Fast Joint Estimation of Time of Arrival and Angle of Arrival in Complex Multipath Environment Using OFDM

Haiyun Xu, Yankui Zhang, Bin Ba, Daming Wang and Xiangzhi Li

Abstract—Joint time of arrival and angle of arrival estimation based on a uniform circle array (UCA) involves problems including high computational complexity in the multi-dimensional spectral search method and lack of applicability in coherent multipath environments. In this paper, we introduce a fast estimation algorithm based on the orthogonal frequency division multiplexing system that uses space-frequency characteristics to reconstruct the virtual array and extend the array aperture. First, we combine the array structure with the signal frequency features to construct the extended virtual array. Then we calculate the channel frequency response covariance matrix via frequency-domain smoothing preprocessing and determine a closed-form solution of angle of arrival using UCA estimating signal parameters via rotational invariance techniques. Finally, using the estimated angle values, we perform a one-dimensional spectral search to determine the time of arrival values. The simulation results indicate that the proposed method provides accurate estimation under the low signal-to-noise ratio conditions in coherent, independent or mixed multipath environments, and provides better performance than the multi-dimensional spectral peak search method, in terms of both computational complexity and estimation accuracy.

Index Terms—Time-of-arrival, angle-of-arrival, space-frequency characteristics, channel frequency response.

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is a multi-carrier digital modulation technology with high frequency band utilization that can effectively counter frequency selective fading. OFDM is widely used in applications such as underwater acoustic communications [1], the 4th generation mobile communications, IEEE 802.11 wireless local area networks [2]. OFDM systems provide users with data service while also providing location based service. Time of arrival (TOA) and angle of arrival (AOA) are significant parameters in location systems, such as radar [3] and indoor navigation [4]. Similar to the time-domain narrowband sources model, the frequency-domain AOA estimation method has been investigated [5] using the multiple signal classification (MUSIC) algorithm, but the performance of this method is restricted by the array aperture size. A method for TOA estimation was proposed in [6] for application to OFDM systems to become more complex.

In this paper, we focus on the joint estimation of the TOA and the AOA in OFDM systems in uniform circle array (UCA). The UCA can get not only two-dimensional (2D) angle estimation but also estimates the robust azimuth angle performance. However, the multipath environment of interest has two possible situations, i.e., independent and coherent, and the existing works in the literature mostly researched on the independent multipath environment, which is the necessary condition for the use of subspace algorithms. In the coherent multipath environment, spatial smoothing techniques [15] can solve the problem for the ULA. However, for the UCA, spatial smoothing was usually applied to perform azimuth angle estimation after the UCA was transformed into a virtual ULA [16, 17]. To estimate the 2D angle, [18] used a delay line, while [19] constructed auxiliary UCAs in the vertical direction. They all calculated multiple covariance matrices and applied spatial smoothing method, but all of these approaches caused systems to become more complex.

We therefore present a joint estimation algorithm that can be applied to coherent, independent or mixed multipath environments. The key aims of our work are to improve the estimation precision and also reduce the complexity caused by use of the spectral search methods. The responses of multiple subcarriers combined with that of the array can be used to reconstruct the signals, but this method cannot be used in multipath environments. While the researchers have applied super resolution algorithms like MUSIC [7], root-MUSIC [8], the propagator method (PM) [9], estimating signal parameters via rotational invariance techniques (ESPRIT) [10], and compressive sensing [11, 12], they produce little enhancement of the estimation accuracy because of signal frequency band limitations.

Joint estimation of the TOA and the AOA using space-time parameter coupling features can improve the precision and also reduce the number of nodes required in location systems. Therefore, the joint estimation algorithms used are significant. The algorithm proposed in [13] solved for the TOA and then estimated the AOA based on the trigonometric geometry of the time delay inequality under the condition of wideband signal conditions, but the performance showed little obvious improvement. In addition, a method was proposed in [14] that provided an extended channel frequency response using a Hadamard product and obtained high-precision joint estimation in OFDM systems. However, with the requirement for a total field-of-view search, the computational complexity of this method was high and the uniform line array (ULA) can only estimate the azimuth angle.
extended virtual array response and perform highly precise estimation. Given that UCA-ESPRIT has no spectral peak searching requirement, we use a closed-form solution and thus reduce the computational complexity. The remainder of the paper is organized as follows. We first briefly present a signal model in Section 2. In Section 3, we explain our estimation approach for the TOA and the AOA, and describe the steps of the proposed algorithm. An analysis of the computational complexity is presented in Section 4. In Section 5, we present the results of the simulations. Finally, we summarize the work in Section 6.

