ADMM-based Optimal Power Control for Cognitive Satellite Terrestrial Uplink Networks

Bin Gao, Min Lin, Member, IEEE, Kang An, Gan Zheng, Senior Member, IEEE, Li Zhao, Xian Liu

Abstract—This paper proposes an underlay power control scheme for cognitive satellite terrestrial uplink networks, where the primary terrestrial mobile network coexists with the secondary satellite communication network. In particular, the schemes are developed to guarantee that both the interference power constraints (IPCs) and interference outage probability constraints (IOPCs) of primary terrestrial users (PTUs) are satisfied. Since the IOPCs of PTUs belong to the chance constraint power constraints (IPCs) and interference outage probability schemes are developed to guarantee that both the interference scheme for cognitive satellite terrestrial uplink networks, alternating direction method of multipliers (ADMM) is introduced to derive the power allocation by alternatively optimizing the subproblems, each of which has the exact closed-form solution. The global convergence of the ADMM is guaranteed. Simulation results demonstrate the effectiveness of the proposed method.1

Index Terms—Cognitive radio (CR), power control, cognitive satellite terrestrial networks, alternating direction method of multipliers (ADMM)

I. INTRODUCTION

Satellite communications have been widely applied in various scenarios, such as navigation, broadcasting, disaster relief, due to its capability of providing seamless connectivity and high-speed broadband access at a low cost, especially in rural and sparsely populated areas that the deployment of wired and wireless terrestrial networks is economically infeasible [1]. However, the continuous growth of emerging broadband satellite applications and services requires large spectrum resources and the licensed spectrum band appears to be insufficient to keep up with the forthcoming demands. To alleviate the spectrum scarcity and congestion, the concept of cognitive satellite terrestrial network (CSTN) has recently been proposed which allows the coexistence of a satellite network with a terrestrial network operating in the same spectrum band. Currently, significant works have been carried out in this area that aims to extend the existing work on cognitive radio (CR) in the terrestrial networks to the hybrid architecture [2].

Among the popular applications of cognitive satellite communication, the case where the terrestrial system serves as the primary network and the satellite system operates as the secondary network has been investigated as a promising network architecture from both academic research and industry trend [3]. Towards this direction, effective power control should be carefully designed to alleviate the mutual interference while ensuring the coexistence of two networks. Vassaki et al. in [4] investigated the optimal power control scheme for the downlink cognitive satellite-terrestrial network based on quality of service constraints. The authors of [5] studied the outage performance of CSTN and demonstrated that tight interference constraint for the satellite interference link leads to a degraded outage performance for the terrestrial user. Lagunas et al. in [6] proposed a weighted sum approach for solving a set of multi-objective optimization frameworks for the power allocation problem of CSTN.

It should be pointed out that the aforementioned works except [6] only considered the single user case, while the approach proposed in [6] could cope with multiple users but only provide a Pareto-optimal solution. Besides, due to channel estimation errors, mobility and feedback delay, the exact perfect channel state information (CSI) of mutual interference link between two systems in CSTN is commonly unavailable.

Under this situation, it is an urgent research challenge to investigate the effect of imperfect CSI on the power control scheme in CSTN. Here, we investigate the global optimal power control in CSTN uplink scenario with multiple users, which aims to maximizing the throughput of cognitive satellite networks while guaranteeing the interference power constraints (IPCs) and interference outage probability constraints (IOPCs).2 The contributions of this paper are outlined as follows:

- We propose an optimal power control approach for CSTN

1Simulation codes are provided to reproduce the results presented in this paper: https://github.com/gaobingaobingaobin/figure3github

2Authors in [19] adopted some similar manipulations such as equivalent substitution of IOPCs but only considered Gaussian channels scenario, which can not be directly applied in the satellite terrestrial scenario involving the Shadowed-Rician fading and Nakagami-m fading models.

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uplink scenario, which is applicable to the cases with an arbitrary number of primary terrestrial users (PTUs) and secondary satellite users (SSUs). It is worth mentioning that our work extends the system models of the related literatures in [3–6] to a more general one.

• From the perspective of PTUs with both delay-insensitive and delay-sensitive services, both IPCs and IOPCs are adopted in order to ensure the long-term and short-term QoS requirements. The joint consideration of above protection mechanisms at PTUs includes the previous works with a single constraint as a special case, and can be employed in both non-realtime applications, such as email, remote login or ftp and realtime applications, such as voice and video transmission.

• An alternating direction method of multipliers (ADMM) based optimization approach is proposed to derive the power control coefficient by alternatively optimizing the subproblems. The closed-form solution for each subproblem is derived, which ensures that the new algorithm is not only low-complexity but also globally convergent.

