An Evolution Model of Group Opinions based on Social Judgment Theory (August 2018)

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This work is supported by the Major Program of the National Natural Science Foundation of China (71490725), the Foundation for Innovative Research Groups of the National Natural Science Foundation of China (71521001), the National Natural Science Foundation of China (71722010, 91546114, 91746302, 71501057), and The National Key Research and Development Program of China (2017YFB0803303).

ABSTRACT:

We integrate social judgment theory and bounded confidence to build an agent-based model for exploring how group opinions form and evolve. Each agent in our model, which is an extended Hegselmann–Krause model, has its own personality (openness and vacillation) and emotion (opinion). In a homogeneous case, we find that agents’ opinions will preserve their initial order. Once a crack forms between two agents at time $t$, then a crack will exist after that time. Despite the fact that the continuous trust function results in a zero infimum of weight, the opinion evolution system will converge. Simulation shows that an increased level of either openness or vacillation can cause the group to reach a consensus easily. Specifically, the higher the level of openness of an agent, the greater the amount of clusters; and the higher the level of vacillation of an agent, the less time it will take for group opinions to converge. Although the initial opinions are randomly distributed, agents with the same levels of vacillation and openness tend to be concentrated during the evolution process.

INDEX TERMS

group opinions; social judgment theory; openness; vacillation; consensus

I. INTRODUCTION

Almost all social interactions are, at least in part, shaped by beliefs and opinions. In reality, individuals’ opinions often change dynamically as they interact with one another. At the macro level, group opinions will gradually shift from disorder to order, and even to consensus. This evolution not only reflects many complex social phenomena, such as the survival of minority opinion[1, 2], opinion polarization[3, 4], and rumor spreading[5], but can also be applied to other fields, such as group decision[6,7], social learning and collective wisdom[8,9,10], and innovation diffusion[11,12].

To explore consensus and opinion dynamics, numerous agent-based models have been built in recent years[13-18]. Among these models, the Hegselmann–Krause (HK) model is one of the most important continuous opinion models based on bounded confidence. Agents update their opinions repeatedly and simultaneously at discrete time steps by taking an average of the opinions that are “close enough” to their own.

1
Dittmer\cite{19} proved that consensus would be reached in finite time if and only if the opinion profile was an $\epsilon$-chain at any time. Lorenz\cite{20} proved that if trust between two agents is mutual, then every agent acquires a little bit of self-confidence, positive weights do not converge to zero, and the HK model will converge. In the HK model, where $n$ is the number of agents in a group, as each positive weight $a_{ij}$ is at least $1/n$ and trust is mutual, the system will converge. Later, authors in \cite{21} found that the upper bound of the convergence time for 1-D HK systems is $O(n^3)$. In \cite{22}, the authors studied HK systems in a noisy environment and found a critical phenomenon: when the noise strength is below a critical value, fragment phenomena in a group will occur. The bounded confidence parameter plays an important role in the HK model, similar to its function in another classical bounded confidence model, namely, the Deffuant model. Bounded confidence parameter is often called the openness or tolerance of an individual\cite{17,23}. A high level of openness always leads to a consensus or few clusters of opinions, whereas a low level often results in a greater number of opinion clusters\cite{18,24,25}. Although the HK model has become one of the most powerful tools available in theorizing about opinion dynamics, in general, it has important inherent limitations\cite{21}. Some numerical problems may arise from the discontinuity of the model. For example, the relative error will be unbounded if the distance is around the threshold\cite{26}. Moreover, the role of the individual’s personality in the evolution of group opinion has not been well studied. Existing research mainly aimed at the openness (tolerance) of individuals, not much on the effects of individual vacillation. In the present study, we are primarily interested in how group opinions form and evolve and how personality impacts the evolution of opinion.

The main contributions of the paper include three aspects. First, inspired by the limitation of the HK model, we introduce a novel agent-based model by combining social judgment theory and bounded confidence principle. We measure homophily between any two agents by defining a continuous trust function. Compared with the classic HK model, our model is more in line with social psychology. In general, each agent should have its own personality in agent-based models\cite{30,31}. Different from the classic HK model, each agent has its own openness and vacillation in our model. Second, in a homogeneous case, we find that agents’ opinions will preserve their initial order. If a crack forms between two agents at time $t$, then a crack will exist after that time. We also find that even if the continuous trust function results in a zero infimum of weight, the system will still converge, an outcome that is quite different from that in literature\cite{20}. Finally, the system is simulated and the influence of the two parameters on the model is analyzed and compared.

This paper is organized as follows. In Section II, we introduce our model and describe how it differs from the original HK model. In Section III, we analyze the basic properties of our model using a mathematical method. In Section IV, we apply simulation to investigate the impact of an agent’s personality on opinion evolution.

