Dynamic compliance analysis of position-based impedance control for the LHDS of legged robot

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ABSTRACT Hydraulic drive mode enables legged robots to have excellent characteristics, such as greater power-to-weight ratios, higher load capacities, and faster response speeds than other robots. Nowadays, highly integrated valve-controlled cylinder, called hydraulic drive unit (HDU), is employed to drive the joints of these robots. However, various robot control issues exist. For example, during the walking process of legged robots, different obstacles are encountered, making it difficult to control such robots because the load characteristics of the ends of their feet change with the environment. Furthermore, although the adoption of HDUs has resulted in high-performance robot control, the hydraulic systems of these robots still have problems, such as strong nonlinearity, complex load characteristics and time-varying parameters. Consequently, robot control is very difficult and complex. In this paper, the leg hydraulic drive system (LHDS) of legged robots is studied. First, the mathematical model of HDU position control system is built. Then, combined with kinematics, statics and dynamics of the leg mechanical structure, the position-based impedance control method is applied to LHDS. Moreover, the serial-parallel composition of dynamic compliance of position-based impedance control is analyzed and three key factors influencing the impedance control accuracy are found, which is the main contribution of our paper. Finally, the key factors proposed in this paper are verified experimentally by using both LHDS and HDU performance test platform. The theories and experiments proposed in this paper provide theoretical foundation for the compensation of position-based impedance control.

INDEX TERMS dynamic compliance, position-based impedance control, hydraulic drive unit (HDU), leg hydraulic drive system (LHDS)

I. INTRODUCTION

Compared with wheeled robots and tracked robots, legged robots are better at adapting to unknown and nonstructural environments. Their unique advantages, such as stepping over obstacles and executing tasks in the wild, have made them a major focus of research in the robotics domain. Compared with electric drive and pneumatic drive, hydraulic drive has greater power-to-weight ratios, higher load capacities, and faster response speeds, which make it appropriate for high performance robots¹-⁴. Some disadvantages of hydraulic motors, such as their large size and low speed creeping, have negative effects on the control of legged robots. Therefore, the valve-controlled hydraulic cylinder, which is also called the hydraulic drive unit (HDU), is the most common joint actuator of hydraulic legged robots⁵-⁶. In the motion of robots, their legs contact with external environment closely and frequently. Thus, the collisions between load environments and foot ends of robots often take place. This generates harmful impacts to the body. To reduce those impacts, the robot’s leg is required to have a certain compliance. Thus, some control methods should be applied to robot’s joint actuator. As a commonly used active compliance control method for legged robots, impedance control of the robot’s leg can be modeled as a second-order mass-spring-damper system; it makes the leg possess certain dynamic compliance. Impedance control is not only applied in electric
driven robots such as Tekken\textsuperscript{[7]}, Scout\textsuperscript{[8]}, KOLT\textsuperscript{[9]}, Cheetah\textsuperscript{[10]} and humanoid Roboray\textsuperscript{[11]}, but as the hydraulic robot becomes more popular, it is also applied in robots such as Bigdog\textsuperscript{[12]}, HyQ\textsuperscript{[13]}, Scalf-1\textsuperscript{[14]}, LWR robot\textsuperscript{[15]}, StarlETH\textsuperscript{[16]}, LS3\textsuperscript{[17]}, JINPOONG\textsuperscript{[18]}; and Atlas\textsuperscript{[19]}. Dynamic compliance is defined as the ratio of the system force variation to the position variation. This ratio is actually a high-order transfer function. For the position control system, dynamic compliance is signified by the ratio of the disturbance force to the output position. When a disturbance force is applied at the system, the ratio increases, and the output position variation decreases; thus, the dynamic compliance decreases. The position control system therefore tends to be an ideal system. For the force control system, dynamic compliance refers to the ratio of the system output force variation to the disturbance position. When a disturbance force is applied to the system, the ratio decreases, the output force variation tends to decrease and the dynamic compliance increases. Therefore, the force control system also tends to be an ideal system. For the dynamic compliance control of the robot leg, the basic principles are depicted as follows. The hydraulic control system is taken as the control inner loop. Then, the dynamic compliance control outer loop is attached to the control system. When the system is under external disturbances, the input signal of the control inner loop can be changed by the control outer loop. Thus, certain dynamic compliance is achieved in the system. However, the self-dynamic compliance of the hydraulic system control inner loop cannot be regarded as ideal due to some practical conditions. More specifically, the overall accuracy of the dynamic compliance control is not only determined by the accuracy of the outer loop but also influenced by the accuracy of the inner loop. Generally, the common inner loop control methods contain the position control closed loop and force closed loop control. This paper adopts the position control in the inner loop for the impedance control, which is a second-order dynamic compliance control.

