Output integral sliding mode fault tolerant control for nonlinear systems with actuator fault and mismatched disturbance

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ABSTRACT In this paper, an adaptive output sliding mode fault tolerant control (AOSMFTC) strategy is proposed for a class of nonlinear systems with actuator fault and mismatched disturbance. A composite observer is proposed first to estimate the state, disturbance and actuator fault efficiency factor. Then by introducing a sliding mode observer, the bias actuator fault is reconstructed. Furthermore, in accordance with the estimated information, the AOSMFTC scheme is presented to tolerate the fault, meanwhile the disturbance is attenuated. The $H_\infty$ performance is considered for the robustness of the system. Finally, a B747-100/200 aircraft model and an electric circuit system are simulated; the simulation results illustrate the effectiveness of the proposed method.

INDEX TERMS Actuator fault, Fault tolerant control (FTC), composite observer, AOSMFTC, mismatched disturbance.

I. INTRODUCTION

Industrial control system is inevitable to suffer from faults caused by actuators, sensors and other components because of long time operation. The actuator malfunctions certainly reduce the nominal performance, result in vibration, destroy the stability of the system and even cause catastrophic accidents to the control system. Fault tolerant control (FTC), known as an imperative part, has played a great important role in guaranteeing the stability of the system. The basic attribute of the FTC is to maintain the functional abilities and performances in the present of faults. Abundant literatures are produced by a variety of fault tolerant control approaches, such as robust control [1], sliding mode control [2], observer based control [3], intelligent learning control [4], and adaptive control [5], and their applications in different areas, such as aircraft system [6], electric vehicle systems [7] and motor systems [8], etc.

For many practical engineering systems, it is hard or too expensive to measure the states of the system, i.e., there exists may unknown parameters in the system. Fault diagnosis (FD), which is known as the first step for FTC, is an efficient method to obtain the location and magnitude of the fault. However, it is difficult to establish the diagnosis mechanism in some conditions. One of the feasible methods is to design an observer and then the controller is constructed based on the estimated information. Some excellent observers have been employed, such as unknown input observer (UIO) [9], state extended observer (SEO) [10] and disturbance observer (DO) [11]. The advantage of the observer is its flexibility, one can construct a proper observer for unknown but critical parameter as required.

It is worth to point out that disturbance exists widely in many control processes, which affects the stability of the system seriously. In other words, both fault and external disturbance exist in the control system. The basic characteristics of the fault and disturbance are their uncertainty, nonlinearity and complexity. Observer is also an actual technique to deal with these unknown factors. Fruitful results have been investigated in the last decades. For instance, in [12], the nonlinear time delay system with...
actuator fault and disturbance was addressed and an adaptive passive robust FTC was designed to accommodate the fault and disturbance. An observer based fault tolerant control scheme was investigated in [13] for a class of linear system with time delay, actuator fault and disturbance, where an intermediate estimator was constructed and the bounded states are proved. In [14], the ball mill grinding circuits system was investigated and a disturbance observer was designed to estimate the strong disturbance, the controller was designed based on the observer to compensate the disturbance. In [15], the author took an insight into the fuzzy DO methodology design for the generic hypersonic vehicle system; the $L_\infty$ performance was proposed to ensure the stability of the system. In [16], the nonlinear stochastic time varying system with randomly occurrence faults and fading channels was investigated, and a time varying estimator is addressed. However, in these results, the actuator fault is only considered one or two patterns; for disturbance, the disturbance distribution matrix and the control input matrix are assumed to satisfy the match condition, i.e., the disturbance distribution matrix can be written as a linear combination of the columns of control matrix. It should be note that mismatched disturbance is more practical in the industrial dynamic systems, mismatched disturbance enter into the system different from the control input channel, i.e., $\text{rank } (B_d)<\text{rank } (B,B_d)$, where $B$ and $B_d$ are the control input matrix and disturbance matrix respectively. In [17], an unknown nonlinear system with actuator fault, sensor fault and mismatched disturbance was addressed; an adaptive DO was designed to reduce the effect of the disturbance. In [18], the disturbance was considered as two parts, the matched disturbance was compensated by a compensator while the mismatched disturbance was attenuated by the variable structure controller. In [19], by constructing a nonlinear disturbance observer (NDO), the sliding mode controller was developed to counteract the mismatched disturbance.

As a functional and powerful control strategy, sliding mode control (SMC) has been widely used because of its insensitivity and robustness to disturbance and uncertainty. The conventional robustness of the SMC only occurs after the sliding motion. To eliminate the reaching phase and ensure the whole sliding motion, integral sliding mode control (ISMC) is proposed [20]. In [21], the ISMC integrated with control allocation (CA) was proposed for the problem of actuator fault and disturbance. In [22], a cooperative tracking problem for a group of nonlinear systems with actuator faults and external disturbance was studied; the ISMC technique was designed to tolerate the fault. In order to acquire fast response, terminal sliding mode control (TSMC) technique is proposed. The TSMC uses the nonlinear sliding surface to enforce the states to reach to the equilibrium point in finite time while maintain the excellent performance, i.e., the high precision and robustness ability [23-24]. However, the above results are failed to consider the disturbance.

