Dual-Use Unimodular Sequence Design via Frequency Nulling Modulation

GUOLONG CUI*1, (Senior Member, IEEE), JING YANG1, (Student Member, IEEE), SHUPING LU2, XIANXIANG YU1, (Student Member, IEEE) and LINGJIANG KONG1, (Senior Member, IEEE)

1School of Information and Communication Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China (e-mail: cuiguolong@uestc.edu.cn; yangjinguestc@163.com; xianxiangy@gmail.com; ljkong@uestc.edu.cn).

2Hangzhou Applied Acoustics Research Institute (HAARI), Hangzhou, China (e-mail: lukeuestc@163.com).

Corresponding author: Guolong Cui (e-mail: cuiguolong@uestc.edu.cn).

This work was supported by the National Natural Science Foundation of China under Grants 61771109 and 61501083.

ABSTRACT This paper is focused on the design of the shared signal for a dual-use system with radar detection and communication functions. To achieve a good aperiodic autocorrelation property of the shared signal for radar detection, the \( l_p \)-norm of the autocorrelation sidelobes is minimized. For communication function, the nulling of one or more frequency bands enables information to be embedded and an energy minimization detection method is introduced to demodulate communication information. Additionally, electromagnetic spectral compatibility and unimodular constraints on the designed signal are considered. The resulting problem is a higher-order, multi-dimensional, and multi-objective constrained optimization, which is efficiently handled through an iteration direct search algorithm. Finally, the performance of the proposed method is assessed and analyzed through numerical simulations in terms of the peak sidelobe level (PSL), average sidelobe level (ASL), and bit error rate (BER).

INDEX TERMS Radar and communication, waveform design, spectrum modulation, peak sidelobe level (PSL), bit error rate (BER).

I. INTRODUCTION

A s a focus of future multi-function electronic system, the integration of radar and wireless communication has already attracted extensive attention [1]. Compared with simple functional superposition of multiple systems, the advantages of the dual-use integrative system are tremendously owing to the uses of the corporate transmit platform and shared signal. On the one hand, it reduces hardware equipments and costs, and provides the potentiality for further miniaturization [2]. On the other hand, the utilization of the shared signal between dual functions increases the exploitation efficiency of radio frequency spectrum and alleviates competition and mutual interference of radio signals. The latter advantage is of particular importance for increasingly crowded spectrum environment [3]–[8] and multi-function system [9].

In order to solve the spectrum coexistence problem, some literatures lower the cross-interference among multiple signals to realize the radar and communications (RadCom) functions [10]. In [11], a null space coexistence mechanism based projection is proposed for guaranteeing the coexistence between co-located multiple-input multiple-output (MIMO) radar and multi-antenna communication systems. In order to overcome the problem of frequency overlap for a wide-band radar system and a multi-carrier communications system, authors readjust their transmission power spectrum through the combined mutual information criterion [12]. In some literatures [13], [14], radar waveforms are optimized to fulfill spectral compatibility with other overlaid radiators, such as military communication system. However, communication performance is not considered. In these cases, the signals can be sent simultaneously, but they need two transmission systems. Another class of methods realizing multi-function is to transmit different signals by time-shared management. In [15], the NASA (National Aeronautics and Space Administration) devises a RadCom system for space shuttle orbiter which employs the same radio frequency (RF) hardware adopting time-shared mode. However, the time-shared pattern results in time blind zone for radar detection. Therefore, both the spectrum coexistence and time-shared cases are not real dual-use system with sharing common platform and signal.

Since the idea of the shared-signal is implemented for
a vehicle-vehicle communications and ranging system [16], there have been many literatures focusing on the sharing signal design for RadCom in one single system platform. These works can be mainly classified into two categories. The first category designs the waveform based on the common communication signal, where orthogonal frequency division multiplexing (OFDM) signal is frequently-used as it has the advantages of high spectrum efficiency, combating multipath, easy synchronization, and so on on [17]- [19]. Early, the capability of applying OFDM signal to netted radar sensing and communications is explored in [17]. In [20], the joint-performance bounds (such as probability of detection and channel capacity) for a dual-use system based on OFDM are discussed. In [21], authors optimize the transmit power allocation of OFDM waveform using information theory to improve the effectiveness of integrated RadCom waveform. The main disadvantage of OFDM signal is large peak-to-average power ratios (PAR). This case is very undesired in radar system owing to the requirement of saturation sending of signal and hardware constraints. However, these methods are difficult to juggle the correlation performance of waveform, which is also vital for radar detection.

The second category approaches the sharing signal design challenge by embedding communication information into radar transmit signal in the form of coding phase, sidelobe amplitude, and so on on [22]- [27]. In [22], the up-chirp and down-chirp signals are utilized for communications and radar sensing functions, respectively, where two signals are required to be orthogonal so that receiver can separate radar and communication echoes. In [23], frequency modulated continuous wave (FMCW) is utilized as the shared-signal in which communication data is embedded through amplitude shift keying mode. More recently, information embedding in radar signal by sidelobe level control is devised in [25]. In [28], authors develop a new sidelobe control technique for RadCom system, where a MIMO system transmitting orthogonal waveforms can better control pre-assigned sidelobe levels while meeting mainlobe performance. To overcome the drawback of only sidelobe communication, a design method of phase-modulation based dual-function signal is presented in [29], which allows information delivery to both sidelobe and main beam regions.