II. SYSTEM MODEL AND POWER CONSTRAINTS

In the considered CSTN operating at uplink mode as shown in Fig. 1, the terrestrial cellular network (e.g. LTE) with M PTUs acting as the primary system shares the spectrum resource with the satellite network (e.g., DVB-SH) with N SSUs, which acts as the secondary system [5]. Herein, the Time Division Multiple Access (TDMA) is considered, which allows several users in both satellite and terrestrial networks to share the same carrier frequency [13], and the underlay CR technique is adopted as the spectrum sharing approach so that the satellite user shares the same spectrum with the terrestrial user simultaneously without deteriorating its communication quality [3] [5]. Specifically, $h_i$ and $g_{im}$ denote the channel coefficient of the $i$th secondary satellite link (iSSL) and the $im$th terrestrial interference link (imTIL), respectively. The weak interference from terrestrial terminal to the satellite is ignored due to the large path loss [14]. The free space loss of iSSL and imTIL are denoted as $L_i^s$ and $L_{im}^p$, respectively. $G_i(\theta_i)$ corresponds to the transmit antenna gain at iSSL, which can be expressed as [15]:

$$G_i(\theta_i) = \begin{cases} G_{i,\text{max}}, & 0^\circ < \theta_i < 1^\circ, \\ 32 - 25 \log \theta_i, & 1^\circ < \theta_i < 48^\circ, \\ -10, & 48^\circ < \theta_i < 180^\circ, \end{cases}$$

where $\theta_i$ is the elevation angle of $i$th SSU and $G_{i,\text{max}}$ represents the maximum transmit antenna gain. $G_{i}(\varphi_{im})$ denotes the equivalent transmit antenna gain for imTIL with off-axis angle $\varphi_{im} = \arccos(\cos(\alpha_{im})\cos(\xi_{im}))$, and $\alpha_{im}$ is the elevation angle of imTIL, and $\xi_{im}$ denotes the angle between the over horizon projected main lobe of the $i$th SSU and the $m$th PTU. Besides, $G_{r,\text{max}}^{iSS}$ is the receive antenna gain of imTIL, and $G_r(\varphi_i)$ denotes the receive antenna gain of iSSL, which can be expressed as [1]:

$$G_r(\varphi_i) = G_{r,\text{max}} \left( \frac{J_1(u_i)}{2u_i} + \frac{36J_3(u_i)}{u_i^3} \right)^2,$$

where $G_{r,\text{max}}$ represents the maximum gain at the onboard antenna boresight, $J_1$ and $J_3$ are the first-kind Bessel function of order 1 and 3, and $u_i = 2.07123 \sin(\varphi_i)$. $\varphi_i$ is the angle between the $i$th SSU and the beam center with respect to the satellite, and $\varphi_i^{3\text{dB}}$ is the 3-dB angle. For brevity, we denote $\psi_{i} = L_i^s G_i(\theta_i) G_r(\varphi_i)$ and $\Phi_{im} = L_{im}^p G_i(\varphi_{im}) G_{r,\text{max}}^{iSS}$ in the rest of the paper, which can be roughly viewed as the antenna gains of iSSL and imTIL, respectively.

A. Cognitive Satellite Link

For the secondary link, we employ the widely-adopted Shadowed-Rician fading model with closed-form expression, which can be used for mobile/fixed terminals operating in various propagation environments [4]. According to [16], the probability density function (PDF) of channel link gain $X_i = |h_i|^2 i \in \{1, \ldots, N\}$ is shown as:

$$f_{X_i}(X_i) = \alpha \exp(-\varpi X_i) F_1(m_i, 1, \delta X_i),$$

where $F_1(\cdot, \cdot, \cdot)$ denotes the confluent hypergeometric function [17] and $\alpha = (2b_i \pi_i / (2b_i \pi_i + \Omega_i))^{\pi_i / 2b_i}$, $\varpi = 1 / 2b_i$, and $\delta = \Omega_i / (2b_i (2b_i \pi_i + \Omega_i))$, with $2b_i$ being the average power of the scatter component, $\Omega_i$ the average power of the line-of-sight (LOS) component and $\pi_i$ the Nakagami fading parameter.

B. Terrestrial interference Link

As for the terrestrial interference link, Nakagami-$m$ fading distribution is considered, which covers a wide range of

![Fig. 1. The uplink system model of CSTN.](image)
fading scenarios for different values of the fading parameter. From [4], the channel link gain of \( Y_{im} = |g_{im}|^2 (i \in \{1, \ldots, N\}, m \in \{1, \ldots, M\} ) \) follows the PDF given by:

\[
f_{Y_{im}}(Y_{im}) = \frac{\varepsilon_{im}^\tau Y_{im}^{\gamma_{im}-1}}{\Gamma(\gamma_{im})} \exp(-\varepsilon_{im} Y_{im}), \tag{4}
\]

where \( \Gamma(\cdot) \) is the Gamma function [17], \( \pi_{im} \) is the Nakagami fading parameter, \( \Omega_{im}^2 \) is the average power and \( \varepsilon_{im} = \pi_{im}/\Omega_{im}^2 \).

