Adaptive Function Expansion 3-D Diagonal-Structure Bilinear Filter for Active Noise Control of Saturation Nonlinearity

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ABSTRACT In this paper, a general function expansion bilinear (FEB) filter with a 3-D diagonal-structure is proposed to deal with the problem of saturation nonlinearity in active noise control (ANC) systems. In an ANC system, the reference microphone and/or loudspeaker may be saturated when the acoustic noise and/or the controller output exceeds the dynamic limits of electronic devices. Such saturation nonlinearity degrades the control performance of the linear and some nonlinear control filters equipped with the filtered-x least mean square (FXLMS) algorithm. In order to tackle the problem of signal saturation, we use a new nonlinear saturation active noise control (NSANC) model and derive a function expansion diagonal-structure bilinear filter (FDBFXLMS) algorithm. The performance of the proposed filters equipped with the associated FEDBFXLMS algorithm is validated through analysis of computational complexity and simulations of various nonlinearities for NSANC systems. Computer simulation results demonstrate that the proposed FEB filter can achieve significant performance improvement in reducing saturation effects in comparison with the diagonal-channel bilinear filter and the recursive second-order Volterra (RSOV) filter based on the FXLMS algorithm, which often outperform the conventional nonlinear Volterra and functional link artificial neural network (FLANN) filters.

INDEX TERMS Active noise control (ANC), bilinear filter, function expansion bilinear filter, 3-D diagonal structure, signal saturation

I. INTRODUCTION

With advancement in control theory and development of low-cost high speed digital signal processors (DSP), the research in the area of active noise control (ANC) has gained a significant attention [1]- [28]. Among feedforward duct ANC systems, the finite impulse response (FIR) filter equipped with an FXLMS algorithm is widely used due to its simple implementation and low computational cost. However, in a real ANC system, the reference noise, primary path and secondary path may exhibit nonlinear distortions [5], [6]. To cope with such nonlinear distortions, the novel nonlinear ANC systems using artificial neural network (ANN) [7], Volterra series [8], FLANN [9] have been developed, which can offer superior performance over the linear FIR controller.

The nonlinearity introduced due to the saturation effect of microphone and loudspeaker is also important when designing the ANC systems. Examples of these saturation phenomena include saturating the reference and error microphone, overdriving the power amplifier or loudspeaker [10]. In such cases, the linear and some nonlinear control filters will suffer from their control performance or even fail. Therefore, many researchers have addressed the saturation problem and proposed various algorithms to tackle them. Costa, Bermudez and Bershad made a statistical analysis of the FXLMS algorithm under the nonlinear saturation effect [10], [11]. Hamidi, Taringoo and Naski modified the cost function in FXLMS algorithm to process the saturation nonlinearity [12]. Kuo, Wu and Gunnala proposed a Fourier analysis method for analyzing the saturation effects in steady
state [13]. The theoretical analysis results have shown that the clipping of a sinusoidal noise produces extra odd harmonics, thus affecting the convergence speed and steady-state solution of the adaptive filter. Whereas, the saturation from the error sensor would slightly reduce the convergence speed, but not affect the optimum solution. Therefore, Kuo and Wu derived a bilinear filter to solve the problems of reference noise saturation [14]. To reduce the computational complexity of bilinear filter, a simplified bilinear filter with a leaky FXLMS (SBLFXLMS) algorithm was presented by Zhao [15]. However, the diagonal-channel feature is not applied in the BFXLMS and SBLFXLMS algorithms. The standard diagonal-channel structure is initially derived to preserve the time invariant property of the cross signal vector by Tan and Jiang [16]. For real applications, Tan, Dong and Du further proposed an efficient channel reduced implementation [17]. In the above bilinear filter and variants, the reference and error signal saturations were considered, however, the loudspeaker saturation was not included in the model. Recently, Sahib and Kamil proposed a pre-distorter based compensation technique to alleviate the loudspeaker saturation nonlinearity in the secondary path [18]. For alleviating both saturation effect from reference noise and loudspeaker, a particle swarm optimization (PSO) based FLANN structure algorithm was proposed by Rout, Das and Panda [19]. The PSO based algorithm did not require the secondary path estimate, thus this saturated model cannot be used for the FXLMS based algorithms. Furthermore, the slow convergence speed may hinder its practical applications. Therefore, one of the objectives in this paper is to established a standard nonlinear saturation active noise control (NSANC) model using the widely used FXLMS algorithm, which includes the reference saturation and SP saturation.

On the other hand, inspired by the feedforward function expansion filters, the function expansion infinite impulse response (FEIIR) filter was designed to further improve the filtering performance and reduce the computational loads, such as the recursive second-order Volterra (RSOV) [20], the recursive FLANN (RFLANN) and recursive even mirror Fourier nonlinear (REMFN) filters [21], [22]. The success of the FEIIR filter motivated us to explore the function expansion bilinear (FEB) filters. In this direction, the bilinear functional link artificial neural network (BFLANN) [23], [24] filter with a standard diagonal-channel structure has been presented and achieved better performance. However, it is not realized that the function expansion bilinear is a generalized 3-D diagonal-channel structure. Therefore, another objective of this paper is to develop a general function expansion bilinear filter with a 3-D diagonal structure using different function expansions.

