ABSTRACT

An optimized design of a sparse code multiple access (SCMA) codebook for uplink wireless communications is presented by dividing an optimized 16-point round quadrature amplitude modulation (QAM) into several subsets. The main goal of the scheme is to maximize the minimum Euclidean distance (MED) and thus reduces the collisions of the information bits on the resources. The final SCMA codebook is obtained with the mapping matrix, which indicates the sub-constellations generating by dividing the mother QAM constellation. Simulation results show that, in a Nakagami fading channel, the optimized SCMA scheme by the proposed design method achieves significantly performance gains approximately 1.0, 1.4, 2.5, 3.5, and 4.0 dB at bit error rate (BER) of $10^{-4}$, respectively, when compared with those of an undivided 16-QAM constellation, a trellis code modulation (TCM) division, an original SCMA codebooks, a low-density signature (LDS), and an irregular LDS (IrLDS) schemes. In addition, at the signal-to-noise ratio (SNR) ranging from 0 to 10 dB, the constellation constrained capacity of the scheme by the proposed method achieves more gains over those of the original SCMA, TCM, the undivided star-QAM, the LDS, and the IrLDS schemes. Thus, it can be combined with the grant-free random access mechanism to obtain rapid and low-cost access in next-generation wireless packet services and other applications.

INDEX TERMS

SCMA, codebook design, constellation segmentation, subset division, minimum Euclidean distance.

I. INTRODUCTION

RECENTLY, fourth-generation (4G) communications with wireless orthogonal multiple access had difficulties efficiently satisfying the rapid development of all multimedia communication services. Consequently, fifth-generation (5G) mobile communications were then emerged and one of the related key services was the broadband Internet of Things (IoT) [1], which requires even much higher spectrum efficiency. Thus, sparse code multiple access (SCMA) [2] was proposed as one of the non-orthogonal multiple access technologies. SCMA is applied in services where numerous users are being simultaneously served on the same resource block, and it achieves massive capacity and connectivity [3]. In an SCMA system, orthogonal frequency division multiplexing is combined with a code division multiple access to enlarge more users in each time-frequency resource in the form of code division [4]. The SCMA-based scheme is also processed with considerably more robustness against the burst interference [5]. The main innovations of this scheme are its low-density spreading spectrum and high dimensional modulation [6]. The key of the SCMA system is the codebook design, which can improve the systematic capacity and the bit error ratio (BER) performance. Similar to a low-density signature (LDS) system, the main issue of the SCMA codebook design is the optimization of the complex multi-dimensional constellation.

The original SCMA codebook design was first proposed and designed for multi-dimensional constellations [7]. It was combined with a constellation rotation and a low-density spreading spectrum sequence. In addition, this study pro-
posed a sub-optimal codebook design that separated the mother constellation from the operator. However, defining layer number was difficult. Thus, in the HUAWEI opening SCMA codebook [8], the mother constellation points were configured on a straight line, and other user codebooks could be obtained by a single phase operation. However, high-order modulations required that the Euclidean distance of a user’s constellation should be large for improved BER performance in decoding. Therefore, a cognitive SCMA system with spectrum hole sensing was proposed to adapt the transmission by using the available subcarriers [9]. In [10], the multi-dimensional complex constellation was obtained by the Turbo trellis-coded modulation (TCM) to increase the minimum Euclidean distance (MED). The permutation operation of this scheme was determined by the phase rotation and the max sum of the interference codeword distance on the resource node. The codebook generated by this method had a lower BER, but larger calculations. Then, an undivided 16-quadrature amplitude modulation (QAM) constellation SCMA codebook was proposed by the new star-QAM constellation in [11], which had better BER performance and lower complexity than that in [7]. With a codebook design scheme at a Gaussian channel in [12], the total constellations of all users on each resource block were initially designed, and then all user constellations were obtained by the subset division of the TCM. The subset division was also combined with the low-density spread sequence to generate the constellation matrix (CM) for the final codebook. However, the BER increased with the growth of the overload, and the decoding complexity increased with the dimension growth of the codebook. In [13], a parallel SCMA system was proposed to reduce the decoding complexity by breaking the long SCMA codeword into several short ones. But it sacrificed the capacity gain for the purpose of low decoding complexity.

In this study, an optimized SCMA codebook design with QAM constellation segmentation is proposed. After comparing the MED of the phase-shift keying (PSK) with that of the QAM constellation, we use the optimized round 16-QAM constellation as the mother constellation. The sub-constellation is obtained by subset division and then combined with a factor matrix to generate a new SCMA codebook for good BER performance. And the proposed codebook design method is mainly used for the uplink SCMA system. The main contributions of this study are listed as follows.

