Finite-time consensus of heterogeneous multi-agent systems without velocity measurements and with disturbances via integral sliding mode control

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ABSTRACT In this paper, the finite-time consensus protocols are designed to solve the problems of unmeasured velocity and disturbances in heterogeneous multi-agent systems, in which the communications among agents are described by a directed graph. The proposed consensus protocols use only position measurements. The velocity of the agents is estimated according to the position measurements of multi-agent systems. The integral sliding mode control is adopted to eliminate the bounded disturbances of the systems and shorten the time to achieve consensus. The sufficient conditions for finite-time consensus are obtained by employing Lyapunov stability theory. Finally, simulations are provided to verify the effectiveness of the proposed schemes.

INDEX TERMS Multi-agent systems, Sliding mode control, Robustness, Graph theory

I. INTRODUCTION

Distributed coordination of multi-agent systems (MAS) has been intensively studied due to its applications in many fields, such as robots, spacecraft, and transportation planning [1]-[3]. A fundamental topic is to develop protocols, which specify the information exchange between agents, such that the group can reach consistency. This problem is known as consensus problem [4]. The extensions of consensus problem include group consensus [35], [36], and resilient consensus [37], etc. The existing results on consensus mainly focus on homogeneous MAS [5]-[9]. That is to say, all agents have the same dynamic behaviors.

However, the agents in the MAS have different dynamic behaviors because of various constraints, so the heterogeneous MAS are more common in practice. The consensus of heterogeneous MAS has been studied in the recent years [10]-[17]. In [12] and [13], the consensus protocols of first-order and second-order MAS have been proposed. Some sufficient conditions that the heterogeneous MAS solve the consensus problems are proposed.

In the studies of distributed coordination problem, most protocols of second-order MAS depend on the availability of feedback information of the full states. But some information is unmeasured because of technology limitations or environment disturbances. For example, velocities of the agents sometimes are not available when vehicles are not equipped with velocity sensors, to save cost, space and weight. It is significant to research the consensus problem of second-order MAS without velocity measurements. In [18] and [19], the authors proposed the consensus algorithms for second-order MAS without...
velocity measurements. Zheng et al. [20] studied the consensus of heterogeneous MAS without velocity measurements. Ren et al. [21] studied the leader-following consensus problem of second-order nonlinear stochastic MAS under unmeasured velocity, and the authors built the distributed observer to estimate the unknown velocity. Mei et al. [22] proposed a distributed control algorithm combined with a distributed filter for the leaderless consensus problem in the case of the unmeasured velocity. Other papers on MAS without velocity measurements are in [23]-[25]. The literatures [18]-[25] paid attention to consensus of MAS with unmeasured velocity in different methods, such as auxiliary system, observer, filter, etc. In [18], the auxiliary system was used in the control scheme. In [19],[21],[24],[25], the observers had been proposed. In [22] and [23], the filters were introduced. However, these papers do not consider the disturbances.

Disturbances widely exist in practical processes, and they are often bad for system performance or even cause system instability. Therefore, there are also some literatures [26]-[28] studied the consensus of MAS in the presence of disturbances, but they do not consider the problem of unmeasured velocity.

Sliding mode control (SMC) is essentially a kind of nonlinear control method. The main difference between this control strategy and other control strategies is that the structure of the controller is variable. It can be purposefully changed in the dynamic process according to the current state of the system, forcing the system to move according to the predetermined sliding mode trajectory. SMC has many advantages, such as fast response, insensitive to uncertain but bounded disturbance, and simple physical implementation. The extensions of SMC have taken place in the last decades which include integral SMC (ISMC) [28]-[30]. The literature [30] firstly proposed the integral sliding mode design concept. The literature [29] proposed the integral sliding manifold design method for linear systems with unmatched constant external disturbances. In [28], the ISMC was used to deal with the disturbance in the homogeneous MAS. Different from [28]-[30], this paper applied ISMC to deal with the bounded disturbances in heterogeneous MAS. The improvement of this paper is that the sliding mode states are designed differently for one-second and second-order agents.

The above literatures considered either the velocity was unmeasured or the disturbance existed. However, the two situations exist simultaneously in reality. Therefore, we consider the consensus of heterogeneous MAS in both situations. The main advantage of this paper is that we design an estimation velocity based on the position information to replace the actual velocity. Moreover, the system is robust to bounded disturbances.

Motivated by the above results, we aim to study the finite-time consensus for heterogeneous MAS without velocity measurements and with disturbances. The main contributions lie in three aspects. Firstly, we propose the novel finite-time consensus protocols for heterogeneous MAS, in which the communications between agents are described by a directed graph. Secondly, based on ISMC, the consensus protocols ensure that the system has good robustness against disturbances. Thirdly, the proposed consensus protocols are designed only by the position measurements.

