Adaption Penalized Complex LMS for Sparse Under-Ice Acoustic Channel Estimations

Yanan Tian¹²³, Xiao Han¹²³, (Member, IEEE), Jingwei Yin¹²³, (Member, IEEE), and Yingsong Li⁴⁵, (Member, IEEE)

¹Acoustic Science and Technology Laboratory, Harbin Engineering University, Harbin 150001, China
²Key Laboratory of Marine Information Acquisition and Security (Harbin Engineering University), Ministry of Industry and Information Technology Harbin 150001, China
³College of Underwater Acoustic Engineering, Harbin Engineering University, Harbin 150001, China
⁴College of Information and Communication Engineering, Harbin Engineering University, Harbin 150001, China
⁵National Space Science Center, Chinese Academy of Science, Beijing 100190, China

Corresponding author: Xiao Han (e-mail: hanxiaol322@hrbeu.edu.cn).

This work was supported by the National Natural Science Foundation of China (No.61631008, No.61471137, No.50509059 and No.51779061), the Fok Ying-Tong Education Foundation, China (Grant No.151007), the National Key Research and Development Program of China (2018YFC140021, 2016YFE0111100), the Heilongjiang Province Outstanding Youth Science Fund (JC2017017), the Fundamental Research Funds for the Central Universities (Grant No. HEUCFM180503), and the Opening Fund of Acoustics Science and Technology Laboratory (Grant No. SSKF2016001).

ABSTRACT Accurate channel information is usually required in under-ice acoustic (UIA) communication, which exhibits a sparse characteristic. A type of norm-constrained least mean square (LMS) algorithm performs well in estimating the real-valued (passband) channels but cannot be directly applied to complex-valued (baseband) channels. This paper generalizes norm-constrained complex LMS. Complex-valued zero-attracting LMS (CZA-LMS) and \( l_p \)-norm LMS (\( C_{l_p} \)-LMS) are first mentioned. These two algorithms render a fixed penalty to each coefficient, which may deteriorate the performance. Then, we propose a complex-valued adaption penalized LMS (CAP-LMS) to further utilize the sparsity of the UIA channels. The adaption penalty is achieved by dividing \( p \)-norm-like constraints into two separate groups according to the \( l_q \)-norm of each coefficient. For the dominant coefficients in the large group, the norm constraint disappears to reduce the estimation bias. For the small coefficients, the adaption penalty aims to accelerate the convergence speed. Simulation results are presented to demonstrate the superior performance of the CAP-LMS algorithm. Data processing results from two under-ice experiments show the feasibility and validity of the proposed algorithms in practical UIA applications.

INDEX TERMS accurate channel information, norm-constrained complex LMS, under-ice acoustic (UIA) communication

I. INTRODUCTION

With the rapid development of various applications in the Arctic in recent years, under-ice acoustic (UIA) communication has become an important issue [1]. UIA communication in the polar regions allows autonomous underwater vehicles (AUVs) to play a role in conducting important research, such as mapping oil spills [2]. At the same time, it is cause for concern that the Arctic sea ice has experienced a gradual decline in its summer coverage, and it is thus necessary to detect and predict the ice melting trend [3]. For the sake of controlling AUVs and monitoring ice decline, there is an urgent demand to research UIA communication [4], [5]. Different from open-water media, under-ice media has a positive gradient with respect to the water depth [6]. The signals scatter from the ice at each reflection, which means that the transmission loss increases, and the propagation conditions become complex and challenging for the receiver [7]. All these features make the signal severely distorted during transmission. To recover the transmitted signal, accurate under-ice channel information is necessary, meaning that channel estimation is an important component of UIA communication.