To achieve the objectives, we analyze the saturation nonlinearities in the ANC system and proposed a NSANC model. In the NSANC model, the reference saturation is approximated by a saturated function, and the saturated secondary path (SSP) is modeled as a linear-nonlinear-linear (LNL) transfer function. We develop three function expansion bilinear forms, including the Volterra [7], trigonometric-based FLANN (TFLANN) [8], and even mirror Fourier nonlinear (EMFN) [24] to show the proposed 3-D diagonal structure. In addition, the associated FEDBFXLSM algorithm using the NSANC model is derived. We analyze the computational complexity and suggest several computational efficient strategies to further reduce the computational loads. Finally, we carry out a number of simulations with a varied degree of saturation nonlinearity both at input signal and the secondary path to evaluate the control performance.

This paper is organized as follows. Section II describes a standard NSANC model. Section III presents the nonlinear function expansion bilinear filter with a 3-D diagonal structure and an associated FEDBFXLSM algorithm. Furthermore, the computational complexity is analyzed. Section IV presents computer simulations to validate the control performance. Finally, conclusions are given in Section V.

II. NSANC model using the FXLMS algorithm

A single channel feed-forward ANC system using an FXLMS algorithm in a duct [4], [5], [14] is illustrated in Fig. 1. As depicted in Fig. 1, the reference noise $x(n)$, collected by the reference microphone, is fed to the adaptive controller. The adaptive controller output signal $y(n)$ is sent to the digital to analog converter (DAC) unit and amplified to drive the loudspeaker. Note that the signal path from $y(n)$ to the cancelling point is referred to the secondary path and is denoted by $s(n)$ while the signal path from the reference noise to the cancelling point is designated as the primary path which is represented as $p(n)$. The noise from the loudspeaker cancels the undesired noise $d(n)$, which arrives from the reference signal location via the primary path. At the cancelling point, the error signal $e(n)$ gathered by the error microphone in addition to the reference noise $x(n)$ is used to update the weight vector of the adaptive controller and to monitor the control performance.

![FIGURE 1. Feed-forward duct ANC system using FXLMS algorithm.](image-url)

As shown in Fig. 1, most of the conventional FXLMS based ANC systems [4]-[9] did not consider the saturation nonlinearity situation. However, in real applications, the microphone may exhibit saturation behavior when the acoustic noise increases beyond the dynamic limits. The saturated error microphone does not affect the optimum solution, thus can be neglected when establishing the NSANC model. The secondary path for a feedforward ANC system usually consists of a digital to analog converter (DAC), a power amplifier, a loudspeaker and an error.
microphone. When the amplified controller output exceeds the dynamic limit of the loudspeaker, the saturation nonlinearity is introduced into the secondary path [19]. It is well known that a linear FIR model can be used to represent the path from the controller output through the power amplifier and the loudspeaker output to the error microphone. When considering the saturated loudspeaker, the saturation nonlinear function can be introduced in modeling the secondary path. Accordingly, Fig. 2. illustrates the model under considering the saturation nonlinearity. As shown in the NSANC model, the reference signal is collected by a microphone exhibiting saturation nonlinearity function. In the SSP, the linear-nonlinear-linear (LNL) block-oriented model is used to match the practical secondary path [26]. The linear transfer function impulse response sequence is represented by the symbol \( L(n) \). The saturation nonlinearity functions for the saturated reference microphone and the loudspeaker are both denoted as the symbol \( N(\cdot) \). Intermediate signals are represented by \( z_i(n) \) and \( z_2(n) \). According to the virtual secondary path concept [26], the estimation of the time-varying SSP can be expressed as:

\[
s(n) = l_1 N'[z_1(n)]l_2
\]  

where \( l_1 \) and \( l_2 \) denotes the time-invariant FIR filters, \( N'[z_1(n)] \) designates the first derivative of the saturation function versus \( z_1(n) \).

**Fig. 2. Feed-forward NSANC using FXLMS algorithm**

**III. Algorithm Development**

In this section, the 3-D diagonal structures for the TFLANN, Volterra and EMFN expansion bilinear filters are designed. The FEDBFXLMS algorithm is derived, and the computational loads for different function expansion algorithms are analyzed.

**A. FUNCTION EXPANSION 3-D DIAGONAL STRUCTURE BILINEAR FILTER**

To clearly show the 3-D diagonal structure, we first discuss the relationship between input \( x(n) \) and output \( y(n) \) for the standard diagonal-channel bilinear filter. The output signal \( y(n) \) can be expressed as [16], [17]:

\[
y(n) = \sum_{j=0}^{N} a_j(n)x(n-j) + \sum_{j=1}^{N} b_j(n)y(n-j) \\
+ \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} g_{ij}(n)x(n-j)y(n-j-i) \\
+ \sum_{i=0}^{N-1} \sum_{j=0}^{N-i} h_{ij}(n)x(n-i-j)y(n-j-1)
\]  

where \( i \) denotes the diagonal channel number and \( j \) designates the time index, \( a_j(n) \) and \( b_j(n) \) are the feed-forward and feedback coefficients, respectively. \( N \) is the memory length of the reference and secondary memory. The \( g_{ij}(n) \) denotes the coefficients for the diagonal channel input signals \( x(n-j)y(n-j-1) \) with \( N \) coefficients, while \( g_{ij}(n) \) the coefficients for the channel input signals \( x(n-j)y(n-j-2) \) with \( N-1 \) coefficients and so on. Similarly, \( h_{ij}(n) \) designates the coefficients corresponding to the channel input signals \( x(n-j-1)y(n-j-1) \) containing \( N \) coefficients and \( h_{ij}(n) \) represents \( N-1 \) coefficients for its diagonal channel input signals \( x(n-j-2)y(n-j-1) \) and so on. There are \( 2N \) time-invariant signal channels for the bilinear section in total.