- The mother constellation in the SCMA codebook is chosen as an optimized round 16-QAM constellation, which is extended from two rings to four rings. The positions of the constellation points are determined by the radius and the angle. Adjusting the radius and angles will balance the mother constellation between the minimum and maximum Euclidean distances of all constellation points, which can slightly improve the BER performance without any complexity increase.
- A new subset division is applied in the proposed mother constellation. Several sub-constellations can be obtained, and the MED is larger than that of the mother constellation. Therefore, it reduces the collision of information bits and has lower computational complexity.
- According to the factor graph, the mapping matrix and the user-subcarrier CM corresponding between the users and the subcarriers are obtained. In the original SCMA codebook, the sub-constellations are generated by rotating and then transposing the mother constellation. However, the new SCMA codebook is generated by the mapping and user-subcarrier constellation matrices. Compared with the original method, the proposed method maximizes the MED between the constellation points, and then improves the systematic SCMA performance and increases the constellation constrained capacity.

The rest of this paper is organized as follows. Section II introduces an SCMA system model. Section III presents the detailed procedures of the optimized SCMA codebook design. In this section, the mother constellation of an optimized round 16-QAM is proposed to maximize the MED for good BER performance. Simultaneously, the sub-constellations divided from the mother constellation are provided to increase the MED. Meanwhile, the optimized SCMA codebook is also produced and then analyzed for possible good performance. Section IV briefly introduces the message passing algorithm (MPA) at the receiver and the constellation constrained capacity with the proposed codebook. In Section V, the simulation results and analyses are presented to verify the good BER performance and constellation constrained capacity. Finally, Section VI concludes the paper.

II. SCMA SYSTEM MODEL

The SCMA encoder is defined as

\[ f : B_{\log_2(M)} \rightarrow \chi, x = f(b), \]

where \( \chi \) is a complex number set with set number \(|\chi| = M\) and \( \chi \subset \mathbb{C}^K \), and \( M \) denotes the number of constellation points for each user on a subcarrier. The superscript “\( \log_2(M) \)” of the set \( B \) in (1) indicates the bit number of the binary combination for each set element. \( K \) denotes the number of subcarriers. The vector \( b \) denotes the set of binary numbers. \( x \) is a sparse vector with an \( K \)-dimensional complex codeword from \( \chi \) and the number of non-zero value in a codeword is \( L (L < K) \). Let \( e \) denote an \( L \)-dimensional complex constellation point, and using (1), the mapping is expressed as

\[ g : B_{\log_2(M)} \rightarrow E, e = g(b), \]

where \( E \) is also a complex number set with \( E \subset \mathbb{C}^L \). The mapping \( g \) denotes the multi-dimensional constellation mapping, which indicates that the binary vector is mapped to a multi-dimensional constellation point set. Thus, the SCMA encoder can be redefined as

\[ f := Vg, \]
where the binary mapping matrix \( V \in \mathbb{B}^{K \times L} \) maps an \( L \)-dimensional complex constellation point to a \( K \)-dimensional SCMA codeword, or the mapping constellation point \( e \) to the non-zero codeword of \( x \). The matrix \( I_L \) is used to represent the mapping matrix \( V \), which is achieved by inserting \( K - L \) all-zero rows in \( I_L \) as in [14]. The generated SCMA codebook contains \( M \) codewords, with each codeword having \( K \) complex values, where there are \( L \) non-zero values. The dimension of non-zero values is determined by the mapping matrix \( V = [V_1, V_2, \ldots, V_j] \) and \( J \) is the user number. \( V_j \) is the mapping matrix of user \( j \), \( j = 1, 2, \ldots, J \).

The diagram of a typical SCMA system is shown in Fig.1, where the MPA is defined as the message passing algorithm in [15]. In this scenario, six users transmitting on four subcarriers are expressed as \((J, M, L, K)=(6, 4, 2, 4)\), where \( J \) denotes the number of users. The codebook of each user is different, but they are all \( 4 \times 4 \) complex matrices. If a single user transmits 2-bit information every time by the binary bits, then there will be four possible combinations of “00”, “01”, “10”, and “11”, corresponding to the four codewords of a codebook. In addition, the codeword is the column vector of the complex matrix. Hence, six users transmit six combinations of \((1, 1), (1, 0), (0, 1), (1, 1), (1, 0), \) and \((0, 0)\) simultaneously, which relate to the \(4, 3, 2, 4, 3, 1\) columns of the codebook, respectively. The different colors in the complex matrix represent different values. The white color respects the zero value, while the other colors respect non-zero values in the matrix. The row of complex matrices is related to the four subcarriers, which represent the corresponding resource blocks.

The signals on subcarrier \( i \) at the receiver can then be expressed as

\[
y_i = \sum_{j=1}^{J} h_{i,j} \cdot C_{i,j}(m_j) + n_i. \tag{4}
\]

In (4), the channel coefficient of user \( j \) with subcarrier \( i \) is denoted as \( h_{i,j} \), \( i \) given ideal channel condition. \( C_{i,j}(m_j) \) denotes the \( i \)-th value in the codeword \( m_j \), which is expressed as the selection of user \( j \). \( n_i \) is the noise on the subcarrier \( i \).

