Planning Solar in Energy-managed Cellular Networks

Mathieu D’Amours, André Girard and Brunilde Sansò

Abstract—There has been a lot of interest recently on the energy efficiency and environmental impact of wireless networks. Given that the base stations are the network elements that use most of this energy, much research has dealt with ways to reduce the energy used by the base stations by turning them off during periods of low load.

In addition to this, installing a solar harvesting system composed of solar panels, batteries, charge controllers and inverters is another way to further reduce the network environmental impact and some research has been dealing with this for individual base stations.

In this paper, we show that both techniques are tightly coupled. We propose a mathematical model that captures the synergy between solar installation over a network and the dynamic operation of energy-managed base stations. We study the interactions between the two methods for networks of hundreds of base stations and show that the order in which each method is introduced into the system does make a difference in terms of cost and performance. We also show that installing solar is not always the best solution even when the unit cost of the solar energy is smaller than the grid cost. We conclude that planning the solar installation and energy management of the base stations have to be done jointly.

Index Terms—Cellular networks, energy management, sleep mode, solar power

I. INTRODUCTION

There is a growing awareness to the fact that the communication sector uses a significant amount of energy especially for the base stations of cellular networks, where energy costs make up a large part of the operating expenses of service providers.

Two broad trends appear in the literature to tackle the wireless energy efficiency problem. On the one hand, there is the search for more energy-efficient transmission devices and technologies [1]. This is mostly done in the physical layer and is thus outside the scope of this paper since we focus here on the network layer. On the other hand, new technology can be used to improve energy use in the base stations, such as sleep mode, where some antennas are turned off during low traffic periods and users can be re-allocated to the active ones.

More recently, the idea of using green energy sources to power base stations has been considered and in particular, the use of solar energy [2] either as a stand-alone source, where the climate permits, or as an addition to conventional grid electricity. In most of that work, there is a common assumption that solar energy is either practically free or at least much cheaper than grid power. From this assumption follows the notion that one should install solar equipment everywhere and that one should use as much solar energy as possible. This of course neglects the capital cost of the solar equipment, which can be significant and might lead to different solutions.

The objective of this paper is to challenge the fact that one can manage the network via sleep mode or provision for solar energy in an independent way. In fact, we show that there is a close relationship between solar planning and sleep mode. Moreover, we find that the order in which the different features are optimized produce a very different outcome. We also find that installing solar equipment everywhere is not always optimal even when the unit cost of solar energy is smaller than the grid cost. Finally, we make the case for a joint network management-solar planning optimization solution.

II. PREVIOUS WORK

A. Base Station Energy Management

It has been known [3], [4], [5] for quite some time that turning off base stations during low traffic demand can yield significant energy savings. This is possible if we can serve users from other active base stations during that period, which is the case for current technology.

Some work [6], [7], [8], [9], [10] uses game theory to compute the base station sleep schedule such that the total power used is minimized subject to constraints on the quality of service received by the users. A similar problem is studied in [11] in the context of a heterogeneous network made up of macro and pico cells and is extended in [12] to the case of femtocells. For 3G networks, the minimum network energy use is computed in [13] where a base station is turned off whenever all of its users can be served by another active base station.

Another energy-management technique is discussed in [14], [15], [16] where base stations can set their transmission power at different values. This is also examined in [17], [18] where coordinated multipoint transmission is used to reach isolated users instead of increasing the base station power. Another cooperative solution is studied in [19] using game theory.

Concerning the network dynamics and savings, there has been a number of models to optimize the operation of base stations in 3G networks [3]. Most of these are real-time algorithms where a quasi-optimal policy is computed given an already existing network. Although these can provide substantial energy savings, they are constrained by the structure of the network in which they operate. A similar model is discussed in [20] where the objective of...
reducing energy use is to minimize two objective functions: the number of installed based stations and the number of users not served by any base station.

The idea that sleep mode management must be integrated at the planning stages was formalized in [21], [22], [23] where it was shown that the joint optimization can bring savings of up to 30% with respect to individual optimization.

### B. Solar-Powered Base Stations

Using solar energy to power base stations has received some attention over the last few years, first for base stations in isolated areas where grid power is either not available or it is very expensive. A simple case is [24] that studies whether a base station can be powered only with solar energy taking into account the traffic load and sleep mode. A statement is made to the effect that capital and operating costs are such that a purely solar power base station is competitive with diesel generation.

The most recent work on using solar power for 3G has been investigated in [2]. The objective is to choose the solar equipment of an UMTS Node B to minimize their net present cost. The hourly load is available as well as the average monthly solar power. The solution method is to compute the power available at each hour for all possible equipment configuration and choose the best one. The energy needed to serve traffic is not taken into account. The only requirement is that the total power available from the batteries and panels should equal the power needed by the base station plus losses. The main conclusion is that solar energy is a realistic option even for 3G technology.

Another aspect of solar energy optimization is to take into account the random variations in solar energy. Stochastic programming was used in [25] to optimize the expected cost of purchasing energy from the grid under uncertainty due to solar energy availability, variable traffic load and on-demand grid prices. The decision variables were the amounts of electricity to buy from the grid during the day.

The work of [26] models the on/off switching strategy for a base station operating only on solar energy. It considers the case of two base stations and proposes a solution based on a robust Bayesian technique assuming perfect information on the traffic.

Network effects can also arise due to the presence of a power distribution smart grid. The energy management for a given base station connected to a smart grid is studied in [27], [28], [29]. A critical upper bound on the batteries’ capacity where no more energy savings can be obtained is investigated in [27]. The charge of the batteries is also studied in [30], where the charge needs to stay within a given range. They model the charge and discharge to optimize the life expectancy of the batteries.

A hybrid solar-grid system for a single base station that is less costly than pure solar is proposed in [31]. An important feature of this work is that the objective is to minimize the total cost made up of the capital cost of panels and batteries plus the operating cost of using grid power. The number of batteries and the size of the solar panels are optimized as well as the energy management. This seems to be one of the rare cases where planning is done for an horizon of many years but not in a network context. This is solved in a two-step process. First, a model is set up for the optimization of the base station in a single year. The multi-year problem is solved by a sequence of single-year optimizations.

### C. Solar and Sleep Mode

The decision to use solar energy or not can be made for each base station separately, based on cost, performance, etc. Such decision has then no impact on other base stations. When both solar power and sleep mode are used, the decision to put a base station in sleep mode means that the users have to be reallocated to other base stations, thus increasing their load. This in turn may impact the decision in these other base stations to use solar energy during that time. Because of this difficulty, there has not been much work done on this topic.

We have found only one reference [32] where both solar and sleep mode are used together. The objective is to minimize the total energy cost, either solar or grid, where decision variables are used to determine the assignment of users to base stations and the use of grid or solar power. There is the usual assumption that “the unit cost of green energy is cheaper than that of the on-grid energy” and the authors also assume batteries with infinite capacity. The solution technique based on a long-term demand forecast is used to compute a target sleep schedule. The main differences with our approach is that the model has no limit on battery storage, the costs are not based on real estimates since only the ratio of solar to grid energy is used and all base stations are assumed to have solar energy available.