Throughout this paper, $I_N$ represents the $N$ dimensional unit array; $[\cdot]^T$ and $[\cdot]^H$ denote a transpose, and a Hermitian transpose, respectively. Finally, diag$[\cdot]$ represents a vector transforming into a diagonal matrix.

II. SIGNAL MODEL

A multipath radio propagation channel is generally modeled as a complex low-pass equivalent impulse response. The response of the $m$th sensor in the $s$th time interval is given by

$$h_{m}^{(s)}(t) = \sum_{k=0}^{K-1} \alpha_k^{(s)} e^{j\beta_k^{(s)}} \delta(t - \tau_k - \xi_{k,m}),$$

(1)

where $K$ is the number of multipath components, $\alpha_k^{(s)} e^{j\beta_k^{(s)}}$ represents the complex attenuation of the $k$th path, $\alpha_k$ is the amplitude, $\beta_k$ is the phase, which is subject to a uniform distribution with density function $U(0, 2\pi)$, and $\tau_k$ is the propagation delay of the $k$th path. The relative delay of the $k$th path and the $m$th sensor can be represented by $\xi_{k,m} = r \sin(\theta_k) \cos(\varphi_k - 2\pi M)/c$, where $r$ is the radius of the UCA, $M$ is the number of sensors used, $c$ is the speed of light and $(\theta_k, \varphi_k)$ is the direction from which the plane wave is impinging. The reference sensors in the UCA are illustrated in Figure 1.

![Fig. 1. Signal arrival to uniform circle array](image)

Assume here that the number of OFDM subcarriers is $L$. If we take the Fourier transform of (1), the channel frequency response of the $l$th subcarrier and the $m$th sensor can be expressed as

$$H_{l,m}^{(s)} = \sum_{k=0}^{K-1} \alpha_k^{(s)} e^{j\beta_k^{(s)}} e^{-j2\pi(f_c+\Delta f_l)(\tau_k+\xi_{k,m})} + n_{l,m}^{(s)},$$

(2)

where $f_c$ is the carrier frequency, $\Delta f$ is the OFDM subcarrier spacing and $n_{l,m}^{(s)}$ is additive white Gaussian noise with power of $\sigma^2$. From (2), the channel frequency response of $l$th subcarrier is given by

$$H_l^{(s)} = A_l^{(s)}(\tau, \xi) \rho^{(s)} + n_l^{(s)},$$

(3)

where

$$\tau = (\tau_0 \quad \tau_1 \quad \cdots \quad \tau_{K-1})^T,$$

$$\xi = (\xi_0 \quad \xi_1 \quad \cdots \quad \xi_{K-1})^T,$$

$$\xi_k = (\xi_{k,1} \quad \xi_{k,2} \quad \cdots \quad \xi_{k,M})^T,$$

$$\rho^{(s)} = (\alpha_0^{(s)} e^{j\beta_0^{(s)}} \quad \alpha_1^{(s)} e^{j\beta_1^{(s)}} \quad \cdots \quad \alpha_{K-1}^{(s)} e^{j\beta_{K-1}^{(s)}})^T,$$

$$A_l^{(s)}(\tau, \xi) = [a_l(\tau_0, \xi_0) \quad \cdots \quad a_l(\tau_{K-1}, \xi_{K-1})]^T,$$

(4)

(5)

(6)

(7)

(8)

(9)

In addition, if the number of time intervals is assumed to be $S$ then (3) can be expressed as

$$H_l = A_l^{(s)}(\tau, \xi) \rho + n_l,$$

(10)

where

$$\rho = (\rho^{(1)} \quad \rho^{(2)} \quad \cdots \quad \rho^{(S)}),$$

$$n_l = (n_l^{(1)} \quad n_l^{(2)} \quad \cdots \quad n_l^{(S)}).$$

(11)

(12)

(13)

