Backtracking Velocity Denoising Based Autonomous in-motion Initial Alignment

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ABSTRACT In the Global Navigation Satellite System (GNSS) denial environment, Strap-down Inertial Navigation System (SINS) has to rely on the body-frame velocity output by autonomous velocity measurement equipments, such as odometer and Doppler Velocity Log (DVL), to implement the in-motion initial alignment. Considering the external velocity noise and the approximation existing in the body-frame velocity aided initial alignment model, the autonomous in-motion alignment for SINS with high precision is a difficult problem. Besides, the position updating of SINS can not be realized only depending on the body-frame velocity. And the position error will negatively impact the precision of the following fine alignment and navigation. In this paper, a backtracking velocity denoising based autonomous in-motion initial alignment is proposed. Forward compass alignment and backward compass alignment are carried out respectively to denoise the external velocity. And body-frame velocity based and navigation-frame velocity based attitude determination are implemented respectively to gradually realize the attitude alignment. The contributions of the work presented here are twofold. First, vehicle velocity during the whole initial alignment process is accurately denoised and determined based on the backtracking compass alignment. Second, high precision position alignment is achieved during the attitude determination. The validity of the proposed method is verified based on field test data.

INDEX TERMS Strapdown inertial navigation system, initial alignment, autonomous

I. INTRODUCTION

As a dead-reckoning navigation method, the strapdown inertial navigation system (SINS) can track the position and orientation of the carrier relative to a known starting point with the measurements provided by the self-contained accelerometers and gyroscopes [1]-[5]. SINS requires to be aligned in the initialization stage [6]-[8]. The purpose of initial alignment including coarse and fine alignment is to obtain an initial strapdown attitude matrix between the body frame and navigation frame and set the misalignments to zero. The alignment procedure often consists of two consecutive stages: coarse alignment and fine alignment. The accuracy of the coarse alignment directly influences the convergence and the convergence speed of the fine alignment. Therefore, the performance of coarse alignment for SINS is quite important.

At present, the initial alignment technology of SINS is relatively mature when the vehicle is stationary or wobbly and the ideal alignment results can be obtained. However, the need for in-motion alignment is more and more urgent for many military applications and commercial aviations. Generally, the typical in-motion coarse alignment approached are gravitational vector observation based initial alignment methods, which use the gravity in the inertial frame as a reference and consider the projection of the gravity in the inertial frame defines a cone whose main axis is the rotational axis of the Earth [9]-[20]. In reference [9], an initial alignment method using the gravity in the inertial frame as a reference is investigated for marine SINS alignment. Reference [10] presents a novel estimation method for fast initial coarse alignment of a ship’s strapdown inertial attitude reference system based on the decomposition of the attitude quaternion into separate Earth motion, inertial rate, and alignment quaternions. In references [11] and [12], the attitude-alignment problem was transformed into a continuous attitude determination problem using infinite vector observations. The INS alignment is heuristically established as an optimization problem of finding the...
minimum eigenvector. This method is known as the optimization-based alignment (OBA) method.

In references [9]-[17], the velocity or position in local level navigation frame output by the Global Navigation Satellite System (GNSS) is essential for in-motion initial alignment of SINS. For the underwater environment and some on-land environment where the GNSS signal is not available, the in-motion initial alignment of SINS is still a difficult problem and has received much attention in recent years. Body-frame velocity based SINS initial alignment methods are studied in Reference [18]-[20]. It means that SINS is aided by autonomous velocity measurement equipments such as odometer or DVL rather than GNSS for initial alignment. In reference [18], the velocity update equation and its integral form in the body frame are studied. And the attitude coarse alignment was then regarded as an optimization-based attitude determination problem between the body frame velocity and the integral form of gravity. Thus, this method can be regarded as the body-frame velocity aided OBA method. Reference [19] proposes a SINS initial alignment scheme aided by the velocity derived from Doppler Velocity Log (DVL).

Inevitably, compared with the navigation-frame velocity aided optimization-based alignment model, there exist more approximations in the body-frame velocity aided optimization-based alignment model. Besides, the measurement noise of autonomous equipments, such as odometer and Doppler velocity log (DVL) is higher than that of GNSS. Therefore, the performance of body-frame velocity aided optimization-based alignment is far behind that of navigation-frame velocity aided optimization-based alignment. Reference [20] indicates that the gyrocompass horizontal alignment loop can denoise the external velocity sources. In reference [21], the validation of the velocity denoising method based on gyrocompass horizontal alignment loop is verified and the denoised velocity is used for SINS initial alignment. Compared with other denoising methods, gyrocompass alignment based velocity denoising method owns some advantages, such as no phase delay and high precision. However, when the initial velocity of the vehicle is not 0, the velocity output of gyrocompass horizontal alignment loop will go through an oscillation process. It means that the denoised velocity can not be used at the very start, that may restrict the applicability of the gyrocompass alignment based velocity denoising method. Besides, during the velocity oscillation, the velocity error will be too large to be used for accurate position updating. Thus, it will be difficult to obtain an accurate position of the vehicle for the following fine alignment and navigation and the navigation precision will be influenced. In view of the problems existing in the body-frame velocity aided SINS in-motion initial alignment, a backtracking denoising based autonomous in-motion alignment method is proposed in this paper. The main idea of this method can be illustrated as follows: forward compass alignment and backward compass alignment are carried out respectively to denoise the external velocity and the velocity outputs of the above two compass horizontal alignment loops are used to calculate the denoised velocity during the whole initial alignment process. Then, based on body-frame velocity and navigation-frame velocity, SINS alignments with different attitude determination models are executed successively, so as to realize an accurate attitude alignment. At the same time, the position updating can be accomplished. The car-mounted experiments verified the validity of the proposed method for accurate attitude alignment and position updating in dynamic environments.