The remainder of this paper is organized as follows. In Section II, some basic ideas of MAS and lemmas are briefly outlined. The main results are presented in Section III. Simulations are given in section IV, and the conclusion is drawn in Section V.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. PRELIMINARIES

In order to study consensus in MAS, some preliminary results about graphs in this paper are introduced here.

Let \( G = \{V,E,A\} \) be a directed graph of order \( n \) with the set of nodes \( V = \{\varsigma_1, \varsigma_2, \ldots, \varsigma_n\} \), the set of directed edges \( E \subseteq V \times V \), and an adjacency matrix \( A = [a_{ij}] \in \mathbb{R}^{n \times n} \). The directed edge \( E_\varsigma \) in graph \( G \) is denoted by the ordered pair of nodes \( (\varsigma_i, \varsigma_j) \), where \( \varsigma_i \) and \( \varsigma_j \) are called the child and parent nodes, respectively. It means that node \( \varsigma_i \) can receive information from node \( \varsigma_j \), but not another way around. The set of neighbors of node \( \varsigma_j \) is denoted by \( N_j = \{\varsigma_i | (\varsigma_j, \varsigma_i) \in E\} \). The graph Laplacian matrix corresponding to \( G \) is defined as \( L = D - A \), in which \( D = \text{diag}\{d_1, d_2, \ldots, d_n\} \) is the in-degree matrix with \( d_j = \sum_{j=1}^{n} a_{ij} = \sum_{j \in N_i} a_{ji} \).

If \( G \) has a spanning tree, the Laplacian matrix \( L \) has the following property. The eigenvalue 0 is algebraically simple and all other eigenvalues are with positive real parts.

Lemma 1 [31]: Consider the non-Lipschitz continuous nonlinear system

\[ x = F(x), \quad F(0) = 0, \quad x \in \mathbb{R}^n. \] (1)
Suppose that there exists a positive definite function $V(x)$ defined in a neighborhood of the origin. Real numbers $c>0$, $0<\alpha<1$, and $\dot{V}(x)+cV^\alpha(x) \leq 0$. Then the system is finite-time stable. The upper bound of the settling time, depending on the initial state $x(0)=x_0$, is satisfied with

$$T(x_0) \leq \frac{V(x_0)^{1-\alpha}}{c(1-\alpha)}$$

for all $x_0$ in some open neighborhood of the origin.

Lemma 2 [32]: For $x_i \in R, i=1,2,\ldots,n$, $0<p<1$, then

$$\left(\sum_{i=1}^{n}|x_i|\right)^p \leq \sum_{i=1}^{n}|x_i|^p \tag{2}$$

B. PROBLEM FORMULATION
In this subsection, we consider a heterogeneous MAS of $n$ agents, which consisted of $m$ second-order agents and $(n-m)$ first-order agents. The communications among agents in the heterogeneous system are described by a directed graph.

Each agent has the dynamics as follows.

$$\begin{align*}
\dot{x}_i &= v_i \\
\dot{v}_i &= u_i + \delta_i, \quad i=1,2,\ldots,m \\
\dot{x}_l &= u_l + \delta_2, \quad l=m+1,m+2,\ldots,n
\end{align*}$$

(3)

where $x_i,v_i,u_i \in R$ represent position, velocity, and control input of second-order agent $i$, $x_l,u_l \in R$ represent position and control input of first-order agent $l$, and $\delta_i, \delta_2$ represent disturbances.

For the system shown in (3), the following assumptions are put forward.

Assumption 1: The digraph has a directed spanning tree.

It is worth noting that the consensusability is related to Assumption 1. That is to say, Assumption 1 is one of conditions in Theorem 1.

Assumption 2: The disturbances $\delta_i, \delta_2$ are bounded, and $\max\left(||\delta_i||,||\delta_2||\right)<k_6$. Here, $k_6$ is the feedback gain in Equation (4), and $k_6$ will be used for the following proof of Theorem 1.

Our goal is to design finite-time consensus protocols via ISMC for the heterogeneous MAS without velocity measurements and with disturbances.

III. MAIN RESULTS
For second-order agents in system (3), the consensus protocol based on ISMC is designed as

$$\begin{align*}
s_i &= \hat{v}_i(0) - \int_0^t \left[\sum_{j\in N_i} a_{ij}(x_j-x_i) + k_2 \hat{v}_i\right] d\tau \\
u_i &= k_1 \sum_{j\in N_i} a_{ij}(x_j-x_i) + k_2 \hat{v}_i - k_6 \text{sgn}(s_i) \tag{4}
\end{align*}$$

where $s_i$ is the sliding mode state, $\hat{v}_i$ is the estimated velocity of $v_i$, and $\hat{v}_i(0)$ is the initial value of $\hat{v}_i$, $\text{sgn}(\cdot)$ represents sign function, and $k_1 > 0, k_2 > 0, k_6 > 0$ denote the feedback gains.

