Direction of Arrival Estimation by Convex Optimization Methods with Unknown Sensor Gain and Phase

LUTAO LIU, (Member, IEEE), YANAN WU
College of Information and Communication Engineering, Harbin Engineering University, Harbin 150001, China (e-mail: liulutao@hrbeu.edu.cn, wyn@hrbeu.edu.cn)
Corresponding author: Yanan Wu (e-mail: wyn@hrbeu.edu.cn.)

This work is partially supported by National Natural Science Foundation of China (61571149) and Fundamental Research Funds for the Central Universities (HEUCF1808).

ABSTRACT Direction of arrival (DOA) estimation performance severely degrades for the reasons that the perturbation from their assumed nominal complex gain values in practical sensor system. In this paper, to mitigate the effects of the uncertainty perturbation, a two-step method for joint DOAs and sensor errors estimation is proposed. In the first step, convex optimization and $l_1$ norm penalties are used to conduct a second order cone framework to estimate DOAs roughly under the assumption that all sensor gains are identity. Secondly, we calibrate the perturbed gains with the DOAs estimated by the first step. The iterative process of the DOAs and gains estimation is taken until convergence. Furthermore, a refinement procedure is addressed to alleviate the effects of the grid limitations to spatial locations. In the present method, no auxiliary calibrated sensors or co-operation signal source is required. And the two-step method can handle both uncorrelated signals and coherent signals, it is also a high resolution method and robust in the condition of low SNR (Signal-Noise Ratio). Several aspects are taken into consideration and numerical experimental results are given to show the advantages of the new method for the DOA estimation.

INDEX TERMS Direction of arrival estimation, Array signal processing, Sparse matrix representation, Gain and phase error blind calibration

I. INTRODUCTION

The need for direction-of-arrival (DOA) estimation arises for the reasons that locating and tracking the signal sources have a great development prospect in civilian and military programs [1]–[4]. Among a large number of high resolution methods, DOA techniques have three main research directions: 1) the most popular subspace-based techniques such as MUSIC (Mutliple signal Classification) [5], exploiting the property that the eigenvectors corresponding to the minimal eigenvalue are orthogonal to the columns of the direction vectors. 2) statistical parametric methods such as the maximum likelihood (ML) principle [6], which have been shown to achieve the best mean squared estimation (MSE) performance. Under the assumption that the initial parameters are chosen accurately, the performance of the ML method is convergence to global minimal at the cost of computational complexity. 3) sparsity-based methods such as $l_1$-SVD (Singular Value Decomposition) [7], which proposes a sparse presentation method using the samples from the measurements with an over-complete basis. SRACV method (sparse representation of array covariance vectors) [8], can obtain the sparsest coefficients from the sample-data covariance matrix to estimate unknown DOAs. The sparse Bayesian inference method [9] developed an iterative algorithm under the assumption that all snapshots of the signals have a Laplace prior information. It is known that the effectiveness and practicability of all these methods depend heavily on the accurate knowledge of elements in the array, including their gains and phases, positions, mutual coupling of the elements, etc. However, in practical, behaviors of the sensors itself are effected due to the environment, thus the response of the array may be not perfect. Therefore, lacking of accuracy of the array manifold deteriorates the performance of some methods, especially for the methods that rely most on the model structure such as the subspace-based methods. Hence, the array manifold is supposed to properly calibrated to alleviate the problems introduced by the imprecise array manifold parameters in DOA estimation.
procedure.