Motivated by [11], [19] and [22], this paper is attempted to solve the FTC problem for a class of nonlinear systems with actuator fault and mismatched disturbance. The main works and contributions can be summarized as follows: 1) Simultaneous state, fault, mismatched disturbance and fault efficiency factor estimation technique is developed by designing a composite observer, with the aid of a compensator, the estimation errors can converge to zero asymptotically. 2) A novel sliding mode surface integrated with estimated information is designed and an observer based controller is constructed. In this controller, two compensators are involved to deal with the actuator fault and mismatched disturbance; 3) The mismatched disturbance is divided into two parts, one part is compensated by the disturbance compensator while the other is attenuated within a $H_\infty$ level; 4) The reaching ability of sliding motion is guaranteed in finite time. The remainder of this paper is organized as follows. Section II states the system description and some assumptions. Section III presents the detail procedure of the proposed observer based control method. In section IV, a B747-100/200 aircraft model and an electric circuit system are used to illustrate the effectiveness of the method. Section V concludes the paper.

II. PROBLEM FORMULATION

Consider the following nonlinear system with external disturbance as

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + g(x,t) + B_d d(t) \\
y(t) &= Cx(t)
\end{align*}
\]

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^q$ is the output of the system, $u(t) \in \mathbb{R}^p$ is the control input, $g(x,t) \in \mathbb{R}^n$ is a nonlinear function, which stands for the uncertainty. $d(t) \in \mathbb{R}^r$ is the external disturbance. $A$, $B$, $C$, and $B_d$ are known matrices with appropriate dimensions.

In this paper, fault may happen on one or several actuators. The actuators fault can be modelled as [12]

\[
u_i^f(t) = (1-\rho_i) u_i(t) + \xi_i \bar{u}_i(t)
\]

where $u_i^f(t)$ and $u_i(t)$ are the $i$th control input signal and actuator input signal. $\rho_i$ is an unknown function represents the actuator efficiency factor and satisfies $0 < \rho_1 \leq \rho_2 \leq \rho_3 \leq 1$, where $\rho_1$ and $\rho_2$ are the lower and upper bound of the efficiency factor, in addition, $\|B_i\| \leq \rho_n$ and $\rho_n$ is a positive constant. $\xi_i$ is a constant and satisfies $\xi_i \in \{0,1\}$, i.e., $\xi_i = 1$ stands for the actuator fault occurs in the $i$th actuator, and $\xi_i = 0$ means that the $i$th
actuator is normal. \( \hat{u}_i(t) \) can be regarded as the stuck fault or bias fault in the \( i \)th actuator.

The actuator fault model can represent the following fault patterns

a) **Stuck fault**: this means that \( u_i(t) \) is locked in a special position or time varying function \( \hat{u}_i(t) \) and dose not response to the subsequent control signal, i.e., \( \rho_1 = 1, \xi_i = 0 \).

b) **Partial loss of effectiveness fault**: this indicates that the response to the control signal is abnormal, i.e., \( 0 < \rho_1 \leq \rho_2 < 1 \), and \( \xi_i = 0 \).

c) **Bias fault**: it is characterized by the actuator floating to nominal control response, i.e., \( 0 < \rho_1 \leq \rho_2 < 1 \) and \( \xi_i = 1 \).

d) **Fault free case**: it occurs when \( \rho_1 = \rho_2 = 0 \) and \( \xi_i = 0 \).

Then the fault model can be described as

\[
\hat{u}_i(t) = \left( I - \lambda(\rho) \right) u(t) + \xi_i \hat{u}_i(t)
\]  

(3)

where \( u(t) \in \mathbb{R}^p \) is the applied control, \( \lambda(\rho) = \text{diag}\{\rho_1, \rho_2, \ldots, \rho_p\}^T \), \( \rho = [\rho_1, \rho_2, \ldots, \rho_p]^T \), \( \xi = \text{diag}\{\xi_1, \xi_2, \ldots, \xi_p\} \), \( \hat{u}(t) = \left[ \hat{u}_1(t), \hat{u}_2(t), \ldots, \hat{u}_p(t) \right]^T \).

**Remark 1**: In the stuck condition, only parts of \( \rho_1 = 1 \), this means that some controllers remain working. In addition, the loss of effectiveness coefficient \( \rho_i \) is a varying parameter and its value depends on the extent of the actuator’s fault.

The dynamic system (1) with actuator fault can be represented as

\[
\dot{x}(t) = Ax(t) + B\left( I - \lambda(\rho) \right) u(t) + B\xi \hat{u}_i(t) + g(x(t)) + B_d d(t)
\]  

(4)

The main objective of this paper is to design a sliding mode controller such that the states of the closed loop system converge to zero asymptotically.