It is known that the finite impulse response (FIR) of the UIA channel exhibits sparse characteristics in nature, which means that most of the energy in the channel is concentrated on a few coefficients, and the remaining coefficients are close to zero [6], [8], [9]. Recently, several studies have been conducted to estimate these sparse channels. One effective...
method is implemented based on compressed sensing (CS) [10-14], such as matching pursuit (MP) and orthogonal matching pursuit (OMP). The MP algorithm is used for sparse channel estimation by selecting an atom that best matches the signal residual from the dictionary matrix until the residual satisfies certain conditions [10]. OMP is proposed to overcome the nonorthogonal problem of MP, which converges faster under the same accuracy requirements [11]. However, these methods may achieve a local optimum rather than a global optimum [12]. In addition, the strict dictionary matrix is not suitable for any signal reconstruction [13]. Another method is the adaptive algorithm with sparse penalties [15-24]. As a typical channel estimation, least mean square (LMS) is widely used due to its low complexity and simple operation [15]. Then, a series of sparse LMS algorithms have been conducted for sparse channel estimation and system identifications. In [16], Gu and Chen proposed zero-attracting LMS (ZA-LMS) which accelerates the convergence speed for near-zero taps by adding an $l_1$-norm on the corresponding coefficients. However, this approach imposes a fixed norm constraint on each tap coefficient, which may deteriorate the estimation performance. To solve this problem, the authors utilized a log function as a sparse penalty to form a reweighted ZA-LMS (RZA-LMS) which controls the convergence speed nonuniformly according to the magnitude of each coefficient. Furthermore, Jin combined the $l_p$-norm with the standard LMS, proposing the $l_p$-LMS [17]. C. Wang improve the performance of $l_p$-LMS by introducing an adaptive zero attractor [18]. To further exploit the sparsity, Wu proposed a nonuniform norm constraint LMS (NNCLMS) to adaptively apply the $l_0$-norm or $l_1$-norm to each coefficient, seeking a tradeoff between the convergence rate and estimation bias [19-21].

The above algorithms are suitable for handling real signals in the passband, but they are not suitable for dealing with complex-valued signals. However, the common UIA signals in a real communication system are usually complex, which preserves the physical characteristics of signals and transformations during the transmission. Therefore, by considering the relationship between the real and imaginary parts or the magnitude and phase, we can make full use of the information carried by the signal. Of course, one can split a complex signal into two parts and then combine them together. However, this method has other disadvantages besides the increased computational complexity [25]. Therefore, it is necessary to develop norm constraint estimation methods in a complex domain for practical applications. In this paper, in light of the $l_p$-norm of complex values, we propose complex norm constraint algorithms termed as complex-valued ZA-LMS (CZA-LMS), RZA-LMS (CRZA-LMS), $l_p$-LMS (CLMS), and adaption penalized LMS (CAP-LMS). The superscripts $\text{t}$, $\ast$, and $\dagger$ stand for transpose, conjugate, and conjugate transpose, respectively. The vectors are represented by a bolded lowercase. As in [26], for $z \in C$, the sign function of $z$ is written as $\text{csgn}(z) \triangleq \text{sgn}[\text{Re}(z)] + j\text{sgn}[\text{Im}(z)]$, where $\text{sgn}(\ast)$ is the sign function of a real scalar and defined as

$$\text{sgn}(x) = \begin{cases} x/|x|, & x \neq 0 \\ 0, & x = 0 \end{cases}.$$  

The $l_p$-norm of $z$ is expressed as $\|z\|^p_p = |\text{Re}(z)|^p + |\text{Im}(z)|^p$.

This paper is organized as follows. In section 2, we give the derivation of the complex LMS method, specifically with sparse penalties. Section 3 theoretically presents the analysis for guaranteeing the convergence of the algorithm. In section 4, the performance of the proposed algorithms, such as the convergence speed and steady-state error, is evaluated over ideal UIA channels. Section 5 discusses the performance of the proposed complex sparse LMS algorithms with actual experimental data to verify the feasibility and effectiveness of the algorithms. Conclusions are presented in section 6.

II. PROPOSED COMPLEX SPARSE LMSs

Consider the system model in the baseband, where the input signal, output signal, and noise are all complex values. A typical adaptive filter structure is shown in Fig. 1.