Considering that a function expansion signals \( \phi[x(n)] \) takes the place of the input signals \( x(n) \), we obtain the 3-D diagonal-structure bilinear filter. The output \( y(n) \) in (2) can be rewritten as:

\[
y(n) = \sum_{j=0}^{N} a_j(n)\phi_j[x(n-j)] + \sum_{j=1}^{N} b_j(n)y(n-j) \\
+ \sum_{k=0}^{N} \sum_{j=0}^{M} g_{ij}(n)\phi_{ij}(x(n-j))y(n-j-i) \\
+ \sum_{k=0}^{N} \sum_{j=1}^{M} h_{ij}(n)\phi_{ij}(x(n-j-1))y(n-j-1)
\]  

where \( k \) designates the basis function number, \( \phi_{ij}[x(n-j)] \) the signal of \( k \)-th function expansion of the signal \( x(n) \) with a delay of \( j \) and \( M \) the total number of basis functions used. The function expansion bilinear filter with a 3-D diagonal-structure can be written in a vector form as follows:

\[
y(n) = A^T(n)[\phi[x(n)]] + B^T(n)y(n-1) \\
+ G^T(u(n) + H^T(n)v(n))
\]

where the signal vectors \( \phi[x(n)], U(n) \) and \( V(n) \) are expressed as:

\[
\phi[x(n)] = [\phi_1[x(n)], ..., \phi_N[x(n)], \phi_0[x(n)]]^T
\]

\[
U(n) = [\phi_1[x(n)]y(n-1), ..., \phi_N[x(n)+N-1]]^T \phi_0[x(n-1)], ..., \phi_N[x(n)+N-1]]y(n-1), \]

\[
V(n) = [\phi_1[x(n-1)]y(n-1), ..., \phi_N[x(n-N)]y(n+i-N-1), \phi_0[x(n-i-1)]y(n-1), ..., \phi_N[x(n-N)]y(n+i-N-1)]
\]

There are also \( 2N \) signal channels for the function expansion bilinear section. The associated coefficient vectors \( A(n), G(n), \) and \( H(n) \) are defined below:

\[
A(n) = [a_0(n), ..., a_N(n), a_1(n), ..., a_1(n-1)]^T
\]

\[
G(n) = [g_{1,1}(n), ..., g_{1,N}(n), ..., g_{N,1}(n), ..., g_{N,N}]^T
\]

\[
H(n) = [h_{1,1}(n), ..., h_{1,N}(n), ..., h_{N,1}(n), ..., h_{N,N}]^T
\]

**Fig. 3 depicts the 3-D diagonal structure and signal elements for the case of \( N=3 \). It can be seen in the TFLANN expansion structure from Fig. 3a that the signal elements in each diagonal channel at its layer are the delayed version of**
the previous one. Every basis function has one-layer elements in the 3-D structure, thus, \( M \) basis functions have \( M \) layers. The diagonal channels from every layer form the rectangle structure in the 3-D diagonal structure. It is obvious that the diagonal-channel bilinear filter is a special case \((M=1)\) of the general 3-D diagonal structure. The TFLANN expansion 3-D diagonal structure is the standard form where the expansion \( \phi[x(n)] \) does not include cross terms.

For the second-order Volterra [8] expansion, as shown in Fig. 3b, the rectangle diagonal channel degenerates approximately into trapezoid channel. The reason is that some signals do not contain complete delayed elements in some layers for the limitation of the memory length \( N \). To clearly show all the signals in the second-order Volterra expansion 3-D structure, we listed the signal \( \phi[x(n)] \), the cross vectors \( u_i(n) \) and \( v_i(n) \) in each layer in Table I. As shown in the last row of Table I, the basis function \( \phi_{M,1}[x(n)]=x(n-1)x(n-N) \) does not contain delayed signal elements, thus, the corresponding cross vector \( v_{M,1}(n) \) have no element. In the Volterra expansion 3-D diagonal structure, the cross vectors \( U(n) \) have \( N(N+1)/(2N+1) \) elements and \( V(n) \) have \((N^3+6N^2+5N)/6 \) elements To provide a fair and real comparison, the constant term in EMFN was not used in the expansion [5]. Thus, the EMFN expansion bilinear signals have a similar 3-D diagonal structure to the Volterra expansion as shown in Fig. 3b.

