A Virtual Negative Inductor Stabilizing Strategy for DC Microgrid with Constant Power Loads

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ABSTRACT In this paper, a novel virtual negative inductor stabilizing strategy is proposed for the dc microgrid with constant power loads. It is known that in the dc-based power system, the constant power load will generate a virtual negative incremental resistance, which may deteriorate the whole system stability. The situation will be more serious for the dc power system with a large line inductance. In the proposed stabilizing strategy, a virtual negative inductor is built on the source-side converter through the droop control method. The built virtual negative inductor counteracts the large line inductance, thus enhancing the system damping effect. Small-signal models of the studied dc microgrid system under the proposed stabilizing strategy are carefully derived. A root-locus based parameter designing approach is proposed for obtaining the optimal parameter value for the stabilizer. An explicit Nyquist stability criterion for the studied dc microgrid system is proposed, with the system minor loop gain carefully derived. Several comparative stability analyses are taken for showing the system robustness to the parameter perturbations. Detailed numerical simulations are also conducted for validating the effectiveness of the proposed stabilizing strategy.

INDEX TERMS DC Microgrid, system stability, active damping, virtual impedance, droop control, virtual negative inductor, constant power loads.

I. INTRODUCTION

In recent years, distributed generations (DGs) from clean and sustainable energy sources near consumers are reshaping the structure of the modern power system. Due to the wide integration of the dc-based renewable energy sources (RESs) and the modern dc loads (e.g., data center and electric vehicles), dc microgrid system is gaining people’s increasing attentions nowadays [1, 2]. Compared with the traditional ac microgrid, the dc microgrid has several advantages, such as, a higher efficiency with less power conversion stages, a higher controllability with no synchronization problem, and an enhanced reliability with a highly reconfigurable structure.

From the system point of view, the dc microgrid is a typical heterogeneous system with diversified type of sources and loads. An effective energy management strategy is required for organizing these diversified components in a coordinated way [3-8]. Such an energy management strategy should include the following functions: maintaining the system stability, regulating the bus voltage, ensuring the dc power quality and pursuing the economic benefit. Due to the complicated nonlinear dynamic characteristics of the constant power loads (CPLs), system stability is a major concern when designing such an energy management strategy [9-17]. Several criterions have been proposed for assessing the stability of the dc microgrid system with CPLs [18-21].

Indeed, the CPL is a type of load fed by a dedicated tightly-regulated converter, which maintains a constant consuming power even when the bus voltage fluctuates. As indicated in [9, 11], the CPL will introduce a negative incremental resistance to the dc bus, which degrades the system damping effect. Moreover, in the case of a large CPL power, a small dc-link capacitance or a large line inductance, the CPL destabilizing effect will be more significant, which may eventually make the system unstable.

To alleviate the instability problem brought by the CPLs, several stabilizing strategies have been proposed. In [13], physical dampers are introduced for increasing the system damping effect, which can be termed as the passive damping method. However, in these passive damping methods, large capacitor or resistor will be implemented. Additional power loss and system weight will be caused by these physical dampers. For solving this problem, several strategies have been proposed for stabilizing the dc microgrid system from the control point of view, such as, the active damping methods...
[22-34], the sliding mode control [35-37], the feedback linearization control [14, 38, 39], and the model predictive control [40]. These stabilizing strategies will not bring any additional physical dampers to the system, hence alleviating the system weight and efficiency problem.

From the side where the stabilizing strategy acting upon, the stabilizing methods mentioned above can be separated into two groups: the load-side stabilizing methods and the source-side stabilizing methods. The load-side stabilizing methods emerged earlier in history [22, 23, 25, 26, 28, 41]. A dedicated stabilizing power is injected to the load for constructing a virtual resistor or a virtual capacitor to enhance the system stability margin. No additional physical damper is required in these active damping methods. However, the injected stabilizing power does impact the load performance, which maybe not preferred for the system with high dynamic requirements. Due to this fact, the source-side stabilizing strategy will be more appealing.

The source-side stabilizing strategy stabilizes the dc microgrid system by modifying the source dynamics, without sacrificing the critical load performance. The source-side stabilizing strategies can be further separated into two groups: the centralized or the decentralized. The active stabilizing method proposed in [24], the sliding mode control [35-37], the global stabilization method [42], the model predictive control [40] and the feedback linearization method [14, 38, 39] mentioned above all belong to the centralized group. With the global system information, the optimal operation and the accurate power sharing control of the dc microgrid system can be easily achieved in these centralized stabilizing strategies. However, the effectiveness of the centralized stabilizing strategy is strongly dependent on the central controller, which may be prone to the single-point-failure problem [28]. The decentralized control strategy, as an alternative, show its advantage in solving this problem.

