Transmit Beamspace-based Unitary Parallel Factor Method for DOD and DOA Estimation in Bistatic MIMO Radar

BAOQING XU1 AND YONGBO ZHAO1,2
1National Lab of Radar Signal Processing, Xidian University, 710071, Xi’an, People’s Republic of China
2Collaborative Innovation Center of Information Sensing and Understanding, Xidian University, 710071, Xi’an, People’s Republic of China
Corresponding author: Yongbo Zhao (e-mail: ybzhao@xidian.edu.cn).

ABSTRACT: In this paper, a novel transmit beamspace-based (TB-based) unitary parallel factor (PARAFAC) algorithm is proposed for direction-of-departure (DOD) and direction-of-arrival (DOA) estimation in bistatic multiple-input multiple-output (MIMO) radar. A modified TB matrix is designed through convex optimization to focus the transmitted energy in the desired spatial sector. Due to the influence of the TB matrix, the tensor model after forward-backward averaging for TB-based bistatic MIMO radar cannot be written as a PARAFAC tensor model. To solve this problem, we have serious requirement for the structure of the TB matrix, which is indispensable in constructing the real-valued PARAFAC model. Based on this precondition, the TB technique can be successfully combined with the unitary PARAFAC (U-PARAFAC) method. Moreover, a look-up table is built to compensate the DOD estimation bias, which actually establishes the mapping relationship for DOD. The Cramer-Rao bound (CRB) on angle estimation in TB-based bistatic MIMO radar is also derived for performance comparison. Simulation results demonstrate the effectiveness of the proposed algorithm.

INDEX TERMS Multiple-input multiple-output (MIMO) radar, angle estimation, transmit beamspace (TB), parallel factor (PARAFAC) decomposition.

I. INTRODUCTION

Compared with traditional phased-array radar, multiple-input multiple-output (MIMO) radar owns many potential advantages [1-3], which has attracted a lot of attention in recent years. MIMO radar can be classified into two types according to array geometry. The first type is called statistical MIMO radar [4] whose transmit and receive antennas are widely spaced and thus can increase spatial diversity. With closely spaced antennas, the second type is called collocated MIMO radar [5]. In this case, the spatial resolution of radar systems is greatly improved due to the increased degrees of freedom (DOFs). This paper puts main focus on the direction-of-departure (DOD) and direction-of-arrival (DOA) estimation problem in bistatic MIMO radar.

The estimation of DOD and DOA in bistatic MIMO radar has received considerable attention recently. In [6], a Capon-based method is proposed for DOD and DOA estimation in bistatic MIMO radar. However, since the solving process is a two-dimensional (2-D) search problem, the computational complexity is huge. In [7], the propagator method (PM) is presented without decomposing the covariance matrix. Although 2-D search is avoided in the PM, additional angle pairing is still needed. To solve this problem, a modified ESPRIT algorithm without angle pairing is presented in [8], which has better estimation performance and lower computational complexity. Dealing with the real-valued signal model, the unitary ESPRIT (U-ESPRIT) algorithm is proposed in [9], which provides good estimation performance. Moreover, with the exploitation of the forward-backward averaging technique [10], maximum of two coherent targets can be separated. All the matrix-based methods [6-9] mentioned above reshape the received data into a matrix, which is used to calculate the signal subspace through singular value decomposition (SVD) or eigenvalue decomposition (EVD).

Since the multidimensional structure of the received data is not considered in matrix-based methods, some tensor-based methods are proposed in [12-17]. The key idea of DOD and DOA estimation in tensor-based methods is based on tensor decomposition [11], which mainly includes higher-order SVD (HOSVD) and parallel factor (PARAFAC) de-
composition. In [12], the unitary tensor-ESPRIT algorithm is extended and the forward-backward averaging technique is extended to the tensor case. Exploiting the multidimensional structure of the received data, the real-valued signal subspace is calculated through HOSVD which has higher precision than traditional matrix-based methods. In [13], the PARAFAC algorithm is proposed, in which the technique of PARAFAC decomposition is directly used to decompose the complex-valued tensor model. Without estimating the signal subspace, the estimation performance of the PARAFAC algorithm is improved especially in low signal-to-noise ratio (SNR). In [14], the trilinear decomposition-based method is proposed. Due to the enlarged virtual receive array aperture, the improved estimation performance is obtained. In [15, 16], the PARAFAC technique is used for angle estimation in the presence of mutual coupling for bistatic MIMO radar. Both the methods in [15, 16] have good performance because the multidimensional structure of the received data is fully utilized. The unitary PARAFAC (U-PARAFAC) algorithm is proposed in [17]. In [17], the real-valued tensor model built in [12] is also proved to follow a PARAFAC model and thus can be decomposed through PARAFAC technique. The deduced U-PARAFAC model exploits the forward-backward averaging technique and does not require estimating the signal subspace. Consequently, different from [9, 12], the U-PARAFAC algorithm still performs well especially in dealing with more than two highly correlated targets.

However, all the methods mentioned above employ full waveform diversity. That is to say, the transmit waveforms are completely orthogonal to each other, which actually sacrifices the transmit coherent gain. In [18, 19], two angle estimation methods combined with the transmit beamspace (TB) technique are proposed in monostatic MIMO radar. While the transmitted energy is focused in the spatial sector of interest, the rotational invariance property is also restored through the TB design. In [20], a tensor-based method is proposed for 2-D DOA estimation in monostatic MIMO radar. The design of TB matrix is based on 2-D transmit array averaging technique and does not require estimating the signal subspace. Unfortunately, all these methods are designed for monostatic MIMO radar and the TB technique is not combined with the transmit beamspace transformation. That is to say, the transmit waveforms diversity cannot be guaranteed perfectly, a look-up table is built to compensate the DOA estimation bias, which actually establishes the mapping relationship for DOD.

4) Specially, the corresponding Cramer-Rao bound (CRB) on DOD and DOA estimation for TB-based bistatic MIMO radar is also derived. When the transmitted energy keeps same, the CRB of the TB-based bistatic MIMO radar is lower than that of the traditional bistatic MIMO radar.