**Assumption 1** [13]: The bias fault function \( \hat{u}(t) \) is bounded, i.e., \( \|\hat{u}(t)\| \leq \delta_d \) and \( \delta_d \) is a known positive scalar.

**Assumption 2** [19]: The external disturbance \( d(t) \) and its derivative are norm bounded, i.e., \( \|d(t)\| \leq d_m \), \( \|\dot{d}(t)\| \leq d_n \), \( d_m \) and \( d_n \) are two known positive scalars.

**Assumption 3**: The nonlinear function \( g(x,t) \) satisfies \( g(x,t) \leq \sigma_m x(t) \) and \( g(a,t) - g(b,t) \leq \sigma_m \|a - b\| \), where \( \sigma_m \) is a known positive scalar, and \( \sigma_m \) is the Lipschitz constant.

**III. MAIN RESULTS**

In this part, the main purpose is to design the control law to guarantee the stability of (4) by two main steps. First, a composite observer is designed to estimate the unknown variables in the system. Then, an observer based sliding mode approach is designed, where a suitable control effort is derived such that the system states can reach the sliding surface in finite time.

**A. OBSERVER DESIGN**

Motivated by [19], in this paper, the observer is designed in the form of

\[
\dot{x}(t) = A\hat{x}(t) + B\left( I - \lambda(\hat{\rho}) \right) u(t) + B\xi \hat{u}_i(t) + L(y(t) - \hat{y}(t)) + B_d \hat{d}(t) + g(\hat{x}(t))
\]  

(5)

\[
\hat{d}(t) = \zeta(t) + L_y y(t)
\]  

\[
\dot{\hat{y}}(t) = -L_C g(\hat{x}(t)) - L_C B\left( I - \lambda(\hat{\rho}) \right) u(t) - L_C A\hat{x}(t)
\]  

\[
- L_C B_d \hat{z}(t) - L_C B_d \hat{L}_y y(t) + \delta(t)
\]

where \( \hat{x}(t) \), \( \hat{d}(t) \), \( g(\hat{x}(t)) \), \( \zeta(t) \), \( \hat{d}(t) \), \( \delta(t) \) and \( \hat{\rho} \) are the estimation of \( x(t) \), \( d(t) \), \( g(x(t), t) \), \( \hat{u}(t) \) and \( \rho \), \( L \) and \( L_1 \) are two observer gains. \( \zeta(t) \) is the intermediate state of the observer, \( \delta(t) \) is a compensator. \( \hat{d}(t) \) and \( v(t) \) are defined by

\[
\delta(t) = \xi^2 L_C B B^T C^T L^T \sigma_d \|\zeta(t) + \kappa_m \| + Q^T \sigma_d \|u(t)\|
\]  

(6)

\[
\sigma_d = \sigma_m L_C C^T L^T + \sigma_m L_C C^T
\]

\[
v(t) = \xi^2 B^T P \sigma_d \|\zeta(t) \| \|
\]  

(7)

where \( \delta_d \) and \( \kappa_m \) are two positive scalars, \( \mu_k = 2 \sum_{i=1}^{p} (\pi_j + e_j) \|(\rho_i - \rho_j)(\rho_i - \rho_j + \mu_a)\| \), \( \pi_j \) and \( e_j \) are designed in (26) and (27) respectively. \( P \) and \( Q \) are two designed positive symmetry matrices.

**Remark 2**: Compared with conventional disturbance observer [19], the proposed observer contains a compensator \( \delta(t) \), which is used to attenuate the effects of the bias actuator fault, diminish the nonlinear uncertainty and the derivative variance. Thus the error of the estimations can converge to zero asymptotically.

In the observer (5), the adaptive laws are designed as follows

\[
\hat{\rho} = \ell_k \begin{cases} 1 & \text{if } \ell_k > 0 \text{ or } \hat{\rho} > \rho_1, \text{ and } \ell_k < 0 \\
0 & \text{otherwise} \end{cases}
\]  

(8)

\[
\dot{\hat{\rho}}_m = \ell_m \begin{cases} 1 & \text{if } \ell_m > 0 \text{ or } \hat{\rho}_m > \rho_1, \text{ and } \ell_m < 0 \\
0 & \text{otherwise} \end{cases}
\]  

(9)

where \( \ell_k = \eta (-\lambda(\hat{u}(t)))^T B^T P \sigma_d (\hat{\rho} - \rho_m) \), \( \ell_m = \eta_m (\hat{\rho} - \hat{\rho}_m) \), \( \eta \) and \( \eta_m \) are two diagonal matrices, \( \hat{\rho}_m \) can be regarded as the output of \( \rho \).