Based on the space-time equivalence, we find that the subcarriers of OFDM signals analogous to array sensors and then apply space signal processing methods to construct a space-frequency extended channel frequency response matrix, which is given by

$$H = \begin{pmatrix}
H_0 \\
H_1 \\
\vdots \\
H_{L-1}
\end{pmatrix}
= \begin{pmatrix}
A_0^{(s)}(\tau, \xi) \\
A_1^{(s)}(\tau, \xi) \\
\vdots \\
A_{L-1}^{(s)}(\tau, \xi)
\end{pmatrix} \rho + n = A(\tau, \xi) \rho + n,$$

(14)

where

$$n = (n_0^T \quad n_1^T \quad \cdots \quad n_{L-1}^T)^T.$$  

(15)

Reconstruction of the extended channel frequency response has two effects. The first is that the virtual bandwidth is extended by $M$ times the actual bandwidth. The other is that the virtual array aperture is extended by $L$ times the real aperture. From a TOA and AOA estimation viewpoint, the increased bandwidth and aperture size will improve the estimation accuracy.
III. THE JOINT TOA AND AOA ESTIMATION

From (14), we can state the covariance matrix as

\[ \mathbf{R}_H = \frac{1}{S} \mathbf{HH}^H = \mathbf{A}(\tau, \xi) \mathbf{R}_\rho \mathbf{A}(\tau, \xi)^H + \sigma^2 \mathbf{I}_M. \]  

where \( \mathbf{R}_\rho \) is a complex attenuation covariance matrix. If the complex attenuations are independent, we find that \( \text{rank}(\mathbf{R}_\rho) = K \) and \( \text{rank} \left( \mathbf{A}(\tau, \xi) \mathbf{R}_\rho \mathbf{A}(\tau, \xi)^H \right) = K \) because \( \text{rank}(\mathbf{A}(\tau, \xi)) = K \), which is the necessary condition for use of the MUSIC algorithm. According to both [14] and [20], eigenvalue decomposition of \( \mathbf{R}_H \) is performed to obtain the noise subspace \( \mathbf{U}_n \) and then the spatial spectrum for joint estimation can then be expressed as

\[ P(\tau, \theta, \phi) = \| \mathbf{U}_n^H \mathbf{A}(\tau, \xi) \|^2. \]  

Using the extended three-dimensional MUSIC (EX-3D-MUSIC) algorithm, we can solve for the values of \( (\tilde{\tau}, \tilde{\theta}, \tilde{\phi}) \), but the 3D spectral peak search has a cost of considerable computational complexity.

When the complex attenuations that are caused by the multipath environment are coherent, we find that \( \text{rank}(\mathbf{R}_\rho) < K \) and the MUSIC algorithm cannot be adapted for use in this coherent situation because \( \text{rank} \left( \mathbf{A}(\tau, \xi) \mathbf{R}_\rho \mathbf{A}(\tau, \xi)^H \right) < K \).

A. **The Smoothing Preprocessing**

To find an algorithm that can be applied to both coherent and independent multipath scenarios without loss of the known information, we define

\[ \mathbf{R}_{\hat{H}} = \frac{1}{LS} \sum_{l=0}^{L-1} \mathbf{H}_l \mathbf{H}_l^H. \]  

This is a smoothing method introduced from [15] to solve the problem of coherent signals identification. First, we define

\[ \mathbf{R}_{\hat{H}_l} = \frac{1}{S} \mathbf{H}_l \mathbf{H}_l^H = \mathbf{A}_l(\tau, \xi) \mathbf{R}_\rho \mathbf{A}_l(\tau, \xi)^H + \sigma^2 \mathbf{I}_M. \]  

Considering the practical situation indicates that the relative time is much smaller than the propagation delay, which means that it can be ignored. We can then establish the resulting equation as

\[ \mathbf{A}_{l+1}(\tau, \xi) \approx \mathbf{A}_l(\tau, \xi) \mathbf{D}, \]  

where \( \mathbf{D} = \text{diag} \left[ e^{-j2\pi \Delta f \tau_1}, \ldots, e^{-j2\pi \Delta f \tau_{K-1}} \right] \). From (20), the matrix defined in (19) is

\[ \mathbf{R}_{\hat{H}_{l+1}} = \mathbf{A}_{l+1}(\tau, \xi) \mathbf{R}_\rho \mathbf{A}_{l+1}(\tau, \xi)^H + \sigma^2 \mathbf{I}_M \]

\[ \approx \mathbf{A}_l(\tau, \xi) \mathbf{D} \mathbf{R}_\rho \mathbf{D}^H \mathbf{A}_l(\tau, \xi)^H + \sigma^2 \mathbf{I}_M. \]  