\section{Model}

In this section, we introduce a novel agent-based model under the framework of bounded confidence. We review the classic HK model and consider a population of $n$ agents indexed in a set $I = \{1,2,\ldots,n\}$. Each agent has a time-dependent real-valued opinion $x_i(t)$.

The opinion of agent $i$ updates as follows:

$$x_i(t+1) = \sum_{j=1}^{n} \frac{a_{ij}}{\sum_{j=1}^{n} a_{ij}} x_j(t)$$

(1)

where $a_{ij}$ is to what extent $i$ can be affected by $j$ if agent $j$’s opinion is close enough to that of $i$’s.

In other words, if $|x_i - x_j| < \epsilon$, then $a_{ij} = 1$;
otherwise, $a_{ij} = 0$. This $a_{ij}$ is a discontinuous function of the distance of their opinions and, as a result, the model will not be robust to the initial opinion values.

From social judgment theory, which was first proposed by Sherif and Hovland [27], every agent has its own opinion region: acceptable region, ambiguous region, and rejective region (Figure 1). When agent $i$ (listener or information receiver) encounters a new opinion from agent $j$ (talker), the former will first judge which region the new opinion locates at. If the talker’s (agent $j$) opinion falls into the receiver’s (agent $i$) acceptable region, then agent $i$ will trust agent $j$ completely. If the opinion of agent $j$ falls into the ambiguous region, then agent $i$ is more or less susceptible to $j$, in which case agent $i$ is partly affected by $j$. If the opinion of agent $j$ falls into the rejective region, then agent $i$ will not be affected by agent $j$ at all.

**Fig. 1. Three regions of agent $i$. The green section represents the acceptable region, the blue sections represent the ambiguous region, and the red sections represent the rejective region.**

### Definition 2.1: Trust Function

Thus, for each agent pair $i$ and $j$, trust function $s: \mathbb{R} \to \mathbb{R}^+$ to measure homophily can be defined as follows:

$$s_{ij}(\tau) = \begin{cases} 1 & \text{if } |\tau| < k \\ \frac{l - |\tau|}{l - k} & \text{if } k \leq |\tau| < l \\ 0 & \text{if } |\tau| \geq l \end{cases} \quad (2)$$

where $\tau$ is the distance of opinion of agents $i$ and $j$, $\tau = x_i - x_j$.

For each agent pair $i$ and $j$, from the above trust function, the weight that $j$ exerts on $i$ is

$$w_{ij} = \sum_{\tau=1}^{\infty} s_{ij}(\tau)$$

Then our opinion evolution model can be written as follows:

$$x_i(t + 1) = \sum_{j=1}^{n} w_{ij}(t) x_j(t)$$

Equation (3) can also be written as a continuous time form:

$$\dot{x}_i = \sum_{j=1}^{n} w_{ij}(x_j(t) - x_i(t)), i = 1, 2, \ldots, n \quad (4)$$

Clearly, the HK model is the case of $l = k$.

Therefore, our model, which is based on social judgment theory, is an extended HK model. Parameter $l$ reflects an agent’s openness. When $k$ is fixed, a large $l$ implies the agent has the ability and willingness to interact with more agents. When $l$ is fixed, a large $k$ implies the agent is willing to completely trust and accept the influence of others. Therefore, parameter $k$ measures the agent’s vacillation, that is, a propensity for changing his/her opinion.

### III. BASIC PROPERTIES OF THE MODEL: ANALYSIS OF HOMOGENEOUS AGENTS

**Property 1:** For any agent pair $i$ and $j$, if there exists $t_1$ that satisfied $x_i(t_1) = x_j(t_1)$, then for any $t > t_1$, $x_i(t) = x_j(t)$.

**Proof:**

As agents $i$ and $j$ have the same dynamics in (3) and once $x_i(t_1) = x_j(t_1)$, then at any time $t > t_1$, agents $i$ and $j$ are affected by others with the same effect. Thus, at any time $t > t_1$, agents $i$ and $j$ will hold the same opinion.

**Property 2:** For any agent pair $i$ and $j$, there exists $t_2 > t_1$ which satisfies

$$|x_i(t_1) - x_j(t_1)| < l$$

and

$$|x_i(t_2) - x_j(t_2)| > l$$

. Thus, for any $t > t_2$, $|x_i(t) - x_j(t)| > l$. 

3
Proof: Without loss of generality, we assume the group’s initial opinions are ordered as $x_i(0) \leq x_2(0) \leq \cdots \leq x_n(0)$.

We denote it by $X(0) = (x_1(0), x_2(0), \cdots, x_n(0))^T$.

$X(t) = A(t)X(0)$. Thus, $x_i(t) = \sum_{j \in I} a_{ij}x_j(0)$, and $a_{ij} \geq 0$, $\vdots x_i(0) \leq x_i(1) \leq x_i(n)$.