![FIGURE 3. Function expansion 3-D diagonal-structure for No.3. (a) TFLANN 3-D diagonal-structure. (b) Volterra 3-D diagonal-structure.](image)

<table>
<thead>
<tr>
<th>Layer</th>
<th>Signals</th>
<th>Number of terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \phi_1[x(n)]=x(n), x(n-1), x(n-2), \ldots, x(n-N) )</td>
<td>( N+1 )</td>
</tr>
<tr>
<td>2</td>
<td>( u_1(x)=x_1(n), \ldots, x_1(n-N) )</td>
<td>( N+1 )</td>
</tr>
<tr>
<td>3</td>
<td>( v_1(x)=x_2(n), \ldots, x_2(n-N) )</td>
<td>( N+1 )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( N+1 )</td>
<td>( u_{N-1}(x)=x_{N-1}(n), x_{N-1}(n-N) )</td>
<td>( 2N+1 )</td>
</tr>
<tr>
<td>( N+2 )</td>
<td>( v_{N-1}(x)=x_{N-2}(n), x_{N-2}(n-N) )</td>
<td>( N )</td>
</tr>
</tbody>
</table>

**B. FEDBFXLMS ALGORITHM**

The function expansion bilinear coefficient vectors \( A(n), b(n), G(n) \) and \( H(n) \) can be combined in a compact weight vector \( w(n) \):

\[
\begin{align*}
w(n) &= \begin{bmatrix} A^T(n), b^T(n), G^T(n), H^T(n) \end{bmatrix} \\
\end{align*}
\]

Similarly, a generalized signal vector \( f(n) \) by combining the signal vectors is defined as

\[
\begin{align*}
f(n) &= \begin{bmatrix} \phi[x(n)], y^T(n), U^T(n), V^T(n) \end{bmatrix} \\
\end{align*}
\]

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where the length of \( f(n) \) is \( M_c + N + M_c N \), \( M_c \) denotes the number of the function expansion signals \( \phi[x(n)] \).

Therefore, the generalized output signal \( y(n) \) can be expressed as:

\[
y(n) = w(n)G^T(n)
\]

Passing the output signal \( y(n) \) through the secondary path acoustically, we can express the cancelling signal \( \tilde{d}(n) \) as

\[
d(n) = s(n) \ast y(n)
\]

Therefore, the residue signal received by the error microphone is expressed as:

\[
e(n) = d(n) - \tilde{d}(n)
\]

where \( d(n) \) is the primary noise at the cancelling point.

For adaptive algorithm development, the cost function is defined as:

\[
\xi(w) = E[e^T(n)]
\]

where \( E[\cdot] \) is the expectation operator.

Using the FXLMS algorithm to minimize the cost function, the update equation is

\[
w(n+1) = w(n) + \mu e(n)f^T(n)
\]

where \( \mu \) is a step size.

For every weight subsection for the FEB filter, the update equations are

\[
\begin{align*}
A(n+1) &= A(n) + \mu_e e(n)\phi(n) \\
b(n+1) &= b(n) + \mu_b e(n)y'(n-1) \\
G(n+1) &= G(n) + \mu_e e(n)U'(n) \\
H(n+1) &= H(n) + \mu_e e(n)V'(n)
\end{align*}
\]

where \( \mu_e, \mu_b \) and \( \mu_c \) are the step sizes which control the convergence speed for updating the coefficients \( A(n) \) and \( b(n) \), and cross term coefficients \( G(n) \) and \( H(n) \), respectively.

The filtered input signal \( \phi'[x(n)], y'(n-1), U'(n) \) and \( V'(n) \) can be written as

\[
\begin{align*}
\phi'[x(n)] &= s(n) \ast \phi[x(n)] \\
y'(n-1) &= s(n) \ast y(n-1) \\
U'(n) &= s(n) \ast U(n) \\
V'(n) &= s(n) \ast V(n)
\end{align*}
\]

where \( s(n) = \lfloor N[z_c(n)] \rfloor \), for the saturated secondary path.

It is important to note that the bilinear structure may contain the local minima which may result in not converging to the global minimum. In addition, the filter has many autoregressive terms, and may suffer from the bounded-input bounded-output (BIBO) stability problem when the initial feedback coefficients are not properly selected [21]-[24]. However, it has been verified by many experiments that, with a careful choice of the step sizes and initial coefficients, the gradient descent algorithms can work well in many situations [14]-[17]. To guarantee the steady state solution, zero initial coefficients often used. In addition, \( \mu_e, \mu_b \) and \( \mu_c \) must satisfy the following condition [17], [23]:

\[
0 < \mu_e, \mu_b, \mu_c < 2\lambda_{max}
\]

where \( \lambda_{max} \) is the maximum eigenvalue of the autocorrelation matrix of the filtered input signals. Therefore, the proposed FEDBFXLS algorithm is depicted in Fig. 4.

C. COMPUTATIONAL COMPLEXITY ANALYSIS

In this section, the computational complexity of the proposed FEDBFXLS algorithm is studied. Note that \( N \) is the memory size of primary and secondary memory, \( L \) is the length of the estimated SSP, \( M \) is the number of basis functions, \( M_c \) is the number of all the function expansion signals, \( P \) is the order of the TFLANN expansion. For different kinds of function expansions, the numbers of basis function and the expansion signals vary significantly. In the TFLANN expansion, \( M_c \) is equivalent to \( M(N+1) \), where \( M = 2P + 1 \). When developing the second-order Volterra and EMFN expansion, specific basis functions and the expansion signals should be analyzed (see section III.A).