### TABLE I
**SUMMARY OF SYSTEMS AND NOTATIONS**

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>number of SSUs</td>
</tr>
<tr>
<td>( M )</td>
<td>number of PTUs</td>
</tr>
<tr>
<td>( B )</td>
<td>bandwidth</td>
</tr>
<tr>
<td>( G_i(\theta_i) )</td>
<td>transmit antenna gain of iSSL (imTIL)</td>
</tr>
<tr>
<td>( G_r(\varphi_i) )</td>
<td>receive antenna gain of iSSL (imTIL)</td>
</tr>
<tr>
<td>( X_i(Y_{im}) )</td>
<td>link power gain of iSSL (imTIL)</td>
</tr>
<tr>
<td>( \sigma_i^2 )</td>
<td>noise power of iSSL</td>
</tr>
<tr>
<td>( L_i^a(L_i^{out}) )</td>
<td>free space loss of iSSL (imTIL)</td>
</tr>
<tr>
<td>( P_i )</td>
<td>underlay transit power of ( i ) th SSU</td>
</tr>
<tr>
<td>( I_m )</td>
<td>IPC constraint limit of ( M ) th PTU</td>
</tr>
<tr>
<td>( \gamma_{im} )</td>
<td>IOPC constraint limit at imTIL</td>
</tr>
</tbody>
</table>

#### C. Power Constraints

To regulate the transmit power \( P_i \) of the \( i \) th satellite user in the long-term duration, average power constraints are commonly employed [3]. Let \( I_m \) be the average transmit power limits. Then, the long-term IPC at the \( i \) th PTU can be described as:

\[
E\left( \sum_{i=1}^{N} \Phi_{im} Y_{im} P_i \right) \leq I_m. \tag{5}
\]

Since the exact CSI of mutual interference link between two systems is much more difficult to be obtained, we consider the outdated CSI of \( Y_{im} \) at imTIL:

\[
\sqrt{Y_{im}} = \rho \sqrt{\hat{Y}_{im}} + \sqrt{1-\rho^2} \sqrt{\check{Y}_{im}}, \tag{6}
\]

where \( Y_{im} \) and \( \hat{Y}_{im} \) denote the current and outdated CSI of imTIL, respectively. \( \hat{Y}_{im} \) represents a Nakagami-\( m \) random variable with unit variance and \( \rho \) is the correlation coefficient given by \( \rho = J_0(2\pi f_d \tau) \), where \( J_0(\cdot) \) is the first-kind Bessel function of order zero, \( f_d \) is the Doppler frequency and \( \tau \) is the delay [18].

Assuming \( \sqrt{\hat{Y}_{im}} \) and \( \sqrt{\check{Y}_{im}} \) are uncorrelated as in [19], the constraint (5) can be written as:

\[
\sum_{i=1}^{N} \Phi_{im} P_i^2 E[Y_{im} P_i] + (1-\rho^2) \sum_{i=1}^{N} \Phi_{im} E[P_i] \leq I_m, \tag{7}
\]

with \( m \in \{1, \ldots, M\} \).

When PTUs require an instantaneous QoS service from the delay-sensitive aspect, the IOPCs would be the more appropriate requirement [19]. Let \( \gamma_{im} \) represents the maximum interference threshold at imTIL, and \( \eta_{im} \) denotes the maximum tolerable interference outage probability. Then, IOPCs can be described as:

\[
Pr(\Phi_{im} Y_{im} P_i > \gamma_{im}) \leq \eta_{im}. \tag{8}
\]

#### III. THE UPLINK OPTIMAL POWER CONTROL UNDER UNDERLAY SCHEME

We aim to maximize the system throughput of CSTN considering both IPCs and IOPCs. The problem can be formulated as:

\[
P_1 : \max_{P_i \geq 0} \mathbb{E} \left[ \sum_{i=1}^{N} B \log_2 \left( 1 + \frac{|Y_i X_i P_i|}{\sigma_i^2} \right) \right] \tag{9a}
\]

s.t. (7) and (8),

\[
B \text{ is bandwidth and } \sigma_i^2 \text{ denotes noise power for } i \text{ th SSU. It is worth mentioning that, although we choose the system throughput as the objective function in this paper, the ADMM-based approach can not only be applied directly to the ergodic capacity model, but also can be easily extended to the energy efficiency model by combining Dinkelbach’s method [20].}

\[
P_1 \text{ is nonconvex because of the probability constraint (8). This intractable constraint can be transformed into an equivalent peak power constraint on the transmit power by the following lemma:}
\]