From (5), one has
\[
\dot{\xi}(t) = L_d CB_d \left( d(t) - \dot{\xi}(t) \right) + L_d CA \left( x(t) - \dot{x}(t) \right) + \delta(t)
\]
\[
+ L_d CB_2 \alpha(t) - L_d C \left( \lambda(\rho) - \lambda(\tilde{\rho}) \right) u(t) + L_d CG(t)
\]

Define \( e_x = x(t) - \dot{x}(t), \) \( e_d = d(t) - \dot{\xi}(t), \) \( \tilde{\rho} = \rho - \hat{\rho}_m, \) \( \rho = \rho - \hat{\rho}_m, \) \( \text{then the following error dynamic system can be obtained:} \)
\[
\begin{align*}
\dot{e}_x &= A_e e_x - B_e \tilde{\rho} u(t) + B_x \xi(t) + B_d e_d \\
- L_e e_x + \tilde{g}(x,t) \\
\dot{e}_d &= -L_d CB_d e_d - L_d CB_2 \tilde{\rho} u(t) - \Gamma e_x + L_d CB_2 \alpha(t) \\
- L_d CG(t) + \tilde{d}(t) - \delta(t) \\
\end{align*}
\]

where \( \tilde{g}(x,t) = g(x,t) - g(\dot{x},t), \) \( \Gamma = L_d CA. \)

According to (8) and (9), the following equations hold
\[
\begin{align*}
\dot{\tilde{\rho}} &= \eta \left( \tilde{\rho} - \hat{\rho}_m \right) + \lambda(u(t)) \begin{bmatrix} B^T \dot{P} \end{bmatrix} e_x + \hat{\rho} \\
\end{align*}
\]

**Theorem 1:** With the assumptions 2, 3, and adaptive laws (8) and (9), \( e_x, e_d, \) and \( \tilde{\rho} \) will converge to zero, if there exist two positive symmetry matrices \( P \) and \( Q \) such that the following condition holds
\[
V(t) = e^T \dot{P} e_x + e_d^T Q e_d + \eta_1^{-1} \tilde{\rho}_m \tilde{\rho} + \eta_1^{-1} \hat{\rho}_m^T \hat{\rho}_m 
\]

From (11), the time derivative of \( V(t) \) can be obtained as
\[
\dot{V}(t) = 2e^T \ddot{P} e_x + 2e_d^T Q e_d + \eta_1^{-1} \dot{\tilde{\rho}} + 2 \eta_1^{-1} \hat{\rho}_m \hat{\rho}_m 
\]

Combining equations (16-20) yields
\[
\begin{align*}
\dot{V}(t) &\leq e^T \dot{P} e_x + e_d^T Q e_d + \eta_1^{-1} \tilde{\rho}_m \tilde{\rho} + 2 \eta_1^{-1} \hat{\rho}_m \hat{\rho}_m \\
&\leq -\left( \left| \eta \right| \right) \left( \left| e_x \right| + \left| e_d \right| \right) + 2 \eta_1^{-1} \tilde{\rho}_m \tilde{\rho} + 2 \eta_1^{-1} \hat{\rho}_m \hat{\rho}_m \\
\end{align*}
\]

From (2), one can obtain that
\[
\begin{align*}
2e^T \dot{Q} L_d CB_2 \tilde{\rho} &\leq e^T \ddot{Q} L_d CB_2 \tilde{\rho} + e_d^T Q e_d + \eta_1^{-1} \tilde{\rho}_m \tilde{\rho} + 2 \eta_1^{-1} \hat{\rho}_m \hat{\rho}_m \\
&\leq -2 \eta_1^{-1} \tilde{\rho}_m \tilde{\rho} + 2 \eta_1^{-1} \hat{\rho}_m \hat{\rho}_m \\
\end{align*}
\]

Due to the control input is bounded, one can check that there exist two matrices \( \phi_i \) and \( \phi_o \) such that the following inequalities hold
where $\|u(t)\| \leq \pi_m$.

From (32), one has

$$V_1 \leq -2\eta^T \varepsilon^T PB_1 e_1 \leq -2\varepsilon^T (P) \sqrt{V_1(t)}$$ (36)

This shows the reachability is satisfied and the sliding motion will take place in finite time. This completes the proof.

Consequently, when the sliding motion take place on the $S$, the error dynamic system can be rewritten as

$$(A - LC)e_x - B\lambda(\hat{p})u(t) + B\xi(\tilde{u}(t) - u(t)) + \tilde{g}(x(t)) + B_d e_d = 0$$ (37)

From (37), it follows that

$$\tilde{u}(t) - u(t) = (B\xi)^T (B\lambda(\hat{p})u(t) - B_d e_d - (A - LC)e_x - \tilde{g}(x(t))) \leq \|B\xi\|^2 \|B_d\| \varepsilon_d + \|B\xi\| \varepsilon_n \|\lambda\| + \|B\xi\| \|\lambda\| + \|A - LC\| \varepsilon_n \|\lambda\|$$ (38)

where $\varepsilon_n \geq (B\xi)^T (B\xi) \geq (B\xi)^T \varepsilon_n \|B\xi\|^2 + \|B\xi\| \|\lambda\| + \|B\xi\| \|\lambda\| + \|A - LC\| \varepsilon_n \|\lambda\|$, $B\xi$ and $\tilde{p}$ will converge to zero, which indicates that $w_m$, $w_d$ and $w_k$ will converge to zero, then $\tilde{u}(t)$ can be approximated by (33). This completes the proof.