Multiplications for calculating the virtual SSP filter coefficients are the same for all algorithms, thus these multiplications are neglected. Besides, the FEDBFXLS algorithm requires four major operations.

1. Generate the function expansion bilinear filter inputs in (4), (5) and (6). The number of multiplications required is dependent on the function expansion models, such as, for generating the function expansion signals \( \phi[x(n)] \), the second-order Volterra and EMFN expansion need \( N+1 \) and \( N \) multiplications, while the TFLANN expansion needs \( 0 \) multiplication; for generating the diagonal channel signals of bilinear section, the TFLANN require \( 2MN \) multiplications; the second-order Volterra and EMFN need \( (N+2)(3N+1)/2 - 1 \) multiplications.

2. Calculate the function expansion bilinear filter output \( y(n) \) given in (3), which requires \( M_c + N + M_c N \) multiplications.

3. Compute the filtered signal elements in (18). The TFLANN expansion bilinear filter requires \( 2MN + M_c N \) multiplications. The Volterra and EMFN expansion bilinear filters require \( [(N+2)(3N+1)/2 + M_c N]L \) multiplications.

4. For updating coefficients given in (17), \( M_c + N + M_c N + 4 \) multiplications are needed.

The comparisons of computational complexity for the proposed FEB filters, bilinear filter and RSOV filter using the FXLMS algorithm are listed in Table II. The TDBFXLMS, VDBFXLMS and FDBFXLMS depict the TFLANN, second-order Volterra and EMFN expansion.
algorithms, respectively. It shows that the DBFXLMS algorithm requires fewer computations, whereas the FEB filters (TDBFXLMS, VDBFXL, and FDBFXLMS) need more computational loads for the same filter length \( L \). However, the FEB filters can achieve better control performance over the standard bilinear and RSOV filters.

Furthermore, some new strategies can be used to significantly reduce the computational load with little sacrificing the control performance for implementation, such as simplified and channel reduced algorithms [15], [17], partial update algorithm [28], and filtered-error structure [26], [30].

**Table II. Comparisons of computational complexity for various algorithms.**

<table>
<thead>
<tr>
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<th></th>
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</thead>
<tbody>
<tr>
<td>Generating inputs</td>
<td>2N</td>
<td>(N+1)^2</td>
<td>2MN</td>
<td>(3N+9N+2)/2</td>
<td>(3N+9N+2)/2</td>
</tr>
<tr>
<td>Control output</td>
<td>N+3N+1</td>
<td>(N+1)+(2N+1)</td>
<td>M+C+N+2</td>
<td>M+N+M+N</td>
<td>M+N+M+N</td>
</tr>
<tr>
<td>Filtered signal</td>
<td>2(N+1)L</td>
<td>[N(N+1)/2+(2N+2)L]</td>
<td>(2MN+M+1)L</td>
<td>[(3N+1)(N+2)/2+M]L</td>
<td>[(3N+1)(N+2)/2+M]L</td>
</tr>
<tr>
<td>Weight update</td>
<td>N+3N+4</td>
<td>(N+1)+(2N+5)</td>
<td>M+C+N+4</td>
<td>M+N+M+N+4</td>
<td>M+N+M+N+4</td>
</tr>
<tr>
<td>Total</td>
<td>2N+8N+2(N+1)L+5</td>
<td>2[(N+1)+2N+1]+[N(N+1)+(2N+2)L]+2(M+N+M+N)+(2M)</td>
<td>2(M+N+M+N)+(3N+1)(N+2)/2+M</td>
<td>2(M+N+M+N)+(3N+2+9N+2)/2+4</td>
<td></td>
</tr>
</tbody>
</table>

**Remark**

- \( M=1; \) \( M=M+(N+1) \)
- \( M=2P+1; \) \( M=M+(N+1) \)
- \( M=(N+2)(N+3)/2-1 \)
- \( M=(N+2)(N+3)/2-1 \)

Multiplications for calculating the virtual secondary path filter coefficients are the same for all algorithms and neglected.

<table>
<thead>
<tr>
<th>Comparison of computational complexity at different ( L ) when ( P=1, N=10 ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
</tr>
<tr>
<td>( 1 )</td>
</tr>
<tr>
<td>( 2 )</td>
</tr>
<tr>
<td>( 3 )</td>
</tr>
<tr>
<td>( 4 )</td>
</tr>
</tbody>
</table>

**FIGURE 5. Saturation nonlinearity function.**

### IV. EXPERIMENTS RESULTS

In order to validate the effectiveness of the proposed filter, we carry out a number of simulations with various cases of saturation nonlinearity under theoretical and measured primary and secondary path models.

Since the bilinear and recursive filters often outperform the conventional Volterra and FLANN filters [16]-[22], we just plot the normalized mean square error (NMSE) to compare the performances achieved by the TDBFXLMS, VDBFXLMS and FDBFXLMS algorithms over the ones obtained from the DBFXLMS, RSOVFXLMS algorithms to show the superior performance of the proposed algorithms. The NMSE is defined as

\[
NMSE = 10\log_{10}\left(\frac{E(e^2(n))}{\sigma_e^2}\right),
\]

where \( \sigma_e^2 \) is the power of the reference noise at the canceling point.