In recent decades, four branches of the research to alleviate the imprecise complex gain errors have been studied. The first branch is algorithms based on the special structure of the manifold. One of the most representative methods is Least-Squared (LS) MUSIC [10], which depends on the facts that the received sample data covariance matrix has a Toeplitz structure, and is suitable to the uniformly spaced linear arrays with unknown complex gains of the elements. Li in [11] analysed the method in [10] and proposed a optimal method with dramatically low complexity. Unfortunately, both of the methods hardly have ability to handle coherent signal sources. Performances of these two LS-based algorithms decrease when radiation signals are coherent and fail if signals are perfectly correlated. The second branch is algorithms based on eigenstructure decomposition methods. A. J Weiss proposed an iterative method in [12], which can estimate the DOA and the complex gain errors under the assumption that the unknown complex gains of the sensors are small, or it would suffer from suboptimal convergence. However, for the reasons that the requirement for joint iteration procedure, the problem of suboptimal convergence is unavoidable. To overcome this problem, Liu intergrated and improved the method in [12] and proposed a combined strategy method [13] for nonlinear array from the perspective of eigen-decomposition of a covariance matrix, in which joint iteration procedures are not necessary and the problem of suboptimal is avoided. The drawback of this method is more than two spatially far separated signals are required. Cao [14] improved Liu’s method for the requirement that the numbers of the signals should be more than two and at least $K (K - 1) + 1$ elements in the array are needed to resolve $K$ radiation signals. With the development of compressive sampling (CS) theory and application, the third branch is studied according to the sparsity property of signals. Methods proposed by Hao [15] fill the gap where the data and the regression matrix are both perturbed and solve regularized total least squares optimization problems under sparsity constraints assuming the distribution of signals known. Han imposed a calibration algorithm [16], combining the eigenstructure method [10] and sparsity total least squares method [15] to resolve array model errors in nested array. Li Yanjun [17] transformed the blind complex gains calibration into a structured bilinear inverse problem in several special cases and derived complexities under the noiseless assumption. Cagdas [18] investigated the problem of blindly calibrating unknown sensor complex gain errors in a compressive sensing framework under noiseassumptions. The sparse estimation (or convex optimization) methods have a wide range of applications, for example, limited number of snapshots, highly correlated signal sources, and limited priori knowledge includes the number of the source number, which are the well-known limitations in subspaced-based methods. Therefore, the sparsity estimation methods have been extensively studied. In addition, the fourth kind of the methods based on auxiliary sensors or co-operation signal sources with known DOAs is applied when the unknown complex gain values and DOAs are no related. Auxiliary signal method in [19] has good performance in calibrating unknown complex gains with the known directions of signal sources. Partly calibrated method [20] also has excellent performance if the numbers of calibrated sensors is more than the number of signal sources. This method has the requirement that the numbers of the signal sources received should be more than one or the method would fail due to the singular property of the data matrix. These two methods can only handle uncorrelated signals.

In this paper, a two-step joint DOA estimation and blind calibration method utilizing the sparsity property of signals has been developed under a convex optimal framework. Starting from the unperturbed models, a perturbed by unknown gain and phase errors array model is addressed. Then, details of the two-step method is introduced, a refinement procedure is applied to reduce the effect that the DOAs are not lie on the grids exactly. On the one hand, the presented method is suitable for both uncorrelated and coherent signals, no covariance matrix singular value decomposition (SVD) operation is applied, thus there is no limitation on signal series. On the other hand, the novel method does not require prior information of auxiliary sensors or co-operation signal source which means it is a blind calibration DOA estimation method. Furthermore, by leveraging the sparsity property under convex optimal framework, the presented method is robust to few samples. The specific organization of the paper are as follows. In section II, the data received model for array is given. Then, we describe how to adapt the DOA estimation problem into a sparsity representation under a convex optimal framework, and address the iterated steps to calibrate uncertainty model errors in section III. In section IV, analysis and simulation of several aspects of the presented method are demonstrated.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters. Superscripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ and $(\cdot)\dagger$ denote transpose, conjugate, complex conjugate transpose and Pseudo-inverse respectively. $I$ is the identity matrix. $\left\| \cdot \right\|_F$ represents the Frobenius norm and $\left\| \cdot \right\|_p$ is the $p$ norm for $p = 1, 2$.

**II. PRELIMINARIES AND PROBLEM STATEMENT**

The advance techniques in DOA estimation is to gather signal information from sampling the spatio-temporal wavefield using sensor arrays. In this section, before describing the perturbed models, it is necessary to present the classical unperturbed models received by array. Assumed $K$ farfield narrowband stationary signals from unknown distinct directions of $\{\theta_1, \theta_2, \theta_3, \cdots, \theta_K\}$ are received by a uniformly linear array (ULA) consisting of $M$ omnidirectional sensors with equal spaced $d (K < M)$. In general, the signal can be expressed as

$$s(t) = u(t)e^{j(2\pi f_0 t + \phi(t))} \tag{1}$$

where $f_0 = c/\lambda$ the center frequency of the signal, $\lambda$ is the wavelength of the radiation source, $c$ is the veloci-
ty of the propagation, \( u(t) \) is the baseband signal. Sensor spacing \( d \) should be less than half of the wavelength to avoid spatial aliasing. For simplicity, we set \( d = \lambda/2 \). Let \( \tau_m = (m-1)d \sin(\theta)/c \), \( m = 1, 2, \ldots, M \) denote the time delay from the signal source to the reference sensor. Under the narrowband assumption, time delays are smaller than the inverse bandwidth and thus it may be represented as a phase shift of the complex envelop

\[
s(t - \tau_m) = u(t - \tau_m) e^{j2\pi f_0(t - \tau_m) + \phi(t - \tau_m)} \approx s(t) e^{-j2\pi f_0 \tau_m}
\]