**Lemma 1.** (8) is equivalent to

\[
P_i \leq P_i^{out} := \min \{ P_i^{out_1}, \ldots, P_i^{out_M} \}, \tag{10}
\]

where \( P_i^{out} = \frac{\gamma_{im}/\Phi_{im}}{\sqrt{1-\rho^2} Q_{\pi_{im}}(\eta_{im})} \) and \( Q_{\pi_{im}}^{-1}(\cdot) \) is the inverse generalized Marcum Q function \( Q_{\pi_{im}}(Y_{im} \sqrt{\frac{2\gamma_{im}^2}{(1-\rho^2)\eta_{im}}}, y) \) with respect to variable \( y \) and \( Q_{\pi_{im}}(\cdot, \cdot) \) defined as [22, Eq. 2-1-122]

\[
Q_{\pi_{im}}(a,b) = \int_0^\infty x^{m-1} e^{-(a^2+b^2)/2} I_{m-1}(ax)dx, \tag{11}
\]

where \( I_m(\cdot) \) is the \( m \)th order modified Bessel function.

**Proof:** See Appendix B.

Then the nonconvex problem \( P_1 \) can be reformulated as a convex problem \( P_2 \):

\[
P_2 : \max_{P_i \geq 0} \mathbb{E} \left[ \sum_{i=1}^{N} B \log_2 \left( 1 + \frac{|Y_i X_i P_i|}{\sigma_i^2} \right) \right] \tag{12a}
\]

s.t. (7) and (10).

\[
(12b)
\]

It is not difficult to verify that \( P_2 \) is a convex optimization problem. Although the famous convex optimization tool CVX can solve it by using polynomial time algorithms, when the number \( N \) of SSUs is large, such generalized polynomial time algorithms become chicken ribs. And some customized convex algorithms which can fully tap the structure of specific problem become urgent. As such, based on the uncoupled structure of both constraint and object function with respect to the power variable in the given power allocation model, we propose
an alternating optimization approach by using a distributed convex optimization technique known as alternating direction method of multipliers (ADMM). ADMM first decomposes the intractable primal problem into $N$ tractable subproblems, which can be effectively solved in closed-form solution in an alternative way.

Before applying ADMM's iterative framework in Appendix A, we first define the Lagrange function of \( P_2 \) as follows:

\[
L_\beta(P, \Lambda) = -\mathbb{E}\left[ \sum_{i=1}^{N} \beta \Psi_i X_i P_i - (\sigma_i^2 + \Psi_i X_i) P_i \right] - \sum_{i=1}^{N} \lambda_i \left( \sum_{i=1}^{M} \Phi_{im} Y_{im} P_i - I_m \right) + \beta \sum_{i=1}^{N} \left( \sum_{j=1, j \neq i}^{M} \Phi_{jm} Y_{jm} P_j - I_m \right)^2,
\]

where \( P = (P_1, \ldots, P_N) \) and \( Y_{im} = \rho^2 Y_{im} + 1 - \rho^2 \), \( \Lambda = (\Lambda_1, \ldots, \Lambda_M) \) and \( \beta \) are respectively the Lagrange multiplier vector and penalty parameter [21]. Since Lagrange function of \( P_2 \) has the same structure for each fading state, we denote \( L_\beta(P, \Lambda) \) with respect to \( P_i \) as \( L_\beta(P_i) \) for a particular fading state. Then, the derivative of \( L_\beta(P_i) \) with respect to \( P_i \) is:

\[
\frac{\partial L_\beta(P_i)}{\partial P_i} = -\frac{B \Psi_i X_i}{\sigma_i^2 + \Psi_i X_i P_i} - \sum_{m=1}^{M} \lambda_i \Phi_{im} Y_{im} + \sum_{m=1}^{M} \beta \Phi_{im} Y_{im} \left( \Phi_{im} Y_{im} P_i + \sum_{j \neq i}^{N} \Phi_{jm} Y_{jm} P_j - I_m \right).
\]

Since \( \frac{\partial^2 L_\beta(P_i)}{\partial^2 P_i} = \frac{B^2 \Psi_i^2 X_i^2}{(\sigma_i^2 + \Psi_i X_i P_i)^2} + \beta \Phi_i^2 Y_i^2 > 0 \), (14) can be depicted as Fig. 2. Now, we derive the solution of

\[
\frac{\partial L_\beta(P_i)}{\partial P_i} = 0.
\]