C. FTC DESIGN

The objective of this subsection is to design an output integral sliding mode control scheme in accordance with the estimated values. The controller is designed in the following form

$$u(t) = u_1(t) + u_2(t)$$ (39)

where $u_1(t) \in R^p$ is used to guarantee the stability of the system and $u_2(t) \in R^p$ is an sliding mode compensator which is used to counteract the observer errors.

The sliding mode surface is designed as follows

$$\sigma(t) = G\xi(t) - G\xi(0) - G\xi_0(0) \int_0^t (\lambda(\hat{p}) + B_d \delta(t)) + B\xi(\tilde{u}(t) - u(t)) + \tilde{g}(x(t)) + g(x(t)) d\tau$$ (40)

where $G$ is an parameter matrix, which is designed in the form of $G = H(CB)^T$, where $H \in R^{P \times P}$ is a designed matrix and

$$(CB)^T = (CB)^T (CB)^T C^T B \xi$$ (41)

From (40), one can deduce that
satisfies. By integrating, are two positive scalars. (52) can be written as

\[
\dot{x} = f(x) + G \dot{z} = (\mathbf{C} \dot{z} + \mathbf{G} \dot{x}) + \mathbf{H} \dot{u}
\]

Then (46) can be rewritten as

\[
\dot{V}_2(t) \leq \sigma(t)^T H \| x - \bar{x} \|^2 - \delta_a | x - \bar{x} |
\]

Next, we will analyze the stability of the system. For (41), when the sliding motion occurs, i.e., \( \dot{\sigma}(t) = 0 \), the equivalent control can be obtained as

\[
u_{eq}(t) = (I - \lambda(\hat{\rho}))^{-1} \left[ -(\mathbf{C})^T \mathbf{A}_e \dot{x} + \lambda(\hat{\rho}) \dot{u}(t) \right]
\]

Substituting (49) into (4) yields

\[
\dot{\mathbf{x}}(t) = \left( I - B(\mathbf{C})^T \mathbf{C} \mathbf{A}_e - (\mathbf{C})^T \mathbf{A}_e \mathbf{C} \mathbf{A}_e - \lambda(\hat{\rho}) \mathbf{C} \mathbf{A}_e \right) \dot{\mathbf{x}}(t)
\]

In this paper, \( u_1(t) \) is designed in the form of

\[
u_1(t) = -\left[ (I - \lambda(\hat{\rho}))^{-1} \left( (\mathbf{C})^T \mathbf{A}_e \dot{z} + (\mathbf{C})^T \mathbf{G} \dot{x} \right) \right]
\]

where \( K_a \) and \( K_m \) are two controller gain matrices. Submitting equation (51) into (50), one has

\[
\dot{\mathbf{x}}(t) = \left( I - B(\mathbf{C})^T \mathbf{C} \mathbf{A}_e \mathbf{C} \mathbf{A}_e \right) \dot{\mathbf{x}}(t) + \left( B_d - B(\mathbf{C})^T \mathbf{A}_e \mathbf{C} \mathbf{A}_e \right) \dot{d}(t)
\]

By taking \( \hat{A} = A - B(\mathbf{C})^T \mathbf{C} \mathbf{A} \), \( \hat{B}_d = B_d - B(\mathbf{C})^T \mathbf{C} \mathbf{A}_e \mathbf{C} \mathbf{A}_e \) and \( \hat{I} = I - B(\mathbf{C})^T \mathbf{C} \), (52) can be written as

\[
\dot{\mathbf{x}}(t) = \left( I - \hat{A} - \hat{B}_d \hat{K}_m \right) \dot{\mathbf{x}}(t) - \hat{B}_d \hat{K}_m \dot{d}(t) - \hat{B}_d \hat{K}_m \dot{d}(t)
\]
\[ \dot{x}(t) = (\tilde{A} - BK_m)x(t) + \tilde{I}g(x(t)) + \tilde{B}_d d(t) 
- BK_m \dot{d}(t) + BK_me_e \] (53)

In this paper, the mismatched disturbance is assumed to be divided into two parts, i.e.,
\[ \tilde{B}_d = [\tilde{B}_{d1} \ \tilde{B}_{d2} \ldots \tilde{B}_{dem}]_m, \] where
\[ \tilde{B}_{d1} = [\tilde{B}_{d_{11}} \ \tilde{B}_{d_{12}} \ldots \tilde{B}_{d_{d1m}}]_m \] for \( i \leq m \) and \( \tilde{B}_{d_{mi}} = 0 \) for \( i > m \). Then the equation \( BK_m = \tilde{B}_{d1} \) is solvable, and (53) can be rewritten as
\[ \dot{x}(t) = (\tilde{A} - BK_m)x(t) + \tilde{I}g(x,t) + B_e \mathcal{H}(t) \] (54)
where
\[ B_e = [BK_m \ \tilde{B}_{d1} \ \tilde{B}_{d2} \ldots d_{dem}]_m \mathcal{H}(t) = [e_1 \ e_{d1} \ \ldots \ d_{dem}]_m. \]