**Notations:** Scalars are denoted by italic letters, vectors by lowercase boldface letters, matrices by uppercase boldface letters and tensors by calligraphic letters. \((\cdot)^T, (\cdot)^H, (\cdot)^*\) and \((\cdot)^{-1}\) denote transpose, Hermitian transpose, complex conjugate and matrix inversion, respectively. Symbols \(\odot, \oplus\) and \(\circ\) denote Kronecker product, Khatri-Rao product and Hadamard product, respectively. \(\text{diag}(\cdot)\) and \(\text{vec}(\cdot)\) denote the diagonalization operation and the vectorization operation, respectively. The notation tr(\cdot) denotes the trace of a matrix. The real part and imaginary part of a complex-valued vector or matrix are denoted by \(\text{Re}\{\cdot\}\) and \(\text{Im}\{\cdot\}\), respectively. The \(M \times M\) identity matrix is denoted by \(I_M\). \(|\cdot||\cdot\|\) represents the Frobenius norm.

**II. TB-BASED TENSOR SIGNAL MODEL**

In this section, the tensor signal model for TB-based bistatic MIMO radar is constructed. To facilitate the understanding, some tensor basics are introduced firstly, which are consistent with [11, 21, 22].

**Definition 1 (PARAFAC decomposition).**

The PARAFAC decomposition of a third-order tensor \(X \in \mathbb{C}^{I \times J \times K}\) can be expressed as

\[
X = \sum_{r=1}^{R} a_r \circ b_r \circ c_r
\]

(1)

where \(\circ\) denotes the outer product [11], \(a_r \in \mathbb{C}^{I \times 1}\), \(b_r \in \mathbb{C}^{J \times 1}\) and \(c_r \in \mathbb{C}^{K \times 1}\) are the \(r\)th column of loading matrices \(A \in \mathbb{C}^{I \times R}\), \(B \in \mathbb{C}^{J \times R}\) and \(C \in \mathbb{C}^{K \times R}\), respectively. The positive integer \(R\) is called the rank of the tensor \(X\).

**Definition 2 (Mode-\(n\) tensor-matrix product).**

The mode-\(n\) product between an \(N\)-order tensor \(X \in \mathbb{C}^{I_1 \times I_2 \times \cdots \times I_N}\) and a matrix \(A \in \mathbb{C}^{J_1 \times I_n}\) is denoted by \(Y = X \times_n A \in \mathbb{C}^{I_1 \times I_2 \times \cdots \times I_{n-1} \times J_1 \times I_{n+1} \times \cdots \times I_N}\), with entries given by

\[
Y_{i_1 \cdots i_{n-1} j_{n+1} \cdots j_N} = \sum_{i_n = 1}^{I_n} X_{i_1 \cdots i_{n-1} i_n j_{n+1} \cdots j_N}
\]

with \(j_n\) varying. The properties of mode product are shown as follows:

\[
X \times_m A \times_n B = X \times_n B \times_m A \quad (m \neq n)
\]

(2)
\[ X \times_m A \times_m B = X \times_m (BA) \] (3)

Consider a bistatic MIMO radar system equipped with \( M \) transmit antennas and \( N \) receive antennas. Both transmit and receive arrays are half-wavelength spaced uniform linear arrays (ULAs). In TB-based bistatic MIMO radar, \( M \) transmit waveforms are not mutually orthogonal and can be expressed as \( \sqrt{E/L} W^* S \), where \( W \in \mathbb{C}^{M \times L} \) denotes the TB matrix and \( L \) (\( L < M \)) is called the number of transmit beams (or the transmit beamspace dimension). \( \sqrt{E/L} \) is a normalization factor that keeps the total transmitted energy fixed at \( E \). \( S = [s_1(t), s_2(t), \ldots, s_L(t)]^T \in \mathbb{C}^{L \times T} \) denotes \( L \) completely orthogonal waveforms, which satisfies \( SS^H = I_L \). \( T \) is the number of samples in single pulse. Assume that there are \( K \) targets located in the far-field and the DOD and DOA of the \( k \)th target are denoted by \( \theta_k \) and \( \varphi_k \), respectively. Then, the received signal of the \( q \)th pulse can be expressed as

\[ Y_q = \sqrt{E/L} B a_q A^T W^* S + N_q, \quad q = 1, 2, \ldots, Q \] (4)

where \( A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_K)] \) and \( B = [b(\varphi_1), b(\varphi_2), \ldots, b(\varphi_K)] \) denote the transmit and receive steering vectors, respectively. \( Q \) denotes the total number of pulses in a coherent processing interval (CPI). The form of the matrix \( A_q \) is given by \( A_q = \text{diag}(c_q) \), where \( c_q = [\alpha_{1,q}, \alpha_{2,q}, \ldots, \alpha_{K,q}]^T \). \( \alpha_{k,q} \) denotes the reflection coefficient of the \( k \)th target in the \( q \)th pulse. As a matter of fact, the TB matrix \( W \) is just used to form the transmit waveforms, which is expressed as \( \sqrt{E/L} W^* S \). \( N_q \) in (4) is the \( N \times T \) zero-mean white Gaussian noise term whose covariance is \( \sigma_n^2 I_N \). The distribution characteristic of the noise term \( N_q \) is not influenced by the TB transformation. Employing the center of the array as the phase reference, the transmit and receive steering vectors of the \( k \)th target can be respectively written as

\[ a(\theta_k) = [e^{-j \frac{M-1}{2} \sin \theta_k}, 1, \ldots, e^{j \frac{M-1}{2} \sin \theta_k}]^T \] (5)

\[ b(\varphi_k) = [e^{-j \frac{N-1}{2} \sin \varphi_k}, 1, \ldots, e^{j \frac{N-1}{2} \sin \varphi_k}]^T \] (6)