In this paper, we develop a novel dual-use radar and communication technique associated with frequency nulling modulation (FNMs) where a fast-time phase-only sequence set is designed as the sharing signal. The transmission of communication information is achieved by FNMs. In the transmitter, we suppose that there are L pre-set frequency bands for the whole radar frequency band, and then null one or multiple bands to embed information symbols. In the communication receiver, a demodulation strategy is introduced using minimum frequency band energy detection, which interprets the transmission information associated with the symbol of minimum band. That is to say, different communication frequency bands represent the corresponding symbols. Meanwhile, in order to guarantee radar detection performance of the shared-signal, we employ the peak sidelobe level (PSL) and average sidelobe level (ASL) indexes to restrain the sidelobe property of waveform. In the optimization process, the $l_p$-norm of aperiodic autocorrelation function associated with $s$ [30] is adopted instead of directly optimizing the two indexes. The resulting problem is tantamount to jointly suppressing the autocorrelation sidelobe and the spectral band energy of the designed polyphase coded waveform under constant modulus constraint. There are many literatures about waveform with desired correlation property under spectral constraint for ultra wide band (UWB) radar system [33]- [36]. On the basis of above analysis, we formulate the waveform design as a higher-order, multi-variable, and multi-objective constrained optimization. Utilizing the idea of iteration direct search algorithm in the previous work [31], [32], we sequentially optimize each code element of the signal while fixing remained code elements. During the solution of each code, we use the bisection approach to search feasible value progressively (refer to the Algorithm 2 in [37]). Numerical results are designed to assess the effectiveness of the proposed strategy.

The rest of the paper is organized as follows. In Section II, we introduce the sharing signal model and give the mathematical version of waveform design problem. In Section III, we employ the iteration direct search algorithm to solve the optimization problem. Section IV analyzes and assesses the performance of the dual-use signal design technique, on simulations, in terms of the PSL, ASL, and Bit Error Rate (BER). Finally, conclusion and discussion are given in Section V.

### A. NOTATION

In the sequel, vectors and matrices are denoted by boldface lower-case and upper-case letters, respectively. Symbols $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^\dagger$ denote the complex conjugate, transpose, and conjugate transpose, respectively. As to the numerical sets, $\mathbb{R}$ is the set of real numbers, $\mathbb{R}^{N \times M}$ is the Euclidean space of $(N \times M)$-dimensional real matrices (or vectors if $M = 1$), $\mathbb{C}$ is the set of complex numbers, and $\mathbb{C}^{N \times M}$ is the Euclidean space of $(N \times M)$-dimensional complex matrices (or vectors if $M = 1$). $I_N$ stands for the $N \times N$ identity matrix, while 0 is the null vector or matrix of proper dimensions. Given a matrix $A = [a_1, \ldots, a_M] \in \mathbb{C}^{N \times M}$, vec $(A) = [a_1^T, a_2^T, \ldots, a_M^T]^T \in \mathbb{C}^{NM \times 1}$, while given a vector $a \in \mathbb{C}^{N \times 1}$, $\text{diag} (a) \in \mathbb{C}^{N \times N}$ indicates the diagonal matrix whose $i$th diagonal element is the $i$th entry of $a$. We write $M \succ 0$ if $M$ is positive definite. The $(k, l)$-entry (or $l$-entry) of a generic matrix $A$ (or vector $a$) is denoted by $A(k, l)$ (or $a(l)$). The acronym s.t. stands for subject to. The symbol $\min f(\bullet)$ denotes the minimization of $f(\bullet)$, we we $s \geq 0$.

The operational sign "mean$(\cdot)$" is defined as the average of vector $a$. ceil$(\cdot)$ and floor$(\cdot)$ denote round toward positive infinity and negative infinity, respectively.
II. PROBLEM FORMULATION

In this section, we introduce the shared signal model for a dual-use RadCom system and formulate the comprehensive optimization problem that is to design the sequence set with the desired waveform correlation property and information transmission function.

In practical radar system, constant-modulus or low peak-to-average power ratios (PAR) is always desirable due to hardware constraints [39]. Herein, we assume that the transmit signal \( s = [s_1, s_2, \ldots, s_N]^T \in \mathbb{C}^{N \times 1} \) is a fast-time phase encoding sequence with unimodular constraint, and \( s_n, n = 1, \ldots, N \) is expressed as

\[
s_n = e^{j\phi_n}, \quad n = 1, \ldots, N,
\]

where \( \phi_n \in [0, 2\pi) \) is the phase of the \( n \)th subpulse waveform.

A. \( l_p \)-NORM OF AUTOCORRELATION SIDELOBES

The aperiodic autocorrelation function associated with \( s \) is defined as [42]

\[
r_k = \sum_{i=1}^{N-k} s_i^* s_{i+k}, \quad k = 0, \ldots, N - 1.
\]

We use the normalized PSL and ASL metrics to evaluate autocorrelation properties of designed waveform, which are defined as

\[
NPSL = 10\log_{10} \left\{ \max_{k=1}^{N-1} \frac{|r_k|^2}{r_0^2} \right\}
\]

and

\[
NASL = 10\log_{10} \left\{ \frac{1}{N-1} \sum_{k=1}^{N-1} |r_k|^2 \right\},
\]

respectively, where the ASL corresponds to the integrated sidelobe level (ISL) [38]. The reason why we use the ASL not ISL is to facilitate the following figures drawing.

The simultaneous optimization of both the PSL and ASL is hard. To this end, we resort to the minimization of \( l_p \)-norm of the autocorrelation sidelobes, which is given by

\[
f_p(s) = \left( \sum_{k=1}^{N-1} |r_k|^p \right)^{1/p}, \quad p \geq 2.
\]

As the value of \( p \) changes, (5) can trade-off between the PSL and ASL [37]. More specifically, the objective function corresponds to the ASL when \( p = 2 \), to the PSL when \( p \to \infty \).