Therefore, just as an example in Fig. 1, the received signal on subcarrier 2 is expressed as

\[
y_2 = h_{2,1}C_{2,1}(m_1) + h_{2,4}C_{2,4}(m_4) + h_{2,5}C_{2,5}(m_5) + n_2. \tag{5}
\]

III. OPTIMIZED SCMA CODEBOOK DESIGN

Generally, an SCMA system has excellent performance and high spectral efficiency due to multi-dimensional constellations and low density signature [16]. Therefore, a new SCMA codebook design can be implemented through a segmentation of the optimized QAM constellation subsets. Alternatively, the optimized 16 round QAM mother constellation can be divided to obtain four subconstellations to maximize the MED. Such a division aims to reduce the possibility of collisions on subcarriers and thus improve the accuracy of decoding. The optimized procedure can be described as follows.

A. OPTIMIZATION MODELING FOR THE SCMA CODEBOOK DESIGN

Dividing the optimized round QAM mother constellation can obtain several sub-constellations to maximize MED. Thus, this method is employed in this study for optimization modeling of the SCMA codebook. Optimization modeling aims to reduce the collision possibility of information bits on the subcarriers and thus improve the decoding accuracy. The key of the SCMA codebook design is an appropriate mother constellation template. Thus, the optimization goal is to maximize the MED between the constellation points in the constellation.

The Euclidean distance is calculated as

\[
d_{\text{min}} = \min\{|a_i - a_j|^2 + |b_i - b_j|^2, i \neq j\}. \tag{6}
\]

where \( a_i \) and \( a_j \) donate the abscissa of the constellation points \( i \) and \( j \), \( b_i \) and \( b_j \) donate the ordinate of the constellation points \( i \) and \( j \), respectively.

Then, the MEDs of the high-order 16-QAM and 16-PSK are calculated respectively as

\[
d_{16QAM,\text{min}} = \sqrt{2}A/3 = 0.47A, \tag{7}
\]

\[
d_{16PSK,\text{min}} = 2A\sin(\pi/16) = 0.39A, \tag{8}
\]

where \( A \) is the maximum amplitude of the modulation.

Therefore, because of the largest MED of the 16-QAM among these modulations, it is chosen as the mother constellation in the proposed scheme. The constellation of a QAM
signal has three major forms, namely, round, uneven round, and square constellations. Different QAM constellations have different MEDs (i.e., $d_{\text{min}}$). Hence, the optimization target is to increase $d_{\text{min}}$ as large as possible by changing the amplitude and phase of the QAM modulation, which is just the design of the mother constellation. Then, in the optimized SCMA codebook design, we propose a new method wherein the optimized round mother QAM constellation is divided to obtain the sub-constellations.

**B. PROPOSED SCMA CODEBOOK DESIGN BY CONSTELLATION SEGMENTATION**

The optimization objective for various QAM modulations is to maximize MED. The optimization procedures can be described as follows. First, the mapping matrix is constructed based on the number of users and subcarriers. After that, the mother constellation is chosen and subsequently optimized by the maximization of the MED. The sub-constellations are then obtained by dividing the mother constellation. Finally, the SCMA codebook is obtained by the mapping and constellation matrices. The detailed procedures are concluded in four main steps and listed as follows.

1) Generation of Mapping Matrix

The mapping matrix determines the relationship between the users and the subcarriers and decides on the MPA detection complexity. The codeword is sparse, and the complexity of the receiver detection is low. The parameters and rules of the mapping matrix design are listed below.

i) $V_j \in B^{K \times L}$, where $B$ denotes a binary matrix.

ii) $\forall i \neq j, V_i \neq V_j$.

iii) $V_j^{(0)} = I_L$, where $V_j^{(0)}$ is the matrix after deleting full-zero rows of $V_j$. The design of the mapping matrix $V_j$ is simplified as inserting $K - L$ full-zero rows within the identity matrix $I_L$.

However, each layer of the mapping matrix ($V_j$) can also be deduced from the factor matrix $F$ [17], [18], which consists of all layers of the mapping matrices and is expressed as

$$F = [\text{diag}(V_1^T), \text{diag}(V_2^T), \ldots, \text{diag}(V_J^T)]. \quad (9)$$

Example 1: The factor graph of an SCMA system with six users and four subcarriers, as shown in Fig. 1, is illustrated in Fig. 2. The related factor matrix is expressed in (10).

$$F = \begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 \\
\end{bmatrix}. \quad (10)$$

In Fig. 2, the user node and the subcarrier node are connected if and only if $F_{i,j} = 1$. Here, $F_{i,j}$ denotes the element of the $i$-th row and the $j$-th column of the matrix; $F_{2,j}$ indicates that subcarrier 2 is used to transmit the information bits of users 1, 4, and 5; $F_{1,3}$ indicates that the information of user 3 is only transmitted on subcarriers 1 and 4.

The position of all-zero rows can be determined according to the $j$-th column of the factor matrix $F$. It is in the position of element “0” in the $j$-th column of the factor matrix $F$. Then the all-zero row in $L$-order identity matrix $I_L$ is inserted in the above position, and the mapping matrix $V_j$ is obtained.