Then, utilizing the orthogonality property \( SS^H = I_L \), (4) is right multiplied by \( S^H \) for matched filtering. Then, the outputs of the matched filters are given by

\[ Y_q = \sqrt{E/L} B a_q A^T W^* S + N_q \]

\[ = \sqrt{E/L} B a_q (W^H A)^T + N_q \] (7)

where \( N_q = N_q S^H \) represents the noise matrix after matched filters. Stacking \( X_q \), \( q = 1, 2, \ldots, Q \), along the third dimension, (7) can be rewritten as an \( N \times L \times Q \) tensor model according to [13]

\[ X = \sqrt{E/L} \sum_{k=1}^K b(\varphi_k) \otimes [W^H a(\theta_k)] \otimes d_k + N \] (8)

where \( d_k \) denotes the \( k \)th column of the matrix \( C = [c_1, c_2, \ldots, c_Q]^T \in \mathbb{C}^{Q \times K} \). \( N \in \mathbb{C}^{N \times L \times Q} \) denotes the noise tensor formed by \( N_q \), \( q = 1, 2, \ldots, Q \). In fact, we have proved in [17] that the tensor signal model (8) can be rewritten as

\[ X = \sqrt{E/L} (I_K \times 1 B \times 2 W^H A \times 3 C) + N \] (9)

where \( I_K \) denotes the \( K \times K \times K \) identity tensor.

III. TB-BASED UNITARY PARAFAC ALGORITHM

In this section, a TB-based unitary PARAFAC algorithm is proposed for DOD and DOA estimation in bistatic MIMO radar. The TB technique is firstly combined with the PARAFAC model in bistatic MIMO radar.

A. TRANSMIT BEAMSPACE DESIGN

In practice, we usually have known the general spatial sector where the targets are located especially in radar tracking problem. Under this condition, it will cause undesirable energy waste when still using the orthogonal transmit waveforms. To focus the transmitted energy in the spatial sector where the targets are most likely to be located, the TB matrix is designed through convex optimization. More importantly, the desired structure of the TB matrix can be obtained by enforcing the constraints in the design process.

Firstly, we define a \( L \times 1 \) \((L < M)\) virtual transmit steering vector

\[ \mathbf{a}(\theta) = [e^{-j \frac{\theta}{2} \sin \theta}, 1, \ldots, e^{j \frac{\theta}{2} \sin \theta}]^T \] (10)

if \( L \) is odd, or

\[ \mathbf{a}(\theta) = [e^{-j \frac{\theta}{2} \sin \theta}, e^{-j \frac{\theta}{2} \sin \theta}, e^{j \frac{\theta}{2} \sin \theta}, \ldots, e^{j \frac{\theta}{2} \sin \theta}]^T \] (11)

if \( L \) is even. The vector \( \mathbf{a}(\theta) \) shares the rotational invariance structure. Our objective is to design an \( M \times L \) matrix \( W \) to minimize the difference between \( W^H a(\theta) \) and \( \mathbf{a}(\theta) \) in the desired spatial sector. Furthermore, the transmitted energy distributed in the out-of-sector should also be minimized. Most importantly, we have serious requirement for the structure of the TB matrix \( W \), which is indispensable in constructing the real-valued PARAFAC model in the following subsection. When \( M \) is even, this can be expressed as the following minimax optimization problem:

\[ \min_{W, \Gamma} \max_j ||W^H a(\theta_j)||, \quad \theta_j \in \Theta, \quad j = 1, 2, \ldots, J \]

s.t. \( \max_i ||W^H a(\theta_i) - \mathbf{a}(\theta_i)|| \leq \beta, \quad \theta_i \in \Theta, \quad i = 1, 2, \ldots, I \)

\[ W = [\Gamma^T, \Pi_L \Pi_H \Pi_L M/2]^T \] (12)

where \( \Theta \in \mathbb{C}^{(M/2) \times L} \). Symbol \( \Theta \) denotes the spatial sector of interest and \( \Theta \) is complement sector of \( \Theta \). The positive parameter \( \beta \) characterizes the worst acceptable deviation between \( W^H a(\theta) \) and \( \mathbf{a}(\theta) \) in \( \Theta \). \( \Pi_L \) denotes an \( n \times n \) exchange matrix having ones on its antidiagonal and zeros elsewhere. If \( M \) is odd, \( W \) can be calculated by solving

\[ \min_{W, \Gamma, g} \max_j ||W^H a(\theta_j)||, \quad \theta_j \in \Theta, \quad j = 1, 2, \ldots, J \]

s.t. \( ||W^H a(\theta_i) - \mathbf{a}(\theta_i)|| \leq \beta, \quad \theta_i \in \Theta, \quad i = 1, 2, \ldots, I \)

\[ W = [\Gamma^T, g, \Pi_L \Gamma_H \Pi_L (M-1)/2]^T, \quad g^* = \Pi_L g \] (13)
where \( g \) is a \( L \times 1 \) vector and \( \Gamma \in \mathbb{C}^{(M-1)/2 \times L} \). The convex optimization problems (12) and (13) can be efficiently solved by using the CVX MATLAB toolbox [23]. The equality constraints in (12) and (13) are indispensable in constructing the real-valued PARAFAC model, which are not used before and will be discussed in the following subsection.

The number of transmit beams \( L \) depends on the width of the desired spatial sector \( \Theta \). We define the nonnegative matrix

\[
\Xi = \int_\Theta a(\theta) a^H(\theta) d\theta
\]  

(14)

The sum of the \( L \) largest eigenvalues of \( \Xi \) should exceed a certain percentage of the total sum of all eigenvalues. In this paper, this certain percentage is chosen as 99\%. This method is called the discrete prolate spheroidal sequences (DPSS)-based method [24], which is usually designed to approximate the quiescent beamspace matrix. The essence of the DPSS-based method for beamspace dimension reduction is to maximize the ratio of the beamspace energy that comes from within the desired spatial sector \( \Theta \) to the total beamspace energy.

