A Novel Method to Design Delay-Scheduled Controllers for Damping Inter-Area Oscillations

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ABSTRACT The main problem of time delay in application of remote signals is that it limits the contribution of the controller devices in damping inter-area oscillations, thus to overcome this problem, we proposed a new time delayed controller in this paper. The controller takes the advantage of delay as a design parameter to solve the SVC contribution limitation problem resulting from delays in feedback loops. To determine the delay and controller parameters, an algorithm is proposed to minimize the rightmost real part of the electromechanical oscillation modes in the design stage such that the system robustness against small variations of time delay is provided. The feasibility of the proposed method is evaluated by conducting a number of simulations on the standard four-machine and 16-machine 68-bus test power systems. The results reveal that compared to the existing methods, the proposed method not only exhibits a better damaging performance, but also introduces a significant delay margin in a certain range of feedback delays.

INDEX TERMS Delay range stability, Delay differential-algebraic equations, Large-scale power systems, Spectral abscissa.

I. INTRODUCTION

The impacts of delay within feedback control loop on control ability, performance enhancement and stability of the closed loop system have been investigated by many researchers [1-2]. The growth of DG penetration in power systems has extended the border of generation areas into the distribution levels. This necessitates broader application of decentralized control systems throughout the power grid. As a fundamental part of the control system, remote measurement devices are expected to be widely used to carry signals from long distances to local control centers. By means of remote measurement systems, electrical data such as phasors of line currents, electric powers, and bus voltages are synchronously available for control and monitoring purposes. Hence, the overall stability of the power system is improved due to existence of broad information about system condition in local control centers. However, delays originated from data measurement and transmission processes are major threats in such applications [3-8]. Several factors including measuring tools, data transmission interfaces, communication links and physical distances between the transmission points affect the incurred time delay [9]. [10] reports inherent time delay values in the range of 100–700 ms. Numerous researches have looked at delayed power systems from different angles to clarify the impact of delay on power system operation and methods of mitigating adverse effects [11-12]; to design supplementary controllers for large-scale power systems in the presence of feedback time delay [13-15]; to determine a maximum delay margin using the linearized power system model [16-17]; and, in [18-19], to compensate the communication network delay time, an adaptive approach is proposed to adjust the weights of the pre-designed compensators based on the goal representation heuristic dynamic programming algorithm. In most cases, the crucial challenge is control system design considering the presence of delays in input signals measured at various locations within the power system. In terms of gaining better control and system stability, giving further delay to the input signal may provide the system with better opportunity to perform. Various design and analysis techniques as well as practical implementations have been presented in [20-22] without providing any application in electric power systems. This study takes the advantage of stabilization effect of delayed feedback signals in designing a supplementary controller.
with the highest ability. The designed supplementary controller is used to manipulate power flow of a FACTS device by controlling its terminal voltage.

Generally, the stability analysis and control of delayed systems is more complicated than delay-less systems. Dynamic representation of delayed systems in state space equations consists of both instant state variables and delayed state variables [23]. Controller design methods can be divided into two main categories. The first category includes methods that yield stability independent of time delay or so called delay-independent stability methods. The second category consists of all the methods that authenticate the conditions in which increasing the delay up to a certain value will guarantee the stability of the system, which are also referred to as delay-dependent stability methods. Since in an actual power system, the amount of delay in feedback loop is limited, the delay-dependent stability methods are more common for designing controllers [24-25]. In this approach assuming no delay, controller is designed to promote stability. However, system should also be stable in the presence of delay. This requirement is met by readjusting controller parameters while considering the maximum permissible value for delay parameter. The maximum value of time delay is computable. The reason lies in the fact that by some assumptions the spectral abscissa of the system becomes a continuous function of delay parameter and hence the upper bound for delay parameter can readily be obtained. This method, however, decreases the capability of the controller in damping oscillations since the controller parameters should be modified to adopt maximum value for delay parameter.

[26] has shown that time delays in the feedback control loop of Power System Stabilizer (PSS) devices do not necessarily imply a deterioration of the dynamic response. Actually, estimating the value of time delay and would a decrease of the gain of the PSS can the effect of the delay on the system dynamic response could be minimized. Despite common approaches, time delay can be viewed in a positive manner when control system design is dealt with. This means that under some circumstances with increasing the time delay of input signals, performance of the control system and its stability would be improved, which is called delay range stability analysis. In electric power systems, measured signals have always time delays, which directly enter into the control loop. This may cause power system to be more vulnerable under unstable conditions. Under these circumstances, a supplementary controller is usually designed to increase distance to instability boundaries of the system. The structure of this controller is usually required to provide a good perception of the system performance and having precise model of it.

In this paper, a new method for designing robust controller based on the premise of delay range stability is proposed. Straightforward design procedure and easy implementation are key benefits of the proposed method. It is important to consider the stability sensitivity in terms of a small change in delay in designing a stabilizer controller. As discussed in [27], a small delay perturbations could bring the system to instability region. Therefore, the controller is so optimized to become robust against a limited time delay variation. The backbone idea and method of obtaining the controller parameters are discussed as well. Delayed power system is modeled in section two. Then, due to its importance, the method of feedback signal selection is described using the model of a single-machine infinite bus system. In the third section, the theoretical foundations and the equations governing the stability analysis of delayed systems are reviewed. Then, the controller’s design technique is described by the optimization of the spectral abscissa in a robust manner. This, in turn, ensures the stability of the system against delay variations. On this basis, the fourth section is dedicated to design a specific controller for a SVC whose complementary action is to damp inter-area power oscillations in a two-area and a five-area power system. A number of nonlinear simulations in MATLAB environment are conducted to show the feasibility and effectiveness of the proposed method. Finally, simulation results along with descriptive explanations are presented in section five.