In previous studies, the first generation of HDU (HDU-1\textsuperscript{st}) was researched and the mathematical model of HDU-1\textsuperscript{st} position control system was built. Parameter sensitivity was also analyzed, which provides a reference for the research of the second generation of HDU (HDU-2\textsuperscript{nd})\textsuperscript{[20,21]}. Moreover, the dynamic compliance composition of the HDU-1\textsuperscript{st} position control system was presented, which improved its disturbance rejection ability\textsuperscript{[22]}. Based on the former works, two problems which influence the position-based impedance control performance are discussed in this paper.

First, in impedance control systems, the inner loop is often considered ideal. Actually, the natural nonlinearity, time-variation and strong coupling of the hydraulic system make the inner loop non-ideal. The characteristics of the inner loop generate non-ideal inner loop dynamic compliance. So, the control performance is not only influenced by the outer loop, also by the accuracy of the inner loop greatly. How that non-ideal inner loop dynamic compliance influences the impedance performance?

Second, because the force sensor of the leg hydraulic drive system (LHDS) is fixed on the end of the HDU, the dynamics including the gravity of the robot’s body, inertia force and friction force influence the force detected on the sensor. If this force is input into the outer loop without compensation, the LHDS will receive an inaccuracy disturbance signal, which finally affects the impedance control performance. The common solution for that problem is adding an inverse dynamics compensator to eliminate the influence of the inaccuracy signal. How the compensation accuracy affects the impedance control accuracy?

The research on the origin of the above two problems is the main contribution in this paper. To solve the two problems, the application of the position-based impedance control for the LHDS equipped with HDU-2\textsuperscript{nd} is studied firstly. Then the serial-parallel composition theory of dynamic compliance is deducted. That theory shows the typical characteristics of impedance control and the essential problems which affect the control performance, which provides possibility for aimed compensation.

II. Introduction of LHDS

Fig. 1 shows the photos of the quadruped robot prototype, HDU-1\textsuperscript{st} with valve controlled symmetrical cylinder and the LHDS using HDU-2\textsuperscript{nd} with valve controlled asymmetrical cylinder\textsuperscript{[20]}

![Photos of HDU-1\textsuperscript{st}, LHDS with HDU-2\textsuperscript{nd} and quadruped robot prototype](image)

**FIGURE 1.** Photos of HDU-1\textsuperscript{st}, LHDS with HDU-2\textsuperscript{nd} and quadruped robot prototype

LHDS is the primary focus of this paper. Fig. 2 shows the mathematical model of the HDU-2\textsuperscript{nd} position control system. Nonlinear factors, such as pressure-flow nonlinearity, friction and initial position of servo cylinder piston rod, are considered
in the model. The details are not discussed in this paper because of space limitations[22].

FIGURE 2. Position control transfer block diagram of HDU-2nd
In Fig.2. X, is input position, \( \dot{X} \), is position tested by position sensor, \( \Delta X \), output position, \( K_d \) is position sensor gain, \( K_{PID} \) is PID controller gain, \( K_a \) is the servo valve amplifier gain, \( K_v \) is servo valve gain, \( K_{co} = K_aK_v \), \( \omega_n \) is natural frequency of servo valve, \( \zeta \) is damping ratio of servo valve, \( K_v = C_v\sqrt{2/p} \) (\( K_v \) is defined as conversion coefficient in this paper), \( X_0 \) is servo valve spool displacement, \( p_i \) is system supply oil pressure, \( p_o \) is inlet cavity pressure of servo cylinder, \( p_f \) is outlet cavity pressure of servo cylinder, \( p_t \) is system return oil pressure, \( Q_t \) is inlet oil flow, \( Q_f \) is outlet oil flow, \( C_p \) is internal leakage coefficient of servo cylinder, \( C_{co} \) is external leakage coefficient of servo cylinder, \( L \) is total length of piston rod, \( L_0 \) is initial length of piston rod, \( V_{21} \) is volume of input oil pipe, \( V_{22} \) is volume of output oil pipe, \( A_{pi} \) is inlet cavity effective piston area of servo cylinder, \( A_{pi} \) is outlet cavity effective piston area of servo cylinder, \( F_{f1} \) is friction attaching to HDU, \( m_i \) is conversion mass, \( B_{pl} \) is damping coefficient.