![Adaptive filter structure](image)

**FIGURE 1. Adaptive filter structure**

$\mathbf{h}(n) = [h_1(n), h_2(n), \ldots, h_M(n)]^\text{t}$ represents the FIR of a UIA channel, where $N$ denotes the channel length. Due to the sparse characteristics of UIA, the energy of $\mathbf{h}(n)$ is mainly concentrated on $K$ dominant nonzero taps, where $K$ is the sparsity. The input sequence $\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-N+1)]^\text{t}$ denotes the most recent transmitted samples at an instant $n$. $\mathbf{z}(n)$ is the additive white Gaussian noise (AWGN) with zero mean, which is assumed to be independent of transmitted signal. The transient error can be expressed as $e(n) = d(n) - \mathbf{h}(n)^\text{t}\mathbf{x}(n)$, where $d(n)$ is the desired signal, and $\mathbf{h}(n)$ represents the estimated channel vector.

As illustrated in [27], the update equation of the complex-valued LMS (CLMS) algorithm is given as $\mathbf{h}(n+1) = \mathbf{h}(n) + \mu(e(n)\mathbf{x}(n))$, where $\mu$ is the step size that controls the convergence speed and the steady-state error. The performance is not optimal without exploiting the sparse property. Combined with the sparse feature of UIA channels, a norm constraint CLMS is proposed with the cost function expressed as:

$$J = \|e(n)\|^2 + \gamma\|\mathbf{h}(n)\|^p_p$$  

where
\[ \| \hat{h}(n) \|_p^p = \sum_{i=1}^{N} [\text{Re}[h_i(n)]]^p + [\text{Im}[h_i(n)]]^p, 0 \leq p \leq 1 \quad (2) \]

The first term in (1) is the cost function of the classic CLMS, and the second term is considered to be a sparse penalty. \( \gamma > 0 \) is a regularization parameter that has an impact on balancing the sparse penalty and estimation error.

Consider two existing cases:

1. When \( p = 1 \),
   \[ \lim_{p \to \infty} \| \hat{h}(n) \|_p^p = \| \hat{h}(n) \| = \sum_{i=1}^{N} [\text{Re}[h_i(n)] + |\text{Im}[h_i(n)]|]. \]
   In this case, the \( l_1 \)-norm is added into the classic CLMS to obtain the complex-valued ZA-LMS (CZA-LMS). Using the complex-valued steepest descent method in [28], the update equation is given as:
   \[ \hat{h}(n+1) = \hat{h}(n) - \mu \nabla_{\hat{h}}(n)J \]
   \[ = \hat{h}(n) + \mu \text{e}(n)\dot{x}^*(n) - \mu \gamma \text{sgn}[\hat{h}(n)] \]
   where \( \nabla_{\hat{h}}(n) \) represents the gradient with respect to \( \hat{h} \).

Equation (3) illustrates that all taps are forced to zero uniformly regardless of whether the tap is close to zero or not, which may degrade the performance. Inspired by a reweighted \( l_1 \)-minimization [29], the complex-valued reweighted ZA-LMS (CRZA-LMS) is proposed with the update equation:
   \[ \hat{h}(n+1) = \hat{h}(n) + \mu \text{e}(n)\dot{x}^*(n) - \frac{\mu \gamma \text{sgn}[\hat{h}(n)]}{1 + \varepsilon \| \hat{h}(n) \|} \]
   \[ (4) \]
   where \( \varepsilon > 0 \) is a parameter that controls the reweighted strength.

2. When \( p = 0 \),
   \[ \lim_{p \to 0} \| \hat{h}(n) \|_p^p = \| \hat{h}(n) \|_0, \]
   Under this condition, the sparse penalty turns into \( l_0 \)-norm, and we obtain the \( C l_0 \)-LMS. It is a nonpolynomial hard problem and can be solved by approximation methods. The update equation becomes the following:
   \[ \hat{h}(n+1) = \hat{h}(n) - \mu \nabla_{\hat{h}}(n)J \]
   \[ = \hat{h}(n) + \mu \text{e}(n)\dot{x}^*(n) - \mu \gamma f_\beta[\hat{h}(n)] \]
   \[ (5) \]
   where
   \[ f_\beta[\hat{h}(n)] = \begin{cases} \beta(1 - \beta) \| \hat{h}(n) \|_0 \text{sgn}[\hat{h}(n)], & \| \hat{h}(n) \|_0 \leq \frac{1}{\beta} \\ 0, & \text{else} \end{cases} \]
   and \( \beta \) controls the constraint strength.