**Theorem 4:** For a given \( H_\infty \) performance level \( \gamma \), the system (54) is asymptotically stable with \( H_\infty \) performance, i.e.,
\[ \int_0^\tau y(t) y(t)\,dt \leq \gamma^2 \int_0^\tau \mathcal{H}(t)\,dt. \] If there exists a symmetric positive matrix \( P_1 \), such that the following condition (55) holds
\[ \Xi_1 = \left[ \begin{array}{cccc} P_1B_e & P_1I & C^* & * \\
* & -\gamma^2 & 0 & 0 \\
* & * & -I & 0 \\
* & * & * & -I \\
\end{array} \right] < 0 \] (55)
where
\[ \Xi_1 = P_1(\tilde{A} - BK_m)^T + (\tilde{A} - BK_m)^T P_1 + \sigma_2^2. \]

**Proof:** Consider the Lyapunov candidate function as
\[ V_3(t) = x(t)^T P_1 x(t) \] (56)
The differential of (56) is
\[ \dot{V}_3(t) = 2x(t)^T P_1 \left( (\tilde{A} - BK_m)x(t) + \tilde{I}g(x,t) + B_e \mathcal{H}(t) \right) 
= x(t)^T \left( P_1(\tilde{A} - BK_m) + (\tilde{A} - BK_m)^T P_1 \right) x(t) 
+ 2x(t)^T P_1 B_e \mathcal{H}(t) + 2x(t)^T P_1 \tilde{I}g(x,t) \] (57)
Note that
\[ 2x(t)^T P_1 \tilde{I}g(x,t) \leq x(t)^T P_1 \tilde{I} + g(x,t)^T g(x,t) \leq x(t)^T \left( P_1 \tilde{I}^2 + \sigma_2^2 \right) x(t) \] (58)
Then (57) can be rewritten as
\[ \dot{V}_3(t) \leq x(t)^T \left( R_1(\tilde{A} - BK_m) + (\tilde{A} - BK_m)^T P_1 \right) x(t) 
+ P_1 \tilde{I} P + \sigma_2^2 x(t)^T g(x,t) \] (59)
Let
\[ J = y(t)^T y(t) - \gamma^2 \mathcal{H}(t)^T \mathcal{H}(t) \] (60)
Then
\[ J \leq y(t)^T y(t) - \gamma^2 \mathcal{H}(t)^T \mathcal{H}(t) + V_2(t) \]
\[ = x(t)^T \left( P_1(\tilde{A} - BK_m) + (\tilde{A} - BK_m)^T P_1 + C^T C + \sigma_2^2 \right) x(t) 
+ P_1 \tilde{I} P x(t) + 2x(t)^T P_1 B_e \mathcal{H}(t) - \gamma^2 \mathcal{H}(t)^T \mathcal{H}(t) \]
\[ = \xi^T \Xi \xi, \] where
\[ \xi = [x(t)^T \ \mathcal{H}(t)^T]^T, \]
\[ \Xi = \left[ \begin{array}{cc} \Psi & P_1 B_e \\
P_1 & -\gamma^2 \end{array} \right]. \]
According to the Schur complement, \( J \leq 0 \). This completes the proof.

**IV. RESULTS ANALYSIS**

**Case 1** In this example, a model of B747-100/200 aircraft which is borrowed from [25] is considered. The states are pitch rate, true airspeed, angle of attack, pitch angle, altitude position and horizontal. For the control purpose, only the first four states have been retained. The inputs are elevator deflection, total thrust and horizontal stabilizer. The dynamic system can be described in the form of (1),
where
\[ A = \begin{bmatrix} -2.98 & 0.93 & 0 & -0.0340 \\
-0.99 & -0.21 & 0.035 & -0.0011 \\
0 & 0 & -2 & 1 \\
0.39 & -5.555 & 0 & 1.89 \end{bmatrix}, \]
\[ B = \begin{bmatrix} -0.032 & 0.5 & 1.55 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
-1.6 & 1.8 & -2 \end{bmatrix}, \]
\[ C = \begin{bmatrix} 1 & 0 & 0 \\
0 & 1 & 1 \\
0 & 1 & 1 \end{bmatrix}, \]
\[ B_d = \begin{bmatrix} -0.032 & 0.8 \\
0 & 0 \\
0 & 0 \\
-1.6 & 1 \end{bmatrix}. \]
The bias fault, disturbance are described as
\[ \hat{a}_1(t) = \begin{cases} 0 & (0 \leq t < 12) \\
0.5 \sin(0.5t) & (12 \leq t < 30) \end{cases}, \]
\[ \hat{a}_2(t) = \begin{cases} 0 & (0 \leq t < 10) \\
3 \sin(t) & (10 \leq t < 15) \\
0.5 & (15 \leq t < 30) \end{cases}. \]
\[ \ddot{x}_1(t) = \begin{cases} 
0 & (0 \leq t < 5) \\
12 \cos(t) & (5 \leq t < 6) \\
0 & (6 \leq t < 10) \\
3 \sin(\alpha) & (10 \leq t \leq 12) \\
0 & (12 \leq t < 30)
\end{cases}, \\
\dot{d}_1(t) = \begin{cases} 
0.5 \sin(2t+1) & (0 \leq t < 5) \\
1 & (5 \leq t < 30)
\end{cases}, \\
\dot{d}_2(t) = \begin{cases} 
0 & (0 \leq t < 5) \\
0.5 \sin(0.3t) & (5 \leq t < 15) \\
1 + 0.3e^{-2t} & (15 \leq t < 30)
\end{cases}. \\
\]