Droop control is a typical decentralized strategy implemented for coordinating multiple parallel-connecting sources in the dc microgrid system [8]. A proportional power sharing between the multiple sources can be achieved by designating different droop coefficients. Based on the droop control, several source-side active damping strategies have been proposed. A virtual impedance based active stabilizing strategy is proposed in [28], which modifies the output impedance of the source converter to match the input admittance of the cascaded CPL. In [43], a nonlinear disturbance observer based stabilizing method is proposed, which utilizes the estimated output current for the feedforward compensation and the feedback droop term. Moreover, a stabilizing method without load performance compromise is proposed in [30]. A virtual resistance effective around the LC resonant frequency is built on the source-side converter and thus can indirectly reduce the source converter output impedance to fulfill Middlebrook’s stability criterion. In [31], a virtual low-pass filtered resistance is constructed in the droop controller, which effectively shape the output impedance of source converters in the high frequency range. However, only the low-pass filter time constant can be tuned in [31], which lacks some flexibility.

For improving the stability of the dc microgrid system, a virtual negative inductor stabilizing strategy is proposed in this paper. A virtual negative inductor is constructed on the output of source-side converter through the droop control method. The constructed virtual negative inductor will counteract the large line inductance, thus enhancing the system damping effect. However, the pure differentiator of the virtual inductor in the proposed stabilizing strategy may bring undesired high frequency noises to the system. For avoiding this problem, a low-pass filtered differentiator is constructed in the proposed stabilizing strategy.

Small-signal model of the studied dc microgrid system is carefully derived, and a root-locus based parameter designing approach is proposed for obtaining the optimal value of the stabilizer parameter. An explicit Nyquist stability condition for the studied dc microgrid system under the proposed stabilizing strategy is specified. The system minor loop gain is then carefully derived from the diagram of the system small-signal model. The stability of the studied dc microgrid system with variations of CPL power, droop coefficient and CPL-side capacitance are carefully analyzed with the help of the Nyquist diagram. Detailed numerical simulations are also conducted for validating the effectiveness of the proposed virtual negative inductor stabilizing strategy.

The following of this paper will be organized as follows: In Section II, the configuration, model and basic control of the studied dc microgrid system are presented. In Section III, the basic principles of the proposed virtual negative inductor stabilizing strategy are described. A root-locus based parameter designing approach is proposed for obtaining the optimal value of the stabilizer parameters. In Section IV, an explicit Nyquist stability condition for the studied dc microgrid system is specified, with the system minor loop gain carefully derived. Comparative stability analyses are taken for the studied dc microgrid system under different parameter variations. In Section V, detailed numerical simulations are conducted for validating the effectiveness of the proposed virtual negative inductor stabilizing strategy. Finally, the conclusions are drawn in Section VI.

II. SYSTEM MODELING AND BASIC CONTROL
The single-line diagram of the typical dc microgrid system is shown in Fig.1, which consists of the battery energy storage system interfaced by the dc/dc bidirectional converter, the PV panel with the boost dc/dc converter, the utility grid with the voltage source converter, the active load and the resistive load (RL).
where, $i_o$ is the output current of two source converters, $R_{dc}$ is the equivalent resistive load.

Assuming that the bidirectional dc/dc source converter is operating in the continuous-current-mode (CCM), we can obtain the dc/dc converter average model as follows:

$$
L_{dc} \frac{di_o}{dt} = v_o - R_{dc} i_o - (1 - d)v_o,
$$

$$
C_v \frac{dv_o}{dt} = (1 - d)i_L - i_o,
$$

and

$$
L_{dc} \frac{di_o}{dt} = v_o - R_{dc} i_o - v_{dc},
$$

where, $d$ is the average duty ratio, $i_L$ is the inductance current, $i_o$ is the output current, $v_o$ is the output voltage, $v_{dc}$ is the dc bus voltage, $L_{dc}$ and $R_{dc}$ are the low voltage side inductance and resistance of the dc/dc boost converter, $C_v$ is the converter output capacitance, $R_v$ and $L_v$ are the line resistance and inductance, respectively.

