliding mode controller has a better convergence performance than other controllers [35]. Therefore, a finite time controller was designed in the present study. The controller was based on the nonsingular terminal sliding mode. However, the chattering phenomenon continued to exist in the controller. Hence, the sigm function was designed in the present study.

The nonsingular terminal sliding surface is given by [23]:

$$s = q_s + \frac{1}{\beta} \text{sg}(\omega) \eta$$

(24)

where $\beta$ represents the positive gain coefficient. $\text{sg}(\omega) \eta$ satisfies:

$$\text{sg}(\omega) \eta = [\|\omega(1)\|^2 \text{sg}(\omega(1)) \, \|\omega(2)\|^2 \text{sg}(\omega(2)) \, \|\omega(3)\|^2 \text{sg}(\omega(3))]$$

(25)

where $p$ and $q$ are positive odd numbers, and these satisfy $1 < p/q < 2$; $\text{sg}(\bullet)$ (namely sigm function) is the improved PD type sigmoid function that would replace the sign function for a vector $\Omega \in \mathbb{R}^i$, which is given by:

$$\text{sgm}(3(i)) = \frac{P[1 - E^{-(\omega(3)(i))}] + D[1 - E^{-(\omega(3)(i))}]}{[1 + E^{-\omega(3)(i)}]} \quad i = 1, 2, 3$$

(26)

At this point, an expanded function of Equation (26) can be given by:

$$\text{Sigm}(3) = [\text{sgm}(3(1)) \, \text{sgm}(3(2)) \, \text{sgm}(3(3))]$$

(27)

where $P$ and $D$ are the proportional and differential coefficients of the sigm function, respectively; $a$ and $b$ are the sigm function coefficients; $E$ represents the natural base number. Indeed, the sigmoid function can alleviate the chattering phenomenon to a certain extent. However, in order to improve its performance in alleviating the chattering phenomenon, a numerator item and differential item are added. It is noteworthy that the addition of a numerator item can increase the sensitivity of the vector $\Omega$. In addition, the time derivative of the vector $\Omega$ becomes a judgment factor with the addition of the differential item. Therefore, when the components of vector $\Omega$ approaches 0, but the time derivative of the components of vector $\Omega$ has larger values, the value of the sigm function is decided through the differential item. On the contrary, when the components of vector $\Omega$ stay away from 0, the value of the sigm function is decided through the proportional and differential items.

The reaching law can be expressed as:

$$T_s = \dot{s} = -k_s \dot{s} - (d + \eta)\text{Sigm}(s)$$

(28)

where $k_s > 0$, $d\text{Sigm}(s) > T_s$ and $\eta > 0$ are the reaching law coefficients.

In view of Equations (9), (24), (25), (26), (27), and (28), the finite-time controller is given by:

$$u = -M_q \beta(q/p)\text{sg}(\dot{q}) \eta^2 q_s + M_q \dot{q}_s + M_4 q_s + I_s T_s$$

(29)

Magnetorquers will supply a compensating torque in case of momentum wheel torque or rotational speed saturation, and the compensation for the magnetic torque can be expressed as [24]:

$$u_w = M \times B$$

(30)

where $M = [M_1 \quad M_2 \quad M_3]$ is the local magnetic field vector; $u_w(i) > 2mNm$ refers to every momentum wheel’s torque saturation condition; $v_w(i) > 6200rpm$ refers to every momentum wheel’s rotational speed saturation condition; $|M(i)| \leq 0.2 Am^2$ and $(B \cdot h_c)/(|\|\|e_c\|\| |B| |h_c|) < 0.707$ represent the magnetorquer’s working conditions. In addition, $k_s$ is the gain coefficient of magnetorquer control law.