Example 2: From (10), there is $L = 2$, where $L$ is the number of non-zero values in the column of the factor matrix. Then the mapping matrices are obtained by $I_2$ and are represented as follows.

$$V_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ \end{bmatrix}, \quad V_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ \end{bmatrix}, \quad V_3 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \end{bmatrix}.$$  

$$V_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ \end{bmatrix}, \quad V_5 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \end{bmatrix}, \quad V_6 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ \end{bmatrix}. \quad (11)$$

Thereby, the maximum number of users is expressed as

$$J_{\text{max}} = C_L^K = \frac{K \cdot (K - 1) \cdots (K - L + 1)}{L \cdot (L - 1) \cdots 1}. \quad (12)$$

Then, the number of all users on a single subcarrier is

$$U = JL/K, \quad (13)$$

where $J \leq J_{\text{max}}$.

Moreover, the overload rate of a user is

$$\lambda = J/K = U/L. \quad (14)$$

Generally, a check matrix of a low-density parity-check (LDPC) code can be used as this class of mapping matrix. Thus, the mapping matrix can be selected from the designed LDPC matrix, just as described in [20].

2) Optimization of the Mother SCMA Constellation

In general, the mother constellation can be any constellation with a large MED. A general optimization can be applied on the multi-dimensional constellation of any constellation with a desirable Euclidean distance. The constellation points in a round QAM constellation are divided into the rings with $R_i$, which is the $i$-th closest to the center of the circle. The first quadrant point can be connected in a straight line to the
third quadrant, and the remaining constellation points are distributed on the abscissa axis.

Example 3: The round constellation of a 16-QAM is shown in Fig. 3(a). It is optimized by extending the two rings into four rings, as shown in Fig. 3(b).

In Fig. 3(b), we set \( |OA| = R_1, |OB| = R_2, |OC| = R_3, |OD| = R_4 \), and let \( \alpha = \frac{R_1}{R_3} = \frac{R_2}{R_4} = 3 \), \( \beta = \frac{R_1}{R_4} = 1.587 \), and \( \theta = 22.5^\circ \) according to [11], [21], [22].

The average signal energy is usually defined as \( E = \frac{1}{M} \sum_{i=1}^{M} \| x_i \|^2 \); thus, there is

\[
E = 0.25(R_1^2 + R_2^2 + R_3^2 + R_4^2).
\]

Based on (6), the standardized MED [23] is calculated as

\[
r = \frac{d_{\text{max}}}{\sqrt{E}} = \frac{\sqrt{0.25(R_1^2 + R_2^2 + R_3^2 + R_4^2) \cos^2 \theta}}{\sqrt{0.25(R_1^2 + R_2^2 + R_3^2 + R_4^2)}} = \sqrt{\frac{4R_1^2}{(1 + \alpha^2)(1 + \beta^2)}}.
\]

Thus the standardized maximum Euclidean distance is

\[
l = \frac{d_{\text{max}}}{\sqrt{E}} = \frac{4R_1}{(1 + \alpha^2)(1 + \beta^2)} = \frac{16\alpha\beta^3}{(1 + \alpha^2)(1 + \beta^2)}.
\]

Based on (16) and (17), when \( \theta \) is determined, a larger \( r \) results in a smaller \( r \) and a larger \( l \). Therefore, \( \alpha = 3 \) and \( \beta = 1.587 \) are good choice for the balance between the MED and the standard maximum Euclidean distance [11]. For general cases, the coordinate set of the optimized constellation points from large to small is written as

\[
S_p = \{ \alpha \frac{\beta^2}{2} - 2x_2, \alpha \frac{\beta^3}{4} - 2x_2, \alpha \frac{\beta^4}{8} - 2x_1, \ldots, \alpha x_2, \alpha x_1, x_2, x_1 \},
\]

where \( S_p \) represents the abscissa axis set of constellation points on the positive horizontal axis, and \( x_2 = R_2 = \beta x_1 = \beta R_1 \). The coordinates of the optimized constellation points in order on the negative abscissa axis are expressed as

\[
S_n = -S_p.
\]

Finally, the coordinates of all constellation points are obtained based on (18) and (19).

3) Generation of sub-constellations by division

Sparse coding is used in practical transmissions to ensure that the SCMA system supports massive users. Simultaneously, each subcarrier transmits user codewords corresponding to different subconstellations to guarantee the decoding accuracy in the receiver. Let \( m \) be the number of bits for a single user to transmit at each time, and the constellation points for occupation is \( M = 2^m \). To avoid decoding confusion, it is required that there is \( df > U \), where \( df \) is the number of sub-constellations and \( U \) is the actual number of users on a single subcarrier. Then the total number of constellation points on each subcarrier is \( MU \). Usually, two sub-constellations are obtained after the first-order division. Then, we perform successive divisions on the subcarriers to further maximize the MED until enough sub-constellations are obtained.