B. FREQUENCY NULLING MODULATION

In general, the used bandwidth for radar detection is markedly wider than that of communication except for ultra-wideband system. Thereby it is a possible way to allocate partial frequency band resource of radar system to achieve communication function. To this end, we develop an information embedding methodology based on frequency nulling modulation (FNM).

Considering the spare communication frequency band (CFB) and necessary frequency isolation belts, we only use \( L \) sub-bands to perform potential message transmission. The basis communication principle is that each usable communication sub-band can represent a symbol, and only current communication sub-band as well as interference frequency band need to be handled by the FNM processing. The communication modulation on energy spectral density (ESD) of the shared signal \( s \) is shown in Fig. 1. As the figure implies, the \( l \)th CFB of the designed sequence should be minimized when the communication information transmitted is \( l \# \), \( l = 1, 2, \ldots, L \).

![FIGURE 1: The ESD diagram of FNM methodology.](image)

According to the spectrum band energy proposed in [14], the energy of the \( l \)th FNM sub-band is given by

\[
E_{L}^{l} = s^\dagger R_{f_1}^{l} (m_1, m_2)s, \quad l = 1, \ldots, L,
\]

with

\[
R_{f_1}^{l} (m_1, m_2) = \begin{cases} f_2^l - f_1^l & m_1 = m_2 \\ e^{2\pi f_2^l (m_1 - m_2)} - e^{2\pi f_1^l (m_1 - m_2)} & m_1 \neq m_2 \end{cases}
\]

where \( f_1^l \) and \( f_2^l \) denote the lower and upper normalized frequencies for the \( l \)th CFB, respectively.

To avoid the electromagnetic interference in a spectrally crowded environment, the designed waveform should have the capacity of spectral compatibility. Based on a cognitive paradigm [40], we assume that the interference frequency band \( \Delta_i = [f_3^i, f_4^i], \quad i = 1, \ldots, I \), where \( f_3^i \) and \( f_4^i \) denote the lower and upper normalized frequencies for the \( i \)th interference radiator, respectively. The total interference spectrum energy is given by

\[
E_I = \sum_{i=1}^{I} s^\dagger R_{f_3^i}^{l} (m_1, m_2)s, \quad i = 1, \ldots, I.
\]

C. OPTIMIZATION CRITERIA

As aforementioned, we are looking for the shared phase-only signal which simultaneously satisfies the requirement of autocorrelation property and communication, under spectrum interference and constant modulus constraint. Taking into
account for these criteria, we formulate mathematically the following constrained minimization problem,
\[
\min_{s} f_{p}(s), \quad s^{\dagger} R_{f_{i_{j}}}^{(i_{j})} (m_{1}, m_{2}) s, \quad \sum_{i=1}^{l} s^{\dagger} R_{f_{i_{j}}}^{(i_{j})} s
\]
\[
\text{s.t. } |s_{n}| = 1, \quad n = 1, \ldots, N.
\]

Based on the scalarization technique, we define the multi-objective optimization problem into a parameterized objective function \( f_{\lambda}(s) \):
\[
f_{\lambda}(s) = \lambda_{1} \left( \sum_{k=1}^{N-1} |r_{k}|^{p} \right)^{1/p} + \lambda_{2} s^{\dagger} R_{f_{i_{j}}}^{(i_{j})} s + \lambda_{3} \sum_{i=1}^{l} s^{\dagger} R_{f_{i_{j}}}^{(i_{j})} s
\]
\[
= \lambda_{1} f_{p}(s) + s^{\dagger} R s,
\]
where \( \lambda_{i} \in [0, 1], \quad i = 1, 2, 3, \lambda_{1} + \lambda_{2} + \lambda_{3} = 1, \) and
\[
R = \lambda_{2} R_{f_{i_{j}}}^{(i_{j})} + \lambda_{3} \sum_{i=1}^{l} R_{f_{i_{j}}}^{(i_{j})}.
\]

Therefore the constrained optimization problem can be written as
\[
\min_{s} f_{\lambda}(s)
\]
\[
\text{s.t. } |s_{n}| = 1, \quad n = 1, \ldots, N.
\]

III. OPTIMIZATION ALGORITHM

In order to better tackle the problem \( P_{S} \), we need to simplify its higher-order term \( f_{p}(s) \), which is equivalent to the minimization of \( \sum_{k=1}^{N-1} |r_{k}|^{p} \). Clearly, the higher-order objective function is too difficult to solve directly. The majorization-minimization (MM) method can reduce the power law through a series of iteration procedure, which can descend the power of the objective function of (5). According to the simplification for \( \sum_{k=1}^{N-1} |r_{k}|^{p} \) in [38], we give the following proposition:

**Proposition III.1.** Let \( \{s^{(m+1)}\}_{m=0}^{\infty} \) be a sequence of constant modular codes. Problem \( P_{S} \) can be simplified by using the following iterative approach:
\[
P_{S}^{(m+1)} \left\{ \min_{S^{(m+1)}} f_{S}^{(m+1)}(s^{(m+1)}) \right\}
\]
\[
\text{s.t. } |s_{n}^{(m+1)}| = 1, \quad n = 1, \ldots, N
\]
\[
\text{with}
\]
\[
f_{S}^{(m+1)}(s^{(m+1)}) = \lambda_{1} \sum_{k=1}^{N-1} a_{k} |r^{(m+1)}_{k}|^{2} + \lambda_{1} \sum_{k=1}^{N-1} b_{k}
\]
\[
\text{Re} \left( (r^{(m+1)}_{k} \ast F^{(m)}_{k}) + P^{(m)} + (s^{(m+1)})^{\dagger} R s^{(m+1)} \right)
\]
\[
\text{where}
\]
\[
P^{(m)} = \lambda_{1} \sum_{k=1}^{N-1} \left( a_{k} |r^{(m)}_{k}|^{2} - (p-1) |r^{(m)}_{k}|^{p} \right),
\]
\[
\lambda_{1} = c^{p} - |r^{(m)}_{k}|^{p} - p |r^{(m)}_{k}|^{p-1} (c - |r^{(m)}_{k}|),
\]
\[
b_{k} = p |r^{(m)}_{k}|^{p-1} - 2 \lambda_{1} |r^{(m)}_{k}|,
\]
\[
c = \left( \sum_{k=1}^{N-1} |r^{(m)}_{k}|^{p} \right)^{1/p}.
\]

Proof: Reference Lemma 10 in [38].