Compared with the standard CLMS, CZA-LMS and \( C l_0 \)-LMS improve the performance by taking the sparsity exploitation as prior information. To make better use of the sparse characteristics and further improve the estimation performance, a new complex-valued adaption penalized LMS (CAP-LMS) algorithm is proposed, motivated by the norm adaptive method proposed in [30].

By directly applying the steepest descent method to (1) with respect to \( \hat{h} \), the update equation is given as:
   \[ \hat{h}(n+1) = \hat{h}(n) - \mu \nabla_{\hat{h}}(n)J \]
   \[ = \hat{h}(n) + \mu \text{e}(n)\dot{x}^*(n) - \frac{\mu k \text{sgn}[\text{Re}\{\hat{h}(n)\}] + j\mu k \text{sgn}[\text{Im}\{\hat{h}(n)\}]}{|\text{Re}\{\hat{h}(n)\}|^{p-1} + |\text{Im}\{\hat{h}(n)\}|^{p-1}} \]
   \[ (6) \]
   where \( k = \mu / 2 > 0 \) is a regularization parameter which balances the sparse penalty and estimation error by combining the effects of \( \mu \) and \( \gamma \).

Substituting \( \hat{h}(n) \) with \( \hat{h}_i(n), \hat{h}_2(n), \cdots, \hat{h}_{N-1}(n), \) we rewrite (6) as:
   \[ \hat{h}_i(n+1) = \hat{h}_i(n) + \mu \text{e}(n)\dot{x}^*(n - i) - \frac{\mu k \text{sgn}(\text{Re}\{\hat{h}(n)\}) + jk \mu \text{sgn}(\text{Im}\{\hat{h}(n)\})}{|\text{Re}\{\hat{h}(n)\}|^{p-1} + |\text{Im}\{\hat{h}(n)\}|^{p-1}} \]
   \[ (7) \]
   where \( 0 \leq i \leq N-1 \). The third item on the right side of (7) is a zero-attraction term. The existence of this item can accelerate the convergence speed for small channel coefficients due to the sparse nature of UIA channels. At the same time, it can be seen that it is similar to \( l_1 \)-LMS and \( l_2 \)-LMS, which will inevitably bring additional estimation errors [31], [32]. Fortunately, we notice that the parameter \( p \) in the third item is adjustable for different \( i \), which has an influence on both the convergence speed and estimation error. Thus, different \( p \) values for different taps should be selected to seek a tradeoff between the sparsity and estimation error.

Differing from (2), let us redefine the vector’s \( l_p \)-norm as:
   \[ \| \hat{h}(n) \|_{p,N} = \sum_{i=1}^{N} [\text{Re}[h_i(n)]]^p + |\text{Im}[h_i(n)]|^p, 0 \leq p \leq 1 \quad (8) \]

We call (8) \( p \)-norm-like for complex values. Then, the cost function becomes:
   \[ J = e(n)^2 + \gamma \| \hat{h}(n) \|_{p,N} \]
   \[ (9) \]

The corresponding update equation related to (9) is given as:
   \[ \hat{h}(n+1) = \hat{h}(n) - \mu \nabla_{\hat{h}}(n)J \]
   \[ = \hat{h}(n) + \mu \text{e}(n)\dot{x}^*(n) - \frac{\mu k \text{sgn}[\text{Re}\{\hat{h}(n)\}] + j\mu k \text{sgn}[\text{Im}\{\hat{h}(n)\}]}{|\text{Re}\{\hat{h}(n)\}|^{p-1} + |\text{Im}\{\hat{h}(n)\}|^{p-1}} \]
   \[ (10) \]
   where \( p = [p_0, p_1, \cdots, p_{N-1}] \) represents different values for different channel coefficients. For large tap coefficients, the parameter should be chosen to lower the sparse penalty in order to reduce the estimation bias. For small coefficients, the effect of \( p_i \) is to enhance the sparse penalty and accelerate the convergence speed. Thus, it is possible to choose different \( p_i \) to achieve a compromise between the estimation bias and sparse intensity.