Suppose that each sensor is assumed to have a normalized gain in all directions, then the steering vector can be written as

\[
a(\theta) = [1, e^{-j2\pi/\lambda d \sin(\theta)}, \ldots, e^{-j2\pi/(M-1) d \sin(\theta)}]^T
\]

(3)

The presence of zero mean Gaussian noise is uncorrelated with the signal, the measurement of the array for one source at time \( t \) is

\[
y(t) = a(\theta) s(t) + n(t)
\]

(4)

where \( t \) is one of \( T \) snapshots. Extension to multiple signals case, the model becomes a superposition form

\[
y(t) = \Psi \Phi (A s(t) + n(t))
\]

(5)

where \( A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_K)] \) is a \( M \) by \( K \) matrix containing the steering vector for \( K \) distinct directions of signals as its columns and \( s(t) = [s_1(t), s_2(t), \ldots, s_K (t)]^T \).

Suppose the DOAs are not time varying in the observation window with \( T \) snapshots, the data model can be written as

\[
Y = AS + N
\]

(6)

where \( Y = [y(1), y(2), \ldots, y(T)] \) is a \( M \times T \) matrix, \( S = [s(1), s(2), \ldots, s(T)] \) and \( N = [n(1), n(2), \ldots, n(T)] \). In the presence of unknown complex gain errors, (5) becomes

\[
y(t) = \Psi \Phi (A s(t) + n(t))
\]

(7)

where \( \Psi = \text{diag}(\psi_1, \psi_2, \ldots, \psi_M) \) is a diagonal \( M \) by \( M \) real-value matrix which represents gain errors and \( \Phi = \text{diag}(e^{j\psi_1}, e^{j\psi_2}, \ldots, e^{j\psi_M}) \) is a diagonal \( M \) by \( M \) complex-value matrix which represents phase errors. For simplicity, a square diagonal matrix \( G = \Psi \Phi = \text{diag}(\psi_1 e^{j\psi_1}, \psi_2 e^{j\psi_2}, \ldots, \psi_M e^{j\psi_M}) \) is used to represent the perturbed matrix since matrix multiplication of two diagonal matrices is equivalent to multiplication of corresponding elements. For simplicity, the perturbed model becomes

\[
Y = G(AS + N).
\]

(8)

So in the perturbed case, the manifold \( A \) multiplied by the perturbed matrix \( G \), means that sensors have different multiplier factor, which corresponds to complex gains of sensors. In the unperturbed model we have the assumption that each sensor has the same complex gain. But in the perturbed model, complex gains of the sensors are not same any more, the greater the difference of the gains is, the more serious effect it has on DOA estimation for array. We can observe that there is \( K \) unknowns and \( M \) equations in equation (5). If the perturbed matrix exists, there are \( K + M \) unknowns and still \( M \) equations in equation (7), thus an over-determined estimate becomes a under-determined estimate.

In practice, correlated signal sources are of great possibility and can not be ignored in the algorithm application. The paper is to propose an efficient algorithm to estimate DOAs under perturbed manifold models, which is robust in the presence of coherent sources. Based on analysis of the existing methods, the two-step algorithm of the blind calibration and DOA estimation is developed, the procedure of the algorithm will be addressed in the following sections.

III. SELF-CALIBRATION AND DOA ESTIMATION PROCEDURES

Considering the perturbed models (8), the unknowns can be divided into two parts, one is DOAs of signal sources received, another is the gain and phase errors (we call it complex gain errors for simplicity in the following part). Inspired by the method in [12], an intuitive idea is whether the two parts can be treated separately. An attractive idea is that DOAs under the unperturbed models is firstly estimated, and then use estimations as determined values in the first step to estimate the model errors. The optimization of DOAs and model errors estimation is achieved by the iteration until the model errors are converged. Details of the procedures are described in the subsections.