**V. CONTROL SYSTEM ANALYSIS**

**A. STABILITY ANALYSIS**

The Lyapunov function can be defined as:

$$Z = \frac{1}{2} \dot{s}^T s$$

(31)
From Equations (8), (9), (24), (25), (26), (27), (28), (29), and (30), the time derivative of the sliding surface can be obtained as:

\[ \dot{s} = \dot{\varphi} + \frac{p}{\beta q} \text{diag}(\mathrm{sg}(\omega)) \left( \frac{L_1^{(1)}}{q} \right) \dot{\varphi} \]

\[ = \dot{\varphi} + \frac{p}{\beta q} \text{diag}(\mathrm{sg}(\omega)) \left( \frac{L_1^{(1)}}{q} \right) I_1^{(-1)} \left[ I, \omega_0 A(q_0) \omega_0 - I, A(q_0) \omega_0 - \omega_0 I, \omega_0 - \omega_0 I, A(q_0) \omega_0 - A(q_0) \omega_0 I, \omega_0 - A(q_0) \omega_0 I, A(q_0) \omega_0 + u + T_s \right] \]

\[ = \dot{\varphi} + \frac{p}{\beta q} \text{diag}(\mathrm{sg}(\omega)) \left( \frac{L_1^{(1)}}{q} \right) I_1^{(-1)} \left[ I, \omega_0 A(q_0) \omega_0 - I, A(q_0) \omega_0 - \omega_0 I, \omega_0 - \omega_0 I, A(q_0) \omega_0 - A(q_0) \omega_0 I, \omega_0 - A(q_0) \omega_0 I, A(q_0) \omega_0 + I, T_s + T_e \right] \]

\[ = \dot{\varphi} + \frac{p}{\beta q} \text{diag}(\mathrm{sg}(\omega)) \left( \frac{L_1^{(1)}}{q} \right) I_1^{(-1)} \left[ \frac{1}{2} (q_s + q_s \omega_0) \varphi I, \omega_0 + \frac{q}{p} \text{diag}(\omega_0) \left( \frac{L_1^{(1)}}{q} \right) I_1^{(-1)} [-k, I, s - (d + \eta) I, \text{Sign}(s) + T_s] \right] \]

\[ = \frac{1}{2} (q_s + q_s \omega_0) \varphi I, \omega_0 + \frac{q}{p} \text{diag}(\omega_0) \left( \frac{L_1^{(1)}}{q} \right) I_1^{(-1)} [-k, I, s - (d + \eta) I, \text{Sign}(s) + T_s] \]

\[ = -\frac{p}{\beta q} \text{diag}(\omega_0) \left( \frac{L_1^{(1)}}{q} \right) I_1^{(-1)} I, s - (d + \eta) I, \text{Sign}(s) + T_s] \]

In view of Equations (24), (25), (26), (27), (31), and (32), the time derivative of the Lyapunov function can be given by:

\[ \dot{Z} = s^T \dot{s} \]

\[ = s^T \left[ -\frac{p}{\beta q} \text{diag}(\omega_0) \left( \frac{L_1^{(1)}}{q} \right) I_1^{(-1)} I, s - (d + \eta) I, \text{Sign}(s) + T_s \right] \]

\[ = -s^T \frac{p}{\beta q} \text{diag}(\omega_0) \left( \frac{L_1^{(1)}}{q} \right) I_1^{(-1)} I, s - (d + \eta) I, \text{Sign}(s) + T_s \]

Since \( p \) and \( q \) are positive odd numbers, and these satisfy \( 1 < p/q < 2 \), therefore:

\[ \text{sg}(\omega_0) \left( \frac{L_1^{(1)}}{q} \right) > 0 \quad (\omega_0(i) \neq 0) \]

Then, it can be observed that \( \text{diag}(\omega_0) \left( \frac{L_1^{(1)}}{q} \right) \) is a positive diagonal matrix, and \( \lambda_{\text{min}}(\text{diag}(\omega_0) \left( \frac{L_1^{(1)}}{q} \right)) \) represents the minimum eigenvalue of the diagonal matrix \( \text{diag}(\omega_0) \left( \frac{L_1^{(1)}}{q} \right) \). In addition, it is known that \( k > 0 \), \( \beta > 0 \), \( \eta > 0 \), and \( d\text{Sign}(s) \leq T_s \).