As mentioned above, since the transmit waveforms in TB-based bistatic MIMO radar are \( \sqrt{E/L} \mathbf{W} \mathbf{S} \), the total transmitted energy can be expressed as

\[
\left( \frac{E}{L} \right) \text{tr}(\mathbf{W}^* \mathbf{S}^H \mathbf{W}^T) = \left( \frac{E}{L} \right) \text{tr}(\mathbf{W}^* \mathbf{W}^T)
\]  

(15)

Hence, to ensure that the total transmitted energy is fixed at \( E \), the obtained TB matrix \( \mathbf{W} \) after solving (12) and (13) should be scaled to satisfy \( \text{tr}(\mathbf{W}^* \mathbf{W}^T) = L \). The transmitted energy distributed in \( \Theta \) for TB-based bistatic MIMO radar can be expressed as

\[
P(\theta) = \left( \frac{E}{L} \right) ||\mathbf{W}^H \mathbf{a}(\theta)||^2
\]  

(16)

B. DOD AND DOA ESTIMATION

Although the real-valued PARAFAC model has been built in [17], the way that how to combine the TB technique and the real-valued PARAFAC model has not been given. The main difficulty is that the structure of the received data for TB-based bistatic MIMO radar is quite different from traditional bistatic MIMO radar due to the influence of the TB matrix. In this case, the tensor model after forward-backward averaging for TB-based bistatic MIMO radar cannot be written as a PARAFAC tensor model anymore. The objective of this subsection is trying to combine the transmit beamforming technique with the real-valued PARAFAC tensor model. To this end, the structure information of the TB matrix is exploited. With the predesigned structure of the TB matrix, it is proved that the transmit beamforming technique can be successfully combined with the real-valued PARAFAC model.

Firstly, an \( N \times L \times Q \) tensor is defined as

\[
\mathbf{X} = X^* \times_3 \mathbf{Π}_N \times_2 \mathbf{Π}_L \times_3 \mathbf{Π}_Q
\]  

(17)

Substituting (9) to (17), we have

\[
\mathbf{\mathcal{Y}} = \sqrt{E/L} (I_K \times_1 \mathbf{B}^* \times_2 (\mathbf{W}^H \mathbf{A})^* \times_3 \mathbf{C}^*) \times_1 \mathbf{Π}_N \\
\times_2 \mathbf{Π}_L \times_3 \mathbf{Π}_Q + \mathcal{N}
\]  

\[
= \sqrt{E/L} (I_K \times_1 \mathbf{Π}_N \mathbf{B}^* \times_2 \mathbf{Π}_L \mathbf{W}^T \mathbf{A}^* \times_3 \mathbf{Π}_Q \mathbf{C}^*) + \mathcal{N}
\]  

\[
= \sqrt{E/L} (I_K \times_1 \mathbf{B} \times_2 \mathbf{Π}_L \mathbf{W}^T \mathbf{A}^* \times_3 \mathbf{Π}_Q \mathbf{C}^*) + \mathcal{N}
\]  

(18)

where \( \mathcal{N} = \mathcal{N}^* \times_1 \mathbf{Π}_N \times_2 \mathbf{Π}_L \times_3 \mathbf{Π}_Q \). In the derivation of (18), we use the fact that \( \mathbf{B} = \mathbf{Π}_N \mathbf{B}^* \). Then, we concatenate \( \mathbf{X} \) and \( \mathbf{\mathcal{Y}} \) along the third dimension and the new \( N \times L \times 2Q \) tensor is given by

\[
\mathbf{Z} = [\mathbf{X} \cup_3 \mathbf{\mathcal{Y}}]
\]  

(19)

Inserting (18) to (19), we obtain

\[
\mathbf{Z} = \sqrt{E/L} [[(I_K \times_1 \mathbf{B} \times_2 \mathbf{W}^H \mathbf{A} \times_3 \mathbf{C}) \cup_3 \\
(I_K \times_1 \mathbf{B} \times_2 \mathbf{Π}_L \mathbf{W}^T \mathbf{A}^* \times_3 \mathbf{Π}_Q \mathbf{C}^*)] + \mathcal{N}
\]  

(20)

where \( \mathcal{N} = [\mathcal{N} \cup_3 \mathcal{N}] \). \([A \cup_n B]\) denotes the concatenation of \( A \) and \( B \) along the \( n \)th mode. From (20), we can find that the tensor \( \mathbf{Z} \) is centro-Hermitian [12] because the sufficient condition \( \mathbf{Z} = Z^* \times_1 \mathbf{Π}_N \times_2 \mathbf{Π}_L \times_3 \mathbf{Π}_Q \) is satisfied.

However, although the tensor \( \mathbf{Z} \) is centro-Hermitian, it cannot be written as a PARAFAC model like [17] without any structure information of \( \mathbf{W} \). The main reason is that the tensor structure of \( \mathbf{Z} \) in (20) is quite different from [17], which causes that the terms \( \mathbf{W}^H \mathbf{A} \) and \( \mathbf{Π}_L \mathbf{W}^T \mathbf{A}^* \) are not equal. Fortunately, the matrix structure of \( \mathbf{W} \) has been properly designed in (12) and (13), which is indispensable in constructing the PARAFAC model. In Appendix A, by using the structure information of the proposed TB matrix, we prove that the following equation holds

\[
\mathbf{Π}_L \mathbf{W}^T \mathbf{A}^* = \mathbf{W}^H \mathbf{A}
\]  

(21)

Then, substituting (21) to (20), (20) can be rewritten as

\[
\mathbf{Z} = \sqrt{E/L} [I_K \times_1 \mathbf{B} \times_2 \mathbf{W}^H \mathbf{A} \times_3 \tilde{\mathbf{C}}] + \mathcal{N}
\]  

(22)

where \( \tilde{\mathbf{C}} = [\mathbf{C}^T, (\mathbf{Π}_Q \mathbf{C}^*)^T]^T \in \mathbb{C}^{2Q \times K} \). Ignoring the noise term, we find that (22) has the same structure with (9). Therefore, we conclude that the tensor \( \mathbf{Z} \) follows a PARAFAC model once (21) holds.