The remaining of the paper is organized as follows. Section 2 derives the model of body-frame velocity based initial alignment for SINS and analyzes the problems existing in autonomous in-motion alignment. Section 3 proposes the backtracking compass alignment based velocity denoising method. Section 4 gives the algorithmic description of the proposed backtracking velocity denoising based autonomous in-motion initial alignment method. Experiment tests are performed for in-motion alignment in Section 5. Discussions and conclusions are made in Section 6.

II. PROBLEM STATEMENT

The coordinate frames used in this paper are defined as follows. Denote by n the local level navigation frame, an orthogonal reference frame aligned with north – east – down (NED) geodetic axes; by b the SINS body frame, an orthogonal reference frame aligned within the IMU axes; by e the Earth frame, an Earth-centered Earth-fixed (ECEF) orthogonal reference frame; by i the chosen inertial frame. Denote by e’ the Earth-fixed frame. The z_e axis is parallel to the Earth’s rotation axis and the x_e axis is in the equatorial plane and points to the meridian of the vehicle initial position. The y_e axis completes the right-handed coordinate system. Denote by e’(0) the Earth inertial frame, which is formed by fixing the e’ frame at the beginning of the coarse alignment in the inertial space. Denote by n0 the navigation inertial frame, which is formed by fixing the n frame at the beginning of the coarse alignment in the inertial space. Denote by b(0) the body inertial frame, which is formed by fixing the b frame at the beginning of the coarse alignment in the inertial space.

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the vehicle initial position. The \( y_e \) axis completes the right-handed coordinate system. Denote by \( e'(0) \) the Earth inertial frame, which is formed by fixing the \( e' \) frame at the beginning of the coarse alignment in the inertial space. Denote by \( n0 \) the navigation inertial frame, which is formed by fixing the \( n \) frame at the beginning of the coarse alignment in the inertial space. Denote by \( b(0) \) the body inertial frame, which is formed by fixing the \( b \) frame at the beginning of the coarse alignment in the inertial space.

Body-frame velocity aided optimization-based alignment model can be derived as follows. The attitude and velocity differential equations of SINS in the \( n \)-frame are respectively known as:

\[
\dot{C}_{en}^n = C_{bn}^n \Omega_{bn}^n \tag{1}
\]

\[
\dot{v}^e = C_{bn}^n f_g^b - (2\omega_b^e + \omega_b^e) \times v^e + g^e \tag{2}
\]

Where,

\[
\Omega_{bn}^b = \Omega_{nb}^b \times , \quad \omega_b^e = \omega_b^e - (C_{en}^n)^T (\omega_e^n + \omega_e^n), \quad g^e = [0 \quad 0 \quad -g]^T, \quad \omega_e^n = [0 \quad \omega_e \cos L \quad \omega_e \sin L]^T
\]

\[
\omega_{en}^e = \begin{bmatrix} -\frac{v_n^e}{R_m + h} & \frac{v_N^e}{R_N + h} & \frac{v_E^e}{R_N + h} \end{bmatrix}^T
\]

\( \omega_{en}^e \) denotes the attitude matrix from the body frame to the navigation frame; \( \dot{v}^e \) the velocity relative to the Earth; \([L, \lambda, h]\) the latitude, longitude and height; \( \omega_b^e \) the body angular rate measured by gyroscopes in the body frame; \( f_g^b \) the specific force measured by accelerometers in the body frame; \( R_m \) the meridian radius; \( R_N \) vector the prime vertical radius.