II. GENERAL MODEL OF DELAYED POWER SYSTEM

A. MATHEMATICAL MODELING

Equation (1) represents general dynamic behavior of a linearized power system with no auxiliary controller.

$$\dot{x}(t) = Ax(t) + Bu(t),$$  \hspace{1cm} (1)

where $x(t) \in \mathbb{R}^n$ represents the state variables, $u(t) \in \mathbb{R}^m$ is manipulated input.

One way of increasing stability and improving system performance is to apply a proper control signal to $u(t)$. Different types of controller can be used for this purpose. In this work, a delay-type controller is considered. The reason here is that the signal received by the controller input is a combination of signals coming from wide area measurement system that are usually associated with delay. Thus, delay-type controller can be used in order to provide stability by adding delay to the input signal before applying it to $u(t)$. As a result, the control law is defined as:

$$u(t) = y_c(t - \tau),$$  \hspace{1cm} (2)

the inherent delay of the proposed controller, $\tau$ is a control parameter to be determined through an optimization process that will be explained in further detail in upcoming sections. The output of the controller, $y_c(t)$ is determined from the associated state space model as given in (3).

$$\dot{x}_c(t) = Fx_c(t) + Gu_c(t),$$  \hspace{1cm} (3a)

$$y_c(t) = Hx_c(t) + Ku_c(t),$$  \hspace{1cm} (3b)

where, $u_c(t)$ is the input vector, $x_c(t)$ is the output and $x_c(t) \in \mathbb{R}^{nc}$ is the vector of states of the controller. Order and structure of the controller are both assumed to be fixed.
and thus the real matrices $\mathbf{G}$, $\mathbf{H}$, $\mathbf{L}$ and $\mathbf{K}$, with proper dimension, represent the controller parameters. The controller input is a simple or a combinational signal measured at any point of the power system. To take into account the delay of measuring system and communication technologies, the relation between the controller input and the power system output should be stated as:

$$u(t) = y(t-h),$$

(4)

where, refer to (1), the generic expression for $y(t)$ – the power system output signal – could be:

$$y(t) = C x(t) + D u(t).$$

(5)

The block diagram of a linearized power system with the proposed controller is schematically shown in Fig.1. From the figure, $h$ that usually varies in a specific range, is the feedback delay in the power system. Substituting (4) into (3) and (2) into (1) and (5), one can obtain (6), which defines the dynamics of the controlled system and is in the form of Delay Differential Algebraic Equations (DDAEs).

\[
\begin{align*}
\frac{\dot{z}_1}{t} &= P z_1 + Q z_2 - t - h + R z_2 - t - h, \\
\frac{\dot{z}_2}{t} &= T z_1 + U z_2 - t - h + V z_2 - t - h,
\end{align*}
\]

(6a)

where

\[
\begin{align*}
P &= \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix}, & Q &= \begin{bmatrix} B & 0 \\ 0 & 0 \end{bmatrix}, & R &= \begin{bmatrix} 0 & 0 \\ 0 & G \end{bmatrix}, & z_1 &= \begin{bmatrix} x \\ x_1 \end{bmatrix}, \\
T &= \begin{bmatrix} C & 0 \\ 0 & H \end{bmatrix}, & U &= \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix}, & V &= \begin{bmatrix} 0 & 0 \\ 0 & K \end{bmatrix}, & z_2 &= \begin{bmatrix} y \\ y_1 \end{bmatrix}.
\end{align*}
\]

Equation (6) actually represents a model for delayed power systems in the form of DDAEs. All coefficients in this equation are matrices with proper dimension. The first step of design is to select the proper input signal which is highly dependent on the application. This will be discussed in detail in the next section.

**FIGURE. 1 State space diagram of the closed-loop power system**

### B. FEEDBACK SIGNAL SELECTION

Feedback signal selection is one of the important steps of a successful design and implementation of supplementary controllers for large-scale power systems. Signal selection based on a linear model is the easiest and most efficient approach. Various methods and criteria have been presented for the selection of the feedback signal, including higher Hankel singular values in [28]; the residue amount and visibility in [13]; and, so on. In many cases, local signals do not demonstrate proper sensitivity to oscillation modes. Therefore, distant measured signals with acceptable values are selected. Control systems are usually designed with the approach of delay-dependent stabilization. Thus, the basis of feedback signal selection also lies in the frame of systems without time delays. However, it may be ineffective, since a linear time-delay system may limit controller capabilities. On the other hand, the proposed control method is designed with the approach of time delay-affected stabilization. Clearly, a linear delay system is matched with an infinite dimensional system, and common criteria for selecting inputs and outputs can only be used for finite-dimensional systems [29]. For linear time-delay systems operating close to equilibrium, methods for linear systems are useful for initial screening for signal selection. Considering the above-mentioned facts, the proposed method of feedback signal selection is described through an example.