III. Application of position-based impedance control method in LHDS
Position-based impedance control schematic of LHDS is as shown in Fig. 3.

FIGURE 3. Position-based impedance control schematic of LHDS
Fig. 3 shows that, in a position-based impedance control system, if there is not a disturbance force applied at the foot end of the robot, the outer control loop will not affect the control process. The trace accuracy of the foot end is determined by the position controller in the inner loop. The detailed mathematical model of the position controller can be found in earlier papers[22].

If there is a disturbance force applied at the foot end, the impedance control outer loop will influence the control
process. The impedance control process can be divided into four steps.

In the first step, force sensor signals are transformed into disturbance force signals. If a disturbance force is applied at the foot end, the force sensors on the knee and ankle joints will detect a force error signal that can be divided into two parts. The first part is the equivalent signal from the disturbance force applied at the foot end. The second part is the equivalent signal from the dynamics of the mechanical structure of the leg. If the force sensor signal is directly taken as the disturbance force signal of impedance control, the control accuracy will be greatly affected. To get the first part, it is necessary to solve the second part by an inverse dynamics solution.

In the second step, the disturbance force signal from the first step is transformed to an input signal variation of the inner loop. If a disturbance force is applied at the knee joint and the ankle joint, the disturbance force on the foot end can be obtained by forward kinematic solution. Then, the position variation can be solved using leg impedance solver. Next, the input signal variation of the inner loop can be obtained by inverse kinematic solution.

In the third step, the input signal in the inner loop is transformed into an output signal. The control signal of the servo valve is given according to the input signal. Then, the output position of the cylinder can be tested.

In the fourth step, the impedance desired position \( \Delta X_{\text{leg}} \) and the impedance actual position \( \Delta X_{\text{leg}} \) of the foot end are calculated to evaluate the impedance control performance.

The impedance desired position \( \Delta X_{\text{leg}} \) can be expressed as follows:

\[
\Delta X_{\text{leg}} = \Delta X_{\text{leg}}^r + \Delta F_{\text{leg}} - \Delta F_{\text{leg}}^r Z_D ,
\]

where \( \Delta F_{\text{leg}}^r \) is the force error signal converted from the dynamics of mechanical structure of leg, which can be solved by inverse dynamics solution. \( \Delta F_{\text{leg}}^r \) is the actual force of the foot end tested by the joint force sensor, \( \Delta X_{\text{leg}}^r \) is the input position of the foot end.

The impedance actual position \( \Delta X_{\text{leg}} \) can be expressed as follows:

\[
\Delta X_{\text{leg}} = \Delta X_{\text{leg}}^r ,
\]

where \( \Delta X_{\text{leg}}^r \) is the actual position of the foot end tested by the joint position sensor. It can be seen from Eq. (2) that \( \Delta X_{\text{leg}}^r \) is composed of two signals. The first is generated by the input signal \( \Delta X_{\text{leg}}^r \), and the other is generated by the input position variation of the disturbance force.

IV. Dynamic compliance analysis of LHDS in position-based impedance control

In this section, the dynamic compliance composition of position-based impedance control is researched. Then, the way that dynamic compliance influences control accuracy is determined through theoretical analysis.

The study starts from the principle of position-based impedance control of HDU, as shown in Fig. 4. Because position variation is used in the system, all input and feedback signals are simplified and represented by variables.

**FIGURE 4. Position-based impedance control transfer block diagram of HDU**

In Fig. 4, \( G_f(s) \) is the transfer function from the disturbance force to the system. \( G_{1p}(s) \) and \( G_{2p}(s) \) are transfer functions of the position control system. The details of the three transfer functions are not provided because of space limitations. \( \Delta F_{\text{s2}} \) can reduce the effect of \( \Delta F_{\text{f}} \) on the system. \( \Delta F_{\text{s1}} \) is the tested force error converted from the disturbance force on the leg. \( \Delta F_{\text{s2}} \) is the tested force error converted from the dynamics of the mechanical structure. \( \Delta F_{\text{k}} \) is the force error calculated by an inverse dynamics solution. \( \Delta X_{\text{D}} \) is the desired input signal of the inner loop. \( \Delta X_{\text{e}} \) is the position error generated from \( \Delta F_{\text{s}} \) in the impedance outer loop. \( \Delta X_{\text{el}} \) is the position error generated from \( \Delta F_{\text{s1}} \) in the impedance outer loop. \( \Delta X_{\text{e2}} \) is
the position error generated from $\Delta F_{xz}$ in the impedance outer loop. $\Delta X'_{xz}$ is the position error generated from $\Delta F_k$ in the impedance outer loop.