In **Proposition III.1**, the objective function (5) is further transformed as the quadratic expression of \( r_{k} \), namely the forth power of transmit signal \( s \). Note that \( r_{k}^{(m)} \) in (14) is constant relative to the current \( r_{k} \) and \( s^{(m+1)} \).

However, Problem \( P_{S}^{(m+1)} \) is still constrained and non-convex. Next, we employ the sequence iteration search algorithm to solve it. The principle of the iteration algorithm in [31] is to sequentially optimize a multivariable objective over one variable keeping fixed the others. To this end, we provide the following proposition:

**Proposition III.2.** At the \( m+1 \) iteration, when the \( n \)th phase code is selected as the specific optimization variable, the minimization problem is given by
\[
P_{S}^{(m+1)} \left\{ \min_{s_{n}^{(m+1)}} f_{S}^{(m+1)}(s_{n}^{(m+1)}), \quad s_{n}^{(m+1)} \right\}
\]
\[
\text{s.t. } |s_{n}^{(m+1)}| = 1, \quad n = 1, \ldots, N
\]
\[
\text{where}
\]
\[
s_{n}^{(m+1)} = \left[ s_{1}^{(m+1)}, \ldots, s_{t}^{(m+1)}, s_{t+1}^{(m)}, \ldots, s_{N}^{(m)} \right]^{T}
\]
\[
\text{and } f_{S}^{(m+1)}(s_{n}^{(m+1)}), s_{n}^{(m+1)} \text{ is dependent on the optimization variable } s_{n}^{(m+1)} \text{ as specified in Appendix A.}
\]

Proof: See Appendix A.

Although the objective function of \( P_{S}^{(m+1)} \) only includes one-dimensional complex variable, it still is non-convex optimization problem due to the unimodulus constraint. Next, we provide Proposition III.3 to make the constrained complex variable into an unconstrained real variable utilizing Euler’s complex variable function formula and trigonometric relationships.
Proposition III.3. Let \( \beta_t = \tan\left(\frac{\Delta t}{2}\right) \), \( \phi_t = \arg\left(s^{(m+1)}_t\right) \). Problem \( P_{s^{(m+1)}, s_t^{(m+1)}} \) is equivalent to the following optimization problem:

\[
P_{\beta_t, s_t^{(m+1)}} \left\{ \min_{\beta_t} f_{\beta_t}(\beta_t; s_t^{(m+1)}) \right\} \quad \text{s.t.} \quad \beta_t \in \mathbb{R} \tag{22}
\]

where

\[
f_{\beta_t}(\beta_t; s_t^{(m+1)}) = \frac{q_t1^2\beta_t^4 + q_t3\beta_t^3 + q_t2\beta_t^2 + q_t1\beta_t + q_10}{(1 + \beta_t^2)^2} \tag{23}
\]

and the coefficients \( q_t1, t = 0, \ldots, 4 \) can be computed by (43).

Proof: See Appendix B.

Assume that \( v^* \) is the optimal value of \( f_{\beta_t}(\beta_t; s_t^{(m-1)}) \) and \( \gamma \) is a real positive variable. If \( \exists \beta_t, f_{\beta_t}(\beta_t; s_t^{(m-1)}) \leq \gamma \), then there exists a point in the feasible set achieving \( v^* \leq \gamma \). Conversely, if \( \forall \beta_t, f_{\beta_t}(\beta_t; s_t^{(m-1)}) > \gamma \), then \( v^* > \gamma \). Based on the iteration of bisection approach we continuously lower the value interval of \( f_{\beta_t}(\beta_t; s_t^{(m-1)}) \) to find the approximate solution of \( v^* \). Define the set \( \Omega \) as

\[
\Omega = \left\{ \beta_t; q_t1\beta_t^4 + q_t3\beta_t^3 + q_t2\beta_t^2 + q_t1\beta_t + q_10 - \gamma (1 + \beta_t^2)^2 > 0 \right\}. \tag{24}
\]

The appendix B in [37] proposes a method to compute the solution set \( \Omega \) of fourth-order polynomial. Obviously, if \( \Omega \neq \mathbb{R} \), the function \( f_{\beta_t}(\beta_t; s_t^{(m-1)}) \leq \gamma \) is feasible.

In the following, a complete algorithm procedure to optimize transmit signal \( s^* \) is summarized in Algorithm 1.