Equation (10) is rewritten as:
\[ \hat{h}(n+1) = \hat{h}(n) + \mu e(n) x^*(n-i) \] 
\[ -\left\{ kp, \text{sgn}(\text{Re}[\hat{h}(n)]) + j kp, \text{sgn}(\text{Im}[\hat{h}(n)]) \right\} \frac{[\text{Re}[\hat{h}(n)]]^\dagger_{-p_i}}{|[\text{Im}[\hat{h}(n)]]^\dagger_{-p_i}|} \]
\[ \text{where } 0 \leq i \leq N-1. \] From the update equation, it can be seen that the channel coefficients can be divided into ‘large’ and ‘small’ groups by introducing a metric of the \( l_1 \)-norm of \( \hat{h}(n) \). At each instant \( n \), taking the expectation on the \( l_1 \)-norm of \( \hat{h}(n) \) and using the classification criteria, we have the following:
\[ m(n) = E[|\hat{h}(n)|] \]

For the ‘small’ group, the effect of \( p_i \) is to speed up the convergence speed of zero or near-zero coefficients. Meanwhile, it is necessary to avoid a particularly strong or weak sparse penalty caused by different small values \( ||\hat{h}(n)|| \) [31]. \( p_i = 1 \) is a general choice with a low implementation complexity. For the ‘large’ group, we hope the sparse constraint will not affect it to reduce the estimation bias. Thus, the optimization of \( p_i \) for the ‘large’ group can be obtained by solving the following:
\[ \arg \min_{p_i} \left\{ p_i, \text{sgn}(\text{Re}[\hat{h}(n)]) + p_j, \text{sgn}(\text{Im}[\hat{h}(n)]) \right\} = 0 \]
subject to \( ||\hat{h}(n)|| \geq m(n) \)

As a result, in order to balance the estimation bias and sparsity, we allocate \( p_i = 1 \) and \( p_j = 0 \) for \( ||\hat{h}(n)|| \leq m(n) \) and \( ||\hat{h}(n)|| \geq m(n) \), respectively. The update equation becomes:
\[ \hat{h}(n+1) = \begin{cases} \hat{h}(n) + \mu e(n) x^*(n-i), & \text{for } ||\hat{h}(n)|| \leq m(n) \\\n\hat{h}(n) + \mu e(n) x^*(n-i) - k \text{sgn}[\hat{h}(n)], & \text{else} \end{cases} \]

Equation (13) illustrates that an adaptive norm constraint is given according to the \( l_1 \)-norm of each coefficient value. For the ‘small’ group, the norm constraint is to accelerate the convergence speed. For the ‘large’ group, the norm constraint disappears to mitigate the estimation error. Compared with CZA-LMS and \( C_{l_1} \)-LMS, the proposed CAP-LMS performs well with respect to the low estimation bias and fast convergence speed.

III. CONVERGENCE ANALYSIS
It is important to choose the proper values to guarantee the convergence of an adaptive algorithm. In this section, the convergence performance of the proposed CAP-LMS is analyzed in detail. Assuming that the optimum FIR is \( h^0 \), we define the coefficient error vector as:
\[ v(n) = \hat{h}(n) - h^0 \]
\[ \text{Thus,} \]
\[ v(n+1) = v(n) + \hat{h}(n+1) - \hat{h}(n) \]
Rewriting (13) as:
\[ \hat{h}(n+1) = \hat{h}(n) + \mu e(n) x^*(n-i) - kg(n) \]
where \( g(n) = [g_0(n), g_1(n), \ldots g_{N-1}(n)] \)