In Fig. 4, it is assumed that the response of control inner loop is very fast. When the voltage error signal of the input end is zero, disturbance force $\Delta F_k$ affects the output of the inner loop $\Delta X_p$ along path (1). At that time, there is a high-order dynamic compliance $Z^1_{zp}$ in the control inner loop, which is expressed as follows:

$$Z^1_{zp} = \frac{-\Delta F_x}{\Delta X_p} = \frac{1}{G_j(s)G_{zp}(s)}$$

$$= m_1 V_1 V_2 s^3 + \left[ \beta_s B_{pi} C_{gi}(V_1 + V_2) + B_{pl} V_1 V_2 \right] s^2$$

$$+ \left[ \beta s B_{pi} + \beta \left( V_1 A_{p1}^2 + V_2 A_{p1}^2 \right) \right] s$$

$$= m_1 V_1 V_2 s^3 + \left[ \beta_s B_{pi} C_{gi}(V_1 + V_2) + B_{pl} V_1 V_2 \right] s^2$$

$$+ \left[ \beta \left( V_1 A_{p1}^2 + V_2 A_{p1}^2 \right) \right] s$$

$$= m_1 V_1 V_2 s^3 + B_{pl} s^2 + \frac{+ \beta_s B_{pi} C_{gi}(A_{p1} + A_{p2})^2}{V_1 V_2 s^3 + \beta \left( V_1 A_{p1}^2 + V_2 A_{p1}^2 \right)}$$

where $Z^1_{zp}$ is called the natural dynamic compliance of the HDU position control system. There exists high-order dynamic links in $Z^1_{zp}$, which means that the disturbance force $\Delta F_k$ affects the output $\Delta X_p$.

However, dynamic compliance is reached with the assumption of a zero-voltage error signal. However, owing to the existence of the position closed loop, the voltage error is not always zero. From Fig. 4, owing to the error at Node I, $\Delta F'$ reduces the effect of the disturbance force $\Delta F_k$ on the system. That is, the error has a positive effect on the system’s disturbance rejection ability. Under those conditions, there exists another high-order dynamic compliance $Z^2_{zp}$, which can be expressed as follows:

$$Z^2_{zp} = \frac{\Delta F_x}{\Delta X_p} = \frac{K_x G_{zp}(s)G_{zp}(s)}{G_j(s)}$$

$$= \frac{K_x B_{pi} C_{gi}(V_1 + V_2) + B_{pl} V_1 V_2}{\beta \left( V_1 A_{p1}^2 + V_2 A_{p1}^2 \right)}$$

$$= \frac{K_x B_{pi} C_{gi}(A_{p1} + A_{p2})^2}{\beta \left( V_1 A_{p1}^2 + V_2 A_{p1}^2 \right)}$$

$$= \frac{K_x B_{pi} C_{gi}(V_1 + V_2) + B_{pl} V_1 V_2}{\beta \left( V_1 A_{p1}^2 + V_2 A_{p1}^2 \right)}$$

$$= \frac{+ \beta_s B_{pi} C_{gi}(A_{p1} + A_{p2})^2}{V_1 V_2 s^3 + \beta \left( V_1 A_{p1}^2 + V_2 A_{p1}^2 \right)}$$

$$= \frac{+ \beta_s B_{pi} C_{gi}(A_{p1} + A_{p2})^2}{V_1 V_2 s^3 + \beta \left( V_1 A_{p1}^2 + V_2 A_{p1}^2 \right)}$$

$$= \frac{+ \beta_s B_{pi} C_{gi}(V_1 + V_2) + B_{pl} V_1 V_2}{\beta \left( V_1 A_{p1}^2 + V_2 A_{p1}^2 \right)}$$

$$= \frac{+ \beta_s B_{pi} C_{gi}(V_1 + V_2) + B_{pl} V_1 V_2}{\beta \left( V_1 A_{p1}^2 + V_2 A_{p1}^2 \right)}$$

$$= \frac{+ \beta_s B_{pi} C_{gi}(V_1 + V_2) + B_{pl} V_1 V_2}{\beta \left( V_1 A_{p1}^2 + V_2 A_{p1}^2 \right)}$$

where $Z^2_{zp}$ is the equivalent dynamic compliance generated by the position control inner loop. $\Delta X_D$ is an input signal and independent with disturbance, so $Z^2_{zp}$ and $Z^2_{zp}$ act on output position $\Delta X_p$ simultaneously. That is, the position variation generated by the two dynamic compliances act on each other and finally effect on the output position $\Delta X_p$. Therefore, the two dynamic compliances are connected in parallel.