The following parameters and initial conditions are provided in this simulation:

- \( a = 0.8 \), \( \beta = 0.001 \), \( \gamma = 1.8 \), \( \delta = 2 \), \( \delta_m = 0.5 \), \( \sigma_m = 0.2 \), \( \sigma_m = 0.5 \),
- \( x(0) = [0.01, -0.01, 0.02, -0.15]^T \), the actuator is loss of effectiveness 20%. By solving (14) and (53), the parameters can be obtained as

\[
P = \begin{bmatrix}
100.1983 & -10.6089 & -9.1207 & 2.2715 \\
-10.6089 & 135.9236 & 49.9467 & 33.4009 \\
-9.1207 & 49.9467 & 57.4948 & 21.8478 \\
2.2715 & 33.4009 & 21.8478 & 31.1199 
\end{bmatrix},
\]

\[
L = \begin{bmatrix}
-2.3022 & 0.4958 \\
0.3292 & -2.5444 \\
-0.8827 & -1.8173 \\
-4.2964 & 7.9461 
\end{bmatrix},
\]

\[
K_n = \begin{bmatrix}
0.3284 \\
-0.4508 \\
0.1316 
\end{bmatrix},
\]

\[
P_1 = \begin{bmatrix}
2.2913 & 0.8277 & -0.7014 & -0.1080 \\
0.8277 & 17.6335 & -7.4630 & -10.0785 \\
-0.7014 & -7.4630 & 13.5274 & -5.0741 \\
-0.1080 & -10.0785 & -5.0741 & 16.2718 
\end{bmatrix},
\]

\[
K_m = \begin{bmatrix}
-2.3200 & -53.6940 & -52.7702 & -50.9463 \\
-3.2397 & -37.3438 & -36.7912 & -35.6127 \\
-1.0775 & 10.9605 & 10.7103 & 10.3439 
\end{bmatrix}.
\]
In this example, an electric circuit system [26] is studied to demonstrate the effectiveness and feasibility of the proposed method. The state equations of the electric circuit system shown as Fig. 9 are obtained as

\[
\begin{align*}
\dot{v}_1 &= -\frac{1}{R_1C_1}v_1 + \frac{1}{R_1C_1}v_2, \\
\dot{v}_2 &= \frac{1}{R_2C_2}v_1 - \left(\frac{1}{R_1C_2} + \frac{1}{R_2C_2}\right)v_2 + \frac{1}{R_2C_2}u, \\
\end{align*}
\]

where \( R_1 = 1\Omega \), \( R_2 = 0.5\Omega \), \( c_1 = 1F \), \( c_2 = 0.001F \).

Define \( x(t) = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \), \( u(t) = u \). Then the system (60) was affected by external disturbance and uncertainty can be described by (1) with \( B_d = \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix} \), \( g(x,t) = 0.5x \). The bias fault, disturbance, fault efficiency factor are assumed as

\[
\begin{align*}
\bar{u}(t) &= \begin{cases} 
0 & (0 \leq t < 10) \\
0.5\sin(0.5t) & (10 \leq t < 20), \\
e^{-t} & (20 \leq t < 30), \\
\end{cases} \\
d_1(t) &= \frac{1}{1+t^2} & (5 \leq t < 30), \\
\end{align*}
\]

\[
\begin{align*}
d_2(t) &= \begin{cases} 
0 & (0 \leq t < 15) \\
0.5\sin(0.5t) & (15 \leq t < 20), \\
1 & (20 \leq t < 30), \\
\end{cases} \\
p(t) &= \begin{cases} 
0 & (0 \leq t < 18) \\
0.5 & (18 \leq t < 30), \\
\end{cases}
\end{align*}
\]