This paper designs the estimated velocity to replace the actual velocity. Inspired by [20], [22], and [33], the estimated velocity is designed by using the position measurements of the systems. $\hat{v}_i$ is given by

$$\ddot{v}_i(t) = -k_4 \hat{v}_i(t) + k_1 \sum_{j\in N_i} a_{ij}(x_j-x_i), \quad i=1,2,\ldots,m \tag{5}$$

where $k_4 > 0, k_5 > 0$.

For first-order agents in system (3), the consensus protocol is designed as

$$\begin{align*}
s_l &= x_l - \int_0^t \left[\sum_{j\in N_i} a_{ij}(x_j-x_i)\right] d\tau \\
u_l &= k_3 \sum_{j\in N_i} a_{ij}(x_j-x_i) - k_6 \text{sgn}(s_i) \tag{6}
\end{align*}$$

where $s_l$ is the sliding mode state, and $k_3 > 0$ is the feedback gain.

Theorem 1: Finite-time consensus in heterogeneous MAS (3) with protocols (4) and (6) is said to be achieved under Assumptions 1 and 2.

Proof: Consider the following Lyapunov function

$$V = V_1 + V_2 \tag{7}$$

where $V_1 = \frac{1}{2} s_i^T s_i, V_2 = \frac{1}{2} \hat{v}_i^T \hat{v}_i$.

Differentiating $V$, yields that

$$\dot{V} = s_i^T \dot{s}_i + \hat{v}_i^T \hat{s}_i$$

$$\begin{align*}
s_i &= \hat{v}_i(0) - \int_0^t \left[\sum_{j\in N_i} a_{ij}(x_j-x_i) + k_2 \hat{v}_i\right] d\tau \\
u_i &= k_1 \sum_{j\in N_i} a_{ij}(x_j-x_i) + k_2 \hat{v}_i - k_6 \text{sgn}(s_i) \tag{4}
\end{align*}$$

(8)

It is worth noting that the agents are not equipped with velocity sensors, and the actual velocity $v_i$ is equal to the estimated velocity $\hat{v}_i$ [20], [22].

According to Eq. (3),
\begin{align*}
\hat{v}_i = \dot{v}_i &= u_i + \delta_i. \\
\text{Hence} & \\
\dot{V} &= s_i^T \left[ u_i + \delta_i - k_1 \sum_{j \in N_i} a_{ij} (x_j - x_i) - k_2 v_i \right] \\
&\quad + s_i^T \left[ u_i + \delta_i - k_3 \sum_{j \in N_i} a_{ij} (x_j - x_i) \right] \\
&= s_i^T \left[ -k_6 \text{sgn}(s_i) + \delta_i \right] + s_i^T \left[ -k_6 \text{sgn}(s_i) + \delta_i \right] \\
&= -k_6 \|s_i\|_1 + \delta_i \|s_i\|_1 - k_6 \|s_i\|_1 + \delta_i \|s_i\|_1 \\
&= -(k_6 - \delta_i) \|s_i\|_1 - (k_6 - \delta_i) \|s_i\|_1. \\
(9)
\end{align*}

From Assumption 2, it is easy to show that
\begin{align*}
\dot{V} &\leq -(k_6 - \delta_i) \|s_i\|_2 - (k_6 - \delta_i) \|s_i\|_2 \\
&= -(k_6 - \delta_i) \sqrt{V_1^{1/2} - (k_6 - \delta_i) \sqrt{V_2^{1/2}}. \\
(10)
\end{align*}

Set
\begin{align*}
c &= \min \left( (k_6 - \delta_i) \sqrt{2}, (k_6 - \delta_i) \sqrt{2} \right). \\
(11)
\end{align*}

Then
\begin{align*}
\dot{V} &\leq -c V_1^{1/2} - c V_2^{1/2}. \\
(12)
\end{align*}

By Lemma 2
\begin{align*}
(V_1 + V_2)^{1/2} &\leq V_1^{1/2} + V_2^{1/2}. \\
(13)
\end{align*}

Then
\begin{align*}
\dot{V} &\leq -c (V_1 + V_2)^{1/2} = -c (V)^{1/2}. \\
(14)
\end{align*}

Thus
\begin{align*}
\dot{V} + c (V)^{1/2} &\leq 0. \\
(15)
\end{align*}

By Lemma 1, we conclude that heterogeneous MAS (3) with protocols (4) and (6) can reach consensus in a finite time.