From Fig. 4, considering the influence of input position $\Delta X_D$, the inner loop position error $E_p$ is expressed as follows:

$$E_p = E_{p1} + E_{p2} = \Delta X_p - \Delta X'_{p} = \Delta X_p - \Delta X'_p - \Delta X_p$$

where $E_{p1}$ is defined as the inner loop position variation generated by the inner loop load force through dynamic compliance $Z_{zp}$. $E_{p2}$ is defined as the variation of the error between the inner loop load position and output position. The load force $\Delta F_{p}$ and input error $E_{p2}$ do not interact. Therefore, the inner loop position error variation generated by $\Delta F_{p}$ is independent with input position $\Delta X_D$. For the convenience of deduction, when the impedance outer loop is not considered, $E_{p2}$ is set to zero, which means $\Delta X_D = 0$. Thus, the inner loop position error variation $- \Delta X_p = E_{p1}$. After the position changes a value of $E_{p1}$, a force $\Delta F_{p}$ is generated by dynamic compliance $Z^2_{zp}$ to eliminate the influence of $\Delta F_{p}$. Thus, the force applied at dynamic compliance $Z^2_{zp}$ is $(-\Delta F_{p} - \Delta F'_{p})$. The two dynamic compliances satisfy the relation of Hooke’s Law in parallel connection. The inner loop overall dynamic compliance is expressed as follows:

$$Z_{zp} = Z^1_{zp} + Z^2_{zp} = -\frac{\Delta F_x}{\Delta X_p}$$

$$= -\frac{\Delta F_{x} - \Delta F'_{x}}{\Delta X_p}$$

$$= \frac{\Delta F_{x}}{E_{p1}}$$

The position variation along path $\Delta X_{e}$ can be expressed as follows:

$$\Delta X_{e}^1 + \Delta X_{e}^2 = \Delta F_{x} = \frac{\Delta F_{x}^1 + \Delta F_{x}^2}{Z_D}$$

Under position $\Delta X_p$, $\Delta F_k$ can be reached through the inverse dynamics relation. We hope that $\Delta F_{k}$ is equal to $\Delta F^2_p$, thus $\Delta F_{k}$ is remove from $\Delta F_{p}$. That means the impedance control outer loop is only influenced by external disturbance forces. The dynamic compliance $Z_i$ between $-\Delta F_{k}$ and $\Delta X_p$ can be expressed as follows:

$$Z_i = -\frac{\Delta F_{k}}{\Delta X_p}$$

After $\Delta F_{k}$ is calculated, position variation $\Delta X_{e}$ can be calculated using impedance characteristics to eliminate position variation $\Delta X_{e}$. Thus, the position variation generated in the impedance control outer loop is expressed as follows:

$$\Delta X_{e} = \frac{\Delta F_{e} - \Delta F_{k}}{Z_D}$$

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Combing Eqs. (5), (6), (8) and (9), the following equation can be obtained:

\[
(\Delta X_r - \Delta X_p - \frac{\Delta F_s}{Z_{sp}} - E_{p2})Z_D = \Delta F_s + Z_k \Delta X_p.
\]  

(10)

The above equation can be simplified to the following form:

\[
-(Z_k + Z_D)\Delta P_p = \Delta F_s(1+Z_D) - (\Delta X_r - E_{p2})Z_D.
\]  

(11)

In the above equation, \( \Delta X_r \) and \( E_{p2} \) is not influenced by disturbance force \( \Delta F_s \). Supposing that \( \Delta X_r = E_{p2} = 0 \), \( Z_{Ap} \) is defined as the overall dynamic compliance of the position-based inner and outer loop, which is expressed as follows:

\[
Z_{Ap} = -\frac{\Delta F_s}{\Delta X_p} = \frac{Z_{sp}Z_D + Z_{sp}Z_D Z_k}{Z_{sp}Z_D + Z_{sp}Z_D Z_k} = \left(1 + Z_k\right) \left(\frac{1}{Z_D + \frac{1}{Z_D}}\right),
\]  

(12)

where \( Z_{sp} \) and \( Z_D \) satisfy Hooke’s Law in serial connection. Thus, \( Z_{Ap} \) is an overall dynamic compliance composed of inner loop dynamic compliance \( Z_{sp} \) and impedance control outer loop dynamic compliance \( Z_{op} \) (\( Z_{op} = Z_D \)) with a time-varied coefficient \( 1 + Z_k / Z_D \) in serial connection. When the impedance control outer loop is not included, \( Z_{Ap} = Z_{sp} \). Thus, the dynamic compliance composition principle of HDU position-based impedance control is shown as Fig. 5 (dynamic compliance is shown as springs).