Therefore, in view of lemma 1, when \( \omega_0(i) \neq 0 \), \( \dot{Z} \) satisfies:

\[ \dot{Z} \leq -s^T \frac{p}{\beta q} \text{diag}(\omega_0) \left( \frac{L_1^{(1)}}{q} \right) I_1^{(-1)} I, s - (d + \eta) I, \text{Sign}(s) + T_s \]

\[ \leq -\frac{p}{\beta q} \lambda_{\text{min}}(\text{diag}(\omega_0) \left( \frac{L_1^{(1)}}{q} \right)) I_1^{(-1)} I, s - (d + \eta) I, \text{Sign}(s) + T_s \]

\[ \leq -\frac{p}{\beta q} \lambda_{\text{min}}(\text{diag}(\omega_0) \left( \frac{L_1^{(1)}}{q} \right)) I_1^{(-1)} (s - (d + \eta) I, \text{Sign}(s) + T_s) \]

\[ \leq 0 \quad \text{(34)} \]
where \( \dot{Z} \leq 0 \) satisfies the Lyapunov function stability condition.

**B. FINITE-TIME ANALYSIS**

Lemma 1 [34]: Suppose that a continuous positive definite function \( Z(t) \) exists, which satisfies the following differential inequality:

\[
\dot{Z}(t) + \mu Z(t) < 0
\]

where \( \mu > 0 \), \( 0 < \nu < 1 \), and \( t_0 \) represents the initial time. Then, \( Z(t) \) converges to the equilibrium point in finite time, and \( Z(t) = 0 \). At this point, the time is given by:

\[
t_1 \leq \frac{Z(t_0)}{\mu(1-\nu)}
\]

where \( Z(t_0) \) is the initial value of \( Z(t) \).

According to Equation (34), when \( \omega_i(i) \neq 0 \), \( \dot{Z} \) satisfies:

\[
\dot{Z} \leq -\frac{P}{\beta q} \lambda_{\min}(\text{diag}(sgn(\omega_j)))\eta||e||
\]

\[
\leq -\eta \frac{P}{\beta q} \lambda_{\min}(\text{diag}(sgn(\omega_j)))\sqrt{2Z}
\]

According to lemma 1, when \( \omega_i(i) \neq 0 \), the time to arrive to the sliding surface can be given by:

\[
t_s \leq \frac{\eta}{\beta q} \lambda_{\min}(\text{diag}(sgn(\omega_j)))
\]

where \( s^0 \) is the initial value of \( s \).

When \( \omega_j(i) = 0 \), the time derivative of \( \omega_j \) is:

\[
\dot{\omega}_j = \frac{1}{2}(q^*_j + q_j, \dot{I}_j)\omega_j, \beta \frac{q}{p} sgn(\omega_j) \eta \dot{s} - k_s - (d + \eta) \text{sgn}(s) + T_j
\]

At this point, \( s(i) \neq 0 \) and

\[
\dot{\omega}_j = -k_s s(i) - (d + \eta) \text{sgn}(s(i)) + T_j(i) \neq 0 \quad i = 1, 2, 3
\]

Hence, \( \omega_j(i) = 0 \) is not attractive, and the time arriving to sliding surface is also finite.

In summary, the finite-time control system is finite time stable.

**VI. RESULTS**

The physical simulations and tests were based on NJUST-2, which was launched at the Cape Canaveral Air Force Station in Florida on April 18, 2017. The International Geomagnetic Reference Field (IGRF-12) model was used to provide the simulated geomagnetic field. A Runge-Kutta algorithm was used for the iteration of attitude dynamics of the satellite. As shown in Figure 3, the ADCS of NJUST-2 includes a CPU, momentum wheels, magnetorquers, magnetometer, and gyroscope. Moreover, it can be observed that three wheels were orthogonal, and the other wheel was skewed. In addition, the three-axis magnetorquer includes one air core coil and two iron core coils.