Combining (16) with (14), we yield:
\[ \hat{h}(n+1) - \hat{h}(n) = \mu e(n) x^*(n-i) - kg(n) \]
\[ = \mu(x^*(n) + z(n)) - kg(n) \]
\[ = \mu x^*(n) - \mu h^*(n) x^*(n) - kg(n) \]
\[ = \mu x^*(n) - \mu (\hat{h}(n) - h^*(n)) x^*(n) - kg(n) \]
\[ = \mu x^*(n) - \mu x^*(n) x^*(n) - z(n) - kg(n) \]
Substituting (17) into (15) gives:
\[ v(n+1) = v(n) - \mu x^*(n) x^*(n) - z(n) - kg(n) \]
Taking the expectation on (18), we obtain the expected value of \( v(n) \) as follows:
\[ E[v(n+1)] = [I - \mu R] E[v(n)] + \mu E[x^*(n) z(n)] \]
\[ kE[g(n)] \]
where \( R = E[x(n) x^*(n)] \)
\[ \text{and the mean of noise is zero, the second term on the right side of (19) equals zero. Then, Equation } \]
\[ E[v(n)] = [I - \mu R] E[v(n)] - kE[g(n)] \]
\[ \text{From the definition of } g(n) \text{, it can be seen clearly that there is a limitation of } E[g(n)]. \]
\[ \text{Thus, the condition for guaranteeing the convergence is that all eigenvalues of the Hermitian matrix } [I - \mu R(n)] \text{ should be less than 1, i.e.,} \]
\[ \mu < \mu \text{ or } 2/\lambda_{\text{max}} \text{, where } \lambda_{\text{max}} \text{ is the maximum eigenvalue of } R \]
\[ \text{Letting } n \rightarrow \infty \text{, Equation } (20) \text{ becomes } E[v(\infty)] = [I - \mu R] E[v(\infty)] - kE[g(\infty)] \text{. Solving this equation yields:} \]
\[ E[v(\infty)] = - k R^{-1} E[g(\infty)] \]
Combining (21) with (14), we can obtain the expected value of the channel coefficient in a steady-state as:
\[ E[\hat{h}(\infty)] = E[v(\infty)] + h^0 = h^0 - \frac{k}{\mu} R^{-1} E[g(\infty)] \]
From (22), we see that it is a biased estimation of the optimum FIR owing to the utilization of the sparse characteristic. When the second term on the right side of the above equation approaches 0 (i.e., the sparse constraint is very weak or even nonexistent), the estimation becomes unbiased.

IV. COMPUTER SIMULATION
In this section, we will compare the performances of the above algorithms’ convergence speed and steady-state error for estimating sparse ULA channels. As seen from the update equations, the performance of CAP-LMS is greatly affected by \( \gamma \) and \( \mu \). From [33], we know that a large \( \mu \) brings a fast convergence speed, but it will also lead to a large steady-state error. In the first simulation, we test the influence with different values of \( \gamma \). The driven signal is a modulated
QPSK signal, while the noise is AWGN, and the signal-to-noise ratio (SNR) is set at 10 dB. The UIA multipath channel has 72 coefficients, where the number of nonzero coefficients equals 5. The position of nonzero taps is randomly distributed, and the values meet $\|h^i\|_1 = 1$. One hundred Monte Carlo simulations are conducted to obtain the results. Let $\mu = 0.01$ be a constant here. The $l_2$-norm of the coefficient error vector $v(n)$ is exploited as a performance metric referred to as the mean square error (MSE), which is defined as:

$$\text{MSE} = 10\log(\|v(n)\|_2^2) = 10\log(\|\hat{h}(n) - h^i\|_2^2)$$  \hspace{1cm} (23)

The effects of $\gamma$ on the MSE is shown in Fig. 2. It is found that $\gamma$ mainly affects the steady-state performance. For $\mu = 0.01$, the MSE decreases with an increase in $\gamma$ within a specific range because the norm constraint is added to the classic cost function and the sparse feature is utilized as priori information. However, with the continuous increasing of $\gamma$, the sparse constraint exerted on the system is so strong that the estimation performance fluctuates greatly. As a result, it is important to choose a proper $\gamma$ to obtain a small MSE for the fixed step size.