Next, the new covariance matrix is given as

\[ \mathbf{R}_{\hat{H}} = \frac{1}{L} \sum_{l=0}^{L-1} \mathbf{R}_{\hat{H}_l}. \]  

We can then rewrite (18) as

\[ \mathbf{R}_{\hat{H}} = \mathbf{A}_0(\tau, \xi) \left( \frac{1}{L} \sum_{l=0}^{L-1} \mathbf{D} \mathbf{R}_\rho (\mathbf{D})^H \right) \mathbf{A}_0(\tau, \xi)^H + \sigma^2 \mathbf{I}_M \]

\[ = \mathbf{A}_0(\tau, \xi) \mathbf{R}_\rho \mathbf{A}_0(\tau, \xi)^H + \sigma^2 \mathbf{I}_M. \]  

Finally, when \( K \leq L \), we determine that \( \text{rank}(\mathbf{R}_\rho) = K \). Regardless of the presence of independent or coherent multipaths, \( \mathbf{R}_{\hat{H}} \) meets the requirements for use of the MUSIC algorithm. We now prove that if \( K \leq L \), then \( \mathbf{R}_\rho = K \) is full rank.

First, we rewrite \( \mathbf{R}_\rho \) as

\[ \mathbf{R}_\rho = \left[ \mathbf{I}_K \mathbf{D} \cdots \mathbf{D}^{L-1} \right] \begin{bmatrix} \mathbf{R}_\rho \quad \mathbf{D}^{-1} \end{bmatrix}, \]

which can be simplified to give

\[ \mathbf{R}_\rho = \mathbf{G} \mathbf{G}^H, \]

where \( \mathbf{G} \) is a \( K \times LK \) block matrix given by

\[ \mathbf{G} = [\mathbf{C} \quad \mathbf{DC} \cdots \mathbf{D}^{L-1} \mathbf{C}], \]

with \( \mathbf{C} \) denoting the Hermitian square root of \( \mathbf{R}_\rho / L \):

\[ \mathbf{CC}^H = \frac{1}{L} \mathbf{R}_\rho. \]

Obviously, the rank of \( \mathbf{R}_\rho \) is equal to the rank of \( \mathbf{G} \). Therefore, we need to prove that \( \text{rank}(\mathbf{G}) = K \). Given that the rank of a matrix is unchanged by its column permutations, it can be verified that

\[ \text{rank}(\mathbf{G}) = \text{rank} \left( \begin{bmatrix} c_{11} \mathbf{d}_1 & c_{12} \mathbf{d}_1 & \cdots & c_{1K} \mathbf{d}_1 \\ \vdots & \vdots & \ddots & \vdots \\ c_{K1} \mathbf{d}_K & c_{K2} \mathbf{d}_K & \cdots & c_{KK} \mathbf{d}_K \end{bmatrix} \right), \]

where \( c_{ij} \) represents the \( i, j \)th element of the \( \mathbf{C} \) and \( \mathbf{d}_k \) is the 1 \( \times \) \( K \) row vector expressed as

\[ \mathbf{d}_k = \left[ 1 \quad e^{-j2\pi \Delta f \tau_k} \cdots e^{-j2\pi (L-1) \Delta f \tau_k} \right]. \]

The rank \( \mathbf{G} = K \), when each row of the matrix \( \mathbf{C} \) has at least one nonzero element and the vectors \( \{\mathbf{d}_1, \ldots, \mathbf{d}_K\} \) are linearly independent. If we assume that all elements of the \( k \)th row of \( \mathbf{C} \) are zeros, this means the \( k \)th signal has zero energy from (27) but this cannot be true. The vectors \( \{\mathbf{d}_1, \ldots, \mathbf{d}_K\} \) can be embedded within a Vandermonde matrix, which is known to be nonsingular. Therefore, it is proved that \( \text{rank}(\mathbf{G}) = K \) and \( \text{rank}(\mathbf{R}_\rho) = K \).