In Equation (1), \( C_{en}^n \) is a time-variant matrix, which can be decomposed as follows:

\[
C_{en}^n(t) = C_{en}^{n(t)} C_{en}^{(t)} \tag{3}
\]

Where,

\[
C_{en}^{n(t)} = C_{en}^{n(t)} \Omega_{en}^{n(t)} \tag{4}
\]

\[
C_{en}^{(t)} = C_{en}^{(t)} C_{en}^{n(t)} \tag{5}
\]

Where, \( C_{en}^{n(t)} \) can be determined according to the initial latitude of the vehicle; \( C_{en}^{(t)} \) can be determined according to the Earth rotation rate and time; the angular motion of the vehicle relative to the inertial space during the initial alignment process is recorded by the matrix \( C_{en}^{n(t)} \), which can be determined according to the output of gyro and time. \( C_{en}^{n(t)}(t) \) records the time-varying of the navigation frame caused by the position Variance of the vehicle. However, when the SINS is aided by the autonomous velocity measurement equipments, the position of the vehicle can not be obtained during the initial alignment process. Considering that when the vehicle is in a low-velocity moving condition, the position of the vehicle will not change obviously in a short time, therefore, \( C_{en}^{n(t)}(t) \) can be approximated as an identity matrix. Thus, the aforementioned matrixes can be given as:

\[
C_{en}^{n(0)} = \begin{bmatrix} 0 & 1 & 0 \\ -\sin L_0 & 0 & \cos L_0 \\ \cos L_0 & 0 & \sin L_0 \end{bmatrix} \tag{6}
\]

\[
C_{en}^{(0)} = \begin{bmatrix} \cos(\omega_e t) & \sin(\omega_e t) & 0 \\ -\sin(\omega_e t) & \cos(\omega_e t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}
\]

\[
C_{en}^{n(t)} \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}
\]

Where, \( L_0 \) denotes the latitude of the vehicle at the beginning of initial alignment.

\( C_{en}^{n(t)} \) can be determined depending on the attitude update equation of SINS. That can be given as:

\[
\dot{C}_{en}^{n(t)} = C_{en}^{n(t)} (\omega_b^e \times) \tag{9}
\]

So the optimization-based alignment focuses on determining \( C_{en}^{n(t)} \), which is a constant matrix. By properly transforming the SINS velocity equations, \( C_{en}^{n(t)} \) determination can be transformed into the well-known Wahba’s problem.

The navigation-frame velocity of SINS can be given as:

\[
v^e(t) = C_{en}^n(t) v^b(t) \tag{10}
\]

Calculating the derivative of equation (10),

\[
\dot{v}^e(t) = C_{en}^n \dot{v}^b(t) + C_{en}^n \dot{C}_{en}^n(t) \dot{v}^b(t) = C_{en}^n(t) \left[ \dot{v}^b(t) + \omega_{en}^e(t) \times \dot{v}^b(t) \right] \tag{11}
\]

Substituting (11) into (2) yields,

\[
C_{en}^n(t) \left[ \dot{v}^e(t) + \omega_{en}^e(t) \times \dot{v}^e(t) \right] = C_{en}^n(t) f^b - (2\omega_e^n + \omega_e^n) \times v^e + g^e \tag{12}
\]

Multiplying \( C_{en}^n(t) \) on both sides,

\[
\dot{v}^b(t) + (\omega_b^e(t) + \omega_b^e) \times \dot{v}^b(t) - f^b = g^b \tag{13}
\]

As \( C_{en}^n(t) \) is unknown during the initial alignment process, so \( \omega_b^e \) can not be solved. Thus, in equation (13), \( \omega_b^e \times \dot{v}^b(t) \) has to be ignored. And then, multiplying \( C_{en}^n(t) \) on both sides of (13) and integrating the equation yields,

\[
\int_{0}^{t} C_{en}^{n(t)}(t) \dot{v}^b(t) dt = C_{en}^{n(t)} \dot{v}^b(t) \bigg|_{0}^{t} - \int_{0}^{t} C_{en}^{n(t)}(t) \dot{v}^b(t) dt \tag{14}
\]

Where,

\[
\begin{align*}
\int_{0}^{t} C_{en}^{n(t)}(t) \dot{v}^b(t) dt & = C_{en}^{n(t)} \dot{v}^b(t) \bigg|_{0}^{t} - \int_{0}^{t} C_{en}^{n(t)}(t) \dot{v}^b(t) dt \\
& = C_{en}^{n(t)} \dot{v}^b(t) \bigg|_{0}^{t} - \int_{0}^{t} C_{en}^{n(t)}(t) \omega_b^e(t) \times \dot{v}^b(t) dt \tag{15}
\end{align*}
\]

Substituting (15) into (14) yields,
\[ C_{b(t)}^{(b)}(t)\Big|_0^t - \int_0^t C_{b(t)}^{(b)} f^b dt = C_{e(t)}^{(e)}(t)C_{a(t)}^{(a)}(t)C_{v(t)}^{(v)} (t)g^a dt \] (16)

Ignoring \( C_{v(t)}^{(v)} \) yields,

\[ C_{b(t)}^{(b)}(t)\Big|_0^t - \int_0^t C_{b(t)}^{(b)} f^b dt \approx C_{e(t)}^{(e)}(t)\int_0^t C_{a(t)}^{(a)}(t)g^a dt \] (17)