Now, we only consider the linear system of the (SMIB) electromechanical model in describing the state space with delay input:

\[
\begin{align*}
\dot{x}_1 &= \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & 0 \end{bmatrix} x_2 + 0 u(t-\tau), \\
\dot{x}_2 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x_2 + 0 y(t-\tau),
\end{align*}
\]

(7a)

where, $x_1$ is the rotor angle deviation, $x_2$ is the rotor speed deviation, $u(t-\tau)$ is the manipulated input, and $\omega_n$ is the non-damped natural frequency. This system has a pair of imaginary poles when no control input is applied. If the desire is to damp out the oscillations by inserting $u(t-\tau)$, selecting feedback signal and its impact on moving the rightmost eigenvalue due to variation of $\tau$ becomes very important. Nevertheless, we assume that either $y_1$ or $y_2$ can be used as a measured variable to design a static output controller. The proportional gain is determined for an incremental delay through an optimization model whose objective is to minimize the spectral abscissa at each step of delay increment. The results indicating minimum spectral abscissa versus delay parameter is shown in Fig.2.

**FIGURE. 2 The spectral abscissa per $\omega_n$ as a function at $\tau \omega_n$**
Fig. 2 reveals that choosing \( y_2 \) as an input for the control system is more beneficial since more damping is introduced when the \( \tau_{\omega h} \) falls in the range of 0 to 0.678 Rad. However, when the \( \tau_{\omega h} \) falls in the range of 0.68 to 2.36 Rad, \( y_1 \) becomes more effective to be used as an input of the control system. Hence, assuming that an appropriate input is available, in order to increase the damping of oscillations in a delayed power system, the best signal to control the parameters of an actuator, such as FACTs, is the angle signal of generators. In this work, the input signal to a SVC supplemental controller has been selected based on a simple and innovative method. The steps to be taken are as follows:

First, consider the representation of the minimal transfer function between each of the output and input signals as follows:

\[
G(s) = \sum_{i=1}^{n} \frac{a_i s + b_i}{s + \sigma_i} + \sum_{i=n+1}^{\infty} \frac{c_i}{s + P_i},
\]

then, for each of the electromechanical dominant poles, create the following set:

\[
Cl_i = \left\{ \frac{a_i}{\text{norm}(a_i, b_i)} \right\}, i = 1, 2, ..., k,
\]

in which, \( k \) is the number of electromechanical and dominant poles. Finally, taking into account the weight of each of the dominant modes in the \( Cl_i \) value, select the input signal for the complementary controller with the lowest \( Cl_i \). It is worth noting that this solution is useful when the approach of time delay-affected stabilization is taken.

III. STRONG STABILITY OF A DELAYED POWER SYSTEM

Generally, the stability analysis of a linear time-delay system is studied by means of measuring spectral abscissa (real part of the rightmost eigenvalues) in spectrum-based analysis. Linear Delay Differential Algebraic Equations (LDDAE) and the Neutral Delay Differential Equations (NDDE) are similar in some aspects of their characteristic equations. In this work, by comparing stability analysis criteria of LDDAEs and NDDEs, a method for stability assessment of LDDAEs is developed and utilized to design the supplementary controller. The stability analysis models are discussed first.

A. STABILITY ANALYSIS OF LDDAE

Suppose a linear time–delay differential system as given in (6). By taking the Laplace transform from both sides, we can set up the characteristic equation as the one given in (10).

\[
\Delta(s) = \left| \begin{array}{cc} D_{11} & D_{12} \\ D_{21} & D_{22} \end{array} \right| = \left| \begin{array}{cc} sI - P & -Qe^{-\gamma \tau} - Re^{-\lambda h} \\ T & I - Ue^{-\gamma \tau} - Ve^{-\lambda h} \end{array} \right|,
\]

where \( | . | \) is determinant operator.

By definition, the system is said to be asymptotically stable if real part of the roots of (10) becomes negative, i.e.

\[
\gamma = \sup Re s_n < 0,
\]

in which, \( \{ s_n \} \) holds the sequence of roots of the characteristic equation that is infinite dimensional, and \( \gamma \) is value of the spectral abscissa. The stability analysis of delayed systems is usually associated with checking stability conditions in a neighborhood of delay variations. This is studied by making a small change in delay parameters of the system to establish a new dynamic model for stability assessment. The following equations are obtained for the system given in (6).

\[
\begin{align*}
\dot{z}_1(t) &= Pz_1(t) + Qz_2(t - \tau) + Rz_2(t - \tau) - d\dot{\sigma} + Mz_1(t) + Nu(t) - dh, \\
\dot{z}_2(t) &= Tz_1(t) + Uz_2(t - \tau) + Vz_2(t - \tau) - dh.
\end{align*}
\]

In which, \( \tau - \delta t > 0 \) and \( h - dh > 0 \) are assumptions.

If system (12) satisfies asymptotic stability conditions, then the time delay system represented by (6) becomes strongly stable. However, it is quite difficult to examine asymptotic stability of the system (12) by repeatedly computing the spectral abscissa against several small changes in time delay parameters. As an alternative, the NDDEs can be rewritten in the form of DDAEs. Therefore, the strong stability conditions of the neutral equations could be applied for DDAEs evaluation. In continuation, the definition and stability conditions of the neutral systems are reviewed so as to propose a fairly new and simple method by which the strong stability of the system (6) can be readily addressed.

A state space representation of a neutral system with zero reference input signal is given in (13):

\[
\frac{d}{dt}z(t) + Az(t) - \tau = Mz(t) + Nz(t - h),
\]

assuming a proper dimension for all coefficient matrices, the stability of (13) as a function of time delay parameter do not always behave monotonically along delay variations. This is mainly due to the algebraic part of the system which may cause the stability to be lost even for a small variation of delay [30]. Referring to (13), the strong stability is achieved when the difference equation which is denoted by \( z(t) - Az(t - \tau) \) becomes exponentially stable. This is equivalent to say that \( \rho(A) < 1 \), where \( \rho(\cdot) \) is the spectral radius computational function. Note that the stability is independent on delay values. For instance, the asymptotic stability of the above-mentioned difference equation, is equivalent to strong stability.