The saturation nonlinearity is assumed as a hyperbolic tangent function \( \tanh(\beta x(n)) \), which can represent the inherent nonlinearity of the saturated microphone and loudspeaker [19], [26]. To show different varying grades of saturation, we vary the value of \( \beta \). A set of input-output response of this form of saturation nonlinearity is obtained by varying \( \beta \) in Fig 5. It is observed that as \( \beta \) increases the function saturates after a smaller range of the input variable.

In Experiment A, we use the varying grades of saturated reference noises, while the secondary path is not saturated. In Experiment B, we consider the nonlinear primary path (NPP) and varying grades SSP with the non-saturated reference noise. In Experiment C, we adopt the saturated reference noise and SSP under the condition of NPP. In Experiment D, the strong and weak saturated tonal signals under the real measured primary and secondary paths are simulated.

Note that the memory size \( N \) is chosen to be 12 for the first three experiments, and the initial filter coefficients are set to zeros in all simulations. The NMSE is computed by averaging the square error signal over 200 different runs with independent sequences of noise. The step-sizes are selected to achieve the maximum convergence speed and still provide the minimum achievable NMSE. In addition, it is assumed that the reference microphone does not pick up a feedback signal from the secondary loudspeaker [13], [21].

**A. VARYING REFERENCE SATURATION GRADRES**

In this experiment, a saturated reference noise under linear primary path (LPP) and non-minimum phase secondary path is used to verify the performance.

The reference noise is the logistic chaotic noise [8] generated by
where $\lambda=4$ and $x(1)=0.9$ are chosen. Fig. 6 shows the original chaotic noise, saturated noise and collected saturated noise. The reference may be saturated when the noise is $1.5x(n)$ or $2x(n)$ which exceeds the dynamic limits of the electronic devices. In such saturation cases, the noise $1.5x(n)$ and $2x(n)$ would transfer through the primary path to the cancelling point, and $\text{tanh}[1.5x(n)]$ and $\text{tanh}[2x(n)]$ are used as the collected references noises.

The primary path $P(z)$ is modeled as FIR filter [8], which is given in $z$-domain by

$$P(z) = z^{-k} - 0.3z^{-6} + 0.2z^{-10}$$  \hspace{1cm} (22)

The secondary path transfer function $S(z)$ and its estimate $\hat{S}(z)$ is taken as the non-minimum phase model [9]

$$S(z) = \tilde{S}(z) = z^{-2} + 1.5z^{-3} - z^{-4}$$  \hspace{1cm} (23)

**FIGURE 6. Reference noises and varying saturation grades.**

Fig. 7a shows the performances of all compared algorithms under the condition of no saturation. It is obvious that there is a significant improved performance of the proposed FEDBFXLM algorithm over the DBFXLMS and RSOVFXLM algorithms. The steady-state NMSEs obtained after adaptation from TDBFXLMS, VDBFXLMS and FDBFXLMS are -44dB, -46dB and -41dB, whereas, the steady-state NMSEs obtained from DBFXLMS and RSOVFXLM are -33.5dB and -38dB. Note that the step sizes are selected as: (1) second-order VDBFXLMS algorithm with $\mu_a=0.0096$, $\mu_b=0.0035$, $\mu_c=0.00001$; (2) second-order DBFXLMS with $\mu_a=0.0046$, $\mu_b=0.0006$, $\mu_c=0.00004$; (3) first-order TDBFXLMS with $\mu_b=0.0006$, $\mu_c=0.00002$; (4) FDBFXLMS with $\mu_a=0.0094$, $\mu_b=0.0006$, $\mu_c=0.00002$; (5) RSOVFXLM with $\mu_a=0.0096$, $\mu_b=0.00028$, $\mu_c=0.00008$, $\mu_d=0.00004$.

Fig. 7b shows the performances under the condition that the noise source is $1.5x(n)$ and the collected noise is $\text{tanh}[1.5x(n)]$, whereas, Fig. 7c shows the performances of $\text{tanh}[2x(n)]$ reference nonlinearity. From Fig. 7b and Fig. 7c, we can see that the increasing saturation nonlinearity will degrade the performance of all controllers. In Fig. 7b, the steady-state NMSEs obtained from TDBFXLMS, VDBFXLMS and FDBFXLMS are -40.5dB, -41dB and -34dB, respectively. In Fig. 7c, the steady-state NMSEs obtained from TDBFXLMS, VDBFXLMS and FDBFXLMS are -34.5dB, -37dB and -27dB, respectively. In addition, for Fig. 7b and Fig. 7c, the step sizes are selected as: (1) second-order VDBFXLMS algorithm with $\mu_a=0.0068$, $\mu_b=0.0032$, $\mu_c=0.00001$; (2) second-order FDBFXLMS with $\mu_a=0.0046$, $\mu_b=0.0006$, $\mu_c=0.00004$; (3) first-order TDBFXLMS with $\mu_a=0.006$, $\mu_b=0.004$, $\mu_c=0.00002$; (4) DBFXLMS with $\mu_a=0.0008$, $\mu_b=0.0006$, $\mu_c=0.00002$; (5) RSOVFXLM with $\mu_a=0.001, \mu_b=0.0008, \mu_c=0.00042, \mu_d=0.00004$. In real ANC system, the reference signal may also be obtained by clipping threshold at the maximum value. When processing the noise of $2x(n)$, the threshold value is assumed to be 1. Fig. 7d shows the NMSE curves from all the algorithms. We can see that all the controllers have an improved performance in this saturation condition in comparison with the $\text{tanh}[2x(n)]$ case. The steady-state NMSEs obtained after adaption using TDBFXLMS, VDBFXLMS and FDBFXLMS algorithms are -44dB, -45dB and -39dB, respectively. Whereas, the NMSEs from the DBFXLMS and RSOVFXLM are -27.5dB and -28dB, respectively. The step sizes are selected as: (1) second-order VDBFXLMS algorithm with $\mu_a=0.012$, $\mu_b=0.004$, $\mu_c=0.00001$; (2) second-order FDBFXLMS with $\mu_a=0.0046$, $\mu_b=0.0006$, $\mu_c=0.00004$; (3) first-order TDBFXLMS with $\mu_a=0.006$, $\mu_b=0.004$, $\mu_c=0.00002$; (4) DBFXLMS with $\mu_a=0.0008$, $\mu_b=0.0006$, $\mu_c=0.00002$; (5) RSOVFXLM with $\mu_a=0.001, \mu_b=0.0008, \mu_c=0.00042, \mu_d=0.00004$. Fig. 6 also shows that the EMFN expansion bilinear filters achieve a poor prediction performance than the Volterra and TFLANN expansion bilinear filters. The reason is that the EMFN expansion bilinear filter lacks the linear basis function $x(n)$.