A. ROUGH ESTIMATION UNDER PERTURBED MODEL

Two aspects are taken into consideration to choose DOA estimation methods. The first is the ability to handle perfectly correlated signal sources, the second is the requirement for high resolution. Even when the complex gain errors are known, some existed high resolution methods do not have the ability to distinguish the perfectly correlated signals because of the rank defect of data model. However, with the development of convex optimal theory and the property that local minimizers are also global minimizers [21], a high resolution method [7] developed by Malioutov, casting the DOA estimation with an overcomplete representation manifold matrix as a sparse representation problem, which does not suffer from accurate initialization and has the ability to resolve correlated signals. Thus, in the first step, we use the \( l_1 \)-SVD processing method in [7] to estimate DOAs.

Starting from the noiseless case, an overcomplete basis of manifold matrix is introduced to represent all possible signal source DOAs. Let \( \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_N \) be the grids of all possible source angles of incidence. The number of the grids \( N_g \) is supposed to be much larger than the number of multiple simultaneous signals \( K \). For instant, in an \( M \)-sensor ULA with the adjacent distance \( d \), the manifold matrix \( A_{oc} \) is a \( M \) by \( N_g \) matrix, composed of all interest angle steering vectors as the columns.

VOLUME 4, 2016
\[
A_{oc} = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
\Delta \sin \hat{\theta}_1 & \Delta \sin \hat{\theta}_2 & \cdots & \Delta \sin \hat{\theta}_N \\
\Delta \sin \hat{\theta}_1 & \Delta \sin \hat{\theta}_2 & \cdots & \Delta \sin \hat{\theta}_N \\
\ldots & \ldots & \cdots & \ldots \\
\Delta \sin \hat{\theta}_1 & \Delta \sin \hat{\theta}_2 & \cdots & \Delta \sin \hat{\theta}_N \\
\end{bmatrix}
\]

where \( \Delta = -j2\pi d/\lambda \). Matrix \( A_{oc} \) in this framework is known but not adaptive and it is an overcomplete basis which does not depend on the DOAs of the incidence sources. Remember that sources come from \( K \) distinct directions, which means under the model

\[
Y = A_{oc}S
\]  

where \( Y \) is a \( M \) by \( T \) matrix, which represents the output measurements of \( M \) sensors. \( S \) is a \( N_g \) by \( T \) matrix represents the coefficient matrix corresponding to the overcomplete basis of manifold matrix. So, in \( S \), there are only \( K \) rows are nonzero and the values in the nonzero rows are the envelop of the baseband signal \( u(t) \) in equation (1). The solution of searching the nonzero values can be regarded as an inverse problem. Note that the nullspace of matrix \( A_{oc} \) is not trivial. An intuitive idea to solve the inverse problem is to search the minimum-norm least squares solutions. But the solution of \( S \) may be not sparse and fails to work. Based on the prior knowledge that \( S \) is sparse, considering a single snapshot of all sensors measurements \( y, s_j \) represents one of the columns of the matrix \( S \), and a meaningful solution to this problem is to solve

\[
\min \|s_1\|_0, \text{ s.t. } y = A_{oc}s_1
\]

where \( \|\bullet\|_0 \) represents the number of nonzero of the vector. It is an NP(Non-deterministic Polynomial) problem, so an alternative solution of (11) can be substituted by

\[
\min \|s_1\|_1, \text{ s.t. } y = A_{oc}s_1
\]

where \( \|\bullet\|_1 \) represents sum of all the module value of nonzero elements in the vector. An equivalent form of equation (12) is

\[
\min \|s_1\|_1, \text{ s.t. } \|y - A_{oc}s_1\|^2_2 = 0
\]

If noise exist, the residual parametric can be set as a regularization parameter \( \beta \) specifying the ability to accommodate noise. Then, equation (13) becomes an appropriate form

\[
\min \|s_1\|_1, \text{ s.t. } \|y - A_{oc}s_1\|^2_2 \leq \beta
\]

The objective function and the constraint item can be represented in an unconstrained form, see [22] for details

\[
\min \|y - A_{oc}s_1\|^2_2 + \zeta \|s_1\|_1
\]