### D. Costs

Much of the work on solar energy assumes that its cost is very small compared with the grid cost and it is very seldom taken into consideration. One exception is the work of [33] that optimizes the sizing of the solar panels installed on macro base stations and the battery banks capacity to minimize the capital cost. The trade-off is between installing the solar systems on macro base stations on the one hand and off-loading traffic on small base stations on the other. The fraction of the total energy requirement that is served by the green sources is given a priori and is not a result of the optimization, which is an important difference with what we do. Also, there is no actual value for the costs and the results are presented as a function of the cost ratio between the two types of energy.

It is also worth mentioning that there is little work trying to estimate realistic costs for the whole solar energy harvesting system. Most papers consider solar panels and batteries but leave out charge controllers and inverters.
which are an important part of the equipment. In fact, not only do they increase the capital cost of the system but they also decrease its efficiency.

In the literature we reviewed, no work considered inverters and charge controllers, with the exception of [3] that takes into account the cost, lifetime and efficiency of the inverters, but not the charge controllers.

Summarizing, we are not aware of any research that integrates solar panels, batteries, inverters and charge controllers with the sleep-mode of the equipment into a long-range planning model.

### III. System Model

The most important feature that sets our model apart from the previous work is that it is a planning model where we choose if and where to install solar equipment. This equipment will be used for a long time, typically many years, so that the model has to be based on a long-term view of the network. We now summarize its most important features.

The model is based on [22] where the network is made up of a given set of base stations that can turn their transmitter on or off during the day in order to save energy. Traffic is generated by so-called traffic test points, devices that aggregate traffic from users. The day is divided into a number of time periods that don’t have to have the same length, e.g., day or night, or hourly during the day and a single night period.

We assume that the base stations, traffic and connection test points are given. The model of [22] is extended by having the possibility of installing solar cells and batteries to power the base stations. The solar cells feed into a battery pack which can then be used to feed the antenna instead of using grid power. The decision to install solar cells is taken once for each base station at the beginning of the study period. At the beginning of each time period, each base station must choose the state, idle or active, and whether the corresponding energy comes from the grid or the batteries.

#### A. Short-term Processes

The operation of the network is driven by external events that occur on different time scales. The amount of solar energy available in a given area varies randomly on the order of minutes, and can also have more regular variations depending on the seasons. Traffic demand also varies randomly, perhaps on the order of minutes. It also has more regular variations depending on the time of day and day of the week.

Because of this, some decisions must be taken on a short term: how to allocate users to base stations, whether to turn antennas on or off and whether to use battery or grid power. These are based on real-time algorithms such as the ones described in [9], [20].

All these short-term processes will have an impact on the decision to install solar equipment or not. As an example, a less efficient battery scheduling would mean that the system would not be able to store and re-use solar energy in an efficient way, which in turn could mean that there would be little incentive to install solar panels. If we wanted to get a truly optimal decision, all these decisions should be designed at the same time. For this, we would need to model all these short-term variations over the multi-year horizon of the planning process. In practice, this will quickly lead to problems of enormous sizes which will be impossible to solve due to memory limitations.

The standard technique when faced with problems with very different time scales is to replace the fast processes by some fixed value, generally the average. In our case, this would lead to an over-simplification since, for instance, replacing the daily variation of sunlight by a single average value for the whole planning horizon would leave out the daily changes in sunlight.

For this reason, we have chosen to define what we call an average day and to model the short-term variations for this average day. If we have hourly measurements of sunlight on an hourly basis over a whole year, in each hour, we use the value of sunlight averages over the whole set of days as the value of sunlight for that period for the average day.

As a consequence, the planning model does not assume anything about a real-time algorithm for the decision variables. Instead, we compute the optimal value in the average day. These could then be used as extra conditions when designing a real-time algorithm. Suppose the model yields some values for connecting the users to the base stations. When we design a real-time scheduling algorithm, these values could be used to specify that a test point is connected most often to a given base station averaged over the planning horizon. This would ensure that the real-time algorithm is tuned to the network structure and to the introduction of solar energy.

#### B. Other Model Features

In addition to the use of the average day, our model integrates some features of a number of previous work into a coherent mathematical formulation. It also has the following features that are different from the state of the art.

First, this is a long-term planning model where the cost of solar equipment is traded off against the operating cost of using grid power. As we mentioned before, adding inverters and charge controllers can change significantly the cost of the solar energy. We also model the batteries costs and limitations, something that is not always done in some models.

The second important difference is that we model a whole network, where it is possible to off-load traffic from one base station to another.

Finally, we formulate the model as a linear integer program so that it can be computed by standard solvers for small enough networks. This allows us to gain insight into the more important factors that affect the decision to implement solar energy or not.
IV. MATHEMATICAL FORMULATION

A. Sets

First we define the following sets:

- \( S \): Installed base stations
- \( I \): Test points
- \( T \): Time periods

The time periods are indexed by \( t = 1 \ldots T \). This is called the time base in the following and each value indicates when the period starts, e.g., a time base \( T = \{0, 8, 16\} \) is made up of 3 periods starting at 0:00, 08:00 and 16:00 hours.

In general, \( j \) refers to a base station, \( i \) to a test point and \( t \) to a time period. These indices are always assumed to run over their whole set \( S \), \( I \) or \( T \) unless otherwise noted.

B. Parameters

These are known network parameters. In some cases, they are readily available and if not, they can be calculated with realistic data as explained in Section [ ]

The objective function is the sum of capital costs and the total value of the operating costs over all days of the study period with

\[
\phi \quad \text{The number of days over the time horizon for the planning, e.g., for a planning horizon of } M \text{ years, } \phi = 365 \times M
\]

\( \Delta_t \): Length of period \( t \).

\( C^S_j \): Installation cost for solar panels, batteries, inverters and charge controllers on base station \( j \). The cost includes capital as well as replacement cost and can depend on the particular base station and type of solar module.

\( C^E_j \): Grid energy cost. We assume that this is constant over the length of the study.

\( E^S_{j,t} \): The amount of electrical energy produced by the solar panels at base station \( j \) during period \( t \).

\( E^0_{j,t} \): Energy needed by the base station in the idle state in period \( t \).

\( E^{1}_{j,t} \): Energy needed by the base station in the active state in period \( t \).

\( E_T^{i,t} \): Energy required by test point \( i \) in period \( t \).

\( N_{bs} \): The number of test points per base stations

\( B^+_{j} \): Maximum battery capacity. This is the total energy that can be stored by all batteries installed in a base station.

\( B^-_{j} \): Minimum battery capacity. This is some fraction of \( B^+_{j} \).

\( k_{i,j} \): Indicator function set to 1 if traffic test point \( i \in I \) is covered by the base station installed in \( j \) and 0 otherwise.

Based on these parameters, we describe the linear model: parameters, variables and constraints corresponding to the base stations and test points.

C. Variables and Constraints

These are the optimization variables that correspond directly to the operation model. They are set to 1 if

- \( z_j \): Solar equipment is installed at base station \( j \)
- \( x^0_{j,t} \): The base station is in the idle state in interval \( t \)
- \( x^b_{j,t} \): The base station uses battery power during interval \( t \)
- \( h_{i,j,t} \): Test point \( i \) is assigned to base station \( j \) in period \( t \)

and 0 otherwise. We also need some intermediate variables to simplify the presentation.