Clearly, this law can govern any other $t > 1$. In other words, under the compromise mechanism, the range of opinion will narrow as time elapses. Without loss of generality, we have $t^* - 1$, and group opinions are sorted as

$$x_i(t^* - 1) \leq x_2(t^* - 1) \leq \cdots \leq x_i(t^* - 1) \leq x_{i+1}(t^* - 1)$$

but $x_{i+1}(t^* - 1) - x_i(t^* - 1) \geq I$.

Thus agent i (including its left) will not interact with $i+1$ (including its right) any longer. At this time, agents will also be divided into two independent subgroups $\{1, 2, \cdots, i\}$,

$\{i+1, i+2, \cdots, n\}$. Owing to the opinions’ range shrinking and the bounded confidence, these two subgroups will not interact at any $t > t^*$.

Property 3: (Order Preservation)

The dynamics do not change the order of opinions; that is, $x_i(t) \leq x_j(t)$ for all $i \leq j$ implies that $x_i(t + 1) \leq x_j(t + 1)$ for all $i \leq j$.

Proof: Here we prove this property by contraction. According to the assumption by contraction, there exists $t_i > t_j$, such that $x_i(t_i) > x_j(t_j)$. Thanks to the continuity, there must exist $t^* \in (t_i, t_j)$, such that $x_i(t^*) = x_j(t^*)$. Then, according to property 2, $x_i(t_i) = x_j(t_i)$, which contradicts $x_i(t_i) > x_j(t_i)$.

Remark: In a connected social network, only neighbors can interact with one another. In this case, for any initial opinions, only individuals with structural equivalence can preserve the initial order.

For heterogeneous agents, if agents i and j have the same levels of l and k (other agents do not necessarily have l and k), this property will still fit these two agents. Specifically, if $x_i(0) < x_j(0)$, then $x_i(t) < x_j(t)$. The proof of this proposition is as the same as before.

Property 4: (Convergence) Group opinions will converge to $X(t^*)$. For any $i, j$, if $x_i(t^*) \neq x_j(t^*)$, then $|x_i(t^*) - x_j(t^*)| > \varepsilon$.

Proof: Here we only prove the convergence of group opinions. If the first proposition is true, then the second proposition will also be true.

First, we assume that a group of initial opinions has already been sorted as $x_i(0) \leq x_2(0) \leq \cdots \leq x_n(0)$.

Denote it by $X(0) = (x_1(0), x_2(0), \cdots, x_n(0))^T$.

Clearly, $x_i(t)$ is a minimum (see property 3) at any time $t$. Thus from (4), $x_i$ is always positive and $x_i(t)$ is a monotonically increasing sequence. Given that $x_i(t)$ has an upper bound, sequence $x_i(t)$ will converge ultimately.

Next, we will prove agent 2 will also converge.

Suppose agent 2 has not converged by time $t_1^*$.

Thus, at time $t_1^* + 1$, agent 2 will not be affected by agent 1. In other words, agent 1 is not agent 2’s neighbor anymore. Hence, from (2, 3), when $t > t_1^*$, $x_2(t)$ will also be a monotonically increasing sequence. Therefore, $x_2(t)$ will also converge.

By repeating the process above, we can obtain that group opinions will converge.

Remark: In the actual situation, the convergence time of agent 2 is not necessarily longer than that of agent 1. It is just a pessimistic hypothesis for this proof.
IV. SIMULATIONS

A. Homogeneous agents

We consider an initial profile with uniformly distributed opinions on a certain opinion space [0,1]. Every agent has identical levels of openness and vacillation (homogeneous agents). Figure 2 shows the impact of openness on the evolution of opinion distribution. During the process of evolution, agents with similar opinions continue to interact so that their final opinions are in full accord. Based on the principle of bounded confidence, there is either one or multiple centrals in a group. Eventually, around these central agents, one or more opinion clusters are formed. The amount of clusters (AOC) decreases with the increase of $l$. For example, the AOC is 4 in the case of $l=0.1$. When $l$ grows to 0.15, 0.2, 0.25, 0.3, and 0.35, the corresponding AOC will decrease to 3, 3, 2, 1, and 1, respectively. In panels (5) and (6), the consensus is reached. Therefore, the higher the level of agents’ openness, the more easily group opinions are aggregated.