It is noted that the tensor \( \mathbf{Z} \) cannot be written as a PARAFAC model if there is no structure information of \( \mathbf{W} \). Since the tensor \( \mathbf{Z} \) is centro-Hermitian, the real-valued tensor can be constructed as [12]

\[
\mathbf{Z} = Z^* \times_1 \mathbf{Q}^H \times_2 \mathbf{Q}^H \times_3 \mathbf{Q}^H \times_3 \mathbf{Q}^H \times_3 Q^H \times_3 Q^H \times_3 Q^H \times_3 Q^H \\
= \sqrt{E/L} [I_K \times_1 \mathbf{B} \times_2 \mathbf{W}^H \mathbf{A} \times_3 \mathbf{C}^*] \times_1 \mathbf{Q}^H \times_2 \mathbf{Q}^H \\
= \sqrt{E/L} [I_K \times_1 \mathbf{B} \times_2 \mathbf{W}^H \mathbf{A} \times_3 \mathbf{Q}^H \times_3 Q^H \times_3 Q^H \times_3 Q^H \times_3 Q^H \times_3 Q^H \times_3 Q^H]
\]  

(23)

where \( \mathcal{N} = \mathcal{N}^* \times_1 \mathbf{Q}^H \times_2 \mathbf{Q}^H \times_3 \mathbf{Q}^H \times_3 \mathbf{Q}^H \times_3 \mathbf{Q}^H \times_3 \mathbf{Q}^H \times_3 \mathbf{Q}^H \times_3 \mathbf{Q}^H \). Here, \( \mathbf{Q} \) is a unitary matrix which is defined as

\[
\mathbf{Q}_{2n} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_n & j \mathbf{I}_n \\ \mathbf{I}_n & -j \mathbf{I}_n \end{bmatrix}
\]  

(24)
if the subscript is even, or
\[ Q_{2n+1} = \frac{1}{\sqrt{2}} \begin{bmatrix} I_n & 0 & jI_n \\ 0 & \sqrt{2} & 0 \\ I_n & 0 & -jI_n \end{bmatrix} \]
if the subscript is odd. Similarly, the real-valued tensor \( Z \) still follows a PARAFAC model without considering the noise term. According to [11], the uniqueness of PARAFAC decomposition for \( Z \) is guaranteed if
\[ kQ_k^H + kQ_j^H w^H a + kQ_k^H = 2K + 2 \]
where \( k \) denotes the Kruskal-rank [11] of \( A \). When the interpolation error in solving (12) and (13) is ignored, there exists \( \overline{A} = W^H A \) in the desired spatial sector, where \( \overline{A} = [\overline{a}(\theta_1), \overline{a}(\theta_2), \ldots, \overline{a}(\theta_K)] \) denotes the virtual transmit steering matrix. In this case, (23) can further be rewritten as
\[ \overline{Z} = \sqrt{E/L}(I_K \times Q_N^H B \times Q_L^H \overline{A} \times Q_{2Q}^H \tilde{C}) + \tilde{N} \]
Up to now, we have constructed the real-valued PARAFAC tensor model for the proposed TB-based bistatic MIMO radar by properly designing the structure of the TB matrix. Interestingly, we can find that although (27) is based on transmit beamspace, it has a similar structure with the real-valued tensor model built in [17]. This is due to the special structure requirement in designing the TB matrix.

For now we have obtained a real-valued PARAFAC model \( Z \). Once the uniqueness condition (26) is satisfied, we can decompose the real-valued tensor \( Z \) by using the PARAFAC decomposition techniques such as the fast dGN method [25]. After that, the real-valued loading matrices \( U \) and \( E \) that include angle information can be obtained. Based on the uniqueness property of PARAFAC decomposition [17], the real-valued loading matrices \( U \) and \( E \) are the results by permuting and scaling the columns of \( Q_N^H \overline{A} \) and \( Q_L^H B \) in (27). As a matter of fact, the terms \( Q_N^H \overline{A} \) and \( Q_L^H B \) are real-valued, which are usually used for real-valued transformation [9, 10, 12, 17]. Then, the DOD and DOA information can be extracted from \( U \) and \( E \), respectively. Neglecting the interpolation error in the desired spatial sector, we have proved in [17] that
\[ \phi_k K_{t1} u_k = K_{t2} u_k \]
\[ \eta_k K_{r1} e_k = K_{r2} e_k \]
where \( \phi_k = \tan(\pi \sin \theta_k/2) \) and \( \eta_k = \tan(\pi \sin \varphi_k/2) \). \( u_k \) and \( e_k \) denote the \( k \)th columns of \( U \) and \( E \), respectively. Matrices \( K_{t1}, K_{t2}, K_{r1} \), and \( K_{r2} \) are given by
\[ K_{t1} = \Re\{Q_N^H J_{L}Q_L^H \}, \quad K_{t2} = \Im\{Q_N^H J_{L}Q_L^H \}, \quad K_{r1} = \Re\{Q_N^H J_{Q}Q_N^H \}, \quad K_{r2} = \Im\{Q_N^H J_{Q}Q_N^H \} \]
where \( J_1 = [0, I_{L-1}] \) and \( J_2 = [0, I_{N-1}] \) are the \((L-1) \times L \) and \((N-1) \times N \) selection matrices, respectively. Since the relationship between \( \phi_k \) and \( \eta_k \) is one-to-one, additional angle pairing procedure is not required anymore. From (28) and (29), \( \phi_k \) and \( \eta_k \) can be calculated through the least squares (LS) method. Then, the estimated \( \hat{\theta}_k \) and \( \hat{\varphi}_k \) can be calculated from the following equations:
\[ \hat{\theta}_k = \arcsin (2\arctan(\phi_k)/\pi) \]
\[ \hat{\varphi}_k = \arcsin (2\arctan(\eta_k)/\pi) \]
where \( \hat{\theta}_k \) and \( \hat{\varphi}_k \) denote the estimated DOD and DOA of the \( k \)th target.