Where,

\[ C_{v(t)}^{(v)}(t) = C_{v(t)}^{(v)}(t) \]
\[ = \begin{bmatrix} -\sin(\omega_x t) & -\sin(\omega_y t) & \cos L_0 \cos(\omega_x t) \\ \cos(\omega_x t) & -\sin L_0 \sin(\omega_x t) & \cos L_0 \sin(\omega_x t) \\ 0 & \cos L_0 & \sin L_0 \end{bmatrix} \] (18)

Defining the parameters,

\[ \alpha(t) = \int_0^t C_{v(t)}^{(v)}(t)g^a dt \] (19)
\[ \beta(t) = C_{b(t)}^{(b)}(t)\Big|_0^t - \int_0^t C_{b(t)}^{(b)} f^b dt \] (20)

Equation (17) can be presented as,

\[ C_{v(t)}^{(v)}(t)\alpha(t) = \beta(t) \] (21)

Thus, \( C_{v(t)}^{(v)}(t) \) determination problem is then transformed into the Wahba’s problem.

\( \alpha(t) \) and \( \beta(t) \) can be determined as follows:

As,

\[ C_{v(t)}^{(v)}(t)g^a = \begin{bmatrix} g \cos L_0 \cos \omega_x t \\ g \cos L_0 \sin \omega_x t \\ g \sin L_0 \end{bmatrix} \] (22)

Thus,

\[ \alpha(t) = \begin{bmatrix} g \cos L_0 \sin \omega_x \Delta t \\ \frac{g \cos L_0}{\omega_x} (1 - \cos \omega_x \Delta t) \end{bmatrix} \] (23)

Where, \( \Delta t \) denotes the integration duration.

In the expression of \( \beta(t) \), \( C_{b(t)}^{(b)}(t) \) is easy to obtain and \( \int_0^t C_{b(t)}^{(b)} f^b dt \) can be solved iteratively.

\[ \int_0^t C_{b(t)}^{(b)} f^b dt = \sum_{k=0}^{N-1} C_{b(t)}^{(b)}(t) f^b dt \]
\[ \approx \sum_{k=0}^{N-1} C_{b(t)}^{(b)}(t) \left[ I + \left( t \int_{t_k} \omega_x^b \right) \right] f^b dt \]
\[ = \sum_{k=0}^{N-1} C_{b(t)}^{(b)}(t) \left[ \Delta v_1 + \Delta v_2 + \frac{1}{2} (\Delta \theta_1 + \Delta \theta_2) \right] ( \Delta v_1 + \Delta v_2 ) + \frac{2}{3} (\Delta \theta_1 \times \Delta v_2 + \Delta v_1 \times \Delta \theta_2) \] (24)

Where, \( \Delta \theta_1 \) and \( \Delta \theta_2 \) denote the angle increments; \( \Delta v_1 \) and \( \Delta v_2 \) denote the velocity increments.

Thus, the attitude determination problem has been transformed is then into a Wahba’s problem. In reference [11] and [19], the optimization based attitude determination methods have been illustrated in detail. Thus, they won’t be covered here.

According to the above analysis, there exist some approximations in the derivation of the body-frame velocity aided optimization based attitude determination model. And in a dynamic environment, the approximations will influence the accuracy of SINS initial alignment to a certain extent. Besides, the body-frame velocity in equation (21) is provided by the autonomous velocity measurement equipments, such as odometer and DVL. The output noise of such equipments is relatively large and that will strongly influence the initial alignment accuracy of SINS. In order to improve the precision of body-frame velocity aided SINS initial alignment, the output of autonomous velocity measurement equipments need to be denoised. As mentioned before, the compass alignment based velocity denoising method has been studied in reference [20] and [21], where the fundamental principle of gyrocompass alignment method has been introduced in detail. As the gyrocompass horizontal alignment loop is an integration loop with feedback controller, it can effectively suppress the noise of the external velocity source and wins some advantages such as no phase delay and high precision. However, it has shortcomings as well. When the initial velocity of the vehicle is not 0, the velocity output of the gyrocompass horizontal alignment loop will inevitably go through an adjustment process before the accurate velocity can be obtained. In order to illustrate this problem more clearly, and odometer aiding gyrocompass horizontal alignment experiment is carried out depending on a segment of car-mounted experimental data, which contains the output of an inertial measurement unit (IMU) composed by three navigation grade laser gyroscopes and three accelerometers, the velocity output of an odometer and the position and velocity output of an GNSS receiver. When the initial velocity of the vehicle is not 0, the velocity output of the gyrocompass horizontal alignment loop is shown in Fig. 1 and the velocity error is shown in Fig. 2.
FIGURE 1. Velocity denoising performance of gyrocompass alignment loop.

FIGURE 2. Regulatory processes of gyrocompass velocity.