Similarly, Figure 3 shows that parameter $k$, which represents the agents’ vacillation, will also affect the opinion evolution. In the case of $l=0.3$ and $k=0.05$, the AOC is 2. When $k$ grows to 0.15, 0.2, 0.25, 0.3 and 0.35, all agents will have the same opinion. The convergence time will also be shorter and shorter as parameter $k$ grows. Therefore, both levels of openness and vacillation can make it easier for group members to reach a consensus.
Figure 4 shows under what condition the system can reach a consensus. Unlike the consensus threshold proposed in literature\cite{28,29}, the critical values here are a combination of $l$ and $k$, as represented by the blue curve in Fig4. Consensus can be reached at the point above the curve but will not be reached if the point is under the curve. A negative slope means an increasing $l$ must reduce $k$. If $l$ is very large (e.g., $l=0.32$), then no matter how small $k$ is, all the agents will always reach a consensus. Moreover, we observe that the curve is very flat. This means it is more difficult to reach a consensus by increasing $k$ than by increasing $l$. Therefore, to some extent, $l$ is more important than $k$ for the opinion evolution system. As $l$ determines the interaction pattern and $k$ reflects the degree of vacillation of the agent or the reflection of the individual's self-confidence and mutual influence of the individual, the interaction pattern is more important than the other factors.

When $k$ and $l$ take the critical value of consensus, convergence time will be very long. For example, if $k$ is 0.1 and $l$ is 0.3 (Figure 4), the convergence time will be 16, which is much longer than that of other cases. With the increase of $k$, convergence time will be shorter and shorter. If $k$ decreases from 0.1 to 0, convergence time will drop to 10 accordingly, and the group will be divided into more and more independent clusters.

Similar to the effect of $k$ on the convergence time of group opinion, the impact of openness on convergence time likewise depends on whether consensus can be reached. When $l$ is small, consensus cannot be reached. Increasing $l$ will not necessarily lead to the increase of convergence time. While $l$ is large enough, such as $l=0.3$, consensus can be reached. In this case, convergence time will be less and less with the growth of $l$.

B. Heterogeneous agents

In a heterogeneous case, the levels of openness and vacillation of each agent are not necessarily identical. Thus, trusts between agents are not mutual. For example, suppose agent A’s opinion is 0.7 and the level of its openness is 0.5. Agent B’s opinion is 0.3 with 0.1 openness. Then, for A, the opinion distance is 0.7 – 0.3 < 0.5, and A will trust B; while for B, as 0.7 – 0.3 > 0.1, B will not trust A. Therefore, A will be attracted by B unidirectionally. Compared with the homogenous case, the asymmetric attraction will make the evolution of opinions more complicated.

In Figure 7, the blue, green, and red plots represent three types of agents with high, medium, and low openness respectively. The figure shows agents with high level of openness are always neutralists, whereas a low level of openness always leads to extremists. In the heterogeneous case, the group opinion no longer has the overall order preservation. However, this property still holds (see the subplot in Figure 7) for agents with the same levels of openness. Agents with high openness are vulnerable to being unilaterally affected by low-level agents. Therefore, they are often attracted by low-level agents in a single
direction, which causes the convergence time of the system to become very long. At the same time, agents with high openness are more likely to gather together than those with low openness and, eventually, become neutralists. By contrast, agents with low-level openness often become extremists eventually.

![Fig. 7. Effect of different levels of openness on opinion evolution](image)

In Figure 8, the blue, green, and red plots correspond to the high, medium, and low vacillation of agents. Under the same level of openness $l=0.3$, the convergence rates of the three types of agents are significantly different (see the subplot in Figure 8). Although the consensus has been formed finally and agents with different $k$ have not become extremists, agents with less vacillation have a longer convergence time. Furthermore, although the initial opinions are randomly distributed, individuals with the same vacillation tend to be concentrated during the evolution process. This outcome is due to the identical dynamics of agents with the same vacillation, and the evolution process is not sensitive to the initial opinions. We also observe that agents with less vacillation are usually at the outermost part of the evolution graph, whereas agents with great vacillation are usually at the innermost part of the evolution graph. This finding implies that agents who always suspect others are more likely to become extremists.

![Fig. 8. Effect of different levels of vacillation on opinion evolution](image)

V. CONCLUSION

Our model is presented based on social judgment theory and the principle of bounded confidence and is an extension of the HK model. Each agent has its own personality (openness and vacillation) and emotion (opinion).

Under the homogeneous assumption, group opinions will have some properties such as order preservation and convergence. Despite the fact that the continuous trust function results in a zero infimum of weight, the opinion evolution system will still converge.

Simulations show that the sensitivities of the process of opinion evolution to the two parameters are different. $l$ determines the interaction pattern, and $k$ reflects the mutual influence of the individual. Therefore, the impact of interaction patterns on opinion evolution is more important than those of other factors.

We observe that a high level of either openness or vacillation of agents can make the group reach consensus more easily. The higher the level of openness of an agent, the more clusters can be obtained; and the higher the level of vacillation of an agent, the less time it takes for group opinions to converge. Although the initial opinions are randomly distributed, agents with the same vacillation and openness tend to gather together during the evolution process. These results can help us to better understand how personality impacts opinion evolution. If we want consensus to be reached quickly, then there should be high openness between group members. Additionally, they should trust and accept others’ opinions.
REFERENCES