\( D_{j,t} \): Energy required by the users assigned to \( j \) in \( t \)

\( E^P_{j,t} \): Energy used by the antennas of \( j \) in \( t \), whether it is in the idle or active state

\( L_{j,t} \): Energy lost because of the limited capacity of the batteries in base station \( j \) in period \( t \)

\( E^B_{j,t} \): The amount of energy available in batteries at base station \( j \) at the beginning of interval \( t \).

These variables are subject to constraints. First we cannot use battery power unless solar equipment has been installed:

\[
x^b_{j,t} \leq z_j \quad \forall j \in S, \forall t \in T.
\] (1)

Next, the user demands are computed from the assignment variables

\[
D_{j,t} = \sum_{i \in I} E^T_{i,t} h_{i,j,t} \quad \forall j \in S, \forall t \in T
\] (2)

where the assignment variables are subject to the constraints

\[
\sum_{j \in S} h_{i,j,t} = 1 \quad \forall i \in I, \forall t \in T.
\] (3)

We also assign the test points only to base stations that are not in idle state:

\[
x^o_{j,t} \leq 1 - h_{i,j,t} \quad \forall i \in I, \forall j \in S, \forall t \in T.
\] (4)

Note that there are no explicit coverage constraints in the model. These can be taken into account in a number of ways. The easiest one would be to add constraints of the form \( h_{i,j,t} \leq k_{i,j} \) which would prevent the allocation of test points to base stations when they are not within the coverage. Most modern solvers would recognize this condition in the pre-solve phase and automatically remove the variables. Another way would be to define the \( h \) variables only for those cases when the test point is within the coverage radius. Finally, one can simply fix the \( h_{i,j,t} = 0 \) in the AMPL script whenever \( k_{i,j} = 0 \). This is the solution we take here.

Next, the next set of constraints describes the energy production and management. First, we can compute the energy used by the antennas as

\[
E^P_{j,t} = E^0_{j,t} x^0_{j,t} + E^{1}_{j,t}(1 - x^0_{j,t}) \quad \forall j \in S, \forall t \in T.
\] (5)

The demand must not exceed the energy available to the antennas in the idle or active state, which yields

\[
D_{j,t} \leq E^P_{j,t} - E^0_{j,t} \quad \forall j \in S, \forall t \in T.
\] (6)

Replacing (2) and (5) in (6), we get

\[
\sum_{i \in I} E^T_{i,t} h_{i,j,t} \leq (E^i_{j,t} - E^0_{j,t})(1 - x^0_{j,t}) \quad \forall j \in S, \forall t \in T.
\] (7)
Next, we must model the assumption that we can use only one of the two energy sources, solar or grid, during a given period and that the decision whether to use solar or not is made at the beginning of each period only. This means that we can use solar energy only if the amount of energy stored in the battery at the beginning of the period plus that produced by the solar panels is at least as large as the required energy during that period without depleting the batteries beyond their minimal value.

In order to simplify the notation, we define $E_{j,t}^P$, the antenna power used in battery mode, as

$$E_{j,t}^P = x_{j,t}^b E_{j,t}^P \quad \forall j \in S, \forall t \in T.$$  

(8)

The constraint on energy use can then be written

$$E_{j,t}^B + E_{j,t}^S - E_{j,t}^P \geq B^- \quad \forall j \in S, \forall t \in T.$$  

(9)

We also have to express the fact that the excess energy remaining at the end of a period is stored in the battery and is available at the beginning of the next one. If we impose the condition that all the energy must be stored, we would have a constraint of the form

$$E_{j,t}^B = E_{j,t-1}^B + E_{j,t-1}^S - E_{j,t-1}^P.$$  

If the value on the right-hand side turns out to be larger than $B^+$, the constraint can be satisfied only by not using solar during that period, which is not a realistic solution. Instead, we write the condition as an inequality constraint

$$E_{j,t}^B \leq E_{j,t-1}^B + E_{j,t-1}^S - E_{j,t-1}^P$$  

so that the excess energy can be used up to the value of $B^+$ but no more. We also define the slack variables $L_{j,t}$ explicitly since they represent the lost energy in that period. We then have the amount of energy stored in the batteries at the beginning of time $t$ as

$$E_{j,t}^B = E_{j,t-1}^B + E_{j,t-1}^S - L_{j,t-1} - E_{j,t-1}^P,$$  

(10)

$$B^- \leq E_{j,t}^B \leq B^+ \quad \forall j \in S, t \in T,$$  

(11)

$$0 \leq L_{j,t} \leq E_{j,t}^S \quad \forall j \in S, \forall t \in T.$$  

(12)

Constraint (12) is due to the fact that the maximum amount of energy we can lose is that produced by the solar panels.

Finally, we need to impose a condition related to the assumption that the energy use during a single day is in a sense representative of the operation of the network over the time horizon. This means that this pattern will repeat itself every day. In that case, the excess energy at the end of the last period is the available energy at the beginning of the next day. For consistency, we impose the condition that

$$E_{j,1}^B = E_{j,T}^B + E_{j,T}^S - L_{j,T} - E_{j,T}^P \quad \forall j \in S.$$  

(13)

The objective function is the sum of the capital cost and the grid cost

$$C = \sum_{j \in S} C^S_j z_j + \phi \sum_{j \in S} \sum_{t \in T} C^E_{j,t} E_{j,t}^P (1 - x_{j,t}^b).$$  

(14)

As written, the problem is not linear since it contains quadratic terms of the form $x_{j,t}^b x_{j,t}^o$ coming from (9) and (16). We can linearize it by defining supplementary variables and constraints

$$w_{j,t} = x_{j,t}^b x_{j,t}^o \quad \forall j \in S, \forall t \in T,$$  

(15)

$$w_{j,t} \leq x_{j,t}^b \quad \forall j \in S, \forall t \in T,$$  

(16)

$$w_{j,t} \leq x_{j,t}^o \quad \forall j \in S, \forall t \in T,$$  

(17)

$$w_{j,t} \geq x_{j,t}^b + x_{j,t}^o - 1 \quad \forall j \in S, \forall t \in T.$$  

(18)

The quadratic term $E_{j,t}^P x_{j,t}^b$, which represents the antenna power used in battery mode, appears in equations (9), (10) and (14). It can be written in terms of $w$ as

$$x_{j,t}^b E_{j,t}^P = E_{j,t}^P w_{j,t} + E_{j,t}^P (x_{j,t}^b - w_{j,t})$$

which is now a linear term.

### D. Linear Optimization Model

We now summarize the complete linear model. In the following, we denote this as problem $PL$. This is

$$\min_{z, x, h, w} \quad C = \sum_{j \in S} C^S_j z_j + \sum_{j \in S} \sum_{t \in T} C^E_{j,t} E_{j,t}^P (1 - x_{j,t}^b)$$

subject to constraints (11), (3), (4), (7), (9), (13) and (16).

### V. Sizing the Parameters

In this section we discuss how to size the parameters of Section IV: the grid and solar equipment costs, the number of solar panels, the batteries that have to be installed to power a micro base station and the time intervals used to approximate the solar and usage profiles. We also show in Figure 1 the small network that will be used later on for explaining some of our results.