B. **UCA-ESPRIT for AOA Estimation**

For \( \mathbf{R}_{\hat{H}} \), the spatial spectrum for joint estimation can be expressed as

\[ P(\tau, \theta, \phi) = \| \mathbf{U}_N^H \mathbf{A}_0(\tau, \xi) \|^2. \]  

where \( \mathbf{U}_N^H \) is the noise subspace obtained by eigenvalue decomposition of \( \mathbf{R}_{\hat{H}} \). Given that a 3D spectral peak search has a high cost in terms of computational complexity, we use the UCA-ESPRIT algorithm proposed in [21] to realize fast estimation. However, the UCA-ESPRIT is only applied to the manifold of the UCA. Consideration of (23) indicates that it can also be defined as

\[ \mathbf{R}_{\hat{H}} = \mathbf{A}_0(0, \xi) \tilde{\mathbf{U}}^H \mathbf{R}_\rho \tilde{\mathbf{U}} \mathbf{A}_0(0, \xi)^H + \sigma^2 \mathbf{I}_M, \]
where \( T = \text{diag} \left[ e^{-j2\pi f_{\tau_0}} \cdots e^{-j2\pi f_{\tau K-1}} \right] \). This shows that the manifold \( A_0(0, \xi) \) in (31) only corresponds to the AOA.

We then take the beam-space transform of (22), which is given by
\[
R_Y = F^H R_H F,
\]
(32)
where \( F^H \) is beam-space transform matrix. Then,
\[
F^H = \mathbf{W}^H C_V V^H,
\]
(33)
where
\[
C_V = \text{diag} \left[ j^{-M} \cdots j^{-1} j^0 j^1 \cdots j^M \right],
\]
(34)
\[
V = \sqrt{M} \left[ w_{-M} \cdots w_0 \cdots w_M \right],
\]
(35)
\[
w_m = \frac{1}{\sqrt{M}} \left[ 1 e^{-j2\pi m/N} \cdots e^{-j2\pi m(N-1)/N} \right]^T,
\]
(36)
\[
W = \frac{1}{\sqrt{M}} \left[ v(\alpha_m) \cdots v(\alpha_0) \cdots v(\alpha_M) \right],
\]
(37)
\[
v(\varphi) = \left[ e^{-jM\varphi} \cdots e^{-j0} \cdots e^{jM\varphi} \right]^T, \quad \alpha_m = \frac{2\pi m}{M}. \quad (38)
\]

We then take the eigenvalue decomposition of \( R_Y \) and calculate the signal subspace \( \bar{U}_S \). Here, we define \( Q \) as a \( K \times K \) full rank matrix, and the signal subspace is expressed as
\[
\bar{U}_S = F^H A_0(\tau, \xi) Q.
\]
(39)
We define
\[
\bar{U}_S' = C_0 W \bar{U}_S,
\]
(40)
where \( C_0 = \text{diag} \left[ (-1)^M \cdots (-1)^1 1 \cdots 1^M \right] \). In addition, we divide \( \bar{U}_S' \) into three parts and define these parts as
\[
\bar{U}_{S1}' = \ldots \bar{U}_{S2}' = \ldots = \bar{U}_{S3}' = \bar{U}_S'.
\]
(41)
Each part above has \( M - 2 \) rows. From [21], we find that
\[
\mathbf{E} \Psi = \Gamma \bar{U}_{S2}'.
\]
(42)
where
\[
\Gamma = \frac{\pi}{\tau} \text{diag} \left[ -(M - 1) \cdots 0 \cdots M - 1 \right],
\]
\[
\mathbf{E} = \left[ \bar{U}_{S1}' \quad \bar{U}_{S3}' \right]
\]
and \( \Psi = \left[ \Psi^T \quad \Psi^H \right] \). From (42), we calculate
\[
\tilde{\Psi} = \left( \mathbf{E}^H \mathbf{E} \right)^{-1} \mathbf{E}^H \Gamma \bar{U}_{S2}'.
\]
(43)
Using (43), we can confirm \( \Psi \), which can also be expressed as
\[
\Psi = Q^{-1} \Phi Q.
\]
(44)
\[
\Phi = \text{diag} \left[ \sin \theta_1 e^{j\varphi_1} \sin \theta_2 e^{j\varphi_2} \cdots \sin \theta_K e^{j\varphi_K} \right].
\]
(45)