![FIGURE 2. MSE performance of CAP-LMS with different values of $\gamma$](image)

The performance of CAP-LMS with different sparsity variations is analyzed in the second simulation. The sparsity of the UIA channels varies from 2 to 16, with the rest of system parameters at the same values as those in the first simulation. First, we choose $\mu$ and $\gamma$ to ensure the same convergence speed to analyze the MSE under different sparsity variations. The parameters are listed as $\mu = 0.012, \gamma = 0.58$. As shown in Fig. 3(a), with the same convergence speed, the MSE is high with large value of $K$. Second, we compare the convergence speed with the same MSE, with the parameters listed as $\mu_{K=2} = 0.018, \mu_{K=6} = 0.01, \mu_{K=10} = 0.0067, \mu_{K=16} = 0.0044, \gamma_{K=2} = 0.52, \gamma_{K=6} = 0.2, \gamma_{K=10} = 0.12, \gamma_{K=16} = 0.07$. Fig. 3(b) shows that the number of iterations required to reach the same steady-state MSE is different at different sparsity levels. The convergence speed is slow with large value of $K$. Clearly, we can arrive at the conclusion that the sparsity has important effects on the performance, where a sparser channel has better performance.

![FIGURE 3. Performance with different sparsity variations](image)

The third simulation is conducted to compare the steady-state error for different algorithms with the same convergence speeds. The number of nonzero coefficients $K$ equals 2, 6, and 10, and the other system parameters are the same as those in the first two simulations. The parameters of each algorithm are selected to guarantee the same convergence speed, which are $\mu = 0.012$ for each algorithm, $\gamma_{CZA-LMS} = 0.01, \gamma_{CZA-LMS} = 0.03, \gamma_{CZA-LMS} = 0.03, \gamma_{CAP-LMS} = 0.035, \epsilon = 10$, and $\beta = 2$. As illustrated in Fig. 4, it is found that CAP-LMS achieves the lowest steady-state MSE compared with that of the other algorithms for different sparsity variations. When the sparsity equals 2, the steady-state error of the CAP-LMS is approximately 4.5 dB lower than that of the $C_{10}$-LMS, and when the sparsity equals 10, the performance is deteriorated but still has a gain of approximately 1.5 dB in comparison with that of $C_{10}$-LMS.
V. EXPERIMENTAL RESULTS

A. Experiment 1

FIGURE 4. Steady-state error of each algorithm with different sparsity variations

The last simulation is conducted to compare the convergence speed of each algorithm under the same steady-state error. The parameters of each algorithm are chosen to guarantee the same steady-state MSE. From the results in Fig. 5, we can see that the CAP-LMS has the fastest convergence rate compared with that of the other complex sparse LMSs. In other words, the proposed CAP-LMS has the best tracking performance when the sparsity varies to achieve the same steady-state error. It is worth noting that the results given in these three figures also show the effect of sparsity on the convergence rate.

FIGURE 5. Convergence speed of each algorithm with different sparsity variations

The explanation of these results is shown as follows: compared with other algorithms, CAP-LMS divides the channel coefficients into a large group and a small group according to the $l_1$-norm of the channel coefficients. No sparse constraints are imposed on the channel coefficients for the large group, while strong constraints are imposed on the small coefficients. This way, the estimation bias decreases when the speed of convergence increases.

V. EXPERIMENTAL RESULTS

A. Experiment 1
On February 9th, 2018, under-ice acoustic communication experiments were conducted in the island of Vladivostok in Russia. The experimental environment is shown in Fig. 6. The average ice thickness is 0.6 m, and the communication distance is 1.2 km. The water depth of the transmitter is 6 m, with a transducer placed at 3 m, while the water depth of the receiver is 6 m, with a hydrophone placed at 1 m. The bandwidth of the transducer is 8-16 kHz.

\[
MSE(n) = 10 \times \log\left[ \frac{1}{L} \sum_{i=-L}^{n} |d(i) - \hat{h}(i-1)u(i)|^2 \right] 
= 10 \times \log\left[ \frac{1}{L} \sum_{i=-L}^{n} |e(i)|^2 \right]
\]

where \(d\) is the received signal and \(L\) is the length of symbol error. The curve of the performance is smooth with large value of \(L\). Additionally, the amount of calculations will increase. In light of the results shown in Fig. 8, the CAP-LMS exhibits the best MSE performance, with a gain of approximately 1 dB compared with that of the \(C_{l_0}\)-LMS.