Since \( \sigma_i^2 + \Psi_i X_i P_i > 0 \), based on (14), solving (15) is equivalent to solve:

\[
\frac{\partial L_\beta(P_i)}{\partial P_i} (\sigma_i^2 + \Psi_i X_i P_i) = 0,
\]

which can be reformulated as:

\[
\Psi_i X_i \phi_i + \frac{\sigma_i^2}{\sigma_i^2 + \Psi_i X_i} P_i + \sigma_i^2 \phi_i - B \Psi_i X_i = 0,
\]

where \( \phi_i = \sum_{m=1}^{M} (\beta \sum_{j \neq i}^{N} \Phi_{jm}^2 Y_{jm} P_j - \beta \Phi_{im} Y_{im} I_m) - \sum_{m=1}^{M} \lambda_m \Phi_{im} Y_{im} \) and \( \phi_i' = \sum_{m=1}^{M} \beta \Psi_{im}^2 \).

Based on the definition \( \Delta_i = \frac{1}{\beta} \left( \Psi_i X_i \phi_i + \sigma_i^2 \phi_i - B \Psi_i X_i \right), \)

\[
\bar{P}_i = \frac{-(\Psi_i X_i \phi_i + \sigma_i^2 \phi_i - B \Psi_i X_i)}{2 \beta \Psi_i X_i}, \]

the quadratic equation (17) is discussed as follows.

Case 1: \( \Delta_i < 0 \). In this case \( \frac{\partial L_\beta(P_i)}{\partial P_i} > 0 \) (shown in Fig. 2(a)), \( L_\beta(P_i) \) is a monotonically increasing function and reaches its minimum value at \( P_i = 0 \) for \( P_i \geq 0 \).

Case 2: \( \Delta_i \geq 0, \bar{P}_i \geq 0 \). In this case (Fig. 2(b)), \( L_\beta(P_i) \) reaches its local minimum value when \( P_i = \bar{P}_i \). Considering 0 is a minimum value in the interval \([0, \bar{P}_i] \), we choose

\[
P_i = \begin{cases} 
\bar{P}_i & \text{if } L_\beta(\bar{P}_i) < L_\beta(0), \\
0 & \text{elsewise}.
\end{cases}
\]

Case 3: \( \Delta_i \geq 0, \bar{P}_i \geq 0 \). In this case (Fig. 2(c)), \( L_\beta(P_i) \) reaches its minimum value when \( P_i = \bar{P}_i \) under the condition \( P_i \geq 0 \).

Case 4: \( \Delta_i \geq 0, \bar{P}_i < 0 \). In this case (Fig. 2(d)), \( L_\beta(P_i) \) is monotonic increasing function reaches its minimum value at \( P_i = 0 \) under the condition \( P_i \geq 0 \).

Given \( \Delta, \beta \) and taking into account the peak constraint (10), the closed-form solution of the minimization of (13) is:

\[
P_i = \begin{cases} 
\min \{ P_i^{out}, \bar{P}_i \} & \Delta_i \geq 0, \bar{P}_i \geq 0, \min \{ P_i^{out}, \bar{P}_i \} > L_\beta(0), \\
0 & \Delta_i \geq 0, \bar{P}_i < 0 < \bar{P}_i, \\
\end{cases}
\]

\[
\text{otherwise}.
\]

Fig. 2. Illustration of different form of the Lagrange function \( L_\beta(P_i) \) and its partial function \( \frac{\partial L_\beta(P_i)}{\partial P_i} \).

With (19) in hand, based on the ADMM algorithm framework in Appendix A, the dual multiplier updating step is:

\[
\lambda_m = P_{R_+} \left[ \lambda_m - \beta \left( \sum_{i=1}^{N} \Phi_{im} Y_{im} P_i - I_m \right) \right].
\]

where \( P_{R_+} \) denotes the projection onto \( R_+ \) under the Euclidean norm [24].

The entire iterative power control algorithm is summarized as Algorithm 1 with \( \varepsilon_0 \) denoting the stopping criterion, namely the tolerance. The global convergence is proved in the following Theorem, which is based on the similar derivation skills as in [25].

**Theorem 1.** The Algorithm 1 can obtain a global optimal solution in a finite number of iterations.

**Proof:** See Appendix C.

The complexity of Algorithm 1 can be roughly estimated through the formation of channel gain matrix and the complexity of solving the power allocation subproblems times the total number of ADMM iterations. The formation of channel gain matrix including \( \Phi_{im} Y_{im} \) and \( \Psi_i X_i \) are on the order of \( NM \). For handling per-iteration computational cost, we notice the most computational cost is \( \phi_i \) which involves the multiplication of channel gain matrix and power vector on the order of \( N^2 M \). Then, we deal with ADMM iteration complexity. By taking into account the \( O(1/k) \) convergence...
Algorithm 1: ADMM Optimal Power Control Algorithm