4) SCMA codebook generation by sub-constellation mapping

In the above division procedure, there are \( df \) subsets generated by the mother constellation, and the set of constellation points is expressed as \( S_{1,1}, S_{2,1}, S_{3,1}, \ldots, S_{df,1} \). The CM is composed of sub-matrix \( CM_{i,j} \), where \( i \) denotes the subcarrier corresponding to the \( i \)-th row of the matrix, and \( j \) denotes the user corresponding to the \( j \)-th column of the matrix. Therefore,

\[
CM_{i,j} = \begin{cases} S_{i,p}, & F_{i,j} = 1 \\ 0, & F_{i,j} = 0 \end{cases},
\]

where \( S_{i,p} \) is the \( p \)-th value after random reordering of the set of \( \{ S_{1,1}, S_{2,1}, S_{3,1}, \ldots, S_{df,1} \} \). Then the element “1” in the factor matrix \( F \) is replaced with \( S_{i,p} \). Combined with the user-subcarrier CM, \( C_{i,j}(m_j) \) is related to the user \( j \) who corresponds to the \( i \)-th value of the codeword \( m_j \). Thus, the CM is expressed as

\[
C_{i,j}(m_j) = \begin{cases} S_{i,p}(m_j), & F_{i,j} = 1 \\ 0, & F_{i,j} = 0 \end{cases}, \quad j \in [1, J].
\]

Finally, the codebook is obtained by all \( C_{i,j}(m_j) \).

C. ANALYSIS OF THE 16-QAM SCMA CONSTELLATION FOR SCMA CODEBOOK DESIGN

The mother constellation is chosen as the optimized round 16-QAM constellation in Fig. 3(b). The proposed division scheme is then shown in Fig. 4. This scheme can be represented in two steps. First, a 16-QAM constellation is divided into two 8-point constellations, which are then further separated into four 4-point sub-constellations \( \{ C_i, i = 1, \ldots, 4 \} \).
In the mother 16-QAM constellation, $R_1 = 1$, $R_2 = 1.5873$, $R_3 = 3$, and $R_4 = 4.7619$. Hence the MED of the mother constellation is 0.39. After division, the MED of the aforementioned sub-constellations $\{C_1, C_2, C_3, C_4\}$ are 2.111, 3.351, 2.111, and 3.351, respectively, according to (6). Thus, the MED of the sub-constellation is obviously increased compared with that of the mother QAM constellation.

From (20), subcarriers and sub-constellations are obtained from Fig.5. For each user, the elements consist of “0” and “1” randomly. Therefore, the aforementioned user subcarrier CM is not unique. The decoding accuracy at the receiver is ensured in case of overload transmission, because different users on the subcarrier correspond to different constellations.

From (21), $C_{i,2}(4)$ indicates that user 2 corresponds to the $i$-th value of codeword 4, which is related to the coordinates of the constellation in the third quadrant. Thus, the first value of codeword 4 is the coordinates of the constellation point in the third quadrant for user 2 on subcarrier 1 ($i=1$), that is, $-0.9238 - 0.3827i$. This value is calculated from Fig. 4 and the above equation (21). Other coordinates can also be calculated similarly by these methods. The second value ($i=2$) is 0. The third value ($i=3$) is the coordinates of the constellation point in the third quadrant for user 2 on subcarrier 3, that is, $-1.4665 - 0.6075i$. The fourth value ($i=4$) is 0. Finally, the value of codeword 4 in the codebook of user 2 is computed as

$$C_{i,2}(4) = \begin{bmatrix} -0.9238 - 0.3827i \\ 0 \\ -1.4665 - 0.6075i \\ 0 \end{bmatrix} . \quad (22)$$

Similarly, $C_{1,2}(m_2)$ denotes the first row of user 2’s codeword. In other words, $C_{1,2}(m_2)$ is the coordinates of the four sub-constellation points of user 2 on subcarrier 1. These coordinates are

$$C_{1,2}(m_2) = [3 \quad 0.9238 + 0.3827i \quad -3 \quad -0.9238 - 0.3827i]. \quad (23)$$

Tab. 1 summarizes and shows the SCMA codebook of an optimized round 16-QAM modulation after division when $(J, M, L, K) = (6, 4, 2, 4)$. And the $i$-th user is briefly represented as $U_i$.

### IV. PERFORMANCE ANALYSES OF THE PROPOSED SCMA CODEBOOK

#### A. PERFORMANCE ANALYSES OF THE MPA AT THE RECEIVER

The MPA receiver adopts the sum-product algorithm in [17]. The processing of information flow for the scheme is shown in Fig. 6, where the resource node can be the subcarrier, and $\mu_{j \rightarrow k}$ is the cost function of the message, which is sent from user node $j$ to resource node $k$. So the message sent from resource node $k$ to check node $c$ is expressed as

![FIGURE 5: Constellations of the different user-subcarriers.](image-url)
where $\Phi(k) \setminus \{c\}$ denotes all user nodes connecting to resource node $k$ except user node $c$. Then, the message transformed from check node $c$ to resource node $k$ is expressed as

$$
\mu_{c \rightarrow k} = \sum \left( \mu_{k \rightarrow c} \cdot \prod_{l \in \Theta(c) \setminus \{k\}} \mu_{k_{l} \rightarrow c} \right),
$$

(25)

where $\mu_{k_{l} \rightarrow c}$ is the cost function of the messages from the variable node $k$ to the check node $c$. $\Theta(c) \setminus \{k\}$ represents all variable nodes connecting to the check node $c$ except the variable node $k$.