By introducing slack states, one can rewrite the neutral system equations as follows:

\[
\begin{align*}
\dot{w}(t) &= Mz(t) + Nu(t), \\
w(t) &= z(t) - Az(t - \tau).
\end{align*}
\]

As it can be seen, the new form is likened to the equations representing the LDDAE. The characteristic equation of system (14) is defined as follows:

\[
\Delta(s) = \left| \begin{array}{cc} D_{11} & D_{12} \\ D_{21} & D_{22} \end{array} \right| = \left| \begin{array}{cc} sI - M - Ne^{-\gamma \tau} \end{array} \right|.
\]
Applying Schur determinant formula to above equation, one can rewrite it as below [31]:
\[
\Delta(s) = \text{det} (D_{zz}) \text{det} (D_{xz} - D_{z2} D_{1z}) ,
\]
where,
\[
\text{det} (D_{zz}) = \text{det} (I - Ae^{-\gamma s}) .
\]

This equation is called the associated model and is used to check the strong stability of the neutral systems. In this regard, by referring to (10), the associated model for strong stability analysis of (6) can be defined in a similar way as:
\[
D_{zz}(s) = I - U e^{-\gamma s} - V e^{-\gamma s} .
\]  
Equation (17) could represent the characteristic matrix for an algebraic time delay system whose difference equation is:
\[
z_2(t) - U z_2(t - \tau) - V z_2(t - h) = 0 .
\]  
It is right to say that system (18) is strongly stable, if the system stability is robustly against delay changes. The following suggestion is made to determine a strong spectral abscissa for system (16):
\[
\gamma_d(\tau, h) = \lim_{\text{norm}(d\tau, dh) \to 0} \text{sup} \text{ Re } s : \text{det} D_{zz}(s) = 0
\]
In fact, (19) gives a function for spectral abscissa computation, which is continuous with respect to any infinitesimal delay perturbations. It also shows that the spectral abscissa is influenced by delay value that calculation is carried out for. It is important to note that the burden of numerical calculation is heavy for a typical neutral system. However, for a real power system where the number of inputs and outputs are often much less than the number of system state variables, the computational burden is reduced. In other words, the dimension of associated model of a typical DDAE system is lower than a neutral system. On the other hand, by referring to the state space equations of a physical system, direct feed through coefficients connecting system inputs directly to the outputs are negligible. In regards to this property, the system represented by (6), for instance, will be retarded time delay system. Consequently, under this circumstance, the spectral abscissa as a function of time delay parameter will be continuous.

General approach in many cases is to use a simplified and reduced order model in designing a phase compensator type of auxiliary controller for delayed large-scale power systems. Thus, sensitivity analysis with respect to time delay parameter is essential in order to examine robustness of the designed controller in maintaining system stability. The stability depends more on displacement of high-frequency modes if delay changes. Feed through coefficients in state space modeling are effective in high-frequency modes and to get an insight into stability state, their sensitivity to delay changes should be studied. This emphasizes the necessity of strong stability analysis when designing controllers for delayed power systems. In this respect, design procedure and successful implementation of the controller are important elements that should be taken into consideration.

To ensure supporting stability conditions, strong stability criteria instead of asymptotic stability of the system (12) have to be noticed. The proposed method for designing controllers for delayed power systems meets strong stability requirements. Inspired from strong stability of neutral systems, a new statement is developed for DDAE systems by considering following theorem:

**Theorem1:** the state space equations of a delayed power system as shown in (6) is strongly stable, if \( \gamma < 0 \) and \( \gamma_d(\tau, h) < 0 \), or the spectral abscissa function at \( \tau \) and \( h \) is strictly negative, i.e.
\[
F = \max(\gamma, \gamma_d) < 0 .
\]

Alternatively, a logical way could use extension of strong stability conditions of neutral systems to the system (6). This, not only opens a way for assessing asymptotic stability analysis of (12), but also provides a bed for successful design of the control system through a straightforward approach.

**B. STRONG STABILITY BASED ON THE SPECTRAL ABSICSSA OPTIMIZATIONS**

As mentioned in the introduction section, this paper addresses design of control parameters, including the controller matrix and the delay in the actuator input, in a way that the spectral abscissa moves to the left of the imaginary axis as much as possible. In general, the power system modeled by (7) can be demonstrated as follows:
\[
z_2(t) = P(k)z_2(t) + Qz_2(t - \tau) + R(k)z_2(t - h),
\]
\[
z_2(t) = T(k)z_2(t) + Uz_2(t - \tau) + V(k)z_2(t - h),
\]
in which, \( k \in F, G, H, K \) stands for the control parameters.

The main goal is to find the parameter \( k, \tau \) such that exponential robust stability against small delay changes is attained in the system. To this end, the following optimization problem is proposed:
\[
\min_{(k, \tau)} F(k, \tau) = \max(\gamma, \gamma_d) ,
\]
The spectral abscissa function indicated in (22) is not always differentiable with respect to \( k \). When characteristic equation of the system with different modes have the same real parts equal to the spectral abscissa, (22) is not also differentiable at that points. However, if the real parts of repeated roots are equal to the spectral abscissa, the objective function is differentiable and Lipschitz continuity is not maintained [32]. Thus, mathematical-based optimization methods are not applicable solely. Instead, the problem can be solved by combining convex optimization methods with heuristic ones. In this work, TDS_STABIL toolbox developed by Wilm Michiels [33] has been utilized to solve the optimization problem defined in (22). This toolbox has been developed in MATLAB with a modification on one of its functions called “STABILIZATION_MAX.m” to provide the possibility of computing optimal value for the controller.
parameters including input delay value. Generally, this method requires the evaluation of the objective function and its sensitivity in differentiable points. Using spectral discretization, the spectral abscissa can be computed followed by Newton corrections. Whenever needed, derivatives are obtained from the sensitivity of individual eigenvalues with respect to the free parameters [34]. Fig. 3 illustrates the different stages of the optimization approach to find spectral abscissa which is robust with respect to delay changes.