The actuators of the NJUST-2 ADCS include four momentum wheels and three-axis magnetorquers, which play different roles, respectively. The control input torque can be mainly produced through the four momentum wheels. At the same time, the three-axis magnetorquer provides a compensating magnetic torque for the wheels, when the speed or control torque of these wheels is saturated. Notably, the test result of wheels at the start-up stage (Fig. 4) indicates that wheel speed can be increased to 6,200 rpm from 0 rpm within 0.5 seconds. Therefore, these wheels can provide an acceleration value of 1,299 rad/s², which corresponds to a control torque value of 4.13 mNm. In order to guarantee enough controllable margins and a long working time for the wheels, the expected maximum control torque value can be set to be 0.05 mNm (Table 1). In addition, the three-axis magnetorquer can provide a rated magnetic moment value of 0.2 Am² (Table 1), which was obtained through experimental test.
### TABLE I
PARAMETERS OF NJUST-2

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass, kg</td>
<td>2.3</td>
</tr>
<tr>
<td>Size, mm³</td>
<td>112.8<em>112.3</em>227.6</td>
</tr>
<tr>
<td>Orbit type</td>
<td>Sun synchronous orbit</td>
</tr>
<tr>
<td>Orbit altitude, km</td>
<td>500</td>
</tr>
<tr>
<td>Orbit inclination angle, deg</td>
<td>97.4</td>
</tr>
<tr>
<td>Orbit period, min</td>
<td>94.5</td>
</tr>
<tr>
<td>Orbit angular velocity, rad/s</td>
<td>0.0011</td>
</tr>
</tbody>
</table>
| Moment of inertia satellite, kg·m² | \[
    \begin{bmatrix}
    3.735 \times 10^{-3} & -1.755 \times 10^{-6} & -4.026 \times 10^{-5} \\
    -1.755 \times 10^{-6} & 1.282 \times 10^{-2} & 1.191 \times 10^{-5} \\
    -4.026 \times 10^{-5} & 1.191 \times 10^{-5} & 1.273 \times 10^{-2}
    \end{bmatrix}
\]                                      |
| Moment of inertia wheels, kg·m² | diag (3.18, 3.18, 3.18, 3.18) \times 10^{-6} |
| Wheels’ maximum speed, rpm  | 6200                                        |
| Wheels’ expected maximum control torque, mNm | 0.05                                         |
| Magnetorquer rated magnetic moment, Am² | 0.2                                          |

### TABLE 2
PHYSICAL SIMULATION PARAMETERS OF NJUST-2

<table>
<thead>
<tr>
<th>Parameters (roll, pitch, yaw)</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attitude maneuver time, s</td>
<td>10</td>
</tr>
<tr>
<td>Attitude maneuver angle, deg</td>
<td>60</td>
</tr>
<tr>
<td>Initial attitude angle, deg</td>
<td>[0.5 \ -0.5 \ 0.5]^T</td>
</tr>
<tr>
<td>Initial angular velocity, deg/s</td>
<td>[0.05 \ 0.05 \ 0.05]^T</td>
</tr>
<tr>
<td>Initial angular acceleration, deg/s²</td>
<td>[0.001 \ 0.001 \ 0.001]^T</td>
</tr>
<tr>
<td>Control period, s</td>
<td>1</td>
</tr>
<tr>
<td>Noise of wheels’ speed, rpm</td>
<td>±6</td>
</tr>
<tr>
<td>Noise of magnetorquer, %</td>
<td>±2</td>
</tr>
<tr>
<td>Noise of magnetometer, nT</td>
<td>±50</td>
</tr>
</tbody>
</table>
| Distribution matrix of wheels, kg·m² | \[
    \begin{bmatrix}
    1 & 0 & 0 \\
    0 & 1 & 0 \\
    0 & 0 & 1
    \end{bmatrix}
\]                                      |
| Residual magnetism disturbance torque, Nm | [1 0.1 1]^T \times 10^{-7} |
| Aerodynamic disturbance torque(ω₀ is orbit angular velocity), Nm | [3\cos(ω₀t) 1.5\sin(ω₀t)+3\cos(ω₀t) 3\sin(ω₀t)]^T \times 10^{-8} |
| Value of coefficient β        | [0.2 \ 0.2 \ 0.2]^T                         |
| Coefficients of sliding surface p and q | 7, 5                                        |
| Coefficients of sign function P, D, a, and b | 1, 0.001, 6\times 10^4, 6\times 10^4        |
| Coefficients of reaching law k1, d, and η | 1.5, 1.3\times 10^{-7}, 1\times 10^{-8}     |
| Coefficient of magnetorquer control law | 0.001                                       |

As shown in Figure 5, it is the physical simulation platform for NJUST-2. The physical simulation platform include ADCS CPU, Dynamics Simulation Computer, Magnetic Torquer, Reaction Wheels, Data Collecting System, Sensors Simulator, Battery and Power Board, etc.