C. COMPENSATION OF THE DOD ESTIMATION BIAS
Up to now, the estimated angle information is obtained according to (30) and (31). It is noted that the term \( W^H a(\theta) \) cannot be exactly equal to \( \overline{a}(\theta) \) when solving (12) and (13), which will inevitably degrade the angle estimation performance. The estimation bias caused by the interpolation error between \( W^H a(\theta) \) and \( \overline{a}(\theta) \) is unexpected for us. Therefore, we must take some measures to compensate the estimation error. An effective way to solve this problem is building a look-up table based on a noiseless tensor model and the estimation bias is only related with the interpolation error in this case. Firstly, we build the noise-free tensor model of (23)
\[ \overline{Z} = \sqrt{E/L}(I_K \times Q_N^H B \times Q_L^H W^H A \times Q_{2Q}^H \tilde{C}) \]
It is indicated from (32) that the term \( Q_N^H B \) including DOA information can be extracted independently, which means that the interpolation error has no effect on the DOA estimation accuracy.

The one-to-one mapping relationship for DOD can be identified based on the following equation
\[ \gamma_i K_{t1} Q_{L}^H W^H a(\theta_i) = K_{t2} Q_{L}^H W^H a(\theta_q) \]
where \( \theta_i, i = 1, 2, \ldots, I \), denotes the true DOD sampled in \( \Theta \). It is noted that the sampling interval should meet the accuracy requirement of DOD. The mapped DOD is given by
\[ \overline{\theta}_i = \arcsin (2\arctan(\gamma_i)/\pi) \]
For each \( i \), the relationship that maps \( \overline{\theta}_i \) to \( \theta_q \) can be determined. Based on this, an offline look-up table can be built to mitigate the influence of interpolation error. When the DOD of the targets \( \theta_k, k = 1, 2, \ldots, K \), are estimated from the noisy model according to (30), more accurate DOD estimation can be achieved through mapping operation.

D. CRB AND COMPLEXITY ANALYSIS
Stacking the received data \( X_q \) in (7) into a vector, we obtain
\[ x_q = \sqrt{E/L} [(W^H A) \odot B] c_q + n_q \]
where \( n_q = \text{vec}(\tilde{N}_q) \). The covariance of the \( LN \times 1 \) noise term \( n_q \) is \( \sigma^2 I_{LN} \). Without considering the normalization factor, (35) keeps the same structure with the signal model of traditional bistatic MIMO radar. Then, according to [26], the CRB on DOD and DOA estimation in TB-based bistatic MIMO radar is derived as follows:
\[ \text{CRB}_{TB} = \frac{\sigma^2 L}{2Q^2E_{\tilde{C}}} \left\{ \text{Re}\{ (G^H P \phi G) \odot H^T \} \right\}^{-1} \]
where $\mathbf{P}_\perp^V = \mathbf{I}_{LN} - \mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H$ denotes the orthogonal projection matrix, and $\mathbf{G} = [\mathbf{G}_t, \mathbf{G}_r]$. Here, $\mathbf{V} = (\mathbf{W}^H\mathbf{A}) \odot \mathbf{B}$ and $\mathbf{G}_t = (\mathbf{W}^H\mathbf{A}) \odot \mathbf{B}_t$, where $\mathbf{A}' = \begin{bmatrix} \frac{da(\theta_1)}{d\varphi}, & \frac{da(\theta_2)}{d\varphi}, & \ldots, & \frac{da(\theta_K)}{d\varphi} \end{bmatrix}$ and $\mathbf{B}' = \begin{bmatrix} \frac{db(\varphi_1)}{d\varphi}, & \frac{db(\varphi_2)}{d\varphi}, & \ldots, & \frac{db(\varphi_K)}{d\varphi} \end{bmatrix}$, respectively. In (36), $\mathbf{H} = \begin{bmatrix} \mathbf{F} & \mathbf{F} \\ \mathbf{F} & \mathbf{F} \end{bmatrix}$, where $\mathbf{F} = C^T C^*/Q$. As a matter of fact, the signal model of traditional bistatic MIMO radar is a special case of (35) with $L = M$ and $\mathbf{W} = \mathbf{I}_M$. Therefore, the CRB on DOD and DOA estimation for traditional bistatic MIMO radar can be written as

$$\text{CRB}_{\text{TB}} = \frac{\sigma^2}{2M}\left\{ \text{Re}\left\{ \left( \mathbf{D}^H\mathbf{P}_\perp^V\mathbf{D} \right) \mathbf{H}^T \right\} \right\}^{-1}$$

(37)

where $\mathbf{P}_\perp^V = \mathbf{I}_{MN} - \mathbf{\Omega}(\mathbf{\Omega}^H\mathbf{\Omega})^{-1}\mathbf{\Omega}^H$ denotes the orthogonal projection matrix and $\mathbf{D} = [\mathbf{D}_t, \mathbf{D}_r]$. Here, $\mathbf{\Omega} = \mathbf{A} \odot \mathbf{B}$, $\mathbf{D}_t = \mathbf{A}' \odot \mathbf{B}_t$ and $\mathbf{D}_r = \mathbf{A} \odot \mathbf{B}'$. Compared with the signal model built in [27], there is only an extra normalization factor $\sqrt{E/L}$ needed to be considered.

The complexity of the proposed algorithm is mainly dominated by the procedure of DOD and DOA estimation. The number of transmit beams $L$ is usually smaller than the number of transmit antennas $M$. When the fast dGN method [25] is adopted to perform PARAFAC decomposition, the complexity of the proposed algorithm is $O(6LNQK + 27K^6)$ for each iteration, which is smaller than $O(6M^3N^3 + 27K^6)$ of the U-PARAFAC algorithm. The complexity of the U-ESPRIT method is $O(M^3N^3 + K^3)$, while the complexity of the PM is $O(MNKQ + K^3)$.