![Dynamic compliance composition schematic of HDU position-based impedance control](image)

**FIGURE 5.** Dynamic compliance composition schematic of HDU position-based impedance control

In Fig. 5, the position-based impedance control system is a system with variable coefficients composed of three dynamic compliances in parallel-serial connection. When disturbance force \( \Delta F_s \) is applied at the system, position variation \( \Delta X_p \) is generated to eliminate it. When the inverse dynamics relation is not included in the system, dynamic compliance \( Z_k = 0 \), the variable coefficient is equal to 1; when the inverse dynamic relation is included in the system, dynamic compliance \( Z_k > 0 \), the variable coefficient is greater than 1. The inverse dynamic relation increases the overall dynamic compliance \( Z_{Ap} \).

The impedance desired position \( \Delta X_{dp} \) can be expressed as follows:

\[
\Delta X_{dp} = \Delta X_r - \Delta X_p = \Delta X_r - \frac{\Delta F_s - \Delta F_s}{Z_D}.
\]  

(13)

The above equation indicates that if \( \Delta X_r = \Delta X_p \), the impedance desired position is only influenced by the external disturbance force, and the influence of \( \Delta F_s \) is eliminated. If the inverse dynamics relation is not considered or the inverse dynamics solution is not accurate, \( \Delta X_r \) will not be equal to \( \Delta X_p \). Thus, the impedance desired position calculated from Eq. (13) is not equal to the impedance desired position generated by disturbance force.

According to Fig. 4 and Eq. (11), the impedance actual position \( \Delta X_{Ap} \) can be expressed as follows:

\[
\Delta X_{Ap} = \Delta X_r - \Delta X_p - (E_{p1} + E_{p2}) = \frac{-\Delta F_s + (\Delta X_r - E_{p2})Z_D}{Z_{Ap} + Z_D + Z_k},
\]  

(14)

where the last part indicates the relationship between impedance actual position and overall dynamic compliance when \( \Delta X_r \) and \( E_{p2} \) is not equal to zero. According to Eq. (13) and (14), the error \( D_p \) between position-based impedance actual position \( \Delta X_{Ap} \) and impedance desired position \( \Delta X_{dp} \) can be expressed as follows:

\[
D_p = \Delta X_{dp} - \Delta X_{Ap} = E_{p1} + E_{p2}.
\]  

(15)

It can be seen from the above equation that, there is a theoretical error between impedance actual position and impedance desired position. The impedance control accuracy is mainly determined by \( E_{p1} \) and \( E_{p2} \). The error is generated in the position control inner loop.

**V. Experiments**

**V.A. Experiments results on LHDS performance test platform**

The experiments are conducted on LHDS. Two working conditions are researched to verify the serial-parallel composition of dynamic compliance. The specific steps are depicted as follows.

First, put the single leg on the ground and push the leg on the top with regular intervals according to the condition of the two characteristics (1. \( Z_{0}^d = Z_D^X \) including stiffness coefficient 6N/mm and damping coefficient 0.5N/mm-s. 2. \( Z_{0}^d = Z_D^X \)
including stiffness coefficient 10N/mm and damping coefficient 0.5N/mm∙s).

Second, in the second condition (\( Z'_d = Z'_d \) including stiffness coefficient 10N/mm and damping coefficient 0.5N/mm∙s), hang the leg in the air statically or moving at a certain frequency. Apply a random disturbance force to the foot end by hand.

1. Experiment results of the first step (put on the ground and pushed on the top)

For the two mentioned conditions, the impedance control response curves of the foot end and the position response curves of the joints are as shown in Fig. 6 and Fig. 7 (the pushing force is mainly along the Y-axis, so only the impedance response curves along Y-axis are shown).

![FIGURE 6. Response curves when stiffness coefficient 6N/mm](image)

2. Experiment results of the second step (hung in the air and disturbed by hand)

1) Static

In this test, the impedance control response curves of the foot end and the position response curves of the joints are as shown in Fig. 8.
FIGURE 8. Response curves when the leg is static

2) X-axis/Y-axis moves sinusoidally with a 5-mm amplitude and 1-Hz frequency

The impedance control response curves and the position response curves of the joints are shown in Fig. 9.