Using (44), we take the eigenvalue decomposition of \( \Phi \) and then calculate the eigenvalue \( \lambda_k (k = 1, \ldots, K) \), which is a function of the azimuth and the elevation angle. The closed-form solutions for \( (\hat{\theta}, \hat{\varphi}) \) are given by
\[
\hat{\theta}_k = \arcsin(|\lambda_k|), \quad \hat{\varphi}_k = \text{angle}(\lambda_k).
\]
(46)

### C. One-dimensional (1D) Search for TOA

We obtain the closed-form solutions for the AOA using UCA-ESPRIT, and combine these solutions with (30) to obtain the TOA search function, which is given by
\[
P(\tau) = \left\| \tilde{U}_N^H A_0(\tau, \xi) \right\|_2^{-2}.
\]
(47)
When the complex attenuations are completely independent, then compared with EX-3D-MUSIC, the smoothing preprocessing reduces the array aperture and the accuracy of TOA provided by the proposed algorithm is lost. Therefore, we present an improved algorithm for the TOA, where the spatial spectrum function can be expressed as
\[
P(\tau) = \left\| \tilde{U}_N^H A(\tau, \xi) \right\|_2^{-2}.
\]
(48)
Using a 1D spectral peak search, we can estimate the \( \hat{\tau}_k \) that corresponds to \( (\hat{\theta}_k, \hat{\varphi}_k) \) and thus accomplish joint estimation of the TOA and the AOA.

### D. Algorithm Steps Conclusion

The main steps for the proposed algorithms can be summarized as follows:

#### Algorithm 1 Joint TOA and AOA Estimation

1) Construct an extended channel frequency response \( H \).
2) Construct the covariance matrix using (18) and calculate the closed-form solution \( (\hat{\theta}, \hat{\varphi}) \) indicated by (46) using UCA-ESPRIT.
3) Perform a 1D spectral peak search of the TOA using (47) and then solve for the corresponding value \( \hat{\tau}_k \).
4) When only independent multipaths are present, calculate the extended channel frequency response covariance matrix \( R_H \), take the eigenvalue decomposition of the covariance matrix to solve the noise subspace \( U_N \) and then solve the \( \hat{\tau} \) using (48).

### IV. Analysis of the Computational Complexity

Here, we analyze the computational complexity of the proposed method using UCA-ESPRIT algorithm and compare it with the complexity of the 3D spectral peak search method based on the MUSIC algorithm.

The complexity of the proposed method can mainly be divided into four parts. The complexities of calculation of the covariance matrix, the eigenvalue decomposition, UCA-ESPRIT and the 1D spectral peak search are \( O(LS M^2) \), \( O(M^3) \), \( O(4M^3) \) and \( O(M(M-K)G_{\tau}) \), respectively, where the \( G_{\tau} \) denotes the number of spectral points in the 1D search. Therefore, the complexity of the proposed algorithm is given by \( O((LS+S)M^2 + M(M-K)G_{\tau}) \). The 3D spectral peak search method has a cost of \( O((LS+M)M^2 + M(M-K)G_{\theta}G_{\varphi}G_{\psi}) \), where the \( G_{\theta} \) and \( G_{\varphi} \) denote the numbers of 2D angle searches.

In addition, in the case when the complex attenuations are all independent, we propose an improved step to obtain a more precise TOA estimation. The additional
complexities of calculation of the extended channel frequency response covariance matrix, the eigenvalue decomposition, and the 1D spectral peak search are \( O(SM^2L^2) \), \( O(M^3L^3) \), and \( O(ML(ML - K)G_r) \), respectively. Therefore, the total complexity is given by \( O((S + ML)M^2L^2 + (LS + 4)M^2 + ML(ML - K)G_r) \). The corresponding cost for the EX-3D-MUSIC method is \( O((S + ML)M^2L^2 + ML(ML - K)G_\theta G_\varphi G_\psi) \). For the sake of clarity, the computational complexities of all these approaches are summarized in Table I. We also compare the complexity of methods versus time intervals (\( S \)), the number of sensors (\( M \)) and the searching step (\( \Delta \tau \) and \( \Delta \theta \)) in Figure 2 (a), (b), (c) and (d), respectively.