In addition, the following steps summarize the design procedure:

Step 1) Build up a detailed model of the power system in MATLAB/PST/SimPowerSystems and obtain linear state-space model as given in equations (1) and (5).
Step 2) Select the feedback signal of the delay-scheduled controller based on the CI index.
Step 3) Obtain the reduced-order model of the whole power system excluding the delay-scheduled controller using the balanced residualization method described in [37].
Step 4) Calculate the control parameters $k$ and $\tau + h$ by solving (22) for a presumed static controller. These are used as initial values to design a fixed-order controller whose state equations are given in (4). It is worth noting that the value of $\tau + h$ is fixed to what obtained from previous stage.
Step 5) Set the value of $\tau$ considering the range of the communication delay, $h$. This can be done based on a trade-off between lower and upper bound of $h$.
Step 6) Verify the feasibility and effectiveness of the designed delay-scheduled controller via nonlinear simulations in MATLAB.
Step 7) Select a new order for the dynamic controller and repeat design procedure going to step 4 provided that some non-satisfactory results obtained from step 6.

The main contributions of the paper can be summarized as follows:

1) Traditionally, delay in wide-area control systems is considered problematic during the controller design procedure, since it deteriorates the system performance, but we note to the stabilizing effect of delay in order to improve the performance of the system and propose a model to design parameters of the controller.
2) The Proposed controller damping rate of inter-area oscillations increases first and then decreases with an increase in communication time delay. This is while the damping rate continuously decreases with an increase in communication time delay in traditional design of controllers.
3) Inspired by NDDE, the paper presents a method to simplify the strong stability analysis in designing a stabilizing controller for WADC in a multi machine power system.

IV. CASE STUDY 1: TWO-AREA SYSTEM
First a typical two area system is used to demonstrate the capability of proposed design approach in damping inter-area power oscillations. The single line diagram of the studied power system is shown in Fig. 4. System characteristics including transmission lines data, generators dynamic specifications and loads information are given in [35]. The structure of the primary system is modified by augmenting a SVC with capacity of 200 MVAR at bus 8 and in the middle of power corridor connecting two areas to each other. The primary function of the SVC is to regulate voltage at bus 8. This regulation is performed by proportionally adjusting the susceptance of the SVC. It is assumed that the capacity of the SVC is partly dedicated to low frequency damping purposes. This needs to define a new supplementary task for the controller that measures and processes remote signals so as to make suitable reference voltage. When reference voltage is applied to the SVC, the overall damping against low frequency oscillations increases. Fig. 4 also illustrates the main concepts of the supplementary controller for the SVC. In Fig. 4, $H(s)$ is the controller that input signals from remote locations and make suitable signals for modulation of the SVC input.
Since remote signals are associated with inherent delays, a delayed controller based on delay stabilization effect is designed to increase system damping. Simulations are carried out by considering base values power as 230 MVA and the system frequency is 60 Hz. It is assumed that every generator is equipped with an AVR whose simplified transfer function has a gain of 200 and a time constant of 0.01 s. They have been studied for two cases; 1) without any PSS, 2) with PSS only on machine G1 and G3.

A. WITHOUT ANY PSS

Without considering the supplementary controller and PSS, small signal analysis of the test system shows that the system has three electromechanical modes. The spectral abscissa of the electromechanical modes is obtained 0.0187 and hence the current state of system is unstable. One solution is to add a supplementary control on SVC as shown in Fig.4 with the aim of improving system performance and damping critical modes. Stabilization due to input delay is only effective when a proper input signal is selected. To select input signals, we have used the methodology described in section II. Therefore, the CI index is first computed for all critical modes of the system. The results are given in Table 1.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>COMPARISON BETWEEN THE RESULTS OF SIGNAL SELECTION FROM THE GENERATOR ANGLE BASED ON CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>CI Index</td>
<td>Modes No. 1</td>
</tr>
<tr>
<td>Rotor Angles</td>
<td>0.0187±j4.018</td>
</tr>
<tr>
<td>G1</td>
<td>0.00322</td>
</tr>
<tr>
<td>G2</td>
<td>0.00120</td>
</tr>
<tr>
<td>G3</td>
<td>-0.17305</td>
</tr>
<tr>
<td>G4</td>
<td>0.00171</td>
</tr>
</tbody>
</table>

Comparing the obtained values for CIs, Table I shows that measuring rotor angles for G2 and G4 are the best. Referring to the first column of Table I, the best signals are the rotor angle of G2 and G4. However, for other critical modes by referring to column 2 and column 3, it is indicated that the best signals could be rotor angle of G2 and G4. Thus rotor angles of Generator 2 and Generator 4 are selected and combined to make a proper input signal to the SVC controller.

To design the controller function, we first need to obtain the open-loop transfer function between the SVC input and outputs that are rotor angles of generators as shown in Fig.4. The PST software package has been used to model the power system [36]. The reason lies in the fact that it has the capability of generating the linearized system matrix. In modeling, generators are defined by their sixth order model. The generators’ excitation system and the SVC voltage regulator models are of the first order. Therefore, the overall dynamic model of the system would be of the twenty-ninth order.