In (15), the first term which represents noise term should be small, and the second term represents the sparsity requirement. An intuitive understanding of \( l_1 \) norm forcing sparsity can be explained in a two-dimension space case. Under two-dimension space case, normalized \( l_1 \) norm is a squared graphic which has one corner at each axis. \( l_1 \) norm is a convex function, when \( l_1 \) norm ball meets the quadratic function, it is extremely likely that the meet-point is at one of corners of the \( l_1 \) norm square graphic, which means the value of one axis is zero and the other one is the value of the meet-points, sparsity is forced. Parametric \( \zeta \) is a regularization term, which balance the solution of the data and the sparsity. The choice of the item \( \zeta \) and the more information can be found in [23]. Extend to multiple sampling, instead of decomposing covariance matrix of data received into signal subspace and noise subspace, singular value decomposition(SVD) of the data is done to obtain its signal source subspace. Mathematically, this translation can be represented in following form

\[
Y = U\Sigma V
\]

where \( U \) is a \( M \) by \( M \) matrix, which represents the left singular vectors. Extracting most of the signal powers, a reduced dimensional matrix

\[
Y_{sv} = U\Sigma D_K = YV^H D_K
\]

In (17), \( D_K = [I_K, 0]^T \) contains two parts, the first part is a \( K \) by \( K \) identity matrix, the second part is a zero matrix. Thus, the output measurement matrix is both dimensional reduced and non-sensitive to noise level. The processing of equation (17) is effective in perfectly correlated case as \( \text{rank}(Y_{sv}) \) is not reduced.

Let \( S_{sv} = \text{SVD}_K, N_{sv} = \text{NVD}_K \) and write the \( l_2 \) norm of each row of \( S \) as a column vector \( \vec{s}^2 \), thus multiple sampling unconstrained form become

\[
\min \|Y_{sv} - GA_{oc}S_{sv}\|^2_F + \zeta \|\vec{s}\|^2_1
\]

If \( G \) is known in (18), only \( S_{sv} \) is an unknown \( N_g \) by \( K \) matrix. Actual DOAs on the grids can be picked up at the nonzero rows, then \( \hat{\theta} = [\hat{\theta}_1; \hat{\theta}_2; \cdots; \hat{\theta}_K] \) is obtained. After DOA estimation, the second procedure is to estimate complex gain errors. According to the estimated DOAs, the manifold can be recovered as

\[
A_{est} = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
\Delta \sin \hat{\theta}_1 & \Delta \sin \hat{\theta}_2 & \cdots & \Delta \sin \hat{\theta}_K \\
\Delta \sin \hat{\theta}_1 & \Delta \sin \hat{\theta}_2 & \cdots & \Delta \sin \hat{\theta}_K \\
\ldots & \ldots & \cdots & \ldots \\
\Delta \sin \hat{\theta}_1 & \Delta \sin \hat{\theta}_2 & \cdots & \Delta \sin \hat{\theta}_K \\
\end{bmatrix}
\]

Then, estimation of source Matrix \( S \) can be obtained by Least Square (LS) method according to (7)

\[
\hat{s}(t) = (GA_{est})^\dagger y(t)
\]  

B. GAIN AND PHASE ERRORS CALIBRATION

Consider DOAs estimated as prior determined parameters, it is easy to construct a suitable cost function to minimize the residual of the measurements for estimating \( G \) in (18). An
appropriate choice of this cost function is

$$C_m = \sum_{t=1}^{T} ||y_m(t) - G_m A_{est}(m,:) s(t)||^2_2$$  \hspace{1cm} (21)$$

where $y_m(t)$ is one of all $T$ measurements of the $m$th sensor in the array. The expression (21) is a convex function, it is a quadratic cost function, relying on the property that the local minimizers are also global minimizers, unknown $G_m$ can be obtained. Then complex gain errors matrix $\hat{G} = \text{diag}([\hat{G}_1, \hat{G}_2, \ldots, \hat{G}_M])$ can be constructed.

After model errors $\hat{G}$ obtained, it is brought back into (18) and DOAs estimation is updated. An iterative operation stops until convergence achieved for DOAs and $\hat{G}$. Simulations have been tested and verified that a accurate DOA estimation will be obtained in no more than five times.

### C. REFINED GRIDS PROCEDURE

As described in (9), an overcomplete basis $A_{oc}$ is constructed in terms of all possible source locations is confined to the setting grid. However, in most cases, the DOA of a radiated source can not lie on the grids exactly, the algorithm may suffer from basis mismatch problem [24]. Therefore, it is necessary to break through the grid limit for better estimation. Inspired by binary searching, we explore a method to refine the grid.