FIGURE 9. Response curves when the leg moves

It can be seen from the curves in Fig. 6 to Fig. 9 that the experiments indicate that impedance actual position can trace impedance desired position well on the condition of different impedance characteristics. The phase angle of the impedance actual position is larger than the phase angle of impedance desired position. For the phase angle in different working conditions, the error between impedance actual and desired positions is as shown in the analysis after Eq. (15). It is obvious that when the speed is zero, the actual position variation $\Delta Y_{fp}$ is larger than desired position variation $\Delta Y_{fr}$, which further proves the serial-parallel composition theory of dynamic compliance.

V-B. Experiments results on HDU performance test platform

In Section V-A, the disturbance force applied on the LHDS is random and uncontrollable. So, the results are difficult to analyzed quantitatively. To solve this problem, the impedance control performance in different disturbance force conditions are researched and analyzed quantitatively to verify the theory results of dynamic compliance. Therefore, the HDU performance test platform is used, the photo of which is shown in Fig. 10.\(^{(22)}\)
Position sensor

Servo valve

Accumulator

Servo cylinder

Force sensor

FIGURE 10. Photo of HDU performance test platform

As it is shown in the above figure, the left part is the system to be tested and the right part is the loading system of disturbance force. In the experiment, the desired position of position-based impedance control is the ratio of disturbance force acting on the tested system to the desired impedance characteristic. While the actual position is the output position tested by position sensor. In order to compare and analyze the dynamic compliance results conveniently under different frequencies and amplitudes, the experimental plan is listed in Table I:

<table>
<thead>
<tr>
<th>Disturbance Form</th>
<th>Disturbance Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response</td>
<td></td>
</tr>
<tr>
<td>Amplitude</td>
<td>500N</td>
</tr>
<tr>
<td>1000N Bias</td>
<td>1500 N Bias</td>
</tr>
<tr>
<td>Sinusoidal</td>
<td>0.5Hz</td>
</tr>
<tr>
<td>Frequency</td>
<td>1Hz</td>
</tr>
<tr>
<td>1Hz</td>
<td>2Hz</td>
</tr>
<tr>
<td>Step Response</td>
<td>at 0.5s input step 1000N</td>
</tr>
</tbody>
</table>

In the experiment, the initial position of the left part HDU is selected to be the middle position of 25 mm. The impedance characteristics parameters adopted in these experiments include stiffness coefficient 100 N/mm and damping coefficient 5 N-mm/s. The experiments are conducted based on the working conditions listed in Table I. The sinusoidal and step response curves obtained are presented in Fig.11 and 12, respectively.

FIGURE 11. Response curves under sinosoidal disturbance force

a) 500N amplitude 1000N bias 0.5Hz

b) 500N amplitude 1000N bias 1Hz

c) 500N amplitude 1000N bias 2Hz

d) 1000N amplitude 1500N bias 0.5Hz

e) 1000N amplitude 1500N bias 1Hz

f) 1000N amplitude 1500N bias 2Hz
According to the experimental curves shown in Fig. 11 and 12, the position-based impedance control can make the system generate a position error when a disturbance force acts on the HDU. Then, the position inner loop forms a new input position, and the system acquires the impedance characteristic.

In order to analyze the control effect quantitatively, the performance indexes of position error under different working conditions are listed in Table II. These indexes are listed as follows: max value of position error = (desired position-actual position)\text{max}, mean value of position error = (desired position-actual position)\text{mean} and middle axis error = (middle axis of desired position-middle axis of actual position). In addition, the performance indexes of phase angle error under different working conditions are listed in Table III. These indexes are listed as follows: max value of phase angle error = (phase angle of desired position -phase angle of actual position)\text{max} and mean value of phase angle error = (phase angle of desired position -phase angle of actual position)\text{mean}.

### Table II. Performance Indexes of Position Error Under Different Working Conditions. (mm, keep three decimal places)

<table>
<thead>
<tr>
<th>Disturbance form</th>
<th>500N</th>
<th>1000N Bias</th>
<th>Sinusoidal Disturbance Force 500N Amplitude 0.5Hz</th>
<th>Sinusoidal Disturbance Force 1000N Bias 2Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>500N</td>
<td>0.148</td>
<td>0.150</td>
<td>0.218</td>
<td>0.252</td>
</tr>
<tr>
<td>1000N</td>
<td>0.149</td>
<td>0.150</td>
<td>0.218</td>
<td>0.252</td>
</tr>
<tr>
<td>1000N Bias</td>
<td>0.148</td>
<td>0.150</td>
<td>0.218</td>
<td>0.252</td>
</tr>
<tr>
<td>1500N Bias</td>
<td>0.148</td>
<td>0.150</td>
<td>0.218</td>
<td>0.252</td>
</tr>
</tbody>
</table>