Order reduction methods are generally used to simplify system equations by reducing number of state variables. Order of the system equations can be reduced by different approaches. Here, we use balanced residualization method to obtain a reduced order model [37]. Unlike the modal residualization method in which the location of mode is used as a criterion to discard ineffective modes, the balanced residualization method takes into account singular value of the residual matrices as a criterion. Due to high sensitivity of fast high frequency modes to delay value, the residual approach is employed so that the impact of high frequency modes are considered in the reduced order system.

The reduced order state space realization is given in (23).

Two structures are examined for the controller to stabilize (23). The first structure is a static controller (nc=0) whose equation is defined as follows:

\[
\dot{x}(t) = Ax(t) + Bu(t),
\]

(23a)

\[
y(t) = Cx(t) + Du(t),
\]

(23b)

where,

\[
y(t) = \begin{bmatrix} \Delta \delta_1(t) \\ \Delta \delta_2(t) \end{bmatrix}.
\]

The rightmost eigenvalues of the system are depicted in Fig.6. The first order of the power system can be generally represented as:

\[
\dot{x}(t) = Ax(t) + Bu(t),
\]

(23a)

\[
y(t) = Cx(t) + Du(t),
\]

(23b)

where,

\[
y(t) = \begin{bmatrix} \Delta \delta_1(t) \\ \Delta \delta_2(t) \end{bmatrix}.
\]

Two structures are examined for the controller to stabilize (23). The first structure is a static controller (nc=0) whose equation is defined as follows:

\[
a(t) = k_1 \delta_2(t - h_1 - \tau) + k_2 \delta_2(t - h_2 - \tau),
\]

(24a)

the second structure is a dynamic controller (nc = 1) whose equations are described as follows:

\[
\dot{x}_1(t) = k_2 x_2(t) + k_3 \delta_2(t - h_1) + k_4 \delta_2(t - h_2),
\]

(24b)

\[
y(t) = k_5 x_1(t) - k_6 \delta_2(t - h_1) - k_7 \delta_2(t - h_2).
\]

(24c)

In our design procedure, first we assume \( h_1 = h_2 = h \) and the static structure for the controller and by solving (22), optimal values for parameters of \( k_2, k_3 \) and \( k_1 = \tau + h \) are determined. To design dynamic controller parameters, delay parameter \( k_1 \) is fixed to what obtained for the static controller, and then using (22) other control parameters \( (k_2 - k_7) \) are determined. The designed controller block diagrams for both structures are shown in Fig.5 where (a) and (b) represent block diagrams of designed controllers with zero and one order, respectively.

In the next stage to evaluate the robustness and stability region of the designed controller with respect to delay parameter variations, upper and lower bands of delay parameter is obtained by developing eigAm.m function [38]. Table II summarizes the results for upper and lower bands, optimal delay parameter and associated spectral absissa for both controllers. Referring to Table II, given value for optimal spectral absissa indicates that all closed loop system eigenvalues in the case of static controller, are always on the left-hand side of spectral absissa with -0.888 and -1.548 for the dynamic controller.

The rightmost eigenvalues of the system are depicted in Fig.6.
and Fig.7 as a function of delay parameter variations, by taking into account the static and dynamic controllers; respectively. With the static controller and for $h + \tau > 0$ ms, the spectral abscissa shows that not only stability would be guaranteed but delay is continuously increased up to 305 ms, for $305 < h + \tau < 577$ ms delay value would be decreased. More precisely, for delay 0-73 ms, inter-area mode is determinative as a rightmost eigenvalue to show the stability condition and this role goes to the local mode between G1 and G2 when delay increases from 73 up to 305 ms. In the range of 305-533 ms, the local mode between G3 and G4 takes a determinative role. For delays more than 577 ms, it is observed that a non-oscillatory mode with a real value $-\infty$ with $h + \tau = 0$ causes the system to go into instability. Since at optimal delay value, 305 ms, inter-area mode has moved more than local modes to the left side of complex plane, it could be concluded that the selected signal is more effective on the inter-area mode although it has a positive effect on local modes. In similar way, Fig.7 shows the loci diagram in terms of delay variations for the controlled system employing a dynamic controller whose order is one.

![Image](image1.png)

![Image](image2.png)

![Image](image3.png)

![Image](image4.png)

**TABLE 2**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>$nc = 0$</th>
<th>$nc = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_l$</td>
<td>Lower bound delays [ms]</td>
<td>0</td>
<td>43</td>
</tr>
<tr>
<td>$\tau_u$</td>
<td>Upper bound delays [ms]</td>
<td>577</td>
<td>626</td>
</tr>
<tr>
<td>$\tau_{DRS}$</td>
<td>Delay- range stability [ms]</td>
<td>577</td>
<td>583</td>
</tr>
<tr>
<td>$\tau_{op}$</td>
<td>Optimal solution delays [ms]</td>
<td>305</td>
<td>305</td>
</tr>
<tr>
<td>$F_{op}$</td>
<td>Spectral abscissa optimization</td>
<td>-0.888</td>
<td>-1.548</td>
</tr>
</tbody>
</table>

**B. WITH PSS ON G1 AND G3**

To demonstrate the performance of the proposed controller to damp the inter-area mode in the presence of other stabilizing controllers, including PSSs, it is assumed that generators G1 and G3 are equipped with a PSS to damp the local oscillation mode. The transfer function of PSS is

$$G_{pss}(s) = \frac{50s}{1 + 0.05s} \left(\frac{1 + 0.05s}{1 + 0.1s}\right)^2,$$

and its output is limited by $\pm 0.2$.