Algorithm 1 : Iteration Direct Search Algorithm for solving \( P_3 \)

Require: \( N, \lambda_i \in [0, 1], i = 1, 2, 3 \), the current CFB \( [f_1^i, f_2^i, f_3^i] \), interference frequency bands \( \Delta_i = [f_3^i, f_4^i], i = 1, 2, \ldots, I, \delta_1, \delta_2 \), where \( \delta_1 (\delta_2) \) is a small positive real number to control inside (outside) iterative convergence;

Ensure: An optimal solution \( s^* \) to Problem \( P_3 \):

1: Compute the spectrum matrix \( R \) using equations (7) and (11):
2: Set \( m = 0; \)
3: Randomly initialize the phase sequence \( s^{(0)} = \left[ s_1^{(0)}, s_2^{(0)}, \ldots, s_N^{(0)} \right] \) by (1);
4: Compute the initial objective value \( f_\delta(s^{(0)}) \);
5: Set \( m = m + 1, t = 1, \) and \( \overline{s}^{(m)} = s^{(m-1)} \);
6: Compute the coefficients \( q_t1, t = 0, \ldots, 4, \) by (43);
7: Set \( l := 0, u(l) = 0, u(l') = f_{\beta_t}(\beta_l; s_t^{(m-1)}) \);
8: \( l := l + 1, \) then \( \gamma(l) = u(l + 1 - u(l')); \)
9: Compute \( N - 1 \) feasible set \( \Omega \);
10: If \( \Omega(l) \neq \mathbb{R} \), \( u(l) = \gamma(l) \) and pick up a feasible solution \( \beta_l; \)

Else, \( u(l) = \gamma(l) \);
11: If \( |u(l) - u(l')| < \delta_1 \)

Set \( s_l^{(m)} = e^{2\arctan(\beta_l)} \); Otherwise, return to Step 7;
12: \( t := t + 1; \) If \( t = N + 1, \) go to next step; Otherwise, return to Step 6;
13: \( s^{(m)} = \overline{s}^{(m)} \), and compute \( f_\delta(s^{(m)}) \);
14: If \( |f_\delta(s^{(m)}) - f_\delta(s^{(m-1)})| / f_\delta(s^{(m-1)}) < \delta_2, \) output \( s^* = s^{(m)} \); Otherwise, return to Step 5.

where

- \( \alpha \) is a constant complex parameter accounting for channel propagation and backscattering effects from the detection target;
- \( v \) is an additive white Gaussian noise with zero mean and covariance \( \sigma_v^2 I_N \), which accounts for the sampling disturbance of radar receiver.

Before the CFAR detection in range profile, the correlation matched filtering of the received radar data using the transmitted signal \( s \) is usually performed to reduce the impact of random noise. More importantly, the good autocorrelation sidelobe property of \( s \), namely low ASL and PSL, can weaken the impact of other false or strong targets.

For a cooperative communication receiver, the received signal down to baseband can be given as

\[
c = \beta s + n \in \mathbb{C}^{N \times 1} \tag{26}
\]

where

- \( \beta \) is the channel coefficient taking into account the propagation environment between the dual-use system and communication receiver;
- \( n \) accounts for internal thermal noise of communication receiver and is modeled as a \( N \times 1 \) vector of additive...
white Gaussian noise with zero mean and covariance \( \sigma^2_I N \).

In the communication receiver, we design a demodulation approach using energy detection of \( L \) CFBs. The amount of energy received on the \( l \)-th CFB can be computed as:

\[
J^l = e^\dagger R_f^l e = |\beta|^2 R_f^l s + \beta* R_f^l n = |\beta|^2 E_l + 2 \text{Re} (\beta* R_f^l n) + n^\dagger R_f^l n.
\]

Further, as shown in Fig. 3, suppose that \( Q \) CFBs are suppressed at the same time, we sort the energy of all CFBs and choose the minimum \( Q \) bands to demodulate the corresponding information.

![FIGURE 2: Operating procedure of the RadCom system and the cooperative communication system.](image)

![FIGURE 3: The demodulation diagram of FNM methodology.](image)

**B. SIMULATION RESULTS**

Next, we evaluate the proposed dual-use shared signal system in terms of radar detection and communication performance. The considered influence factors include the values of \( p \), communication band width, communication band location, compatible spectral interference, and multi-band communication. A specific random phase-coded unimodal sequence with \( N = 128 \) as the initial sequence and two stopping crite-

\[
\delta_1 = \frac{|w_2 - w_1|}{w_2} < 10^{-5} \quad \text{and} \quad \delta_2 = \frac{|f\lambda(s^{(m)} - f\lambda(s^{(m-1)})|}{f\lambda(s^{(m-1)})} < 10^{-5}, \quad \text{are adopted for all the following simulations.}
\]

1) **Simulation 1**: Selection of \( p \)

Since the parameter \( p \) significantly impacts the correlation properties of the shared waveform, it’s of great importance to select an appropriate value of \( p \) in applying the technique. Without any spectral requirement, the NPSL and NASL are assessed with \( p \) increased from \( 2, 100 \). The NPSL and NASL of the initial sequence are \(-15.87 \text{dB} \) and \(-27.38 \text{dB} \), respectively.

The performance curves of the NPSL and NASL under different \( p \) values are shown in Fig. 4(a). The curves show that a larger \( p \) leads to a decreased NPSL as well as an increased NASL. More specifically, the iteration trends of correlation functions when \( p = 2 \) and 10 are depicted in Fig. 4(b). The convergent tendencies of the NPSL and NASL for different \( p \) are very similar along with the increasing of iteration times. The curve of \( p = 10 \) achieves a lower NPSL as well as a higher NASL compared with \( p = 2 \), which is consistent with Fig. 4(a). Besides, the NPSL performance when \( p = 2 \) deteriorates in the tail of the iteration curve because the objective function only corresponds to the NASL.

2) **Simulation 2**: Selection of \( \lambda \)

To illustrate the impact of \( \lambda \) on the performance of the dual-use RadCom functions, we increase \( \lambda_1 \) from 0 to 1 and set \( \lambda_3 = 0 \) with \( p = 10 \). Assume that \( f^1 = 0.6 \) and \( f^2 = f^1 + \Delta f \) where \( \Delta f = 0.02 \) is the width of CFB.