**IV. SIMULATION RESULTS**

In this section, several simulations are conducted to assess the performance of the proposed algorithm. The PM [7], the U-ESPRIT algorithm [9], the unitary tensor-ESPRIT algorithm [12], the U-PARAFAC algorithm [17] are tested for performance comparison. Moreover, the CRB of the proposed TB-based bistatic MIMO radar (36) and the CRB of traditional bistatic MIMO radar (37) are also tested. The number of transmit and receive antennas is $M = 8$ and $N = 4$, respectively. The number of targets is known as prior, which is usually estimated by using the Akaike information criterion (AIC) [28] and minimum description length method (MDL) [29]. The total transmitted energy of both the TB-based and traditional bistatic MIMO radar is fixed at $E = M$, which ensures the comparability of all the methods. The evaluation criterion is the root mean squared error (RMSE) which is defined as

$$\text{RMSE} = \sqrt{\frac{1}{2PK} \sum_{k=1}^{K} \sum_{p=1}^{P} \left[ \hat{\theta}_{k,p} - \theta_k \right]^2 + \left[ \hat{\varphi}_{k,p} - \varphi_k \right]^2}$$

(38)

where $\hat{\theta}_{k,p}$ and $\hat{\varphi}_{k,p}$ denote the estimated DOD and DOA (after mapping operation) of the $k$th target for the $p$th Monte Carlo trial, respectively. The total number of Monte Carlo trials is $P = 400$. For traditional bistatic MIMO radar, $M$ orthogonal waveforms are transmitted with $T = 1024$ and take the following expressions

$$s_m(t) = \sqrt{\frac{1}{T}} e^{j2\pi \frac{m}{T} t}, \quad m = 1, 2, \ldots, M$$

(39)

In the proposed TB-based bistatic MIMO radar, only the first $L$ orthogonal waveforms in (39) are used to form the transmit waveforms, which are given by $\sqrt{E/L}\mathbf{W}^*\mathbf{S}$.

Before the angle estimation, the TB matrix $\mathbf{W}$ is designed firstly. In all the simulations, the desired spatial sector is set to be $\Theta = [0^\circ, 30^\circ]$. Setting $\beta = 0.05$ and $L = 6$, then the TB matrix can be calculated according to (12) and (13). It is noted that the obtained $\mathbf{W}$ should be scaled to ensure $\text{tr} (\mathbf{W}^H\mathbf{W}^T) = L$, which keeps the total transmitted energy fixed at $E$. Fig. 1 shows the transmitted energy distribution for the TB-based and traditional bistatic MIMO radar, which can be expressed as $(E/L) ||\mathbf{W}^H\mathbf{a}(\theta)||^2$ and $(E/L) ||\mathbf{a}(\theta)||^2$ respectively. For both the TB-based and traditional bistatic MIMO radar, the total transmitted energy is $E$. As can be seen from Fig. 1, the proposed TB-based bistatic MIMO radar achieves the transmit coherent gain in the desired spatial sector successfully, while traditional bistatic MIMO radar has flat transmit gain in all directions.

**Fig. 1.** The transmitted energy distribution for the TB-based and traditional bistatic MIMO radar.

Based on the TB matrix $\mathbf{W}$ obtained above, the necessity of mapping operation is evaluated in the first simulation. Assume that there are three uncorrelated targets located at $(\theta_1, \varphi_1) = (5^\circ, -20^\circ)$, $(\theta_2, \varphi_2) = (10^\circ, 20^\circ)$ and $(\theta_3, \varphi_3) = (25^\circ, 40^\circ)$, respectively. The SNR of all three targets is fixed at 5 dB and the number of pulses is 50. Fig. 2 shows the estimation results of the proposed algorithm before taking the mapping operation. The number of Monte Carlo trials is 400. We observe from Fig. 2 that DOD estimation accuracy of the proposed algorithm is seriously degraded by the interpolation error. Fig. 3 shows the estimation results of the proposed algorithm after the mapping operation. As demonstrated in Fig. 3, the DOD and DOA of the targets are estimated correctly. The DOD estimation error is compensated successfully based on the established mapping relationship. Comparing Fig. 3 with Fig. 2, it can be found that the
DOD estimation error can be reduced greatly by using the look-up table.

Fig. 2. Estimation results of the proposed algorithm before the mapping operation.

Fig. 3. Estimation results of the proposed algorithm after the mapping operation.

In the second simulation, \( K = 2 \) uncorrelated targets are located at \((\theta_1, \varphi_1) = (5^\circ, -40^\circ)\) and \((\theta_2, \varphi_2) = (20^\circ, 60^\circ)\), respectively. Fig. 4 demonstrates the RMSE versus SNR with the number of pulses fixed at \( Q = 50 \). The CRB of traditional bistatic MIMO radar and the CRB of TB-based bistatic MIMO radar are also plotted for comparison. When plotting the CRB derived in (36) and (37), the transmitted energy is fixed at \( E = M = 8 \) and the number of transmit beams is \( L = 6 \). With SNR fixed at 5 dB, the RMSE versus the number of pulses is shown in Fig. 5. The RMSE curve of the proposed algorithm is most close to the CRB derived in (36). As can be seen from Figs. 4 and 5, the CRB of the TB-based bistatic MIMO radar is lower than the CRB of traditional bistatic MIMO radar. This means that more accurate estimation can be achieved in the TB-based bistatic MIMO radar. The estimation accuracy of the proposed algorithm is significantly improved compared with other methods. In Fig. 6, we compare the CPU time of the tested methods versus the number of pulses. By comparison of Fig. 5 and Fig. 6, the proposed algorithm has better estimation performance than the U-PARAFAC method with lower computational complexity. However, this does not mean that the computational complexity of the proposed algorithm is lower than the unitary tensor-ESPRIT algorithm in all cases, it heavily depends on the number of iterations.

Fig. 4. RMSE performance comparison versus SNR for two uncorrelated targets.

Fig. 5. RMSE performance comparison versus the number of pulses for two uncorrelated targets.