In Fig. 1 the green solid line represents the velocity output of the odometer, which contains large high frequency noise. The blue solid line represents the reference velocity provided by SINS/GNSS integrated navigation. The red dot line represents the velocity output of the gyrocompass horizontal alignment loop. As shown in Fig. 1, the initial velocity of the vehicle is not 0. And at the beginning of compass alignment, the velocity output of the gyrocompass horizontal alignment loop can not accurately track the reference velocity. Fig. 2 presents the velocity error of the gyrocompass horizontal alignment loop respect to the reference velocity, according to which, at the very start, the compass velocity will go through an oscillation process, the time elapsed on which is usually under 100s. During this stage, due to the larger error, the compass velocity is hard to be used for attitude alignment or position updating. In view of this problem, a backtracking compass alignment based velocity denoising method is proposed in this paper. The main idea of this method can be shown in Fig. 3.

III. BACKTRACKING COMPASS ALIGNMENT BASED VELOCITY DENOISING METHOD

As shown in Fig. 3, based on a segment of SINS raw data, forward compass alignment and backward compass alignment are carried out respectively. As the adjustment time of gyrocompass horizontal alignment loop is less than 100s, the velocity of forward compass horizontal alignment loop and backward compass horizontal alignment loop can be regarded as available velocity after 100s. V1 and V2 denote the latter half of the velocity output of forward compass horizontal alignment loop and backward compass horizontal alignment loop respectively. Flipping the sign of V2 and reversing the order, we get V2'. Then, the velocity information during the whole alignment process can be obtained by combing V2' and V1.

In order to verify the validity of the backtracking compass alignment based velocity denoising method. Choose a total 300s segment of SINS and Odometer data. Resolve the body-frame velocity of SINS based on forward compass horizontal alignment loop and backward compass horizontal alignment loop respectively. The results are shown in Fig. 4.
In Fig. 4, the blue solid line represents the velocity output of the forward compass horizontal alignment loop, which is V1 in Fig. 3. The green solid line represents the velocity output of the backward compass horizontal alignment loop, which is V2’ in Fig. 3. The red dot line represents the velocity reference obtained by SINS/GPS integrated navigation. As shown in Fig. 4, at the beginning of SINS initial alignment, the velocity error of forward compass horizontal alignment loop is too large to be used for optimization based alignment introduced in Section I. And within 100s, the velocity accurately tracks the reference velocity. While the velocity output of backward compass horizontal alignment loop is the polar opposite, which accurately tracks the reference velocity at the beginning and velocity error appears at the end.

For clarity, the velocity errors of forward compass horizontal alignment loop, backward compass horizontal alignment loop and the proposed backtracking compass alignment based velocity denoising method are respectively shown in Fig. 5.

In Fig. 5, the blue solid line represents the velocity error of the forward compass horizontal alignment loop; the green solid line represents the velocity error of the backward compass horizontal alignment loop; and the red solid line represents the velocity error of the proposed backtracking compass alignment based velocity denoising method. As shown in Fig. 5, the velocity errors of the forward and backward compass horizontal alignment loops tend to converge after oscillation. The proposed method can effectively realize velocity denoising and accurately determine the velocity during the whole initial alignment process. So as to provide accurate velocity for optimization based alignment.

IV. BACKTRACKING VELOCITY DENOISING BASED AUTONOMOUS IN-MOTION INITIAL ALIGNMENT

In view of the problems existing in autonomous in-motion initial alignment, such as the large noise of the external velocity source, model error of the body-frame optimization based alignment and accurate position updating is hard to be realized, a Backtracking velocity denoising based autonomous in-motion initial alignment method is proposed in this paper. The main idea can be shown in Fig. 6.

In Fig. 6, $V_{od}^b$ denotes the body-frame velocity output by the odometer, $V_{comp}^b$ the denoised body-frame velocity obtained by the backtracking compass alignment, $V_{comp}^n$ ( i = 1, 2 ) denotes the navigation-frame velocity decomposed by $V_{comp}^b$. The module “Attitude Determination I” represents a body-frame velocity based alignment process. “Attitude Determination II” and “Attitude Determination III” represent the navigation-frame velocity based alignment process. [ $\theta_{01}$, $\gamma_{01}$, $\phi_{01}$ ] denotes the [pitch, roll, yaw] at the beginning of the initial alignment, determined by “Attitude Determination I”; [ $\theta_{02}$, $\gamma_{02}$, $\phi_{02}$ ] denotes the [pitch, roll, yaw] at the beginning of the initial alignment, determined by “Attitude Determination II”. [ $\theta$, $\gamma$, $\phi$ ] denotes the [pitch, roll, yaw] at the end of the initial alignment, determined by “Attitude Determination III”. As shown in Fig. 6, firstly, the odometer velocity is denoised by backtracking compass alignment. Thus, the accurate body-frame velocity $V_{comp}^b$ can be obtained, which is then transferred to module “Attitude Determination I”, where the body-frame velocity aided attitude determination is carried out to calculate the attitude of SINS at the start time of initial alignment. Depending on [ $\theta_{01}$, $\gamma_{01}$, $\phi_{01}$ ] and the output of gyros, the attitude of SINS during the alignment process can be calculated and the body-frame velocity
$V_{comp}^n$ can be decomposed to navigation-frame, then we get $V_{comp}^n$ based on which, navigation-frame velocity aided attitude determination is carried out in module “Attitude Determination II” and the initial attitude $[\theta_{02}, \gamma_{02}, \varphi_{02}]$ can be obtained. As, compared with body-frame speed aided attitude determination, there exists less approximation in the derivation of navigation-frame speed aided attitude determination, the precision of $[\theta_{02}, \gamma_{02}, \varphi_{02}]$ is expected to be higher than that of $[\theta_{01}, \gamma_{01}, \varphi_{01}]$. Thus, it is foreseeable that the precision of $V_{comp}^n$, which is obtained depending on $[\theta_{02}, \gamma_{02}, \varphi_{02}]$ and attitude updating, will be higher than that of $V_{comp}^n$. As the precision of external velocity source will directly influence the optimization based alignment, the improvement of the precision of navigation-frame velocity will benefit for the alignment precision in “Attitude Determination III”. The attitude output of “Attitude Determination III” is $[\theta, \gamma, \varphi]$, which regarded as the attitude determination result of the proposed initial alignment method. At the same time, as the denoised body-frame velocity is accurately decomposed to navigation-frame, the SINS position can be updated during the attitude alignment process.