As shown in Figure 2, the complexities of 3D-MUSIC and EX-3D-MUSIC are high because of the huge number of spectral points, particularly under small spectral step conditions. In contrast, both the proposed method and the improved proposed method use UCA-ESPRIT to solve for the AOA and simply perform a 1D spectral peak search to solve for the TOA. In addition, the complexity of ESPRIT is much less than that of MUSIC. Therefore, both methods also effectively reduce the computational complexity. When compared with the proposed method, the improved proposed method pays the higher cost of increased complexity caused by the extended channel frequency response to obtain more precise TOA estimation.

V. Simulation Results

This section reports the results of performance simulation experiments in which the proposed algorithm using UCA-ESPRIT is compared with 3D-MUSIC. Furthermore, we also compare the improved algorithm under independent complex attenuations conditions with EX-3D-MUSIC. The Cramer-Rao bound (CRB) \([22]\) is plotted as a benchmark. The simulations are based on an OFDM system with \( K = 64 \) subcarriers, guard duration \( T_G = 1.6\mu s \), fast Fourier transform period \( T_{FFT} = 3.2\mu s \), bandwidth \( B = 80\text{MHz} \), and carrier frequency \( f_c = 2.4\text{GHz} \). The UCA has \( M = 15 \) sensors and the radius of array is defined as

\[
r = \frac{c}{f_c (4 \sin \frac{\pi r}{M})}. \tag{49}
\]

We select the spectral steps of \( \Delta \theta = 0.05^\circ \) and \( \Delta \tau = 0.001\text{ns} \). To measure the accuracy of these algorithms, we define the root mean square error (RMSE) as

\[
\text{RMSE} = \sqrt{\frac{1}{QK} \sum_{i=1}^{Q} \| \lambda - \hat{\lambda}_i \|^2}, \tag{50}
\]

where \( Q \), \( \lambda \) and \( \hat{\lambda}_i \) are the number of Monte Carlo simulations, the \( i \)-th real values and the estimated values, respectively.

1) Performance under the low signal-to-noise ratio (SNR) conditions

We consider three possible situations. First, assuming that the number of coherent multipath components is two, the delays of these are 3.5ns and 13.5ns, their azimuth angles are 0\(^\circ\) and 30\(^\circ\), and their elevation angles are 30\(^\circ\) and 45\(^\circ\), respectively. Second, we add an independent multipath component with an azimuth angle of -20\(^\circ\), an elevation angle of 20\(^\circ\) and a time delay of 23.5ns. We use the proposed algorithm in these two situations. Third, we assume that all three multipaths are independent and we use the improved proposed algorithm. We select \( Q = 100 \) and determine the distributions of the TOA and the AOA under the conditions of SNR = -5dB and \( S = 500 \). These three situations are shown in Figure 3-5, respectively. The figures demonstrate that the corresponding algorithm solves for the parameters successfully in all situations and that the estimated values are all concentrated around the real values. Additionally, the proposed algorithms perform well under low SNR conditions and are robust.

![Fig. 3. Estimated distributions under the condition of SNR=-5dB](a) elevation and azimuth; (b) elevation and TOA)

![Fig. 4. Estimated distribution under the condition of SNR=-5dB](a) elevation and azimuth; (b) elevation and TOA)

![Fig. 5. Estimated distribution under the condition of SNR=-5dB](a) elevation and azimuth; (b) elevation and TOA)
TABLE I

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>The proposed algorithm</td>
<td>(O((LS + 5M)M^2 + M(M - K)G_r))</td>
</tr>
<tr>
<td>3D-MUSIC</td>
<td>(O((LS + M)M^2 + M(M - K)G_rG_w))</td>
</tr>
<tr>
<td>Improved proposed</td>
<td>(O((S + ML)M^2L^2 + (LS + 4)M^2 + ML(ML - K)G_r))</td>
</tr>
<tr>
<td>algorithm</td>
<td>(O((S + ML)M^2L^2 + ML(ML - K)G_rG_w))</td>
</tr>
</tbody>
</table>

Fig. 2. Complexity comparison: (a) versus time intervals when \(M = 15, \Delta \theta = 0.05^\circ\) and \(\Delta \tau = 0.001\)ns; (b) versus the number of sensors when \(S = 500, \Delta \theta = 0.05^\circ\) and \(\Delta \tau = 0.001\)ns; (c) versus \(\Delta \theta\) when \(M = 15, S = 500\) and \(\Delta \tau = 0.001\)ns; (d) versus \(\Delta \tau\) when \(M = 15, S = 500\) and \(\Delta \theta = 0.05^\circ\).