The check and variable nodes participate in the processing of the MPA decoding. In each decoding iteration, a message arriving on one (input) edge is directly sent to other (output) edges. The output information from the user is compared with the received information from the variable node, and then the accurate decoding is given. In one iteration, the complexity of MPA is calculated as $L \cdot M^{d_{j}}$. Thus, the entire complexity is proportional to

$$
C = \lambda \cdot L \cdot M^{d_{j}},
$$

(26)

where $\lambda$ is the number of decoding iterations. From (26), the complexity of MPA is increased with the growth of the decoding iterations.

**B. CONSTELLATION CONSTRAINED CAPACITY OF THE SCMA WITH THE PROPOSED CODEBOOK**

In a Gaussian channel, the SCMA constellation constrained capacity is expressed by the mutual information of the input $x$ and output $y$. For user 1, this capacity is expressed as

$$
I(x_{1}, y) = H(y) - H(y \mid x_{1}),
$$

(27)

where there are

$$
H(y) = - \int p(y) \log_{2}(p(y)) dy,
$$

(28)

$$
p(y) = \frac{1}{M^{6}} \sum_{x} p(y \mid x),
$$

(29)

$$
H(y \mid x_{1}) = \frac{1}{M} \sum_{i=1}^{M} H(y \mid x_{1} = x_{1}(i))
$$

$$
= - \frac{1}{M} \sum_{i=1}^{M} \int p(y \mid x_{1} = x_{1}(i)) \log_{2}\{p(y \mid x_{1} = x_{1}(i))\} dy,
$$

(30)

$$
p[y \mid x_{1}(i)] = \frac{1}{M^{6}} \sum_{x \neq x_{1}} p(y \mid x - x_{1}, x_{1}(i)).
$$

(31)

Based on the factor matrix in (4), the conditional probabilities are deduced as follows:

$$
p(y \mid x) = p(y_{1} \mid x) p(y_{2} \mid y_{1}, x),
p(y_{3} \mid y_{1}, y_{2}, x) = p(y_{3} \mid y_{1}, y_{2}, z_{x}),
p(y_{1} \mid x_{11}, x_{12}, x_{13}) = p(y_{2} \mid x_{21}, x_{24}, x_{25}),
p(y_{3} \mid x_{32}, x_{34}, x_{36}) = p(y_{4} \mid x_{43}, x_{45}, x_{46}),
$$

$$
CN(y_{1} : x_{11} + z_{12} + x_{13}, \sigma^{2}) CN(y_{2} : x_{21} + x_{24} + x_{25}, \sigma^{2}),
$$

$$
CN(y_{3} : x_{32} + x_{34} + x_{36}, \sigma^{2}) CN(y_{4} : x_{43} + x_{45} + x_{46}, \sigma^{2})
$$

(32)

where $CN$ denotes the complex Gaussian distribution [24], on condition that $x_{ij} = h_{i,j} \cdot C_{i,j}(m_{j})$. $x$ denotes the codewords of users, and $x_{1}$ denotes the codewords of all users except user 1, $\sigma^{2}$ is the variance of noise $n$. Finally, the constellation constrained capacity of a single user can be obtained as

$$
t_{1} = - \frac{|\Delta_{123} + z_{1}|^{2} - |\Delta_{145} + z_{2}|^{2}}{|\Delta_{246} + z_{3} - |\Delta_{356} + z_{4}|^{2}} ,
$$

(34)

$$
t_{2} = - \frac{|\Delta_{123} + z_{1}|^{2} - |\Delta_{145} + z_{2}|^{2}}{|\Delta_{246} + z_{3} - |\Delta_{356} + z_{4}|^{2}} ,
$$

(35)

$$
I(x_{1}, y) = \log_{2}(M) - \frac{1}{M^{6}} \sum_{x} E \left\{ \log_{2} \left( \sum_{x \neq x_{1}} \frac{\exp(t_{1} / \sigma^{2})}{\exp(t_{2} / \sigma^{2})} \right) \right\},
$$

(36)

where $t_{1}$ and $t_{2}$ are temporal variable expressed in (34) and (35). $\Delta$ is equal to the sum of codewords part of the received signal minus the codewords for corresponding users. For example, $\Delta_{123}$ is the sum of codewords difference from user 1, 2 and 5. The expectation $E(\cdot)$ is calculated with respect to the complex Gaussian distributions of $z_{1}$, $z_{2}$, $z_{3}$, and $z_{4}$, where $z_{i}$ is with the complex Gaussian distribution $N(y_{i} ; \Delta, \sigma^{2})$.