The procedure of the proposed backtracking velocity denoising based autonomous in-motion initial alignment method can be presented as follows.

Initialization: Denote the initial attitude matrix as $C_b^w$, which is provided by the gyrocompass horizontal alignment loop at the start of SINS alignment. $n^i$ denotes the computational frame of gyrocompass horizontal alignment loop. Set the initial velocity and attitude to 0.

**Step 1:** velocity updating and velocity denoising based on forward compass alignment

\[
\delta p_{E,k+1} = \delta p_{E,k} - T_{K_E} \delta \gamma_{E,k+1}^n (25)
\]

\[
\omega_{N,k+1} = \omega_{N,k+1}^n + \gamma_{N,k+1}^n (26)
\]

\[
v_{E,k+1} = v_{E,k} + T_s \gamma_{E,k+1}^n (27)
\]

\[
\delta p_{N,k+1} = \delta p_{N,k+1} - T_{K_N} \delta \gamma_{N,k+1}^n (28)
\]

\[
\omega_{E,k+1} = \omega_{E,k+1} + \gamma_{E,k+1}^n (29)
\]

Step 3: velocity combination after denoising

\[
\delta p_{E,k+1} = \delta p_{E,k} - T_{K_E} \delta \gamma_{E,k+1}^n (30)
\]

\[
\omega_{N,k+1} = \omega_{N,k+1}^n + \gamma_{N,k+1}^n (31)
\]

\[
v_{E,k+1} = v_{E,k} + T_s \gamma_{E,k+1}^n (32)
\]

\[
\delta p_{N,k+1} = \delta p_{N,k+1} - T_{K_N} \delta \gamma_{N,k+1}^n (33)
\]

\[
C_{b,k+1} = C_{b,k}^w + (I + T_s (\omega_{E,k+1}^b - \omega_{E,k+1}^n) - \omega_{E,k+1}^b) \times (34)
\]

\[
V_{comp,k+1}^n = [v_{E,k+1}^n; v_{N,k+1}^n; 0] (35)
\]

\[
V_{comp,k+1}^b = C_{b,k+1} V_{comp,k+1}^n (36)
\]

**Step 2:** attitude updating and velocity denoising based on backward compass alignment

\[
\delta p_{E,k+1} = \delta p_{E,k} - T_{K_E} \delta \gamma_{E,k+1}^n (37)
\]

\[
\omega_{N,k+1} = \omega_{N,k+1}^n + \gamma_{N,k+1}^n (38)
\]

\[
v_{E,k+1} = v_{E,k} + T_s \gamma_{E,k+1}^n (39)
\]

\[
\delta p_{N,k+1} = \delta p_{N,k+1} - T_{K_N} \delta \gamma_{N,k+1}^n (40)
\]

\[
\omega_{E,k+1} = \omega_{E,k+1} + \gamma_{E,k+1}^n (41)
\]

\[
v_{E,k+1} = v_{E,k} + T_s \gamma_{E,k+1}^n (42)
\]

\[
\delta p_{N,k+1} = \delta p_{N,k+1} - T_{K_N} \delta \gamma_{N,k+1}^n (43)
\]

\[
v_{E,k+1} = v_{E,k} + T_s \gamma_{E,k+1}^n (44)
\]

\[
\omega_{E,k+1} = \omega_{E,k+1} + \gamma_{E,k+1}^n (45)
\]

\[
C_{b,k+1}^w = C_{b,k}^w (I - T_s (\delta p_{E,k+1}^n - \delta p_{E,k+1}^w) - \omega_{E,k+1}^n) \times (46)
\]

Where, $(V_{comp}^b)^1$ denotes the former half of $V_{comp}^b$ after reversing and negating; $(V_{comp}^b)^2$ denotes the latter half of $V_{comp}^b$.