**Remark 1:** The settling time is computed as follows.
\begin{align*}
T &\leq \frac{\left[ \frac{1}{2} \right]}{c \left( \frac{1}{2} \right)}. \\
(16)
\end{align*}

According to (11), the (16) can be continued as
\begin{align*}
T &\leq \frac{2 V_0^{1/2}}{\min \left( (k_6 - \delta_i) \sqrt{2}, (k_6 - \delta_i) \sqrt{2} \right)}. \\
(17)
\end{align*}

Synthesizing (4), (6), and (7), one has
\begin{align*}
V_0 &= \frac{1}{2} x_i^T(0) x_i(0). \\
(18)
\end{align*}

Then
\begin{align*}
T &\leq \frac{\left[ x_i^T(0) x_i(0) \right]^{1/2}}{\min \left( (k_6 - \delta_i), (k_6 - \delta_i) \right)}. \\
(19)
\end{align*}

**Remark 2:** The schemes proposed in this paper are designed based on ISMC. The existing consensus protocols [4], [20], [34] widely applied for MAS are written as follows.
\begin{align*}
u_i &= k_1 \sum_{j \in N_i} a_{ij} (x_j - x_i) + k_2 v_i \\
(10)
\end{align*}

\begin{align*}
u_i &= k_1 \sum_{j \in N_i} a_{ij} (x_j - x_i) \\
l &= m + 1, m + 2, \cdots, n
\end{align*}

where $k_1$, $k_2$, and $k_3$ are the feedback gains. $k_1 > 0$, $k_2 > 0$, $k_3 > 0$. $v_i \in R$ is given by (5).

**IV. NUMERICAL SIMULATIONS**

In this section, simulations are given to verify the effectiveness of the proposed schemes. The heterogeneous MAS is composed of six agents, where the agents 1-4 are governed by second-order integrators and the agents 5-6 are governed by first-order integrators.

The direct topology of the system is described by Figure 1.

![Directed graph with six agents](image)

**FIGURE 1:** Directed graph with six agents.

The adjacency matrix $A$ is given by
\begin{align*}
A &= \begin{bmatrix}
0 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 \\
\end{bmatrix}
\end{align*}

$D = \text{diag} \{3, 3, 1, 1, 2, 2\}$, then
\begin{align*}
L &= \begin{bmatrix}
3 & -1 & 0 & -1 & -1 & 0 \\
-1 & 3 & -1 & 0 & -1 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 \\
-1 & -1 & 0 & 0 & 2 & 0 \\
0 & -1 & -1 & 0 & 0 & 2 \\
\end{bmatrix}
\end{align*}

The eigenvalues of $L$ are 0, 0.6972, 1.3820, 3.6180, and 4.3028. Then Assumption 1 holds.

The initial states are chosen as $x(0) = \{8, 5, 2, -4, 1, -5\}$. The values of the parameters are designed by $k_1 = 10$, $k_2 = 5$, $k_3 = 30$, $k_4 = 9$, $k_5 = 3$, $k_6 = 5$. We
assume the disturbances \( \delta_1 = \delta_2 = 4 \sin(t) \). The simulation results are illustrated in Figures 2-5. Figures 2 and 3 depict the trajectories of position errors \((\bar{x}_i - \bar{x})\) and velocity errors \((\bar{v}_i - \bar{v})\) under the consensus protocols (20) in Remark 2. Here, \( \bar{x} = \frac{1}{n} \sum_{j=1}^{n} x_j \), \( \bar{v} = \frac{1}{m} \sum_{j=1}^{m} v_j \).

Figures 4 and 5 depict the trajectories of position errors \((\bar{x}_i - \bar{x})\) and velocity errors \((\bar{v}_i - \bar{v})\) under the consensus protocols (4) and (6). Compared with the protocols (20), we can see that the consensus protocols based on ISMC can suppress the disturbances effectively and reach consensus more quickly. The protocols (4) and (6) given in this paper can still achieve perfect performance in the case of unmeasured velocity and disturbances.

According to Eq.(19),

\[
T \leq \frac{[x'_i(0)x_i(0)]^{1/2}}{\min((k_o - \delta_1),(k_o - \delta_2))} = \frac{[26]^{1/2}}{\min((5 - 4\sin(t)),(5 - 4\sin(t)))} = \sqrt{26} \approx 5.099
\]

From the Figures 4 and 5, it can be seen that the settling time in numerical simulations is in the time range.
V. CONCLUSION
In this paper, the finite-time consensus protocols for heterogeneous MAS without velocity measurements and with disturbances have been studied under a directed graph. The estimated velocity is designed to replace the actual velocity. The ISMC is adopted to suppress the disturbances. The performance and effectiveness of the theoretical results are illustrated by numerical examples. Future work will consider the problem of time-varying topologies.

REFERENCES


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