As shown in Figure 6, it is the flow diagram of physical simulation process. Dynamic simulation computer is used to construct the whole CubeSat dynamic system. Its input includes the speed of momentum wheels and the feedback information of magnetic torquer, and its output includes the actual attitude information of the CubeSat. Actuators are now fully physical access. The feedback information of magnetic torquer and the speed information of momentum wheels are collected through the data collecting system, and then the updated attitude information is carried out by the dynamic simulation computer. At present, the sensors simulator is used in
the physical simulation to simulate the CubeSat sensor's attitude measure information.

**FIGURE 5. NJUST-2 attitude control system and its physical simulation platform**

**FIGURE 6. The flow diagram of physical simulation process**

In Figures 7-10, it can be observed that in the process of 100 and 60 degrees’ maneuver, the control torques all satisfy the expected maximum limitation, and obviously, comparing with quintic polynomial path planning, Beta DE path planning has a smaller attitude angle error. In the process of 60 degrees’ maneuver, the maximum angular velocity is -2.7 degrees/s for quintic polynomial path planning, however, the maximum angular velocity is -1.8 degrees/s for Beta DE path planning. So comparing with quintic polynomial path planning, Beta DE path planning decreases 0.9 degrees/s in the aspect of angular velocity.

For quintic polynomial path planning, it just has one path segment. Therefore, it can decrease angular velocity just by means of increasing maneuver time. However, increasing maneuver time is unwanted for attitude maneuver missions.

For Beta DE path planning algorithm, it can decrease angular velocity as far as possible, in the meantime, it can also satisfy CubeSat’s limited actuator’s performance through the time optimization of the constant speed period using Beta DE. Additionally, the stages of acceleration and deceleration can guarantee a smooth and continuous angular acceleration during the maneuver process.

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FIGURE 7. The physical simulation results of Beta DE path planning (60 degrees' maneuver)

FIGURE 8. The physical simulation results of quintic polynomial path planning [12-15] (60 degrees’ maneuver)
FIGURE 9. The physical simulation results of Beta DE path planning (100 degrees’ maneuver)

FIGURE 10. The physical simulation results of quintic polynomial path planning [12-15] (100 degrees’ maneuver)
VI. CONCLUSION

Despite the fact that the study of ADCS for CubeSat remains mainly focused on the three-axis stabilized earth oriented control, the CubeSat needs to perform more significant missions in the future, such as rendezvous, docking, staring imaging, deep-space exploration, space debris tracking, etc. These missions require the CubeSat to possess good attitude agility, in order to achieve the attitude large angle maneuver.

The present study provides an attitude maneuver control algorithm for an agile CubeSat, which can decrease angular velocity in the attitude maneuver process as far as possible to improve system stability under the condition of satisfying CubeSat’s limited control capability, then achieve a stable attitude large angle maneuver just by using momentum wheels and a magnetorquer. The physical simulations and tests verify the validity of the finite-time controller based on Beta DE path planning, and some concluding remarks are listed, as follows:

1. An optimal path planning algorithm based on quintic polynomial with three path segments using Beta DE is proposed, in order to decrease angular velocity in the attitude maneuver process as far as possible to improve system stability under the condition of satisfying CubeSat’s limited control capability.

2. In the path planning optimization algorithm, the variation and crossover coefficients in Beta DE all obey the beta distribution. This not only guarantees the suitable value of coefficients in different stages, but also produces a random number to maintain the group’s diversity.

3. In order to obtain a more accurate attitude tracking model, the momentum wheel model should be more detailed, in which the situation that the CubeSat angular velocity has influence on the wheel control input torque should be considered. Because the angular velocity cannot be ignored in attitude maneuver process.

4. The sign function or saturation function is replaced by the sign function, which can alleviate the chattering phenomenon in the control process.

References


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