1. **Input:** \( P^1 = [P^1_1, \ldots, P^1_N] = 0, \lambda^1 = 0, \) the maximum iteration number \( J \) and the error tolerance \( \varepsilon_0 \).
2. **for** \( k = 1 : J \) **do**
3. calculating \( P^{k+1} \) using (19) \( \forall i \in \{1, \ldots, N\} \);
4. updating \( \lambda^{k+1} \) via:
5. if \( \|P^{k+1} - P^k\| / \|P^k\| \leq \varepsilon_0 \)
6. break;
7. end

---

rate of ADMM with \( k \) being the iteration number, the ADMM requires \( O(1/\varepsilon_0) \) iterations in order to achieve an \( \varepsilon_0 \)-accuracy solution [28]. Hence, the computational complexity of Algorithm 1 is \( O(N^2M + NM) \).

IV. NUMERICAL RESULTS

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>5</td>
</tr>
<tr>
<td>( M )</td>
<td>2</td>
</tr>
<tr>
<td>( B )</td>
<td>50 MHz</td>
</tr>
<tr>
<td>( G_{t, \text{max}} )</td>
<td>42.1 dB</td>
</tr>
<tr>
<td>( G_{r, \text{max}} )</td>
<td>52.1 dB</td>
</tr>
<tr>
<td>( \theta )</td>
<td>[1°, 0.5°, 0.7°, 0.6°, 0.2°]</td>
</tr>
<tr>
<td>( \vartheta )</td>
<td>[79°, 138°, 68°, 116°, 33°]</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>[0.1°, 0.1°, 0.1°, 0.1°, 0.1°]</td>
</tr>
<tr>
<td>( \eta )</td>
<td>[49°, 118°, 122°, 29°, 21°]</td>
</tr>
<tr>
<td>( \sigma_i^2 )</td>
<td>-130 dB</td>
</tr>
<tr>
<td>( \gamma_{\text{IM}} )</td>
<td>1 dB</td>
</tr>
</tbody>
</table>

To evaluate the performance of the proposed scheme, numerical results are presented in this section. In the simulations, we consider the simulation parameters as shown in Table 2 unless otherwise stated [1]. Besides, three shadowing scenarios of the satellite link are considered, namely, Infrequent Light Shadowing (ILS), Frequent Heavy Shadowing (FHS) and Average Shadowing (AS). The typical values of satellite channel parameters can be obtained from Table III of [16]. The simulation result presented here is obtained by taking average over 10000 rounds simulations by running the software of MATLAB R2013b.

Fig. 3 compares the convergence trajectories of the proposed ADMM for different penalty factor vector \( \beta \). It is shown that ADMM is not sensitive to the choice of \( \beta \). All kinds of ADMM with different \( \beta \) parameter selection will be convergent after 300 iterations. Fig. 4 shows the SSUs’ convergence behavior over iteration numbers. The transmit power selection of SSU converges to a stationary power in about 130 iterations. While the transmission power of SSU still needs some iterations to adjust before the final convergence point arrives as shown in Fig. 3.

Fig. 5 depicts the system throughput of the satellite user versus IPC limit for different shadowing scenarios of the satellite link. The results indicate that the system throughput would increase when the satellite link experiences the weaker shadowing conditions. It is observed that the final sum rate under IOPCs is less than the solution without IOPCs by about 15% in higher threshold of IPCs while almost equal in lower IPCs. The theoretical reason is that IPCs in lower value interval are tighter than IOPCs. Hence, the problem with IOPCs does not trigger IOPCs.
influence on when increasing with respect to \( \hat{\rho} \). Dedicated way, while ADMM’s step size is directly determined method yields strongly fluctuating results. This is because it. We try to explain such a phenomenon by using (27).

Fig. 7 plots the system throughput solution. However, system throughput is higher increasing with respect to \( \eta \). This is as expected because \( \hat{\rho} \) increases faster than \( \eta \). This is as expected because \( \hat{\rho} \) is less than the turning point of (1 \( \rho \)).

\( \begin{aligned}
\text{APPENDIX A} \\
\text{ADMM}
\end{aligned} \)

We briefly illustrate the essence of ADMM, the standard form of which applies to the following convex problem [21]:

\[
\begin{align*}
\min_{i=1}^{N} & \theta(x_i) \\
\text{s.t.} & \sum_{i=1}^{N} A_i x_i \leq d, \quad x_i \in \mathcal{X}_i \subseteq \mathbb{R}^m, d \in \mathbb{R}^m.
\end{align*}
\]