#### A. Equipment Cost

We model four types of solar equipment: the solar panels, denoted by $S$, charge controllers, by $C$, inverters, by $I$ and batteries, by $B$. These should not be confused with the set notation used elsewhere in the paper. Each equipment type $K \in \{S, C, I, B\}$ has a number of specific parameters and the following common parameters:

- $N^K$ The number of equipment units installed in the base station
- $L^K$ The lifetime of each unit
- $C^K$ Cost of one unit
- $R^K$ The number of times the equipment has to be replaced during the study period

The number of replacements is given by

$$R^K = \left\lceil \frac{\phi L^K}{C^K} \right\rceil,$$  

(19)
where \( \phi \) is the length of the planning horizon. The cost of solar equipment is then given by
\[
C_{j}^{S} = \sum_{K} C_{j}^{K} N_{j}^{K} R_{j}^{K}.
\]
\( (20) \)

The number of solar panels and batteries \( N^{S} \) and \( N^{B} \) is fixed by the user. \( N^{I} \) and \( N^{C} \) are then calculated so that the inverter power and the charge controller current match the solar panels power and batteries voltage.

To obtain a realistic cost, the equipment parameters have to be computed from real data. Here, the solar panels are monocrystalline silicon [34] and the batteries are flooded lead acid [35]. The battery Depth of Discharge is the lower limit for the batteries’ charge. It is set at 50\%, which is a common trade-off between the lifetime and efficiency. More information on the inverters and charge controllers can be found in [36], [37].

The values used in this paper are all in the low price range. Also, power cables and possibly other devices are not taken into consideration which is one of the reasons why this model is somewhat biased in favor of solar energy. Table I summarizes the values for the parameters.

### B. Solar Energy

We compute \( E_{j,t}^{S} \), from the electrical output power \( W_{j}^{S} \), of a solar module that, at some instant \( t \), is directly proportional to the solar radiation \( G(t) \) at that time [24]. In particular, for the maximum value \( G_{\pi} \), which depends on the region where the module is installed, and a given module surface area \( A \), we get
\[
W_{j}^{S} = G \pi A \eta^{S}.
\]
\( (21) \)

The electrical power \( W_{i,t}^{S} \) produced in a given period \( t \) is given by
\[
W_{i,t}^{S} = W_{j}^{S} \pi_{i,t}^{S},
\]
\( (22) \)

with \( \pi_{i,t}^{S} \) as the fraction of maximal solar radiance during the period of the day \( t \) (See Section V-D for a description of \( \pi_{i,t}^{S} \)).

The total energy produced by a module during that period is
\[
E_{j,t}^{M} = W_{i,t}^{S} \Delta t.
\]
\( (23) \)

If we install \( N_{j}^{S} \) solar modules, the total solar energy they produce can be computed using (21), (22) and (23) and is given by
\[
E_{j,t}^{S} = N_{j}^{S} E_{j,t}^{M}
= N_{j}^{S} G A \pi_{i,t}^{S} \Delta t \prod_{k} \eta_{k}^{K}
\]
\( (24) \)

where \( \eta_{k}^{K} \) is the efficiency of each solar equipment \( K \in \{S,C,I,B\} \).

### C. Test Point Energy

We assume that in the busiest time of the day with test points at their maximum load, there is enough base stations to feed the test points. That means that, at that time, all of the energy available in the set of base stations is used. The difference between the base stations energy in thee active and idle states is the energy needed to power the test points.

\[
E_{i,t}^{T} = \frac{1}{N_{bs}^{tp}} \left( E_{j,t}^{1} - E_{j,t}^{0} \right) \pi_{i}^{U} f_{i}
\]
\( (25) \)

where \( \pi_{i}^{U} \) is the fraction of the maximum traffic of an user for a period \( t \). We randomize each test point traffic with a Gaussian distribution \( f_{i} \) with a mean of 1 and a scale of 0.2 truncated to the interval \([0,2]\).

### D. Network Parameters

The small network shown in Figure 1 is made up of 4 micro base stations [11] and 12 test points. We have not used macro base stations because the base power in the idle mode is larger than the amount of power that can be generated by any reasonable amount of solar panels. In that case, the decision to use solar power depends only on the ratio of solar to grid cost and not on the network management, which makes the solar planning problem trivial.

For this reason, we use only micro base stations because they only need 94 and 39 watts of power in the active and idle states. Table I shows the different parameters that are used for the macro and micro base stations which are taken from [11], [22].

In Figure 1 we can see that the coverage areas of the base stations need to overlap to some extent to be able to reassign some of the test points to different base stations when some base stations are in sleep mode.

The variation of solar illumination and traffic demand over the day is modeled by the parameters \( \pi_{i}^{S} \) and \( \pi_{i}^{U} \). They represent the fraction of the peak value that is present in period \( t \). They have been taken from [24].
With 4 and 5 panels per base station, the solution they produce too little energy as compared with their panels, the best solution is not to install any because base stations will need 57.04 MWh of electricity. In that period, the grid costs of the network over 20 years. We present in Table III both the solar and slots which yields the more accurate results as discussed in section V-F. These results are computed for a time base of 24 and compute an optimal solution of problem PL in each installed on a base station. We vary the number of panels everywhere should be the most economical solution. As we will see later, it turns out that this is due to the use of sleep mode. Instead, these are chosen at the outset based on These results are computed for a time base of 24 slots which yields the more accurate results as discussed in section V-F. We present in Table III both the solar and grid costs of the network over 20 years. In that period, the base stations will need 57.04 MWh of electricity.

When the base stations are equipped with only 3 solar panels, the best solution is not to install any because they produce too little energy as compared with their cost. With 4 and 5 panels per base station, the solution is to install solar equipment everywhere and most of the harvested energy is stored in the batteries without losses. This is because the effective solar energy that is used to power the base stations, shown in column Solar Used of Table III is close to the total solar energy that could be generated, as shown in the last column Installed. We see that the minimum cost is achieved with 6 solar panels. If the base stations are equipped with more panels, the optimal solution is to deploy solar equipment only in 3 of the 4 base stations. If we install more than this, the total energy produced by the panels will be larger than what is actually needed and some of it will be lost. The solar energy actually used becomes stable at a value close to 43 MWh because the losses just keep accumulating with the increase of available energy.

The same procedure has been applied for the batteries and we concluded that, for a set of batteries at different capacity and price, a single 2568 VAh battery is enough to store the solar energy without being too expensive. Therefore, in all the following, we assume that base stations have 6 solar panels and a single 2568 VAh battery. The number of inverters and charge controllers is set so that their nominal power is greater or equal than the power of the solar panels and batteries.

### E. Solar Equipment Sizing

Our model does not optimize the number of solar panels and batteries that are to be installed on the base stations. Instead, these are chosen at the outset based on the following considerations. First we try to estimate a good value for the number of solar panels that should be installed on a base station. We vary the number of panels and compute an optimal solution of problem PL in each case. These results are computed for a time base of 24 slots which yields the more accurate results as discussed in section V-F. We present in Table III both the solar and grid costs of the network over 20 years. In that period, the base stations will need 57.04 MWh of electricity.