The following parameters and initial conditions are provided in this simulation, \( \beta_u = 0.8 \), \( \delta_a = 0.001 \), \( \gamma = 1.8 \), \( \delta_k = 2 \), \( \delta_m = 0.5 \), \( \sigma_m = 0.2 \), \( \sigma_m = 0.5 \), \( x(0) = [2 \ 3]^T \). By solving (14) and (53), the parameters can be obtained as

\[
\begin{bmatrix}
P \\
L \\
L_1 \\
K_n \\
K_m
\end{bmatrix} =
\begin{bmatrix}
5.2603 & -5.1638 \\
-5.1638 & 5.1550 \\
385.0464 & 388.4274 \\
0.1567 & 3.2254 \\
-28.3229 & -28.0877 \\
28.0877 & 29.5799 \\
-1.6595 & -1.6503
\end{bmatrix},
\]

indicates that the disturbance can be replaced by the estimated values when designing the controller.

Case 2 In this example, an electric circuit system [26] is studied to demonstrate the effectiveness and feasibility of the proposed method. The state equations of the electric circuit system shown as Fig. 9 are obtained as

\[
\begin{align*}
\dot{v}_1 &= -\frac{1}{R_1C_1}v_1 + \frac{1}{R_1C_1}v_2, \\
\dot{v}_2 &= \frac{1}{R_2C_2}v_1 - \left(\frac{1}{R_1C_2} + \frac{1}{R_2C_2}\right)v_2 + \frac{1}{R_2C_2}u, \\
\end{align*}
\]

where \( R_1 = 1\Omega \), \( R_2 = 0.5\Omega \), \( c_1 = 1F \), \( c_2 = 0.001F \).

Define \( x(t) = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \), \( u(t) = u \). Then the system (60) was affected by external disturbance and uncertainty can be described by (1) with \( B_d = \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix} \), \( g(x,t) = 0.5x \). The bias fault, disturbance, fault efficiency factor are assumed as

\[
\begin{align*}
\bar{u}(t) &= \begin{cases} 
0 & (0 \leq t < 10) \\
0.5\sin(0.5t) & (10 \leq t < 20), \\
e^{-t} & (20 \leq t < 30), \\
\end{cases} \\
d_1(t) &= \frac{1}{1+t^2} & (5 \leq t < 30), \\
\end{align*}
\]

\[
\begin{align*}
d_2(t) &= \begin{cases} 
0 & (0 \leq t < 15) \\
0.5\sin(0.5t) & (15 \leq t < 20), \\
1 & (20 \leq t < 30), \\
\end{cases} \\
p(t) &= \begin{cases} 
0 & (0 \leq t < 18) \\
0.5 & (18 \leq t < 30), \\
\end{cases}
\end{align*}
\]

The following parameters and initial conditions are provided in this simulation, \( \beta_u = 0.8 \), \( \delta_a = 0.001 \), \( \gamma = 1.8 \), \( \delta_k = 2 \), \( \delta_m = 0.5 \), \( \sigma_m = 0.2 \), \( \sigma_m = 0.5 \), \( x(0) = [2 \ 3]^T \). By solving (14) and (53), the parameters can be obtained as

\[
\begin{bmatrix}
P \\
L \\
L_1 \\
K_n \\
K_m
\end{bmatrix} =
\begin{bmatrix}
5.2603 & -5.1638 \\
-5.1638 & 5.1550 \\
385.0464 & 388.4274 \\
0.1567 & 3.2254 \\
-28.3229 & -28.0877 \\
28.0877 & 29.5799 \\
-1.6595 & -1.6503
\end{bmatrix},
\]
Figs. 10 and Fig. 11 show the trajectories of the electric circuit system under the proposed method and the method in [26]. It can be seen that the stability of the system can be guaranteed under the two methods. However, compared with the method in [26], the robustness of the proposed method in this paper is better and the response speed is faster. The response of the control input of the system is given in Fig. 12; it can be observed that the both two controllers work in the presence of fault, and the performance of propose method is better than the method in [26].

Figs. 13–15 characterize the bias faults, disturbance, fault efficiency factor and their estimations. From Figs. 13–15, it can be concluded that the estimation errors are small, which imply that the proposed observer can estimate the external disturbance, fault and fault efficiency factor well, and the estimated information can be utilised to design the controller.

**V. CONCLUSION**

In this paper, the FTC problem of a class of nonlinear system with actuator fault and mismatched disturbance is investigated. The partial loss of effectiveness fault and bias fault are contained. In order to achieve the fault and disturbance compensation, a new observer based robust control method is proposed. Concretely, firstly, a composite observer is proposed to estimate the state, disturbance, bias fault and fault efficiency factor simultaneously. Then, in accordance with the estimated information, an integral
sliding mode controller including a fault compensator, disturbance compensator and an adaptive law is presented. Furthermore, the disturbance is divided into two parts, where the matched part is compensated by the compensator and mismatched part is attenuated within $H_\infty$ index. Finally, the simulation results for the B747-100/200 aircraft and an electric circuit system show the effectiveness of the proposed method.

REFERENCES