**B. VARYING LOUDSPEAKER SATURATION GRADES**

In this experiment, we assume a NPP and LNL model SSP to further evaluate the performances.

In the primary path, the relationship between the cancelling point signal $d(n)$ and the reference signal $x(n)$ is defined as [28],

$$d(n) = x(n) + 0.8x(n-1) + 0.3x(n-2) + 0.4x(n-3) - 0.8x(n)x(n-1) + 0.9x(n)x(n-2) + 0.7x(n)x(n-3)$$  \hspace{1cm} (25)

The $l_i(z)$, $N(z)$ and $\tilde{l}_i(z)$ in $z$-domain are represented as

$$l_i(z) = 1 - 0.6z^{-1} + 0.05z^{-2}$$

$$N[z(n)] = \text{tanh}[\beta z_3(n)]$$

$$\tilde{l}_i(z) = 1 + 0.2z^{-1} + 0.05z^{-2}$$  \hspace{1cm} (26)

The noise is chosen as a uniform random number generated by $x(n)=(\text{rand}-0.5)$. Fig. 8 depicts the control curves of all controllers under two different SSPs: $\text{tanh}[2z_3(n)]$ and $\text{tanh}[3z_3(n)]$. When processing the $\text{tanh}[2z_3(n)]$ nonlinearity secondary path, the step sizes are selected as: (1) second-order VDBFXLMS algorithm with $\mu_a=0.012$, $\mu_b=0.004$, $\mu_c=0.00001$; (2) second-order FDBFXLMS with $\mu_a=0.002$, $\mu_b=0.002$, $\mu_c=0.0008$; (3) first-order TDBFXLMS with $\mu_a=0.0032$, $\mu_b=0.002$, $\mu_c=0.00002$; (4) DBFXLMS with $\mu_a=0.0005$, $\mu_b=0.002$, $\mu_c=0.0006$; (5) RSOVFXLM with...
\( \mu_1 = 0.0046, \mu_2 = 0.0016, \mu_0 = 0.00032, \mu_0 = 0.00032. \) For the \( \tanh[z_2(n)] \) nonlinearity secondary path, the step sizes are selected as: (1) second-order VDBFXLMS algorithm with \( \mu_a = 0.0042, \mu_b = 0.0012, \mu_c = 0.0004; \) (2) second-order FDBFXLMS with \( \mu_a = 0.0046, \mu_b = 0.0006, \mu_c = 0.00004; \) (3) first-order TDBFXLMS with \( \mu_a = 0.006, \mu_b = 0.0002, \mu_c = 0.00002; \) (4) DBFXLMS with \( \mu_a = 0.0005, \mu_b = 0.002, \mu_c = 0.0006; \) (5) RSOVFXLMS with \( \mu_a = 0.0046, \mu_b = 0.0016, \mu_0 = 0.00032, \mu_0 = 0.00032. \)

The results in Fig. 8a and Fig. 8b show an improvement in the performance of the proposed VDBFXLMS and FDBFXLMS algorithms compared with the DBFXLMS and RSOVFXLMS algorithms. Since the primary path contain cross terms of reference signal which is not included in the FLANN filter, the TFLANN expansion bilinear filter perform poor than the other FEB filters. The varying nonlinearity grades of loudspeaker saturation have little impact on all adaptive controllers when the reference noise has no saturation.
C. BOTH SATURATION OF REFERENCE AND LOUDSPEAKER

In this experiment, both saturations of reference and loudspeaker are taken for investigation.

The reference noise is generated using 1.2(rand) which generates the uniformly distributed random numbers between 0 and 1.2. The reference noise is saturated by \( \tanh[1.2x(n)] \).