When the base stations are equipped with only 3 solar panels, the best solution is not to install any because they produce too little energy as compared with their cost. With 4 and 5 panels per base station, the solution is to install solar equipment everywhere and most of the harvested energy is stored in the batteries without losses. This is because the effective solar energy that is used to power the base stations, shown in column Solar Used of Table III is close to the total solar energy that could be generated, as shown in the last column Installed. We see that the minimum cost is achieved with 6 solar panels. If the base stations are equipped with more panels, the optimal solution is to deploy solar equipment only in 3 of the 4 base stations. If we install more than this, the total energy produced by the panels will be larger than what is actually needed and some of it will be lost. The solar energy actually used becomes stable at a value close to 43 MWh because the losses just keep accumulating with the increase of available energy.

The same procedure has been applied for the batteries and we concluded that, for a set of batteries at different capacity and price, a single 2568 VAh battery is enough to store the solar energy without being too expensive. Therefore, in all the following, we assume that base stations have 6 solar panels and a single 2568 VAh battery. The number of inverters and charge controllers is set so that their nominal power is greater or equal than the power of the solar panels and batteries.

### F. Time Quantization

We now define a good time base to use in our model. The number of slots of the time base has three important effects. First, having a larger number of slots means that the network can adapt more accurately to the changes in demand or illumination, which will lead to less costly solutions. On the other hand, switching the base stations on and off can reduce the lifetime of the equipment so that the smaller the number of transitions the better. Also, from the point of view of calculating a solution, a larger time base increases the number of variables and constraints and will increase the solution time.

In order to estimate the effect of the time base, we first consider two cases with uniform interval sizes, one with one-hour and the other with three-hour intervals. We then

<table>
<thead>
<tr>
<th>Solar Panel</th>
<th>Cost (USD)</th>
<th>Area (m²)</th>
<th>Efficiency (%)</th>
<th>Power (W)</th>
<th>Degradation rate per year (%)</th>
<th>Lifetime (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>112</td>
<td>1.62</td>
<td>18.03</td>
<td>280</td>
<td>0.5</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Battery</th>
<th>Cost (USD)</th>
<th>DoD (%)</th>
<th>Efficiency (%)</th>
<th>Voltage (V)</th>
<th>Capacity (VAh)</th>
<th>Degradation rate per year (%)</th>
<th>Lifetime (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>345</td>
<td>50</td>
<td>90</td>
<td>6</td>
<td>428</td>
<td>3</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inverter</th>
<th>Cost (USD)</th>
<th>Efficiency (%)</th>
<th>Power (W)</th>
<th>Lifetime (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>90</td>
<td>2000</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Charge Controller</th>
<th>Cost (USD)</th>
<th>Efficiency (%)</th>
<th>Current (A)</th>
<th>Lifetime (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>95</td>
<td>60</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE I: Solar Equipment Parameters

<table>
<thead>
<tr>
<th>BS</th>
<th>Active power (W)</th>
<th>Idle power (W)</th>
<th>Radius (m)</th>
<th>Capacity (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macro</td>
<td>965</td>
<td>450</td>
<td>1230</td>
<td>210</td>
</tr>
<tr>
<td>Micro</td>
<td>94</td>
<td>39</td>
<td>850</td>
<td>70</td>
</tr>
</tbody>
</table>

### TABLE II: Base stations parameters

The illumination profile used to model the harvested solar energy is that of the city of Palermo, Italy. The industrial grid cost is set at 0.22$/kWh which is representative of this country’s real electricity pricing [38]. The total cost of a solar system calculated with (20) is $2197 for the cost of $0.16/kWh, which is lower than the grid cost. Based of the city of Palermo, Italy. The illumination profile used to model the harvested solar energy is that of the city of Palermo, Italy. The industrial grid cost is set at 0.22$/kWh which is representative of this country’s real electricity pricing [38]. The total cost of a solar system calculated with (20) is $2197 for the micro base station. This is an underestimation because the cable and labor costs of the installation are not considered. Note also that we can compute the total energy produced by the panels based on the solar profile and the other panel parameters. In the present case, this yields a value of 14 MWh over a 20-year horizon. This yields an equivalent cost of $0.16/kWh, which is lower than the grid cost. Based on these values, one might conclude that installing solar panels everywhere should be the most economical solution. As we will see later, it turns out that this is not the best solution due to the use of sleep mode.

<table>
<thead>
<tr>
<th>Solar Panel</th>
<th>Cost (USD)</th>
<th>Area (m²)</th>
<th>Efficiency (%)</th>
<th>Power (W)</th>
<th>Degradation rate per year (%)</th>
<th>Lifetime (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>112</td>
<td>1.62</td>
<td>18.03</td>
<td>280</td>
<td>0.5</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Battery</th>
<th>Cost (USD)</th>
<th>DoD (%)</th>
<th>Efficiency (%)</th>
<th>Voltage (V)</th>
<th>Capacity (VAh)</th>
<th>Degradation rate per year (%)</th>
<th>Lifetime (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>345</td>
<td>50</td>
<td>90</td>
<td>6</td>
<td>428</td>
<td>3</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inverter</th>
<th>Cost (USD)</th>
<th>Efficiency (%)</th>
<th>Power (W)</th>
<th>Lifetime (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>90</td>
<td>2000</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Charge Controller</th>
<th>Cost (USD)</th>
<th>Efficiency (%)</th>
<th>Current (A)</th>
<th>Lifetime (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>95</td>
<td>60</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE I: Solar Equipment Parameters

<table>
<thead>
<tr>
<th>BS</th>
<th>Active power (W)</th>
<th>Idle power (W)</th>
<th>Radius (m)</th>
<th>Capacity (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macro</td>
<td>965</td>
<td>450</td>
<td>1230</td>
<td>210</td>
</tr>
<tr>
<td>Micro</td>
<td>94</td>
<td>39</td>
<td>850</td>
<td>70</td>
</tr>
</tbody>
</table>

### TABLE II: Base stations parameters

The illumination profile used to model the harvested solar energy is that of the city of Palermo, Italy. The industrial grid cost is set at 0.22$/kWh which is representative of this country’s real electricity pricing [38]. The total cost of a solar system calculated with (20) is $2197 for the micro base station. This is an underestimation because the cable and labor costs of the installation are not considered. Note also that we can compute the total energy produced by the panels based on the solar profile and the other panel parameters. In the present case, this yields a value of 14 MWh over a 20-year horizon. This yields an equivalent cost of $0.16/kWh, which is lower than the grid cost. Based on these values, one might conclude that installing solar panels everywhere should be the most economical solution. As we will see later, it turns out that this is not the best solution due to the use of sleep mode.

The number of slots of the time base has three important F. Time Quantization

We now define a good time base to use in our model. The number of slots of the time base has three important effects. First, having a larger number of slots means that the network can adapt more accurately to the changes in demand or illumination, which will lead to less costly solutions. On the other hand, switching the base stations on and off can reduce the lifetime of the equipment so that the smaller the number of transitions the better. Also, from the point of view of calculating a solution, a larger time base increases the number of variables and constraints and will increase the solution time.