**TABLE I**

<table>
<thead>
<tr>
<th>System Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_s$</td>
<td>source voltage</td>
<td>100 V</td>
</tr>
<tr>
<td>$v_{bus}$</td>
<td>dc bus voltage nominal reference</td>
<td>200 V</td>
</tr>
<tr>
<td>$L_n$</td>
<td>input inductance of source converter</td>
<td>2 mH</td>
</tr>
<tr>
<td>$R_n$</td>
<td>input resistance of source converter</td>
<td>0.04 Ω</td>
</tr>
<tr>
<td>$C_v$</td>
<td>output capacitance of source converter</td>
<td>2200 μF</td>
</tr>
<tr>
<td>$R_v$</td>
<td>line resistance of source</td>
<td>0.1 Ω</td>
</tr>
<tr>
<td>$L_v$</td>
<td>line inductance</td>
<td>0.1 mH</td>
</tr>
<tr>
<td>$C_{eq}$</td>
<td>CPL-side equivalent capacitance</td>
<td>2200 μF</td>
</tr>
<tr>
<td>$R_{eq}$</td>
<td>line resistance of CPL</td>
<td>0.1 Ω</td>
</tr>
<tr>
<td>$L_{eq}$</td>
<td>line inductance of CPL</td>
<td>0.1 mH</td>
</tr>
<tr>
<td>$R_{dc}$</td>
<td>equivalent resistive load</td>
<td>60 Ω</td>
</tr>
<tr>
<td>$P_{conv}$</td>
<td>nominal power of dc/dc converter</td>
<td>5 kW</td>
</tr>
<tr>
<td>$f_s$</td>
<td>switching frequency</td>
<td>10 kHz</td>
</tr>
</tbody>
</table>

Classical dual-loop controller is utilized here for regulating the dc/dc converter output voltage, as shown in Fig. 3. The controlling function can be expressed from Fig. 3 as follows:

$$d = k_p \left[ k_{pi} (v_e - v_o) + k_{pi} \left( v_e - v_o \right) \cdot dt - i_L \right] + k_i \left[ k_{pi} (v_e - v_o) + k_{pi} \left( v_e - v_o \right) \cdot dt - i_L \right] \cdot dt,$$

where, $d$ is the PWM duty ratio, $k_{pi}$ and $k_{pi}$ are the inductance current PI controller parameters, $v_o$ and $i_L$ are the output voltage PI controller parameters, $v_e$ and $i_L$ are the output voltage and inductance current of the source converter.
FIGURE 3. Basic control structure of the bidirectional dc/dc converter.

Substituting (5) into (4), we can obtain the closed-loop transfer function of the inductance current loop and output voltage loop as follows:

\[ G_{cl1}(s) = \frac{(k_v s + k_i) V_o}{L_in s^2 + k_i V_o + k_v} \]  

\[ G_{cl1}(s) = \frac{(k_v s + k_i)(1 - D)}{C_o s^2 + k_v (1 - D) s + k_i (1 - D)} \]  

It should be noted that the output voltage closed-loop transfer function has been simplified here by neglecting the dynamics of the inductance current loop.

As shown in (6) and (7), both the inductance current loop and output voltage loop can be taken as the typical second-order system. According to the classical control theory of the second-order system, the PI controller parameters can be directly determined by designating the natural frequency and damping ratio of the system. With the main circuit parameters shown in Table I, the natural frequency of the inner current \( \omega_n \) is selected to be 2000 rad/s and the damping ratio \( \xi_n \) is 0.5. Moreover, for ensuring the effectiveness of the cascaded controller, the natural frequency of the output voltage loop \( \omega_c \) should be designed to be much lower than that of the inner current loop, which is selected to be 400 rad/s in this paper. The damping ratio for the voltage loop \( \xi_c \) is designed to be 0.5 for ensuring a fast dc bus voltage regulating dynamics. The final parameter designing results of the inner loop controllers are listed in Table II.

III. VIRTUAL NEGATIVE INDUCTOR STABILIZING STRATEGY

In this section, basic principles of the proposed virtual negative inductor stabilizing strategy are first described by analyzing a simplified circuit of the studied dc microgrid system. Small-signal models of the studied dc microgrid under the proposed stabilizing strategy is then derived for the following parameter designing and stability analysis. A root-locus based parameter designing approach is proposed for obtaining the optimal value of the proposed stabilizer parameters.