For the LNL model SSP, \( N(\cdot) \) is chosen as

\[
N[z(n)]=\tanh[2z(n)]
\]

The comparisons of control performances in terms of the NMSE under the saturated reference noise and loudspeaker are shown in Fig. 9. The proposed TDBFXLMS, VDBFXLMS and FDBFXLMS algorithms result in lower NMSE than the DBFXLMS and RSOVFXLMS algorithms. The steady-state NMSEs obtained from TDBFXLMS, VDBFXLMS and FDBFXLMS are -15.5dB, -16dB and -15dB, respectively. Whereas, the NMSEs from DBFXLMS and RSOVFXLMS are -11.5dB and -11dB, respectively. The step sizes are selected as: (1) second-order VDBFXLMS algorithm with \( \mu_a=0.0048, \mu_b=0.00028, \mu_c=0.00001 \); (2) second-order FDBFXLMS with \( \mu_a=0.026, \mu_b=0.00002, \mu_c=0.00001 \); (3) first-order TDBFXLMS with \( \mu_a=0.0082, \mu_b=0.0006, \mu_c=0.0001 \); (4) DBFXLMS with \( \mu_a=0.008, \mu_b=0.002, \mu_c=0.0006 \); (5) RSOVFXLMS with \( \mu_{a1}=0.0086, \mu_{b1}=0.0016, \mu_{b2}=0.00032, \mu_{b3}=0.00032 \).

D. SATURATED TONAL NOISE UNDER MEASURED PRIMARY AND SECONDARY PATH

In this experiment, the measured primary and secondary paths used in [1] are adopted to evaluate the performance in the practical application.

Fig. 10 shows the amplitude and phase responses for the measured primary path and secondary path. It is apparent that the primary path and the secondary path contain significant magnitude responses over a broadband frequency range.

The reference signal consists of three sinewaves at normalized frequencies of 0.01, 0.02, and 0.08 and is normalized to have a unit power [14], [17]. The signal to noise power ratio (SNR) at the noise cancelling point was set to 40dB. In addition, the reference signal is assumed to have weak and strong saturation and obtained by clipping threshold at 90% and 50% of the maximum signal value. The memory size \( N \) is chosen to be 30.

Fig. 11 shows the control performances for controlling the weak and strong saturated nonlinear narrow band noise using the measured primary and secondary paths. All the proposed algorithms achieve an improvement control performance over the DBFXLMS and RSOVFXLMS algorithms. As shown in Fig. 11a, the steady-state NMSEs obtained after convergence from the TDBFXLMS, VDBFXLMS and FDBFXLMS are -35.5dB, -38dB and -37dB, respectively.
Whereas, the steady-state NMSEs obtained from the DBFXLMS and RSOVFXLMS are both -30dB. For the strong saturation in Fig. 11b, the steady-state NMSEs from the TDBFXLMS, VDBFXLMS and FDBFXLMS are -32dB, -34dB and -33dB, respectively, whereas, NMSEs from DBFXLMS and RSOVFXLMS are -24dB and -25.5dB, respectively. Fig 10 also shows that the increasing saturation nonlinearity degrades the performance which is consistent with the results in Experiment A.

The step sizes for the weak saturation are chosen as: (1) second-order VDBFXLMS algorithm with $\mu_0=0.0008$, $\mu_0=0.0006$, $\mu_0=0.00004$; (2) second-order FDBFXLMS with $\mu_0=0.0005$, $\mu_0=0.0002$, $\mu_0=0.00004$; (3) first-order TDBFXLMS with $\mu_0=0.0012$, $\mu_0=0.0004$, $\mu_0=0.00004$; (4) DBFXLMS with $\mu_0=0.0005$, $\mu_0=0.0002$, $\mu_0=0.00002$; (5) RSOVFXLMS with $\mu_0=0.0002$, $\mu_0=0.0001$, $\mu_0=0.00012$. The step sizes for the strong saturation are chosen as: (1) second-order VDBFXLMS algorithm with $\mu_0=0.0008$, $\mu_0=0.0004$, $\mu_0=0.00006$; (2) second-order FDBFXLMS with $\mu_0=0.0005$, $\mu_0=0.0002$, $\mu_0=0.00006$; (3) first-order TDBFXLMS with $\mu_0=0.0012$, $\mu_0=0.0008$, $\mu_0=0.00004$; (4) DBFXLMS with $\mu_0=0.0005$, $\mu_0=0.0002$, $\mu_0=0.00002$; (5) RSOVFXLMS with $\mu_0=0.0002$, $\mu_0=0.0001$, $\mu_0=0.00012$.

V. CONCLUSIONS

In this paper, a general adaptive function expansion bilinear filter with a 3-D diagonal structure has been proposed to solve the problem of saturation nonlinearity using the new NSANC model. Three function expansion bilinear filters, including the Volterra, trigonometric-based FLANN, and EMFN functions, are developed. The associated function expansion diagonal-structure bilinear FXLMS algorithm (FEDBFXLMS) is derived. The performances of different function expansion bilinear controllers have been compared according to the computational complexity and via computer simulations. Furthermore, some computational reduction strategies are suggested to achieve efficient implementation. Computer simulations validate that the proposed FE filters have an obvious improvement in reducing saturation effects than the standard bilinear and RSOV filters using the FXLMS algorithm.

REFERENCES