In order to estimate the effect of the time base, we first consider two cases with uniform interval sizes, one with one-hour and the other with three-hour intervals. We then

<table>
<thead>
<tr>
<th>BS</th>
<th>Active power (W)</th>
<th>Idle power (W)</th>
<th>Radius (m)</th>
<th>Capacity (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macro</td>
<td>965</td>
<td>450</td>
<td>1230</td>
<td>210</td>
</tr>
<tr>
<td>Micro</td>
<td>94</td>
<td>39</td>
<td>850</td>
<td>70</td>
</tr>
</tbody>
</table>
solve problem PL for a medium sized network of 27 test points and 9 base stations with these two time bases. The results are presented in the first two rows of Table IV. From this, we can see that having smaller time slots does improve the quality of the solution as expected but takes longer to solve. The first time base has 8 time slots and takes 0.28 second to be solved to optimality with a total cost of 24.7 k$. With 24 time slots, the optimization takes 0.66 second but has a lower cost of 22.97 k$.

We can strike a compromise between the computation time and the quality of the solution if we choose a time base better suited to the actual traffic and solar radiation profiles. We can then choose the number of time slots desired and do a best fit of the slot sizes with respect to the actual profiles. We can see in figures 2 and 3 the traffic and illumination profiles along with the fitted time base shown in (26).

\[ T = \{0, 9, 10, 13, 15, 18, 19, 20\}. \] (26)

This time base is relatively small and we can see from the third line of Table IV that the cpu time of 0.1 seconds is much smaller than that of the 24-slot base with a total cost of 22.97 k$, which is the same cost as with 24 time slots.

Based on these results, we will be using 8 time slots for the results of section VI-C. For the results of VI-E, we use 24 time slots to have a clearer view of the changes in the solution for the different optimizations.

VI. NETWORK RESULTS

We now present more detailed results for six networks of increasing size. In each case, we consider a number of scenarios to look at the interaction between the solar installation and the use of sleep mode. Each scenario requires the solution of a variation of problem PL where some variables are fixed to some value and the optimization is on the set of remaining variables.

A. Scenarios

First we consider the base case where neither solar power nor sleep mode is used. This scenario corresponds to most present networks and can be used as a comparison point. The next two scenarios correspond to the use of either sleep mode or solar power but not both. We call these single technology scenarios. Then we consider two scenarios for introducing both technologies in a network, one after the other. We call these sequential scenarios. In both cases, the end result is a network using both...
solar power and sleep mode. The difference is in the way we evolve the current network to its final configuration. The final scenario is when we plan the network with both technologies available from the start. We call this the joint scenario.

1) Problem P1: Base Network: In this scenario, there is no solar equipment or sleep mode available and the base stations are always on. We design the network by solving a restricted version of problem PL where the variable \( z_j \) and \( x_{j,t} \) are fixed to 0 so that the only optimization is on the assignment variables \( h_{i,j,t} \).

2) Problems P2 and P3: Single Technology: Problem P2 is the case when solar is not available so that we fix the \( z_j = 0 \) and optimize over the other variables \( x_{j,t} \) and \( h_{i,j,t} \). In case P3, when solar is available but not sleep mode, we fix the \( x_{j,t} = 0 \) and solve again for the remaining variables.

3) Problem P4: Sequential Scenario: Sleep Mode First: In this first sequential scenario, we design the network in two steps by solving a different special case of problem PL each time. At first, we don’t use solar energy so that we fix the \( z_j = 0 \) and optimize the network over the scheduling variables \( x_{j,t} \) and the test point assignment \( h_{i,j,t} \). Next, to model the introduction of solar equipment in the network, we solve a new special case of PL based on the results of the first step where sleep mode has already been planned. For this, the variables \( x_{j,t} \) and \( h_{i,j,t} \) are fixed at their current values and the optimal values of the \( z_j \) variables are recomputed by solving this restricted version of PL.

4) Problem P5: Sequential Scenario: Solar First: In this scenario, the sequence is inverted: solar power is introduced first so that we fix the \( x_{j,t} = 0 \) and optimize the network over the scheduling variables \( x_{j,t} \) and the test point assignment \( h_{i,j,t} \). Next, to model the introduction of solar equipment in the network via the placement of solar equipment via the introduced first so that we fix the \( x_{j,t} = 0 \) and solve again for the remaining variables.

5) Problem PL: Joint Optimization: Finally, we get to the scenario when both technologies are optimized at the same time by solving the full problem PL described in Section VI-D. This is obviously the best option and we can compare the benefit of optimizing the two options at the same time to the previous cases.

### B. Solution Algorithm

In this paper, problem PL and its variants are solved using Gurobi with the default options and the Ampl preprocessor. Once reduced by Ampl, the largest network we have solved has 42408 rows, 32959 columns and 125082 binary variables. We don’t need to develop a specialized algorithm since this allows us to study the interaction of solar energy and sleep mode in sufficiently large networks.

For large cases, we were unable to solve the joint PL problem from a cold start. We found that Gurobi would generate a large search tree trying to find a feasible solution and would run out of memory even on a machine with a large memory. Incidentally, the same thing happened with the Cplex solver. Nevertheless, it is possible to solve these cases to a reasonable accuracy if one starts the optimization with a known feasible solution. This could be the solution of any one of the P1 to P5 problems and in all the following, the solution of PL is always computed with one of these solutions as the starting point. The cpu time is the total time used by all of the processors. Some values for the three larger networks are shown in Table VI along with the optimality gap at the final solution. The first column shows which of the restricted problems is being solved. The Solar and Sleep headings indicate whether a technology, solar or sleep mode, is available (yes) or not (no). For the sequential scenarios of Sections VI-A3 and VI-A4, a starred entry means that this was the first technology introduced. Finally, the remaining columns show the total cpu time needed and the relative optimality gap. It is quite clear from these results that using Gurobi is a realistic option for off-line dimensioning of networks of up to 300 base stations. Larger networks will require heuristic techniques specifically tailored to the problem.

### C. Sequential Scenarios

We now examine in Table VI the effect of sequential scenarios where we optimize one technology after the other. As before, the results for each network are grouped in blocks of five lines. The first boxed line shows the number of test points and base stations and the next four, the total cost for each scenario. The cost saving relative to the cost of P1 is also shown in parenthesis for each scenario. The first of these lines is the cost of the

**TABLE V: CPU time (sec).** * indicates option available first.
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2018.2877040, IEEE Access

**TABLE VI: Optimal network cost (k$) for sequential algorithms: Effect of order**

<table>
<thead>
<tr>
<th>Sleep First</th>
<th>Solar First</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P1</strong></td>
<td><strong>P5</strong></td>
</tr>
<tr>
<td><strong>Base</strong></td>
<td><strong>Joint</strong></td>
</tr>
<tr>
<td><strong>Step 1</strong></td>
<td><strong>Step 2</strong></td>
</tr>
<tr>
<td><strong>P1</strong></td>
<td><strong>P5</strong></td>
</tr>
<tr>
<td><strong>Base</strong></td>
<td><strong>Joint</strong></td>
</tr>
<tr>
<td><strong>Step 1</strong></td>
<td><strong>Step 2</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sleep First</th>
<th>Solar First</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P1</strong></td>
<td><strong>P5</strong></td>
</tr>
<tr>
<td><strong>Base</strong></td>
<td><strong>Joint</strong></td>
</tr>
<tr>
<td><strong>Step 1</strong></td>
<td><strong>Step 2</strong></td>
</tr>
</tbody>
</table>

**TABLE VI: Optimal network cost (k$) for sequential algorithms: Effect of order**

base configuration obtained by solving P1 without sleep mode or solar power. The next two lines represent the two scenarios for sequential optimization P4 and P5. The column marked Sleep First is for the case where the sleep mode is optimized first. In that case, Step 1 is to optimize the sleep mode and Step 2, the solar installation. For column, labeled Solar First, solar installation is optimized first, which is Step 1, and then sleep mode, in Step 2. Finally, the last line of the block shows the total cost when both options are optimized together by solving PL.