**Step 4:** Attitude determination aided by body-frame speed. Vehicle attitude at the start of alignment process is denoted by $[\theta_{01}, \gamma_{01}, \varphi_{01}]$.

**Step 5:** According to $[\theta_{01}, \gamma_{01}, \varphi_{01}]$ and attitude tracking, navigation-frame velocity $V_{comp}^n$ can be obtained by composing $V_{comp}^b$. Then, attitude determination aided by navigation-frame speed can be carried out and SINS attitude at the start of alignment process can be determined, which is denoted by $[\theta_{02}, \gamma_{02}, \varphi_{02}]$.

**Step 6:** According to $[\theta_{02}, \gamma_{02}, \varphi_{02}]$ and attitude tracking, attitude determination aided by navigation-frame speed will be carried out and SINS attitude at the end of alignment process can be determined, which is denoted by $[\theta, \gamma, \varphi]$.

Remark: Actually, the proposed in-motion alignment method solves the problems existing in autonomous initial alignment in a phased solution. “Attitude Determination I” focuses on suppressing the influence of external velocity noise on the body-frame velocity aided initial alignment. “Attitude Determination II” mainly deals with the inaccuracy of the alignment model. “Attitude Determination III” mainly addresses the negative influence of inaccurate velocity decomposition and position updating on the autonomous alignment. It is remarkable that the improvement of velocity decomposition precision will benefit for optimization based attitude determination and position updating. Conversely, the improvement of attitude precision will promote more accurate velocity decomposition. That will be a stable negative feedback, which will be favourable to the attitude determination and position alignment.
V. PERFORMANCE EVALUATION THROUGH FIELD TEST

In this section, the performance of the backtracking denoising based autonomous in-motion alignment method is evaluated using field test data. The used field test data was collected from a car-mounted navigation-grade SINS which is equipped with a triad of ring laser gyroscopes with a drift rate \(0.007^/\text{h} (1\sigma)\) and quartz accelerometers with a bias of \(5 \times 10^{-4} \text{g} (1\sigma)\). In the experiment test, except for the SINS, a single-antenna GPS receiver and an odometer are also equipped on the car. The SINS output raw data of gyroscopes and accelerometers at 125 Hz. A GPS antenna is installed outside the cabin on the top of the car and provides velocity and position information at 1Hz. The pulse equivalent of the used odometer is \(0.08194 \text{m}\). The update rate of the odometer is 125Hz. The integrating results of the SINS and GPS are used as the attitude and velocity reference.

After the experimental instruments and equipments are powered on, the car keeps static for 20 min before maneuvering. The attitude and position outputs of INS/GPS integrated navigation system with static initial alignment can be used as the reference attitude and position. When the vehicle is in motion, the outputs of SINS and odometer are collected for the autonomous in-motion alignment experiments.

In order to intuitively illustrate the implementation of the proposed backtracking velocity denoising based autonomous in-motion initial alignment method, the initial alignment process of the proposed method is divided into three stages: Stage 1: “Attitude Determination I”, body-frame velocity aided attitude determination; Stage 2: “Attitude Determination II”, navigation-frame velocity \(V_{\text{comp1}}\) aided attitude determination; Stage 3: “Attitude Determination III”, navigation-frame velocity \(V_{\text{comp2}}\) aided attitude determination.

Choose a total 300s data segment from the experimental data for the initial alignment experiment, which contains the raw data of gyro and accelerometers, speed and position output by integrate GPS/INS and forward speed output by odometer. During the experiment, the variation of the forward velocity and attitude of the vehicle can be shown in Fig. 7-Fig. 9.

As shown in Fig. 7, Fig. 8, and Fig. 9, during the initial alignment experiment, the forward velocity of the vehicle varies in a range from 12 m/s to 22 m/s. The yaw angle ranges from 25° to 65°. And the level attitude ranges from -2° to 2°. The above experimental environment can represent a typical on-board situation. Thus, the experimental results can effectively evaluate the performance of the proposed alignment method. As mentioned above, the initial alignment process is divided into three stages and the experimental results can be shown in Fig. 10-Fig. 12.

![Figure 7. Forward velocity of the vehicle during the alignment process.](image1)

![Figure 8. Yaw angle of the vehicle during the alignment process.](image2)

![Figure 9. Level angle of the vehicle during the alignment process.](image3)

![Figure 10. Pitch errors of various stages of initial alignment.](image4)
FIGURE 11. Roll errors of various stages of initial alignment.

FIGURE 12. Yaw errors of various stages of initial alignment.