**V. NUMERICAL SIMULATIONS AND RESULT ANALYSES**

In this section, we give the simulations of the SCMA schemes by the proposed divided optimization codebook design, the SCMA codebook in [7], the undivided 16 star-QAM constellation SCMA codebook in [11], and the SCMA codebook of
TABLE 2: Simulation Parameters for the SCMA schemes.

<table>
<thead>
<tr>
<th>Parameter items</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel type</td>
<td>Nakagami channel with fading coefficient 2, Rayleigh channel with fading coefficient 1</td>
</tr>
<tr>
<td>Number of symbols/frame</td>
<td>4096</td>
</tr>
<tr>
<td>Total simulation cycles</td>
<td>1000</td>
</tr>
<tr>
<td>Number of users</td>
<td>6</td>
</tr>
<tr>
<td>Number of subcarriers</td>
<td>4</td>
</tr>
<tr>
<td>Number of iterations</td>
<td>15</td>
</tr>
<tr>
<td>Overload factor</td>
<td>150%</td>
</tr>
<tr>
<td>Code rate</td>
<td>1/2</td>
</tr>
</tbody>
</table>

FIGURE 7: BER performance comparison among different SCMA codebooks.

The proposed design method is also compared with those of the SCMA, the LDS, and the irregular LDS (IrLDS) in [24] to verify its good performance in capacity improvement. Tab. 2 presents the simulation parameters given the SCMA scheme parameters of \((J, M, L, K)\) as \((6, 4, 2, 4)\), respectively.

With the above simulation parameters, the experimental results of the BER comparison among different SCMA codebooks in a Nakagami fading channel are shown in Fig. 7.

Fig. 7 reveals that the BER performance of the new designed SCMA codebook is obviously superior to that of the SCMA codebook in [7]. At BER of \(10^{-4}\), the system performance is significantly improved by 2.5 dB. When compared with the SCMA codebook of an undivided 16-QAM constellation in [11], the codebook of a 16-QAM constellation based on TCM subset division in [12], the LDS and IrLDS schemes in [24], the SCMA scheme of the proposed codebook also improves considerably in performance gains by as much as 1.0, 1.4, 3.5 and 4.0 dB at BER of \(10^{-4}\), respectively. Also in more experiments, the BER performances of the proposed SCMA codebooks in Nakagami fading channels with fading coefficients greater than 2 are still better than those in Rayleigh fading channels with fading coefficient of 1. The improvements can be explained as follows. The MED is 2.111, which is much larger by using the optimized 16-QAM round constellation as the mother constellation. It is then divided to achieve the aforementioned sub-constellations. Based on (6), the MEDs in [7], [11], [12] can be calculated, with values of 1, 1.765, and 1.414, respectively. The MED of the proposed scheme is about 1.196 times of the largest one (i.e., the MED in [11]). In summary, the comparison of MEDs from different codebooks are listed in Tab. 3 with relationship between MED and SNR at BER of \(10^{-4}\). And the item “N/A” in the table indicates that it is hard to be calculated.

The performance improvement can likewise be obtained as follows. First, the mother constellation in the proposed scheme is chosen as an optimized round 16-QAM constellation and is then followed by a new subset division. After that, several sub-constellations can be obtained, whose MEDs are larger than those of the mother constellation. Given that the larger MED directly contributes to an improved BER performance, the proposed scheme thus obtains the best performance among these schemes. The information bits of users are mapped to the sparse codewords transmitted on the resource blocks, and different users correspond to different codebooks, which reduces the collision of information bits and thus guarantees the decoding accuracy. Therefore, the SCMA schemes by the proposed codebook possess all the advantages of the codebooks in [11] and [12], especially in the case of high signal-to-noise ratios (SNRs), for better BER performance.

In the proposed scheme, the mother constellation selects \(\alpha = R_4/R_2 = R_5/R_1 = 3\) and \(\beta = R_2/R_1 = R_4/R_3 = 1.587\). From (16) and (17), a larger \(\alpha\) results in a smaller \(r\), but the reverse is the outcome for \(l\). Therefore, \(\alpha = 3\) is a suitable choice. To verify this result, the BER performance of different schemes in both Nakagami and Rayleigh channels with the parameters of \(\alpha = 2, \alpha = 3,\) and \(\alpha = 4\) are compared in Fig. 8.

At the same SNR, the scheme with \(\alpha = 3\) achieves the best BER performance, whereas the other schemes with \(\alpha = 2\) and \(\alpha = 4\) have rather poor BER performances in a Nakagami fading channel or a Rayleigh channel. From (16) and (17), compared with the scheme of \(\alpha = 2\), the scheme of \(\alpha = 4\) has a smaller MED, but its maximum Euclidean distance is still larger. Additionally, the scheme with \(\alpha = 2\) has a larger MED, but its maximum Euclidean distance is comparatively smaller. However, \(\alpha = 3\) reaches a balance between minimum and maximum Euclidean distances. The

TABLE 3: Comparison of MEDs from the different codebooks.