An interesting point is that order does matter when doing sequential optimization. For all cases tested, planning solar equipment before the sleep schedule yields better results than doing it in the opposite order. Recall that in scenario VI-A3 the sleep mode is optimized first without solar equipment. The sleep schedule is then frozen and the solar equipment is planned. Clearly, this will not be very effective for two reasons. First, having the solar equipment available when planning the sleep schedule should afford more flexibility and thus a lower network cost. Also, the sleep schedule that has been optimized without solar will probably not be very efficient in a network where solar equipment is available. Based on this, it is intuitively clear that we should plan solar equipment before planning the sleep schedule. What is not so clear, however, is the size of the cost difference between the two approaches, which is provided by the results of Table VI. We see that doing the planning in the wrong order can decrease the savings by a few percent and that his effect increases with network size.

Another conclusion is that the results of problem P5, where solar equipment is optimized first, are very close to those of the joint optimization PL, which is the truly optimal solution. A simple and accurate heuristic could then be to optimize first the solar installation and then add the dynamic operation of base stations without having to do the joint scenario PL.

Finally, starting from row 1 to 4 of each block, we can see from the results that the savings offered by the two technologies are clearly additive irrespective of the order in which they are planned.

### D. Solar Energy

Here, we discuss the results regarding the use of solar energy in the different networks. The installed and used solar energy is examined for each of the optimization problems. In section VI-D1 it is done for the small network and section VI-D2 presents a summary of the results for the bigger networks.

1) **Detailed Solar Use for the Small Network:** To demonstrate how the network makes use of solar energy, we present in Table VII more results for the small network of Figure 1 with 12 test points and 4 base stations, a 24-slot time base and where all the solutions are optimal. For this small network, the results from P5 and PL are identical. It is not common to have an optimal solution when the solar installation is solved first but it might happen in small networks.

In that table, the last column labeled Ant energy is the total energy needed by all the antennas over the study period. It depends on the total energy required by the users and how many base stations are in sleep mode. The Inst solar column is the total amount of energy that the installed solar equipment can produce while the column Solar used is the amount of solar energy that was effectively used by the network. The difference between the Inst solar and Solar used columns is the amount of lost energy due to battery overflow (13).

In that table, problems P1 and P3 correspond to the two cases where there is no sleep mode. As expected, they have a larger energy requirement than the other four cases where sleep mode is used. Out of these four cases, P3 is the one that uses the largest amount of solar energy. Here, the antennas need the most energy and sleep mode is not available so that the solution is to install as much solar equipment as possible, leading to a large solar usage (col Solar used) with a correspondingly large capital cost (col Total solar cost).

Conversely, the smallest solar usage is that of P4, when sleep mode is planned first, without solar, and then the solar equipment is installed. Using sleep mode reduces considerably the amount of energy needed by the antennas (col Ant energy) but the amount of solar energy effectively used (col Solar used) also decreases significantly, with a corresponding decrease in the capital cost (col Total solar.
<table>
<thead>
<tr>
<th>Pb No</th>
<th>Solar Avail</th>
<th>Sleep Avail</th>
<th>Cost (k$)</th>
<th>Cost saving relative to P1 (%)</th>
<th>Total Solar Cost (k$)</th>
<th>Grid (k$)</th>
<th>Unit Solar Cost ($/kWh)</th>
<th>Solar used</th>
<th>Inst solar</th>
<th>Ant energy</th>
<th>Cost saving relative to P1 (%)</th>
<th>Total Solar Cost (k$)</th>
<th>Grid (k$)</th>
<th>Unit Solar Cost ($/kWh)</th>
<th>Solar used</th>
<th>Inst solar</th>
<th>Ant energy</th>
<th>Cost saving relative to P1 (%)</th>
<th>Total Solar Cost (k$)</th>
<th>Grid (k$)</th>
<th>Unit Solar Cost ($/kWh)</th>
<th>Solar used</th>
<th>Inst solar</th>
<th>Ant energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>no</td>
<td>no</td>
<td>14.49</td>
<td>-</td>
<td>14.49</td>
<td>NA</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>65.9</td>
<td>-</td>
<td>14.49</td>
<td>NA</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>65.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>no</td>
<td>yes</td>
<td>12.55</td>
<td>13.4</td>
<td>0</td>
<td>12.55</td>
<td>NA</td>
<td>0</td>
<td>0</td>
<td>57</td>
<td>-</td>
<td>12.55</td>
<td>NA</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>57</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td>yes</td>
<td>no</td>
<td>11.2</td>
<td>22.7</td>
<td>8.79</td>
<td>2.42</td>
<td>0.16</td>
<td>54.9</td>
<td>56.1</td>
<td>65.9</td>
<td>-</td>
<td>8.79</td>
<td>2.42</td>
<td>0.16</td>
<td>54.9</td>
<td>56.1</td>
<td>65.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P4</td>
<td>yes</td>
<td>yes*</td>
<td>10.02</td>
<td>30.9</td>
<td>6.59</td>
<td>3.43</td>
<td>0.159</td>
<td>41.5</td>
<td>42.1</td>
<td>57</td>
<td>-</td>
<td>6.59</td>
<td>3.43</td>
<td>0.159</td>
<td>41.5</td>
<td>42.1</td>
<td>57</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P5</td>
<td>yes*</td>
<td>yes</td>
<td>9.995</td>
<td>31</td>
<td>8.79</td>
<td>1.21</td>
<td>0.169</td>
<td>51.6</td>
<td>56.1</td>
<td>57</td>
<td>-</td>
<td>8.79</td>
<td>1.21</td>
<td>0.169</td>
<td>51.6</td>
<td>56.1</td>
<td>57</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PL</td>
<td>yes</td>
<td>yes</td>
<td>9.995</td>
<td>31</td>
<td>8.79</td>
<td>1.21</td>
<td>0.169</td>
<td>51.6</td>
<td>56.1</td>
<td>57</td>
<td>-</td>
<td>8.79</td>
<td>1.21</td>
<td>0.169</td>
<td>51.6</td>
<td>56.1</td>
<td>57</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE VII: Solar use, small network, energy in MWh

Fig. 4: base stations activation, sleep optimization first

cost). The downside is that because there is less solar equipment available, the grid cost increases (col Grid) so that the total cost (col Cost) is still relatively large.

The important point to note is that neither of these two solution is optimal and the solution that uses the most solar energy is definitely not the best one. The solutions of problems P5 and PL strike a balance between the reduction of grid energy on the one hand and the ensuing capital cost and yield a value for the amount of solar energy used that is between the ones of P3 and P4. This shows that any solar optimization technique must take into account the capital cost of the solar equipment before deciding to deploy this technology and that one cannot assume that it comes for free.