By searching for the saddle point of the following augmented Lagrangian function:

\[
L_\beta (x_1, \cdots, x_N, \Lambda) = \sum_{i=1}^{N} \theta(x_i) - \Lambda^T \left( \sum_{i=1}^{N} A_i x_i - d \right) + \frac{\beta}{2} \left( \sum_{i=1}^{N} A_i x_i - d \right)^2,
\]

where \( \theta_i \) are closed, proper and convex functions. \( \lambda \) and \( \beta \) are dual variable and augmented factor, respectively. The saddle point of \( L_\beta (x_1, \cdots, x_N, \lambda) \) can be found by performing an alternating procedure which starts from arbitrary initial values \( x_1^0, \cdots, x_N^0 \) and \( \lambda_0 \), and iteratively updates entries according to

\[
\begin{aligned}
x_1^{k+1} &= \arg \min_{x \in \mathcal{X}} L_\beta (x_1, \cdots, x_N, x_1^k, \lambda^k), \\
& \vdots \\
\lambda^{k+1} &= \Pi_{\mathbb{R}_+^m} \left[ \lambda^k - \beta \left( \sum_{i=1}^{N} A_i x_i^{k+1} - d \right) \right],
\end{aligned}
\]

until convergence, where \( \Pi_{\mathbb{R}_+^m} \) denotes the projection onto \( \mathbb{R}_+^m \) under the Euclidean norm.

\( \text{APPENDIX B} \)

\( \text{PROOF OF LEMMA 1} \)

The joint PDF of \( Y_{\text{im}} \) and \( Y_{\text{im}} \) is given by [26]

\[
f_{Y_{\text{im}}, Y_{\text{im}}}(x, y) = \frac{4 \pi y}{\Gamma(\pi_{\text{im}}) \eta_1 \eta_2 (1 - \rho^2)} \left( \frac{\gamma_{\text{im}}}{\xi_{\text{im}}} \right)^{\pi_{\text{im}}} \left( \frac{1}{(1 - \rho^2)^{\frac{\gamma_{\text{im}}}{\xi_{\text{im}}}}} \right)^{\pi_{\text{im}} - 1} \\
\cdot I_{\pi_{\text{im}} - 1} \left\{ \frac{2 \rho \beta y}{\sqrt{\eta_1 \eta_2 (1 - \rho^2)}} \right\},
\]

\( \text{V. CONCLUSIONS} \)

In this paper, we propose the ADMM-based optimal power control schemes for underlay cognitive satellite terrestrial uplink networks with multiple satellite users and multiple terrestrial users. It was shown that ADMM-based approach ensures a global convergence with closed solution in each iteration. To further employ in non-realtime and realtime applications, we consider IPCs and IOPCs together to protect the operations of PTUs. Our findings suggest that the small values of the interference outage probability can cause rigid constraint on SSUs’ peak transmit power and lead to a decrement of throughput less than the same model with perfect CSI, i.e., \( \rho = 1 \).
while

\[ f_{Y \mid Y_i}(x) = \frac{2}{\Gamma(\pi_{im})} \eta_{im}^2 \pi_{im} x^{2\pi_{im}-1} e^{-x/\eta_{im}}, x \geq 0. \quad (25) \]

Based on (24) and (25), the conditioned PDF is given by

\[
f_{Y \mid Y_i}(y|x) = \frac{2\eta_{im}^{2\pi_{im}-1} y^{\pi_{im}-1} e^{-(\eta_{im}^2 + \eta_{im} y^2)/(\eta_{im}^2 (1 - \rho^2))}}{x^{\pi_{im}-1} \eta_{im} (1 - \rho^2)^{(\pi_{im}-1)/2}}, \quad I_{\pi_{im}-1} \left( \frac{2 \rho x y}{\sqrt{\eta_{im}^2 (1 - \rho^2)} \right). \quad (26) \]

Then, the conditioned cumulative distribution function (CDF) is achieved as

\[
Pr(\Phi_{im} Y_i P_i \geq \gamma_{im} | Y_i) = F_{\gamma_{im}} \left( \frac{\gamma_{im}}{\pi_{im} \eta_{im}} \right) = f_{Y_i \mid Y_i} dY_i = Q_{\pi_{im}} \left( Y_i \sqrt{\frac{2 \rho^k}{(1 - \rho^2)^{\pi_{im}/2}} \frac{\gamma_{im} \rho_{im}}{\pi_{im} \eta_{im}^2 (1 - \rho^2)^{\pi_{im}/2}}}, \right) \quad (27)\]

where (27) follows from [22, Eq.(2-1-122)]. Since Marcum Q is a strictly decreasing function with respect to y, we can easily get

\[
P_i \leq \frac{\sqrt{\pi_{im} / \Phi_{im}}}{\sqrt{1 - \rho^2}} \frac{1}{\eta_{im}} Q_{\pi_{im}}(\pi_{im}(\eta_{im})^{-1}) \quad (28)\]

The proof is complete.