B. Experiment 2

Another UIA communication experiment was conducted in the Songhua River in January 2018. The water depth of the experimental region is approximately 6 m, the ice thickness is approximately 50 cm, and the air temperature is around -22°C. The distance between the transducer and receiving array is approximately 800 m. The transmitted signals are modulated by QPSK. The transmitted waveform is pulse-shaped using a raised-cosine filter with a roll-off factor of 1. The center frequency is 3 kHz, the symbol rate is 1000 symbols/s, and all signals are sampled at 48 kHz; 11000 symbols are transmitted during this experiment.
this channel is more complicated. Clearly, this channel also exhibits a sparse characteristic. The steady-state error with same convergence speed is shown in Fig. 10. Although the performance improvement of CAP-LMS is not as obvious as that in experiment 1, it still achieves the lowest MSE compared to that of all the previously mentioned algorithms, which also verifies the superiority of the CAP-LMS algorithm.

VI. CONCLUSION

Under-ice acoustic communication has broad application prospects. Accurate channel information plays an important role in UIA communication. To effectively estimate the sparse UIA channel at the baseband, norm constraint complex LMS algorithms are proposed in this paper. Unlike the CZA-LMS and $C_I$-LMS which impose fixed constraints on channel coefficients, the CAP-LMS divides the channel coefficients into two groups according to the $l_1$-norm of the channel coefficients. No constraints are imposed on the large channel coefficients, which reduces the estimation error; strong constraints are imposed on the small channel coefficients to accelerate the convergence speed. The simulated results show that the performance of CAP-LMS is improved with respect to the convergence speed and steady-state error. Under-ice experiment results confirm the feasibility and validity of the proposed algorithms in practical applications.

REFERENCES


Yanan Tian received her B.S. degree from the College of Information and Communication Engineering, Harbin Engineering University, Harbin, China, in 2017. She is now working towards her Ph.D. degree in the College of Underwater Acoustic Engineering, Harbin Engineering University, China. Her research is interested in underwater acoustic communication.

Xiao Han received his B.S, M.S, and Ph.D. degree in underwater acoustic engineering from the Harbin Engineering University, China in 2011, 2014, and 2016, respectively. Currently he is a lecture in the College of Underwater Acoustic Engineering, Harbin Engineering University, China. His research interests include ocean acoustics and underwater acoustic communication.

Jingwei Yin received his B.S, M.S, and Ph.D. degree in underwater acoustic engineering from the Harbin Engineering University, China in 1999, 2006, and 2007, respectively. He is a visiting professor of the Russian Far Eastern Federal University (FEFU). He is currently a professor and dean of the College of Underwater Acoustic Engineering, Harbin Engineering University, China. He is the deputy director of the water acoustic branch of the China Acoustic Society. He is also a director of Key Laboratory of Marine Information Acquisition and Security (Harbin Engineering University), Ministry of Industry and Information Technology. He has published a monograph, more than 100 papers, and more than 20 inventions. His current research interests include underwater acoustic engineering, underwater acoustic communication, and polar acoustics.

Yingsong Li received his B.S. degree in electrical and information engineering in 2006, and M.S. degree in Electromagnetic Field and Microwave Technology from Harbin Engineering University, 2006 and 2011, respectively. He received his Ph.D. degree from both Kochi University of Technology (KUT), Japan and Harbin Engineering University, China in 2014. He is a visiting scholar of University of California, Davis from March 2016 to March 2017. Now, he is a full professor of Harbin Engineering University from July 2014. He is also a visiting professor of Far Eastern Federal University (FEFU) and KUT. He is a senior of Chinese Institute of Electronics (CIE), member of IEEE. He is an Associate Editor of IEEE Access and AEÜ - International Journal of Electronics and Communications. Dr. Li also serves as a reviewer for more than 20 journals. Recently, his recent research interests are mainly in underwater communications, signal processing, compressed sensing and antennas.