In the third simulation, the estimation performance for closely spaced targets is assessed. Two closely spaced targets are located at \((\theta_1, \varphi_1) = (5^\circ, 20^\circ)\) and \((\theta_2, \varphi_2) = (8^\circ, 23^\circ)\), respectively. Fig. 7 shows the RMSE versus SNR with \( Q = 50 \). When the targets are closely spaced, the steering vectors of the targets are almost collinear and thus results in the ill-conditioned signal subspace. This will degrade the estimation performance for the subspace-based methods. The superior performance of the proposed algorithm is attributed to the combination of the TB technique and PARAFAC tensor model. Although both the proposed algorithm and the U-PARAFAC algorithm do not require to estimate the signal subspace, the proposed algorithm performs better than the U-PARAFAC algorithm due to the transmit coherent gain.

In the fourth simulation, the estimation performance for two highly correlated targets is considered. Two correlated
targets are respectively located at \((\theta_1, \varphi_1) = (5^\circ, 0^\circ)\) and \((\theta_2, \varphi_2) = (20^\circ, 30^\circ)\) with the correlation coefficient being 0.99. In Fig. 8, the results that the RMSE versus SNR are shown with \(Q = 50\). It can be observed from Fig. 8 that the proposed algorithm has the lowest RMSE compared with other methods. As mentioned above, the forward-backward averaging technique can deal with maximum of two coherent targets. This is why the performance of the U-ESPRIT algorithm, the unitary tensor-ESPRIT algorithm, and the U-PARAFAC algorithm still remains relatively stable in this case. On the contrary, the PM fails to estimate the DOD and DOA accurately when dealing with correlated targets. 

In the fifth simulation, three highly correlated targets are considered, which are located at \((\theta_1, \varphi_1) = (5^\circ, -40^\circ)\), \((\theta_2, \varphi_2) = (15^\circ, 0^\circ)\), and \((\theta_3, \varphi_3) = (25^\circ, 40^\circ)\), respectively. The correlation coefficients between the first two targets, as well as the first and the last targets are set to be 0.99. Fig. 9 shows the RMSE versus SNR with \(Q = 50\). In accordance with [17], the U-PARAFAC method has excellent performance in dealing with more than two highly correlated targets. Even in this case, the proposed method still has better estimation performance.

V. CONCLUSION

In this paper, a novel TB-based unitary PARAFAC algorithm is proposed for angle estimation in bistatic MIMO radar. A TB matrix is designed through convex optimization to focus the transmitted energy in the desired spatial sector. Most importantly, the desired structure of the TB matrix can be obtained by solving the optimization problem, which is indispensable in constructing the real-valued PARAFAC model. Based on this precondition, the TB technique is successfully combined with the U-PARAFAC method. The DOD estimation bias is compensated by building a lookup table. Specially, the corresponding CRB on DOD and DOA estimation in TB-based bistatic MIMO radar is also derived. Numerical simulations are conducted to validate the improved performance of the proposed algorithm.

APPENDIX A THE DERIVATION OF (21)

In Appendix A, we prove that the term \(\Pi_L W^T A^*\) in (18) is equal to \(W^H A\) by using the structure information in (12) and (13).

According to the equality constraints in (12) and (13), the
obtained \( \mathbf{W} \) can be expressed as
\[
\mathbf{W} = \left[ \Gamma^T, \Pi_L \Gamma^H, \Pi_M (M/2) \right]^T
\]
if \( M \) is even, or
\[
\mathbf{W} = \left[ \Gamma^T, \mathbf{g}, \Pi_L \Gamma^H, \Pi_M (M-1)/2 \right]^T
\]
if \( M \) is odd. Without loss of generality, the following proof is given with the assumption that \( M \) is odd. In this case, there exists \( \mathbf{g}^* = \Pi_L \mathbf{g} \) according to (13). Inserting (41) to the term \( \Pi_L \mathbf{W}^T \mathbf{A}^* \), we have
\[
\Pi_L \mathbf{W}^T \mathbf{A}^* = \Pi_L \mathbf{W}^T \Pi_L \Pi_M \mathbf{A}^* = \Pi_L \Gamma^T, \mathbf{g}, \Pi_L \Gamma^H, \Pi_M (M-1)/2 \Pi_M \Pi_M \mathbf{A}^*
\]
\[
= \Pi_L \Gamma^T, \mathbf{g}, \Pi_L \Gamma^H, \Pi_M (M-1)/2 \Pi_M \mathbf{A}
\]
In the derivation of (42), we use the fact that \( \Pi_M \mathbf{A}^* = \mathbf{A} \). Moreover, we have
\[
\Pi_L \Gamma^T, \mathbf{g}, \Pi_L \Gamma^H, \Pi_M (M-1)/2 \Pi_M = \begin{bmatrix} 0 & 0 & \Pi_M (M-1)/2 \\ 0 & 1 & 0 \end{bmatrix}
\]
\[
= \begin{bmatrix} \Gamma^H \mathbf{g}^*, \Pi_L \Gamma^T, \Pi_M (M-1)/2 \end{bmatrix} = \mathbf{W}^H \mathbf{A}
\]
Then, inserting (43) to (42), (42) can be rewritten as
\[
\Pi_L \mathbf{W}^T \mathbf{A}^* = \mathbf{W}^H \mathbf{A}
\]
Up to now, we have proved that the equation \( \Pi_L \mathbf{W}^T \mathbf{A}^* = \mathbf{W}^H \mathbf{A} \) holds with the structure information of \( \mathbf{W} \).

REFERENCES

BAOQING XU received the B.S. degree from Qingdao Technological University, Qingdao, China, in 2014. He is currently pursuing the Ph.D degree with the Department of Electrical Engineering, Xidian University. His research interests include parameter estimation and adaptive signal processing.
YONGBO ZHAO was born in XinXiang City in Henan Province, China. He received the M.E. and Ph.D. Degrees, both in electrical engineering from Xidian University, Xi’an, China, in 1997 and 2000, respectively. He is now a Professor with National Laboratory of Radar Signal Processing, Xidian University. His research interests include adaptive signal processing, array signal processing, MIMO radar and advanced radar concepts.

***