Before the refine procedure, assumed that the DOA estimation is around the true value, we refine each DOA estimation separately. Choosing one rough grid $\hat{\theta}_1$ of potential source DOA, the task of refined DOA estimation is to searching the biggest nonzero spectrum value in the neighborhood of $\hat{\theta}_1$ for better performance. We divide the searching interval into $Q$ subgrids, where $Q$ is set to even integer for simple. And the spacing of the grid is $\epsilon$. For instance, choosing $Q = 4$, the possible source DOA should be in $[p_1, p_2, p_3, p_4, p_5] = [\hat{\theta}_1 - 2\epsilon, \hat{\theta}_1 - \epsilon, \hat{\theta}_1 + \epsilon, \hat{\theta}_1 + 2\epsilon]$, see Figure1. the refined grid manifold matrix can be written as

$$A_{rg} = \begin{bmatrix} 1 & e^{-\Delta \sin p_1} & e^{-2\Delta \sin p_2} & \cdots & e^{-\Delta \sin p_5} \\ e^{\Delta \sin p_1} & 1 & e^{2\Delta \sin p_2} & \cdots & e^{\Delta \sin p_5} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e^{(M-1)\Delta \sin p_1} & e^{(M-1)\Delta \sin p_2} & \cdots & 1 & e^{(M-1)\Delta \sin p_5} \end{bmatrix}$$  \hspace{1cm} (22)$$

Using the final iteration result of gain errors matrix in (21), denoted as $G_{est}$, the objective function based on equation (18) can be rewritten as

$$\min \| Y_{sv} - G_{est} A_{rg} S_{sv} \|_F^2 + \zeta \| \hat{G} \|_1.$$  \hspace{1cm} (23)$$

Figure 1 illustrates the details of the refine procedure when $Q = 4$. Consider one of the rough DOAs value before the refinement as the reference point (marked as a red line), then we extend $Q/2$ grids around the reference point in either side, and compute the spectrum values of the $Q + 1$ points according to equation (23). The grid position corresponding to the biggest value of the spectrum values is the updated reference grid point. Five grid points have labelled from $p_1$ to $p_5$ in the beginning. If the reference point still fall into point $p_3$ after the first updated, the new searching interval will be shorten to $[p_2, p_4]$, see case 1 in Figure 1.

![Figure 1. Illustration of equal space refinement.](image)

Repeat to divide interval $[p_2, p_4]$ to $Q$ equal grids and compute $Q + 1$ spectrum values. If the reference point fall into point $p_2$ after the first update, the new searching interval becomes $[p_1, p_3]$, see case 2. In the situation that the reference point is chosen on the edge point$(p_1$ or $p_5$), the searching will expend to new segment. For example, the reference point just lies on the $p_3$, see case 3. The new shorten interval become $[p_4, p_6]$, which is also divide into $Q$ equal grids in the same way around $p_5$. As stated above, the refined DOA can be obtained with the searching interval decreasing. An important issue in this procedure is that the grid can not be set infinitely small, otherwise the columns of the basis become perfectly correlated, and the basis fails to work. So the minimum grid length depending on the requirement of the estimation should be set before the refined procedure.

If minimum grid length is $\epsilon$, then the number of iterations of the refine procedure in the method is no more than

$$\delta = \left\lfloor \log_4 \frac{Q\epsilon}{\epsilon} \right\rfloor$$  \hspace{1cm} (24)$$

Thus, after the refined step above, the more accurate result of DOA estimation will be obtained.

### IV. EXPERIMENTAL RESULTS

In this section, several experimental results are given to show the performance of the proposed method. We consider a ULA with $M = 7$ sensors, in which the distance between adjacent elements is half-wavelength of incident narrow-band signals.

Three equal power signals impinge from different DOAs $\theta_1 = -18.87^\circ$, $\theta_2 = 6.12^\circ$, $\theta_3 = 32.25^\circ$. The coarse grid is uniform with $1^\circ$. So number of sampling is $Ng = 181$, scanning angles from the boresight to endfire, [$-90^\circ$, $90^\circ$]. As stated above, none of the impinging angles lie on the grid exactly. Gain errors are assumed as a uniform distribution with interval $[0.5, 1.5]$ and phase errors as a uniform distribution with interval $[-10^\circ, 10^\circ]$. It should be noted that sensor gain and phase uncertainties should not exceed this scope or
the performance of the proposed method will decline. The regularization parameter $\zeta$ is chosen according to the noise level, see [7]. The proposed algorithm tabulated as follows.