As it can be seen in Table II, when the system is under the sinusoidal disturbance force with bias, max value of position error, the mean value of position error and the bias error are obvious when the frequency changes. Besides, the values of these parameters increase as the amplitude of disturbance force increases. For example, when the sinusoidal disturbance frequency is 0.5Hz, amplitude 500N and bias 1000N, the mean error is 0.148mm and the mean error is 0.065mm. Under the same amplitude, when the frequency is 2Hz, the max/mean error are 0.150mm and 0.065mm. Under the same frequency 2Hz, when the amplitude is 1000N and the bias is 1500N, the max/mean error are 0.252mm and 0.102mm. When the step disturbance force is 1mm and 2mm, the max error is 0.33mm and 0.77mm respectively. The mean error is 0.07mm and 0.17mm. Due to the error between the bias of actual and desired position, the phase angle error between two curves is relatively greater. For example, when the disturbance force is frequency 0.5Hz, amplitude 500N and bias 1000N, the max value of phase angle error between the actual and desired position reaches to 41.5°. And the mean error of phase angle error reaches to 12.6°. In addition, when the system is under the step disturbance force, as the step disturbance force increases, the max value mean value of position error increase. For example, when the step disturbance force is 1000N and 1500N, the max values of position error are 0.252mm and 0.343mm respectively. The mean values of position error are 0.112mm and 0.175mm respectively.

As it can be seen in Fig.11, Table. II and Table. III, when the HDU is under the sinusoidal disturbance with bias, it is obvious that there is relatively larger error between the bias of actual and desired position curves. Therefore, this makes the relatively larger position and phase angle deviation between actual and desired position. Besides, the amplitude range of actual position is always larger than that of desired position.

As it can be seen in Fig.12, Table. II and Table. III, when the HDU is under the step disturbance force, there are certain position error between actual and desired position. When it comes to the steady state, the step of actual position is larger than the desired one.

According to Eqs. (3) and (4) in the manuscript, the numerator, which represents the dynamic compliance of the position control system inner loop, contains the constant items. Therefore, when the disturbance force remains constant, the position control system generates an inner loop error \( E_p \). This...
error is the main factor affecting the impedance control performance when the disturbance force is constant. Comparatively speaking, the trace error $E_{pl}$ between the input and output positions of the position inner loop in Eq. (15) slightly affects the impedance control performance. The sinusoidal disturbance force with bias is composed of the constant force with bias and the sinusoidal force without bias. Thus, $E_{pl}$ is the reason for the error between the biases of the actual and desired position curves.

The above discussions analyze the dynamic compliance theoretically. It also can be shown in Fig.5 in the manuscript. To show it more vividly, every dynamic compliance can be regarded as a “spring” and every spring has its “stiffness coefficient”. According to Hooke’s law, in a position-based impedance system, the “inner loop spring” and the “outer loop spring” are connected in series. When a constant disturbance force is applied on the system, the “inner loop spring” is pressed, which makes the “stiffness coefficient” of the whole spring system smaller than the “stiffness coefficient” of the outer loop spring (impedance control desired stiffness coefficient). Therefore, the whole system shows the characteristic that, the actual variation position (the ratio of disturbance force to the “total stiffness coefficient”) is larger than the desired variation position (the ratio of disturbance force to “outer loop stiffness coefficient”). The above analysis is also accordant to this paper’s conclusion.

VI. Conclusions

The system dynamic compliance includes four parts in serial and parallel connection, two of which are in the inner loop, the others are in the outer loop. The inner loop dynamic compliance includes the natural dynamic compliance and the equivalent dynamic compliance. The two dynamic compliance is connected in parallel. The outer loop dynamic compliance includes the equivalent dynamic compliance of impedance characteristics and the equivalent dynamic compliance of inverse dynamics compensation. The four dynamic compliances of both inner loop and outer loop are connected in series with time-variable coefficients. Through the dynamic compliance analysis, two factors influence the impedance control performance. They are control error generated by the disturbance and control error generated by the position closed loop control.

Future work: Based on the contributions of this paper, the further research is mainly about the following parts:

1) This paper studied the position-based impedance control, how the performance of the force-based impedance control is? The difference of dynamic compliance between force-based and position-based control is an interesting research;

2) Design compensation controllers respectively for the inner loop disturbances and the force closed loop control to improve the impedance control performance.

References