To better represent the performance of CFB suppression, the mean CFB level \( (M_{com}) \) is defined as

\[
M_{com} = \sum_{k=1}^{K_2} 10 \log_{10} \left( \frac{|\text{FFT}(s, K)|^2}{N} \right), \quad k = 1, \cdots, K,
\]

where \( \text{FFT}(s, K) \) denotes \( K \) Fast Fourier Transform (FFT) points of transmit signal \( s \), and \( k_1 = \text{ceil}(f^1 * K) \) and \( k_2 = \text{floor}(f^2 * K) \) are respectively defined as the lower and
upper frontiers. Set $K = 2^{10}$ to ensure enough FFT points for narrow CFB width.

Fig. 5 illustrates how $\lambda$ affects the NPSL and $M_{\text{com}}$. Generally, a larger $\lambda$ results in a better NPSL performance and a worse CFB suppression thus resulting in a decreased NPSL at the cost of an increased $M_{\text{com}}$ in the figure, which reflects the compromise function of $\lambda$ between the NPSL and CFB suppression.

3) Simulation 3: Effect of Different CFB
We design a scene to test the impacts of different CFB suppression, i.e., delivering different communication message, on the autocorrelation property and communication function. Set $\lambda_3 = 0$, $\lambda_1 = 10^{-12}$. The lower frontiers of 8 usable CFBs are set to $\{0.1, 0.2, 0.5, 0.6, 0.68, 0.76, 0.82, 0.9\}$ with $\Delta f = 0.02$. If the communication message is 6#, the CFB suppressed is $[0.76, 0.78]$.

Figs. 6(a)-(b) show the autocorrelation function levels and ESDs of the optimized sequences with different CFBs 1#, 4#, 8# without spectrum interference. We observe that all the autocorrelation curves are approximate and the ESDs show the notch, respectively, corresponding to the frequency band $[0.1, 0.12], [0.6, 0.62], [0.9, 0.92]$ as expected.

Table 1 gives the concrete NPSL and $M_{\text{com}}$ results with different CFB. It can be seen that the transmission of different communication messages almost has no impact on correlation property of sharing signal, which is expected when the dual-use system works.

To analyze the performance of the demodulation strategy based on energy detection in Fig. 3, we devise $10^7$ Monte-Carlo experiments under each CFB and given SNR value.
### TABLE 1: The NPSL and $M_{com}$ for different CFBs 1#, 4#, and 8#.

<table>
<thead>
<tr>
<th>CFB</th>
<th>NPSL (dB)</th>
<th>$M_{com}$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1#</td>
<td>-28.82</td>
<td>-52.66</td>
</tr>
<tr>
<td>4#</td>
<td>-28.92</td>
<td>-50.02</td>
</tr>
<tr>
<td>8#</td>
<td>-28.92</td>
<td>-59.76</td>
</tr>
</tbody>
</table>

The other settings are same as Fig. 6. In Fig. 7, the BER curves for three kinds of optimized sequences without interference are presented, which clearly shows that the transmission of different symbols has little impact on the BER when other conditions are determined.

![Figure 7: The BER versus SNR of different CFBs without spectral interference.](image)

### TABLE 2: The NPSL and $M_{com}$ for different CFB widths.

<table>
<thead>
<tr>
<th>Bandwidth</th>
<th>0.005</th>
<th>0.01</th>
<th>0.02</th>
<th>0.05</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPSL (dB)</td>
<td>-30.03</td>
<td>-29.64</td>
<td>-28.92</td>
<td>-26.37</td>
<td>-22.49</td>
</tr>
<tr>
<td>NASL (dB)</td>
<td>-33.48</td>
<td>-32.49</td>
<td>-32.38</td>
<td>-30.51</td>
<td>-29.19</td>
</tr>
<tr>
<td>$M_{com}$ (dB)</td>
<td>-59.77</td>
<td>-57.88</td>
<td>-48.66</td>
<td>-43.47</td>
<td>-32.68</td>
</tr>
</tbody>
</table>

Specifically, the evolution curves of the BER versus SNR with three kinds of CFB widths 0.01, 0.02, and 0.05 are plotted in Fig. 8. Interestingly, a lower BER under the same SNR is achieved for a larger CFB width as a wider usable CFB means more energy accumulation and leads to a bigger difference between the current CFB and other usable CFBS. However, a larger CFB width means more occupancy of limited bandwidth.

![Figure 8: The BER versus SNR with different CFB width = 0.01, 0.02, and 0.05.](image)

### 5) Simulation 5: Effect of Spectrum Interference

In this subsection, we utilize the optimized sequences to test the impacts of delivering different communication message in the presence of interference spectrum. The assumed normalized compatible frequency band is located at $[0.3, 0.4]$, and $\lambda_1 = 10^{-12}$, $\lambda_2 = \lambda_3 = 0.5 \times (1 - 10^{-12})$. We suppose that there are 8 usable CFBS same as Fig. 6. Fig. 9 shows the autocorrelation function levels and ESDs of the optimized sequences with different CFBs 1#, 4#, 8# considering spectrum interference. We observe that all the correlation curves the CFB suppression levels for three kinds of communication information are also approximate. Table 3 gives the concrete NPSL and $M_{com}$ results. It again confirms that the transmission of different communication messages almost has no impact on correlation property of sharing signal with spectrum interference. However, the performance of autocorrelation sidelobe and communication band suppression is worse compared with the scene in Fig. 6 for compatibility with spectrum interference.