In column Unit Solar Cost, the solar price in dollars per kilowatt hour is simply the total cost of the solar equipment installed divided by the amount of solar energy that was used. This turns out to be 0.169$/kWh for the best solutions of P5 and PL, which is smaller than the 0.22$/kWh grid cost so that solar is an economically viable option.

Similar results hold for the other networks. A summary of the results for the larger networks is shown in section VI-D2.

2) Optimization and Solar Installation: We now present other results regarding the installation of solar equipment on the base stations. The goal here is to show that installing solar everywhere need not be the best solution even when solar energy is cheaper than grid energy. For this purpose, we use a model where solar is installed everywhere. This is called problem PFS, a variant of PL where the variable $z_j$ is set to 1.

Table VIII shows the number of base stations that have solar installed in the column Solar base stations where problem PFS has been solved to optimality. We see that solar is installed on most, but not all base stations even though solar energy is cheaper than the grid. This is still true for bigger networks where the solar energy rates are between 0.16$/kWh and 0.17$/kWh depending on the network.

Further information is provided in column Total Cost where the objective function value is shown. For the first
three networks, the total cost is smaller when some base stations do not use solar energy. For the two larger networks, the results are not conclusive because the solutions are not optimal, with gaps of 3% and 4%.

At last, column Solar Energy Losses presents the energy that is lost due to the limited capacity of the batteries. It is a ratio of the energy that is effectively used to power the base stations over the total installed solar energy. This result shows that having too much solar equipment in the network can lead to more losses.

E. Network Dynamics

We now examine in detail how the presence or absence of either solar or sleep mode can affect the operation of the network both from the point of view of the activation of the base stations and of the use of solar energy. For this, we consider the two scenarios of Sections VI-A3 and VI-A4 where the small network is optimized sequentially.

1) Sleep Optimization First: First we examine the case where we optimize sleep mode first and then solar. We can see in Figure 6 the activation of the base stations during the day.

Base stations No 2 and 3 are always on because they need to serve some test points that are only covered by them. Base station 0 is almost always in the sleep mode except during the two traffic peaks from 10:00 to 12:00 and 16:00 to 18:00, as shown in Figure 2. Base station 1 is almost always on except during the peak period at 16:00 where the traffic is taken up by base station 0.

This behavior is to be compared with the use of solar energy shown in Figure 7. We see that base station 0 never uses solar while 1, 2 and 3 use solar almost all of the time. This is a direct consequence of the optimization procedure. Because we optimize the sleep schedule first without solar power, the network has fewer opportunities to turn off the base stations and tries to compensate by using as much solar power as possible. This is consistent with the results of Table VII.

2) Solar Optimization First: Next, we consider the case where the network is optimized first for solar equipment and then for the sleep schedule. In the present case, this solution is also the optimal solution where both technologies are optimized together.

We can see in Figure 3 the base station activation schedule. This is strikingly different from Figure 6. Base stations 2 and 3 are always on because they need to serve some test points that are only covered by them. Base station 0 is almost always in the sleep mode except during the two traffic peaks from 10:00 to 12:00 and 16:00 to 18:00, as shown in Figure 2. Base station 1 is almost always on except during the peak period at 16:00 where the traffic is taken up by base station 0.

This behavior is to be compared with the use of solar energy shown in Figure 7. We see that base station 0 never uses solar while 1, 2 and 3 use solar almost all of the time. This is a direct consequence of the optimization procedure. Because we optimize the sleep schedule first without solar power, the network has fewer opportunities to turn off the base stations and tries to compensate by using as much solar power as possible. This is consistent with the results of Table VII.

1) Measurements: In addition to the visual representation of the assignments from Figure 8, we compute two measures to evaluate the assignation of the test points for each time slot: the dispersion and the standard deviation of the assignment vector. With this measurement, it is easier to see how balanced the network is throughout the day for the every optimization problem.

The dispersion is simply the difference between the maximum and minimum numbers of test points assigned to a single base station. Define the number of test points assigned to a given base station $j$ at time $t$ as the degree

$$
\Delta_{j,t} = \sum_{i \in I} h_{i,j,t} \quad \forall j \in S, \forall t \in T. \quad (27)
$$

At time $t$, the dispersion $D_t$ is

$$
D_t = \max_{j \in S} \Delta_{j,t} - \min_{j \in S} \Delta_{j,t} \quad \forall t \in T. \quad (28)
$$

Next, we compute the standard deviation $\sigma_t$

$$
\sigma_t = \frac{1}{|S|} \sqrt{\sum_{j \in S} (\Delta_{j,t} - N_{bs}^{tp})^2} \quad \forall t \in T, \quad (29)
$$

where $|S|$ is the total number of base stations and $N_{bs}^{tp}$ the number of average test points connected per base station.
This average value is equivalent to the ratio of test points over installed base stations, which is an input parameter of our model described in section IV-B.

2) Assignment results: Figure 8 shows the map of the network where the links represent the assignment of a test point to a base station for the joint problem (PL). Four time periods have been chosen to give an idea of the network dynamics: 10:00, 16:00, 17:00 and 18:00. Figure 8a shows the network when traffic is at its peak. Next, we can better see the interaction between base stations 0 and 1.
in figures 8b, 8c, and 8d, where a few test points are served alternatively by both base stations in order to switch them off more often.

We also present the dispersion in Figure 9, where the different profiles for each of the six problems are shown. Note that problems P2, P3 and P4 and problems P5 and PL have the same measure of dispersion. The same goes for the σ represented in Figure 10. Both of these figures are very similar and the same conclusions can be made with either one of these. In the peak period shown in 8c, all of the base stations need to be activated. This peak period is translated into a smaller dispersion and standard deviation as shown at times 10:00 and 11:00 in figures 9 and 10. This is also true, yet less striking, in the second peak starting at 15:00 and ending at 17:00. On the other hand, the network is less balanced with a higher dispersion and σ during night time because of the sleep mode of the base stations. In other words, with a lower traffic, some base stations are put to idle mode and will not have any test points connected which results in a greater dispersion and σ.

At last, we focus on the different test point assignments for the six problems. For the basic problem P1, since the network is not large, the assignment is therefore more randomized with a higher dispersion and σ obtained in off-peak periods. For the other five problems, the maximum value of the two measurements is lower. The minimum stays the same because, in high traffic, the network is well balanced for every problem. P2-P3-P4 and P5-PL only differ during the off-peak before 9:00 and after 20:00. In these hours, the optimal solution of P5-PL has a slightly greater dispersion and σ.

Altogether, we see that the networks with both sleep mode and solar energy are much more flexible and can change the base stations much more often than the other cases. This is of course the root of the better efficiency of these networks as expected.

VII. Conclusion

In this paper, we investigated the relationship between installing a solar harvesting system to power base station of a cellular network and the energy management under varying demand. For this, we presented a solar installation planning model that takes into account the hourly dynamics of the cellular network. We challenged the belief that solar energy can be considered free and that should always be installed everywhere in the network. This was done by explicitly modelling solar panels, batteries, inverters and charge controllers, as well as the cellular network demand and energy-management. We found that the solar installation and the energy-management of the base stations are so tightly interrelated that even the order in which the technologies are introduced can have an important impact on network cost and network performance. Finally, we show that installing solar equipment everywhere need not be the best solution even when the unit cost of solar power is smaller than that of the grid.