**TABLE 1.** Framework of blind calibration and DOA estimation.

<table>
<thead>
<tr>
<th>Input</th>
<th>$Y$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>$G_{\text{est}}$, DOAs.</td>
</tr>
</tbody>
</table>

**Step 1** initial $G_{\text{est}}(0) = I_{M \times M}$, set $A_{oc}$ according to the grid and array structure; iteration times $i$, refined grids times $j$.

**Step 2** for $i = 1, 2, 3, \ldots$ do

**Step 3** Estimate DOAs use (18) upon convergence

**Step 4** $i = i + 1$

**Step 5** Update $G_{\text{est}}(i)$ use (21)

**Step 6** end for

**Step 7** for $k = 1$ to $K$ do

**Step 8** for $j = 1$ to $\delta$ do

**Step 9** Refine DOA estimation use (23)

**Step 10** end for

**Step 11** end for

**A. EFFECTIVENESS OF THE PROPOSED METHOD**

Firstly, the estimation of the uncorrelated signals is shown. The total number of snapshots $T$ is 100. The root mean squared error (RMSE) of uncalibrated estimation, calibrated estimation without refinement and the refined estimation are given for 500 trials in Figure 2.

It can be seen that the proposed method give better performance of DOA estimation and do alleviate the perturbed problem.

![FIGURE 2. RMSE for uncorrelated sources with DOAs of $-18.87^\circ$, 6.12$^\circ$, 32.25$^\circ$. Snapshots $T = 100$.](image1)

In Figure 3, we compare the proposed method with joint iteration method [12], LS method [10], single auxiliary signal source (SAS) method [19], partly calibrated method (PCM) [20]. Joint iteration method [12] is not shown in Figure 3 because the method is failed in the larger phase perturbation situation.

![FIGURE 3. Comparison among the presented method, LS-MUSIC method, SAS method, PCM method for three uncorrelated signal sources with DOAs of $-18.87^\circ$, 6.12$^\circ$, 32.25$^\circ$. Snapshots $T = 100$.](image2)

The larger RMSE of LS method may be caused by two reasons. The first reason is that when the number of the snapshots is not enough, the signal subspace and the noise subspace are not completely independent in the eigen-expansion based method. The other one is limitation of the searching step, the angles can not lie on the grid exactly. SAS method has better performance than the proposed method when SNR is lower than 0dB, but the SAS method performance remains when the SNR is beyond -2dB. In addition, the proposed algorithm is a blind estimation method without co-operation radiation source for the calibration. So the proposed method is suitable for more wide practical application compared with SAS method. PCM has the requirement that the number of the unperturbed sensors should more than the numbers of the signals received, so four unperturbed sensors in the array are required in the simulation. Its advantages lie in less sensors needed to calibrated. It can be seen that the proposed method still has better DOA performance close to that of

![FIGURE 4. RMSE for strongly correlated sources with DOAs of $-18.87^\circ$, 6.12$^\circ$, 32.25$^\circ$. Snapshots $T = 100$.](image3)
PCM method compared with SAS and LS when SNR is larger than 2dB.

Figure 4 gives the RMSE of the proposed method versus SNR with correlated sources existence. We set correlation coefficient between the second signal of $\theta_2 = 6.12^\circ$ and the third from $\theta_3 = 32.25^\circ$ to 0.99, which means two sources are almost perfectly correlated. Other parameters are same as the uncorrelated case. It can be seen that the proposal algorithm can successfully estimate the DOAs and complex gain errors in the situation of impinging correlated sources.

**B. ROBUSTNESS TO THE NUMBER OF THE SNAPSHOTS**

According to the analysis of eigen-expansion based method, the performance of MUSIC-like estimation methods depend heavily on SNR and the number of the snapshots. By the sparsity representation framework, robust to the limitation of the snapshots is one of the merits of our method since no covariance matrix is computed, and there is less impact whether signal subspace and noise subspace are separated well. In Figure 5, the behavior of the robustness to the number of the snapshots is illustrated in SNR=0dB. Both cases of uncorrelated and strongly correlated sources are taken into consideration. As the parameters are the same to IV-A, it can be observed that the presented method still have ability to obtain good estimation from perturbed data. Certainly, the RMSE improves as the the number of the snapshots increases. And the performance for uncorrelated case is better than for correlated case.

![Figure 5](image)

**FIGURE 5.** Robustness to the numbers of the snapshots.

**C. COMPARISON OF MODEL ERRORS AND CALIBRATED RESULTS**

The aspect of model errors estimation in our method is illuminated and we show the comparison of the model errors and calibrated results after iteration. The model errors of the array are generated randomly, which list in table 2.