### TABLE 3: The NPSL and $M_{com}$ for different CFBs 1#, 4#, and 8# with spectrum interference.

<table>
<thead>
<tr>
<th>CFB</th>
<th>With Spectrum Interference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NPSL (dB)</td>
</tr>
<tr>
<td>1#</td>
<td>-22.46</td>
</tr>
<tr>
<td>4#</td>
<td>-22.46</td>
</tr>
<tr>
<td>8#</td>
<td>-22.42</td>
</tr>
</tbody>
</table>

### 6) Simulation 6: Dual-Use Performance with Multi-CFB

In this subsection, we investigate the performance of multi-CFB communication. This scenario enables delivering multiple communication messages simultaneously. We still suppose that there are $L = 8$ usable communication bands like Fig. 6. And 2 CFBS of $\{4#, 5#\}$, 3 CFBS of $\{1#, 4#, 5#\}$, and 4 CFBS of $\{1#, 4#, 5#, 8#\}$ are suppressed, respectively. The $M_{com}$ for multiple CFBS are computed by the
FIGURE 9: Correlation property curves and ESDs for different CFBs 1#, 4#, 8#: (a) correlation function with interference frequency band [0.3, 0.4]; (b) ESDs with interference frequency band [0.3, 0.4].

average of all the $M_{com}$. The NPSL and $M_{com}$ curves for different numbers of CFB are shown in Fig. 10. In order to better show the change trend, the results with zero and one CFB (4#) are also plotted. We can observe that the NPSL and $M_{com}$ increase as the number of CFB increases. Specially, the change spans of NPSL and $M_{com}$ from one CFB to four CFBs reach 4 dB and 12 dB, respectively.

FIGURE 10: The NPSL and $M_{com}$ for different numbers of CFB.

In Fig. 11, the BER curves versus SNR for multiple CFBs are shown. The setup of multi-CFB is the same as that of Fig. 10. When SNR is small, it is of a higher BER for a smaller number of CFBs suppressed at the same time.

FIGURE 11: The BER versus SNR for multiple CFBs.

V. CONCLUSION AND DISCUSSION

In this paper, we have introduced and accessed a new design methodology of sharing signal for dual-use radar and communication functions, which achieves good correlation properties (NPSL and NASL) for radar detection and employs the partial frequency nulling modulation to transmit communication messages. Besides, the unimodular and possible spectrum interference were taken into account. In the communication receiver, we devised a demodulation strategy based on energy minimum detection of usable communication frequency bands. An optimization method based on the iteration direct search algorithm was presented to solve the higher-order, multi-objective, and non-convex problem.

At the analysis stage, we have employed random sequence to evaluate the performance of the proposed dual-use RadCom technique. The analysis results reveal that the parameters $p$ and $\lambda_1$ allow to trade-off the NPSL, NASL, and communication band suppression. When selecting wider communication width, we have gotten better BER performance. However, this case results in worse correlation performance of sharing signal. It is worth noting that the transmission of different communication messages, namely suppressing different communication bands, has almost no impact on the BER when other conditions are determined. Moreover, we have accessed the performance of multiple communication bands which can deliver multiple symbols for one pulse. Although multi-band case means the greater communication efficiency, the NPSL and $M_{com}$ also increase along with the increasing number of communication bands.

A possible future work might concern the shaping of local ambiguity function [43] and the beampattern design of the co-located MIMO radar [44] to realize the RadCom functions.
APPENDIX A PROOF OF PROPOSITION III.2

To find \( f_{s_{t}^{(m+1)} (s_{-t}^{(m+1)})} \) over the optimization variable \( s_{t}^{(m+1)} \), let

\[
r_k \left( s_{t}^{(m+1)} \right) = \sum_{i=1}^{N-k} \left( s_{t}^{(m+1)} \right) \ast s_{t+k} + \sum_{i=1}^{N-k} \left( s_{t}^{(m+1)} \right) \ast 1_{A} (t + k) + \sum_{i=1}^{N-k} \left( s_{t}^{(m+1)} \right) \ast \left( s_{t}^{(m+1)} \right) \ast 1_{A} (t - k)
\]

\[
= at_{k} s_{t}^{(m+1)} + b_{t} \left( s_{t}^{(m+1)} \right) \ast c_{t} k = 1, \ldots, N - 1,
\]

and

\[
\left( s_{t}^{(m+1)} \right) \ast R_{s_{t}^{(m+1)}} = \sum_{n=1, n \neq t}^{N} \left( s_{t}^{(m+1)} \right) \ast R_{n, s_{t}^{(m+1)}}
\]

\[
+ \sum_{n=1, n \neq t}^{N} \left( s_{t}^{(m+1)} \right) \ast R_{n, s_{t}^{(m+1)}}
\]

\[
= \left( \sum_{n=1, n \neq t}^{N} s_{t}^{(m+1)} \right) \ast R_{n, s_{t}^{(m+1)}}\ast
\]

\[
+ \sum_{n=1, n \neq t}^{N} \left( s_{t}^{(m+1)} \right) \ast R_{n, s_{t}^{(m+1)}}
\]

\[
= 2Re \left( \epsilon_{t} s_{t}^{(m+1)} \right) + f_{t}
\]

where \( s_{t,0}^{(m+1)} = \left[ s_{t}^{(m+1)}, \ldots, s_{t-1}^{(m+1)}, 0, s_{t+1}^{(m+1)}, \ldots, s_{t}^{(m+1)} \right]^{T} \).

\[
1A (x) = \begin{cases} 
1 \text{ If } x \in A, \\
0 \text{ Otherwise} 
\end{cases} \quad A = \{1, 2, \ldots, N\},
\]