When PSSs are present on machines G1 and G3, the system becomes stable, and much more damped than the previous case; and there is only one poorly damped electromechanical mode. In order to determine the reduced-order model, by choosing the deviation of G2 and G4 rotor angle as the output and SVC reference as the input, balanced residuazalization method is applied to obtain a reduced-order model of the original 35th order test system that the result showed that it could be eighth-order. The designed controller block diagrams for both structures are shown in Fig.8 where (a) and (b) represent block diagrams of designed controllers with zero and one order; respectively.

Table 3 summarizes the results for upper and lower bands, optimal delay value and associated spectral abscissa for both controllers. The rightmost eigenvalues of the system are
depicted in Fig. 9 and Fig. 10 as a function of input delay variations, by taking into account the static and dynamic controllers, respectively. Tests results have shown that without considering PSSs the proposed approach can significantly enhance the overall stability by improving the damping performance of local and inter-area modes. In addition, the existence of PSSs not only does not interactions with the proposed controller action, it is observed that the margin of delay and convergence rate have increased considerably.

**TABLE 3**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>nc = 0</th>
<th>nc = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_L )</td>
<td>Lower bound delays [ms]</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \tau_u )</td>
<td>Upper bound delays [ms]</td>
<td>819</td>
<td>873</td>
</tr>
<tr>
<td>( \tau_{DRS} )</td>
<td>Delay-range stability [ms]</td>
<td>819</td>
<td>873</td>
</tr>
<tr>
<td>( \tau_{op} )</td>
<td>Optimal solution delays [ms]</td>
<td>430</td>
<td>430</td>
</tr>
<tr>
<td>( F_{op} )</td>
<td>Spectral abscissa optimization</td>
<td>-1.473</td>
<td>-1.603</td>
</tr>
</tbody>
</table>

### C. NONLINEAR SIMULATION

For these two cases, a nonlinear simulation is conducted when a three-phase short circuit occurs at bus 7 for a maximum time of 50 ms. In order to evaluate the simulation results, inter-area oscillations are observed by measuring angle difference between the first and the third generators. However, to inspect local oscillations, difference angle between the fourth and the third generators is used. On the other hand, to examine maximum permissible input signal amplitude in terms of delay, the signal \( u \) is measured during all simulations. It is obvious that the amount of time delay in remote signals is uncertain but limited. The communication link and the physical distance between the points have a significant effect on the time delay. There are various types of communication links such as telephone lines, optical fibers, power-line communications, and microwave and satellite links. We assume that the controller receives the output signal through a Power Line Communication (PLC) with a delay in the range of 150 to 350 ms. If we take 250 ms as the average delay, we can create a 55 ms time delay in the SVC input according to the results obtained in the previous section for optimal time delay value in case without any PSS. Hence, the overall value of total delays including of SVC input delay and measuring feedback signals would be nearly 305 ms. In similar way, for case with PSS on machine G1 and G3, we can create a 180 ms delay in the SVC input, and the simulation results shown in Fig. 14 to Fig. 16. In order to demonstrate the robustness and efficiency of the proposed controller, the simulation results are illustrated in Fig. 11 to Fig. 13. As one can see, employing both the proposed controllers exhibit a proper performance and an acceptable robustness against delay changes in the feedback. It can also be seen that the peak of the control signal decreases as the delay increases. In other words, the performance is also improved in terms of actuator saturation. It is seen in Fig. 10 to Fig. 15 that when \( h \) is increased from 150 to 250 ms, the convergence rate of states of the closed-loop system increases.
FIGURE 11 State evolution where \( n_c = 0, \quad \tau = 55 \text{ ms} \) and 
\( h \in [150, 250, 350] \text{ ms} \), without any PSS

FIGURE 12 State evolution where \( n_c = 1, \quad \tau = 55 \text{ ms} \) and 
\( h \in [150, 250, 350] \text{ ms} \), without any PSS

FIGURE 13 Control input evolution with zero and one controller and without any PSS

FIGURE 14 State evolution where \( n_c = 0, \quad \tau = 180 \text{ ms} \) and 
\( h \in [150, 250, 350] \text{ ms} \), with PSS on G1 and G3

FIGURE 15 State evolution where \( n_c = 1, \quad \tau = 180 \text{ ms} \) and 
\( h \in [150, 250, 350] \text{ ms} \), with PSS on G1 and G3

FIGURE 16 Control input evolution with zero and one controller and with the PSS on G1 and G3
However, for $h$ is increased 250 to 350 ms, the corresponding convergence rate decreases, indicating that for $h=250$ ms, we achieve the highest convergence rate of the closed loop system.

Now is the time to demonstrate the control effectiveness of the proposed control system in the presence of the PSS. Figures 14-16 show that in a wide range of feedback delay values, the overall performance of the electromechanical oscillations control system can be improved with the participation of the proposed controller. By comparing the amplitude of the output signal of the proposed controller shown in Figures 16 and 14, it can be concluded that the control effort by the proposed controller in SVC is reduced due to the PSS on G1 and G3. This is due to the fact that when the PSSs are used to damp the local oscillations of the test system, the output amplitude of the proposed controller is reduced. In additional, as was shown in Fig.6 and Fig.7, a worse case condition is achieved when the maximum permissible value of the overall delay is about 577 ms for the static and 623 ms for the dynamic controllers for case without any PSS, respectively. While when the PSSs are in operation, the highest time delay can be increased to 821 and 873 ms for static and dynamic controllers, respectively.