<table>
<thead>
<tr>
<th>Sensor index</th>
<th>Gain</th>
<th>Phase(degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.25</td>
<td>-1.8</td>
</tr>
<tr>
<td>2</td>
<td>1.27</td>
<td>-9</td>
</tr>
<tr>
<td>3</td>
<td>1.33</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>1.38</td>
<td>-4.5</td>
</tr>
<tr>
<td>5</td>
<td>1.14</td>
<td>-3</td>
</tr>
<tr>
<td>6</td>
<td>0.78</td>
<td>1.5</td>
</tr>
<tr>
<td>7</td>
<td>1.29</td>
<td>-3.9</td>
</tr>
</tbody>
</table>

It can be seen that all sensors are perturbed more or less. To show the calibration results clearly, we normalized the complex gains of the seven sensors in the array and divide them into real and imaginary parts. The more closer the distance between the preset value and calibrated value, the better the algorithm does. The first sensor is identity due to the normalize operation. The results of the real and imaginary parts are shown in As the simulation results, the complex gain errors can be well calibrated and the effectiveness of the calibration method is verified. Secondly, the calibration ability among four methods mentioned above are compared. The following RMSE of the complex gain values is computed as the following

$$\text{RMSE}_{cv} = \sqrt{\frac{\sum_{m=1}^{M_c} \sum_{e=1}^{E} (\hat{e}_m - v_m)^2}{M_c E}}$$  \hspace{1cm} (25)$$

where $M_c$ represents the number of sensors calibrated in the simulation and $E = 500$ is the number of the trials. Figure 6 and 7 respectively.

![Figure 6](image)

**FIGURE 6.** Calibration results comparison of real part.

The proposed method and the LS-MUSIC method need to calibrate 7 sensors, SAS method needs to calibrate 6 sensors, PCM method needs to calibrate 3 sensors. $v$ is represented as the initial perturbed values, and $\hat{v}$ as the calibrated values. Because the perturbed values are complex values, both normalized real part and imaginary part are taken into consideration. It can be seen that the proposed method has a good performance in unknown perturbation calibration in Figure 8 and Figure 9.
The proposed method has a better performance than other three methods in real part, and a little less in imaginary part than SAS method and PCM method. The number of sensors in the method to be calibrated is more than the two methods may cause this result. The performance of the calibration is in agreement with the result shown in Figure 3.

![Calibration results comparison of imaginary part.](image)

**FIGURE 7.** Calibration results comparison of imaginary part.

![Calibration ability comparison of normalized real part among four methods.](image)

**FIGURE 8.** Calibration ability comparison of normalized real part among four methods.

**V. CONCLUSION**

In this paper, we proposed novel strategies to solve the sources location problem by array with model errors. By analysing existed DOA estimation and the sparsity representation framework of perturbed model errors, a two-step blind calibration and DOA estimation method is developed to alleviate the model errors perturbation problem for good DOA performance through iterations. And we give a DOA refinement process to mitigate the effects of the limitation of grid. To further illustrate the advantages of the presented method, we verify that it is suitable to either uncorrelated signal sources or strongly correlated sources and the robustness to the limitation of snapshots.

**ACKNOWLEDGEMENTS**

The authors would like to thank the anonymous reviewers and the Associate Editor for their valuable comments and suggestions, which have greatly improved the quality of this paper.

**REFERENCES**


---

**LUTAO LIU** was born in Harbin, China in 1978. He received the B.A. degree in Electrical Engineering in Southeast University of China in 2000. In July 2003, he received the MSC degree in Telecommunication Engineering from Harbin Institute of Technology, China. In July 2005, he received the MSC degree in Microelectronics from Delft University of Technology, Netherlands. In July 2011, he received the Ph.D. degree in Telecommunication Engineering from Harbin Engineering University, China. In 2013, he was a visiting scholar at Signal Processing and Communication (SPAC) Laboratory, Stevens Institute of Technology, USA. At present, he is an associate professor in College of Information and Telecommunication, Harbin Engineering University. His research interests are in the general area of signal processing for telecommunication.

---

**YANAN WU** was born in Changchun, China in 1992. She received the B.A. degree in Measuring and Control Technology and Instruments from Chang’an University, Xi’an, China, in 2015. She is currently working toward the Ph.D. degree in College of Information and Telecommunication, Harbin Engineering University, China. Her research interests include array signal processing and their applications.

---

* * *