So the objective function of \( P_{S_{t}^{(m+1)}} \) can be rewritten as

\[
\left| r_{k} \left( s_{t}^{(m+1)} \right) \right|^{2} = \left| a_{tk} e^{j \phi_{tk}} + b_{tk} e^{-j \phi_{tk}} + c_{tk} \right|^{2}
\]

\[
= u_{tk} \beta_{t}^{2} + v_{tk} \beta_{t}^{2} + v_{tk} \beta_{t}^{2} + v_{tk} \beta_{t}^{2} + u_{tk} \beta_{t} + u_{tk} \beta_{t}
\]

(33)

APPENDIX B PROOF OF PROPOSITION III.3

With reference to (1), \( s_{t} = e^{j \phi_{t}}, \phi_{t} \in [0, 2\pi) \), let \( \beta_{t} = \tan \left( \frac{\beta_{t}}{2} \right) \). Then we can obtain

\[
\left| r_{k} \left( s_{t}^{(m+1)} \right) \right|^{2} = \left| a_{tk} e^{j \phi_{tk}} + b_{tk} e^{-j \phi_{tk}} + c_{tk} \right|^{2}
\]

\[
= u_{tk} \beta_{t}^{2} + u_{tk} \beta_{t}^{2} + u_{tk} \beta_{t}^{2} + u_{tk} \beta_{t} + u_{tk} \beta_{t}
\]

(34)

and

\[
Re \left\{ r_{k} \left( s_{t}^{(m+1)} \right) \ast R_{k}^{(m)} \right\}
\]

\[
= v_{tk} \beta_{t}^{2} + v_{tk} \beta_{t}^{2} + v_{tk} \beta_{t}^{2} + v_{tk} \beta_{t}^{2} + v_{tk} \beta_{t}^{2}
\]

(35)

where

\[
u_{tk} = \frac{a_{tk} + b_{tk} + c_{tk}^{2}}{1 + \beta_{t}^{2}}
\]

(36)

with

\[
a_{tk} = Re (a_{tk}), a_{tk} = Im (a_{tk})
\]

(37)

\[
b_{tk} = Re (b_{tk}), b_{tk} = Im (b_{tk})
\]

(38)

\[
c_{tk} = Re (c_{tk}), c_{tk} = Im (c_{tk})
\]

(39)

\[
\begin{align*}
u_{tk} & = \bar{a}_{tk} + \bar{b}_{tk} + \bar{c}_{tk} \\
u_{tk} & = 2(\bar{a}_{tk} - \bar{b}_{tk}) \\
u_{tk} & = 2(\bar{a}_{tk} - \bar{b}_{tk}) \\
u_{tk} & = 2(\bar{a}_{tk} - \bar{b}_{tk})
\end{align*}
\]

(40)

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2018.2876644, IEEE Access
Similarly, (30) can be transformed as

\[
2\text{Re} \left( e_{t_i}^{(m+1)} \right) + f_t
= 2\text{Re} \left( (e_{t_i} + je_{t_i}) \left( \cos(\phi_{t_i}) + jsin(\phi_{t_i}) \right) \right) + f_t
= 2e_{t_i} \left( 1 - \tan^2 \left( \frac{\phi_{t_i}}{2} \right) \right) - 2\text{Re} \left( \frac{2\tan \left( \frac{\phi_{t_i}}{2} \right)}{1 + \tan^2 \left( \frac{\phi_{t_i}}{2} \right)} \right) + f_t
= w_{12} \beta_t^2 + w_{11} \beta_t + w_{10}
\]

where \( e_{t_i} = \text{Re}(e_{t_i}), e_{t_i} = \text{Im}(e_{t_i}) \), and

\[
\begin{align*}
t_{l0} &= f_t + 2e_{t_r} \\
t_{l1} &= -4e_{t_i} \\
t_{l2} &= f_t - 2e_{t_r}
\end{align*}
\]

Let \( a = \lambda_1 [a_1, \ldots, a_k, \ldots, a_N]^T, b = \lambda_1 [b_1, \ldots, b_k, \ldots, b_N]^T \), \( u_{t_i} = [u_{t_i1}, \ldots, u_{t_ik}, \ldots, u_{t_N}]^T; i = 0, \ldots, 4, v_{t_i} = [v_{t_i1}, \ldots, v_{t_ik}, \ldots, v_{t_N}]^T; i = 0, \ldots, 4 \). Based on the transformation of \( |r[k]|^2, \text{Re} \{r[k] F^{(m)}(k) \} \), and \( s^t R_s \), the objective function \( f(x; \beta_t; s^{t(m+1)}) \) can be recast as

\[
f(x; \beta_t; s^{t(m+1)}) = q_4 \beta_t^4 + q_3 \beta_t^3 + q_2 \beta_t^2 + q_1 \beta_t + q_0
\]

where

\[
\begin{align*}
q_0 &= a_0 u_{t_0}^* + b_0 v_{t_0}^* + w_{t_0}^* + P(m) \\
q_1 &= a_1 u_{t_1}^* + b_1 v_{t_1}^* + w_{t_1}^* + P(m) \\
q_2 &= a_2 u_{t_2}^* + b_2 v_{t_2}^* + w_{t_2}^* + w_{t_0}^* + 2P(m) \\
q_3 &= a_3 u_{t_3}^* + b_3 v_{t_3}^* + w_{t_3}^* + w_{t_1}^* \\
q_4 &= a_4 u_{t_4}^* + b_4 v_{t_4}^* + w_{t_4}^* + w_{t_2}^* + w_{t_0}^* + 2P(m)
\end{align*}
\]

ACKNOWLEDGMENT
This work was supported by the National Natural Science Foundation of China under Grants 61771109 and 61501083.

REFERENCES