<table>
<thead>
<tr>
<th>Codebooks</th>
<th>MED</th>
<th>SNR at BER of (10^{-4}) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed SCMA</td>
<td>2.111</td>
<td>10.0</td>
</tr>
<tr>
<td>SCMA with 16-QAM</td>
<td>1.765</td>
<td>11.0</td>
</tr>
<tr>
<td>SCMA with TCM division</td>
<td>1.414</td>
<td>11.4</td>
</tr>
<tr>
<td>Original SCMA</td>
<td>1.0</td>
<td>12.5</td>
</tr>
<tr>
<td>LDS</td>
<td>N/A</td>
<td>13.5</td>
</tr>
<tr>
<td>IrLDS</td>
<td>N/A</td>
<td>14.0</td>
</tr>
</tbody>
</table>

The improvements can be explained as follows. The MED is 2.111, which is much larger by using the optimized 16-QAM round constellation as the mother constellation. It is then divided to achieve the aforementioned sub-constellations. Based on (6), the MEDs in [7], [11], [12] can be calculated, with values of 1, 1.765, and 1.414, respectively. The MED of the proposed scheme is about 1.196 times of the largest one (i.e., the MED in [11]). In summary, the comparison of MEDs from different codebooks are listed in Tab. 3 with relationship between MED and SNR at BER of \(10^{-4}\). And the item “N/A” in the table indicates that it is hard to be calculated.

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The proposed scheme optimizes a round 16-QAM constellation by extending two rings to four rings, which is taken as the mother constellation. By adjusting the radius and angles, the mother constellation is then compromised between the minimum and maximum Euclidean distances. Given that the minimum and maximum Euclidean distances affect the collision probability of the information bits and the inter-symbol interference, which determines the final performance, the proposed scheme can thus achieve comparatively excellent performance.

Fig. 9 also shows the comparison of BER performance between the schemes with the parameters of $\beta = 1.4$, $\beta = 1.587$, and $\beta = 1.8$ in both Nakagami and Rayleigh channels.

From Fig. 9, the BER of $\beta = 1.587$ in both channels are smaller than those of others. This finding is attributed to the larger $\beta$, which results in a smaller standard MED $r$ and a larger maximum Euclidean distance $l$, which is also based on (16) and (17). However, it is required in the proposed scheme that the minimum and the maximum Euclidean distance should be as large as possible. So $\beta$ should not be increased infinitely. Here, the choice of $\beta = 1.587$ leads to the compromise between the minimum and the maximum Euclidean distances and thus achieves better BER performance.

Different from the general constellation constrained capacity calculation, the capacity of the SCMA is expressed by the mutual information. Based on equations (24) to (30), the simulation results of constellation constrained capacity using the Monte Carlo integration method are shown in Fig. 10.

Fig. 10 compares the single-user constellation constrained capacity of the proposed SCMA scheme and other existing schemes. In Fig. 10, the constellation constrained capacity of the new proposed SCMA scheme is larger than those of the original SCMA, the LDS, and the IrLDS schemes in [24]. At the low SNR of 0 dB, the constellation constrained capacity of the new scheme is almost equal to those of existing SCMA methods. As the SNR increases, the proposed SCMA is superior to other schemes. At the high SNR of 10 dB, the constellation constrained capacity also improves as much as 0.05, 0.10, 0.10, 0.15 and 0.4 bit/symbol, respectively, when compared with SCMA methods with 16-QAM, TCM, original SCMA, LDS, and IrLDS in [24]. The main reason lies in the fact that we directly optimize the mother constellation and divide it into sub-constellations to achieve better performance instead of leveraging vector transform as the design metric. However, in the original SCMA codebook, the sub-constellations are just generated by rotating and then transposing the mother constellation. Simultaneously, the sparseness of the codewords leads to more users transmitting information. Thus, the information bits from more users are
carried at each resource block carrier for even larger amounts of transmission throughput.

VI. CONCLUSION

This paper mainly introduces a new SCMA codebook design method with large constellation constrained capacity, good decoding accuracy, and low complexity. The main procedures of the codebook design are as follows. 1) Generate a mapping matrix. 2) Determine the mother constellation by maximizing the MED criterion and then performing division to obtain the sub-constellation. 3) Combine the sub-constellation with the mapping matrix and then generate the new SCMA codebook.

The main innovations are the optimized 16-QAM round mother constellation design, the subset division method to obtain sub-constellations, and the mapping matrix to generate the new SCMA codebook. Thus, the proposed method maximizes the MED among user constellation points and improves the decoding. The proposed codebook significantly improved the SNR gains compared with the current ones. Compared with the SCMA codebooks of an undivided 16-QAM constellation, and an TCM division codebook, the performance of the codebook by the proposed method is also enhanced greatly. It also has higher constellation constrained capacity than those of the current SCMA, LDS and IrLDS schemes. Therefore, the proposed scheme can be efficiently used in the SCMA codebook design for the 5G wireless communications, future IoT, and other applications.

REFERENCES

S. Liu et al.: Optimized SCMA Codebook Design by QAM Constellation Segmentation with Maximized MED

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