In Fig. 10-Fig. 12, the green solid line represents the attitude errors at Stage 1; the blue solid line represents the attitude errors at Stage 2; and the red solid line represents the attitude errors at Stage 3. As shown in Fig. 10-Fig. 12, the level attitude precision of the three stages is similar. However, the precision of yaw angle is increased gradually. The improvement of yaw angle is produced for two main reasons: (1) The position precision during the alignment process is improved. As the proposed backtracking compass alignment based velocity denoising method can accurately determine the velocity of SINS during the whole initial alignment process, the position of SINS can be updated at the very start of initial alignment. Thus, the position precision can be guaranteed during the alignment process. (2) The decomposition precision of velocity is gradually improved. As the model precision of navigation-frame velocity aided attitude determination is higher than that of body-frame velocity aided attitude determination, the yaw precision of Stage 2 will be higher than that of Stage 1. And then, the decomposition precision of velocity from body-frame to navigation-frame could be improved. That virtually improves the observation precision of attitude determination method and will benefit for the precision of initial alignment. The experimental results reasonably agree with the analysis in Section 1.

In order to verify the advantages of the proposed backtracking denoising based autonomous in-motion alignment method compared with traditional in-motion alignment methods, the following three alignment methods were carried out based on the aforementioned experimental data and the results are shown in Fig. 13-Fig. 15.

Scheme 1: ADIA algorithm aided by odometer velocity without denoising.
Scheme 2: ADIA algorithm aided by denoised body-frame odometer velocity.
Scheme 3: the proposed backtracking denoising based autonomous in-motion alignment method.

FIGURE 13. Pitch errors by different alignment schemes.

FIGURE 14. Roll errors by different alignment schemes.

FIGURE 15. Yaw errors by different alignment schemes.

In Fig. 13-Fig. 15, the green solid line represents the attitude errors of Scheme 1; the blue solid line represents the attitude errors of Scheme 2; the red solid line represents the attitude errors of Scheme 3. As shown in Fig. 13-Fig. 15, the attitude precision of Scheme 2 and Scheme 3 is similar. However, the yaw angle precision of Scheme 3 is obviously better than that of Scheme 2. In Fig. 15, the yaw
angle errors of Scheme 1, Scheme 2 and Scheme 3 at 300s are respectively 19.1°, 1.83° and 0.31°. Thus, it can be concluded that noise of the external velocity source and the model approximation of optimization based attitude determination method will seriously influence the precision of SINS in-motion initial alignment and the proposed method can effectively denoise the external velocity noise and gradually decompose it to the navigation-frame, so as to realize an accurate in-motion initial alignment. At the same time, the position of SINS during initial alignment could be updated with high precision. Thus, the accurate initial position for fine alignment or navigation can be obtained. In the above experiment, the result of position updating by the proposed method is shown in Fig. 16-Fig. 19.

In Fig. 16 and Fig. 18, the blue solid line represents the calculated position by the proposed method; the red dot line represents the reference position by INS/GPS integrated navigation. As shown in Fig. 16 and Fig. 18, the calculated position coincides to the reference position well. Fig. 17 and Fig. 19 present the position errors. After analysis, the positioning error of the proposed method is lower than 1% of the total distance. It means that the proposed method can realize relatively accurate position updating.

Under the above test conditions, 6 more experiments are carried out and the results are shown in Tab. I.

As shown in Tab. I, among the 6 sets of experiments, the peak and mean of pitch errors of SINS initial alignment are 0.012° and 0.0063° respectively. The peak and mean of roll errors of SINS initial alignment are 0.017° and 0.012° respectively. And the peak and mean of yaw errors of SINS initial alignment are 0.56° and 0.25° respectively. The peak and mean of the ratios between the positioning error and the total distance are 1.32% and 0.937%. Although the alignment results will be influenced by the dynamic environments, the precision of attitude and position of all sets of experiments can satisfy the requirements for SINS coarse alignment. Therefore, it can be concluded that the proposed backtracking denoising based autonomous in-motion alignment method can effectively solve the problems existing in SINS autonomous in-motion initial alignment and realize an accurate attitude and position alignment.
VI. CONCLUSIONS

It is essential for SINS to realize a rapid and accurate initial alignment. In view of the problems existing in body-frame velocity aided in-motion alignment, a backtracking velocity denoising based autonomous in-motion initial alignment method is proposed in this paper. The backtracking compass alignment method is studied to denoise the external velocity. And three attitude determination processes are executed successively to gradually deal with the problems such as inaccuracy of the alignment model and inaccurate velocity decomposition and position updating. The performance of the proposed method is evaluated by car-mounted field test data. The experimental results show that, in the cases of relatively high dynamic and large external velocity noise, the proposed autonomous in-motion alignment method can realize an accurate attitude determination and position alignment in 300s. The level attitude accuracy is better than 0.015°. The yaw angle accuracy is better than 0.3°. And the ratio between positioning error and total distance is better than 1%. Therefore, it can be concluded that the proposed initial alignment method can effectively solve the problems of body-frame velocity aided autonomous in-motion alignment and achieve a high performance SINS initial alignment.

REFERENCES