2) Performance versus SNR

Consider again the last two situations used in simulation 1. We compare the proposed algorithm with the 3D spectral peak search method under both coherent and independent multipath component conditions. We then compare the improved algorithm with EX-3D-MUSIC, the TOA estimation method introduced in [5] and the AOA estimation method introduced in [7]. The RMSE performance versus the SNR, which ranges from \(-15\)dB to 20dB at 5dB intervals, is shown in Figure 6-7.

Figure 6 shows that the RMSE of the proposed method is lower than that of 3D-MUSIC. When the complexities are compared, the complexities of the proposed method and 3D-MUSIC are approximately \(O(1.17 \times 10^9)\) and \(O(5.87 \times 10^{13})\), respectively. Therefore, the proposed method using UCA-ESPRIT not only reduces the complexity but also obtains a value that has been estimated with high precision. Because the proposed algorithm applies the smoothing processing losing virtual array aperture, there is a gap between the RMSE of it and CRB.

When only independent multipaths are presented, as shown in Figure 7, the proposed algorithm enhances the precision compared with the conventional TOA and AOA estimation methods, which demonstrates that the joint estimation methods can improve the accuracy. However, it also obtains an RMSE that is bigger than that of EX-3D-MUSIC because it is applied to a smaller array aperture. The EX-3D-MUSIC obtains the RMSE much closer to CRB, but it has huge complexity of \(O(2.98 \times 10^{17})\). Therefore, we present an improved proposed algorithm. Because it has the same step as proposed algorithm to resolve AOs, its RMSE is equal to that of proposed algorithm. The Figure 7 (b) shows that the improved algorithm enables to improve the accuracy of TOA compared with proposed algorithm, with an RMSE that is closer to that of EX-3D-MUSIC and CRB. Moreover, the complexity of improved proposed algorithm increases to \(O(2.43 \times 10^{10})\), but has been greatly reduced compared with that of EX-3D-MUSIC.

3) Performance versus the number of time intervals

The number of time intervals is varied in the range from \(S = [20, 50, 100, 200, 500, 1000, 2000, 5000]\) when SNR = 15dB and the results are shown in Figure 8-9. As the number of time intervals increases, the RMSE decreases, but the decline gradually reaches a plateau. The other conclusions are the same as those from simulation 2. Figure 8 shows that the proposed algorithm is superior to 3D-MUSIC in terms of accuracy and complexity. And Figure 9 presents that the RMSE of proposed algorithm is higher than that of EX-3D-MUSIC but with much lower complexity. Moreover, the improved proposed algorithm improves the accuracy of TOA close to that of EX-3D-MUSIC and much closer to CRB compared with proposed algorithm.

VI. CONCLUSIONS

For joint estimation of the TOA and the AOA in a UCA and with the aim of solving problems in complex multipath environments, we propose an estimation algorithm that is applicable to OFDM systems. We combine the channel frequency responses of the sensors with those of the subcarriers to use more of the available information and then perform frequency-domain smoothing preprocessing to meet the conditions of the subspace algorithms, regardless of whether coherent or independent multipath conditions applied. In this paper, the algorithm model and the process are first introduced. The
complexity of the proposed algorithm is then analyzed. Finally, the simulation experiments are presented. The results of the theoretical analysis and the simulations show that the proposed algorithm can estimate the parameters under coherent, independent or mixed multipath conditions and is robust under low SNR conditions. When compared with the spectral peak search method, the proposed algorithm using UCA-ESPRIT has greatly reduced the complexity and provides higher precision for the estimated values when the number of spectral points is restricted. In the case where there are only independent multipaths, we propose an improved method for TOA that enhances the accuracy without too much of an increase in complexity.

REFERENCES


Fig. 8. Performance comparison versus time intervals (a) azimuth and elevation; (b) TOA

Fig. 9. Performance comparison versus time intervals (a) azimuth and elevation; (b) TOA

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