V. CASE STUDY 2: 16-MACHINE 68-BUS SYSTEM

To further explore the feasibility of the proposed approach in a high-order and more practical system, the delayed controller design for SVC in a 68-bus 16-machine test system shown in Fig. 16 is presented here. This system is a reduced order equivalent of the inter-connected New England test system (NETS) and New York power system (NYPS), with five geographical regions out of which NETS and NYPS are represented by a group of generators whereas. The detailed description of this test system including network data, parameters of generators, excitation systems, and PSSs are given in [39]. For this test system, a 350-Mvar SVC is installed at the bus 51 to maintain its voltage profile. Generators G1 to G12 have fast static excitation (ST1A), their gain is 200, while G13 to G16 have manual excitation as they are area equivalents instead of being physical generators. In the case, the transfer function of all PSSs installed on machines G1 to G12 is

$$G_{PSS}(s) = \frac{300s}{1 + 15s} \left(1 + \frac{0.15s}{1 + 0.04s}\right)^2,$$

and minimum and maximum output limit is $-0.05$ and $0.2$ [40].

A. DELAY CONTROLLER DESIGN

The small signal analysis of the test system shows that it is stable, and there are only two poorly damped inter-area modes and all the local modes get properly damped, the results can be seen in Fig. 18. To select the input signals to the SVC supplementary control, the CI index is first computed for the critical modes of the system. The results are shown in Fig. 19.
FIGURE. 20 Supplemental controllers for SVC in bus 51

\[
\begin{align*}
\Delta \delta_{51}(t-h_{51}) &= -0.529 \\
\Delta \delta_{53}(t-h_{53}) &= 1.311 \\
\Delta \delta_{54}(t-h_{54}) &= -0.246 \\
\Delta \delta_{55}(t-h_{55}) &= 1.749
\end{align*}
\]

\[ e^\gamma (\tau - h) \rightarrow u(t) \]  

FIGURE. 21 The real part of rightmost electromechanical modes as function of \( \tau + h \) for the designed controllers

\[
\begin{align*}
\Delta \delta_{43}(t-h_{43}) &= -0.072 \frac{s+3.93}{s+2.32} \\
\Delta \delta_{44}(t-h_{44}) &= 0.0746 \frac{s+2.32}{s+2.32} \\
\Delta \delta_{45}(t-h_{45}) &= -0.345 \frac{s+14.7}{s+2.32} \\
\Delta \delta_{51}(t-h_{51}) &= 2.738 \frac{s+4.49}{s+2.32}
\end{align*}
\]

B. NONLINEAR SIMULATION

To assess the effectiveness of the proposed control system, a nonlinear simulation is conducted when a three-phase short circuit 75 ms at bus 45, the simulation results are illustrated in Fig.22 to Fig. 23 as a function of \( h \). As one can see, employing both proposed controllers exhibit a proper performance and an acceptable robustness against delay changes in the wide-area signal. Therefore, in order to damp out inter area oscillations for a high-order and more practical system, like 5-area power system, using delay as a design parameter not only can increase the delay margin of the closed loop system, but also provides the good damping performance.

VII. CONCLUSION

In this work, the approach of delay-scheduled stabilization is studied and shown that by means of this approach a delayed input supplementary controller could be designed to damp out dynamic oscillations in a power system. In this study the supplementary controller has been installed on an SVC device.

FIGURE. 22 Responses of the 5-area power system to fault under different time delays in feedback (\( nc = 0, \tau = 70 \text{ ms} \)).

FIGURE. 23 Responses of the 5-area power system to fault under different time delays in feedback (\( nc = 1, \tau = 70 \text{ ms} \)).

FIGURE. 24 Responses of the SVC output to fault under different time delays in feedback (\( \tau = 70 \text{ ms} \)).
An eigenvalue based framework is developed to design the controller based on the optimization of the strong spectral abscissa. The input delay and the utilization factor are important parameters in the improvement of damping and delay margins. It is determined that increasing the delay margins and the decay rate of low-frequency oscillations in a delayed power system is possible by adjusting the delay in the SVC input. Simulation results in different cases showed that by entering a proper delay to the actuator (SVC) input, the performance of the system to damp out power oscillations will be improved.

In addition, the advantages of using delay as a design parameter were investigated for damping multiple inter-area oscillation modes, and it was shown that in the case where the design of WADC without delay is not easy, the stabilizing delay effect can be exploited. Further studies will focus on coordinating design and tuning of WADCs for damping multiple inter-area oscillation modes by using results of such a controller in the power system with multiple random delays.

### APPENDIX A

The state space representation of the 8th-order reduced linear model of uncontrolled the 2-area power system is as follows:

\[
\begin{bmatrix}
0.010 & -3.66 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
3.66 & 0.010 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & -0.219 & -6.115 & 0.720 & 0.023 & 0.183 & 0.000 \\
0.000 & 0.000 & 6.1039 & -1.084 & 0.131 & 2.640 & 0.054 & 0.000 \\
0.000 & 0.000 & 0.6617 & -2.388 & -5.053 & 0.313 & 2.261 & 0.000 \\
0.000 & 0.000 & 0.0163 & -0.026 & -0.163 & 0.023 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & -0.095 & 0.1695 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.1573 & -0.386 & -1.892 & -0.220 & 0.682 & 0.000
\end{bmatrix} \begin{bmatrix}
0.000 \\
0.000 \\
0.000 \\
0.000 \\
0.000 \\
0.000 \\
0.000 \\
0.000
\end{bmatrix}
\]

### REFERENCES