APPENDIX C

PROOF OF THEOREM 1

Prior to entering the proof process, two important lemmas are first introduced. One is the necessary and sufficient condition of strongly convex function [23]:

**Lemma 2.** Let X be a convex set in a real vector space and let f : X → R be a function. f is µ-strongly convex if and only if for any \( f - \frac{\mu}{2} \| \cdot \|^2 \) is convex.

The other is the convergence condition of N block splitting ADMM inspired by [25]:

**Lemma 3.** For the coupled convex problem (21), suppose \( \theta_i \) to be \( \mu_i \)-strongly convex. For any

\[
0 < \beta \leq \min_{1 \leq i \leq N} \left\{ \frac{2 \mu_i}{3(N - 1) \| A_i \|^2} \right\}, \quad (29)\]

the sequence \( \{x_i^k, \ldots, x_i^N\} \) generated by the ADMM converges to a global optimal solution.

**Proof:** Based on (21) and Lemma 4.3 in [25], after some simple manipulations, we obtain the inequality:

\[
\|y^{k+1} - \gamma^k\| \leq \|y^k - \gamma^k\|^2 - 2\mu_i \sum_{i=1}^{N} (x_i^{k+1} - x_i^k)^2

- \beta \sum_{i=1}^{N} \|A_i x_i^{k+1} - d\|^2 + 3\beta(N - 1) \sum_{i=1}^{N} \|A_i x_i^{k+1} - A_i x_i^k\|^2, \quad (30)\]

where \( y^k = \{x_1^k, \ldots, x_N^k, \lambda^k\} \) and \( y^* = \{x_1^*, \ldots, x_N^*, \lambda^*\} \).

According to the inequality \( \|A_i x_i^{k+1} - A_i x_i^k\|^2 \leq \|x_i^{k+1} - x_i^k\|^2 \), it follows from (30) that

\[
\|y^{k+1} - \gamma^k\|^2 \leq \|y^k - \gamma^k\|^2 - \sum_{i=1}^{N} l_i (x_i^{k+1} - x_i^k)^2

- \beta \sum_{i=1}^{N} A_i x_i^{k+1} - d\| \|^2, \quad (31)\]

with \( l_i = 2\mu_i - 3(N - 1)\beta \|A_i\|^2 \). Considering the condition (29), we get

\[
\|y^{k+1} - \gamma^k\|^2 \leq \|y^k - \gamma^k\|^2 \leq \cdots \leq \|y^0 - \gamma^0\|^2. \quad (32)\]

Thus, we conclude that the sequence \( \{y^k\} \) is bounded. Meanwhile, By summing (31) from \( k = 0 \) to \( \infty \), we have

\[
\sum_{k=0}^{\infty} \left\{ \sum_{i=1}^{N} l_i (x_i^{k+1} - x_i^k)^2 + \beta \sum_{i=1}^{N} A_i x_i^{k+1} - d\|^2 \right\} \leq \sum_{k=0}^{\infty} \|y^{k+1} - \gamma^k\|^2 \leq \|y^0 - \gamma^0\|^2 \quad (33)\]

which means

\[
\lim_{k \to \infty} (x_i^{k+1} - x_i^k)^2 = 0, \quad (34)\]

\[
\lim_{k \to \infty} \sum_{i=1}^{m} A_i x_i^{k+1} - d\|^2 = 0. \quad (35)\]

Thus, after some simple manipulations based on cluster point theory, we can conclude that the sequence \( \{y^k\} \) converges to \( y^* \). This completes the proof.

Now we begin the proof of Theorem 1. According to Lemma 2 (Proposition 10.6 in [23]), if the convexity of function \( f(P_i) = r_i(P_i) - \mu_i P_i^2 / 2 \) is proved, \( \mu_i \)-strong convexity of \( r_i(P_i) \) is self-evident. It is easy to get \( \nabla^2 f(P_i) = (\psi_i x_i)^2 - \mu_i \). Considering \( \Phi_{im} Y_i P_i < I_m, \forall m \in \{1, \ldots, M\} \) (refer to (5)), we obtain

\[
\nabla^2 f(P_i) \geq 0, \quad (36)\]

with \( \mu_i = \frac{(\psi_i x_i)^2}{(\psi_i x_i)^2 + \min_{1 \leq i \leq M} (r_i(P_i) - \mu_i P_i^2 / 2)} \). Then from Theorem 2.1.4 in [27], the function \( r_i(P_i) - \mu_i P_i^2 / 2 \) is convex. Therefore, the \( \mu_i \)-strongly convexity of \( r_i(P_i) \) is proved. Combining with Lemma 2, the algorithm 1 will converge to a global optimal solution. The proof is complete.

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