TABLE II

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 )</td>
<td>natural frequency of inductance current loop</td>
<td>2000 rad/s</td>
</tr>
<tr>
<td>( a_0 )</td>
<td>natural frequency of output voltage loop</td>
<td>400 rad/s</td>
</tr>
<tr>
<td>( \xi_n )</td>
<td>damping ratio of inductance current loop</td>
<td>0.5</td>
</tr>
<tr>
<td>( \xi_c )</td>
<td>damping ratio of output voltage loop</td>
<td>0.5</td>
</tr>
<tr>
<td>( k_v )</td>
<td>proportional coefficient of PI inductance current controller</td>
<td>0.02</td>
</tr>
<tr>
<td>( k_c )</td>
<td>integral coefficient of PI inductance current controller</td>
<td>40</td>
</tr>
<tr>
<td>( k_v )</td>
<td>proportional coefficient of PI output voltage controller</td>
<td>1.76</td>
</tr>
<tr>
<td>( k_c )</td>
<td>integral coefficient of PI output voltage controller</td>
<td>704</td>
</tr>
<tr>
<td>( R_{drop} )</td>
<td>droop coefficient for the source converter</td>
<td>0.4</td>
</tr>
<tr>
<td>( L_{drop} )</td>
<td>virtual negative inductor of the proposed stabilizing strategy</td>
<td>0.46 mH</td>
</tr>
<tr>
<td>( \tau )</td>
<td>low-pass filter time constant of the proposed stabilizing strategy</td>
<td>1.0 ms</td>
</tr>
</tbody>
</table>

A. BASIC PRINCIPLES

It is well-known that most of the dc-based power systems suffer from the constant power load (CPL) instability problem. Moreover, as indicated in [15], if the number of the voltage regulating unit is small, the dc-link capacitance is small or the line inductance is large, the destabilizing effect of the CPL will be more serious.

For illustrating this fact, a simplified equivalent circuit for the studied dc microgrid system is depicted here, as shown in Fig. 4.

The overall impedance of the equivalent circuit shown in Fig. 4 can be expressed as follows

\[ Z(s) = \frac{1}{R_{drop} + R_c + L_c s} / \frac{1}{C_eq s} / R_{CPL} / R_{dc} \]

\[ = \frac{R_{drop} + R_c + L_c s}{a_3 s^2 + a_2 s + a_0} \]

where,

\[ a_0 = 1 + (R_{drop} + R_c)(1/R_{dc} + 1/R_{CPL}) \]

\[ a_1 = C_eq (R_{drop} + R_c) + L_c (1/R_{dc} + 1/R_{CPL}) \]

\[ a_2 = C_eq L_c \]

\[ R_{CPL} = \frac{v_{eq}^2}{P_{CPL}} \]
For ensuring the system asymptotical stability, the overall impedance \( Z(s) \) should have no right half plane (RHP) poles, where the following necessary conditions need to be satisfied according to the Hurwitz stability criterion:

\[
a_i > 0, \quad i = 1, 2, 3
\]

Solving (8), (9), (10) and (11), we can obtain the following results

\[
\begin{align*}
P_{\text{CPL}} &< v_{eq}^2 \cdot \left( \frac{1}{R_{\text{drop}} + R_L} + \frac{1}{R_L} \right) \\
P_{\text{CPL}} &< C_{eq} \left( R_{\text{drop}} + R_L \right) v_{eq}^2 / L_{eq} + v_{eq}^2 / R_{dc}
\end{align*}
\]

(12)

As indicated in (12), a decrease in the line inductance \( L_{eq} \) will generate a large allowable CPL power, and therefore the system stability margin can be enhanced.

However, the line inductance is physically determined by the length and wiring way of the electric cable, which cannot be modified arbitrarily by users.

Inspired by this fact, we tried a different new way for stabilizing the studied dc microgrid system. A virtual negative inductor stabilizing strategy is proposed here. Different from the existing way, the virtual negative inductor is built on the source-side converter through the droop control method, as shown in Fig. 5.

The proposed virtual negative inductor will be built on the output side of the source converter through the droop method by modifying the conventional constant droop coefficient as follows

\[
Z_{\text{drop}}(s) = R_{\text{drop}} + (-L_{\text{drop}}) \cdot s
\]

(13)

where, \( R_{\text{drop}} \) is the conventional droop coefficient, and \( L_{\text{drop}} \) is the proposed virtual inductor.

As shown in (13), there is a pure differential operator in the droop coefficient of the proposed stabilizing strategy, which may bring undesired high-frequency noises to system. For solving this problem, a low-pass filtered differentiator is replaced here as an alternative. The modified droop coefficient can be then expressed as:

\[
Z_{\text{drop}}(s) = R_{\text{drop}} + (-L_{\text{drop}}) \cdot s / (\tau s + 1)
\]

(14)

where, \( \tau \) is the time constant of the low-pass filter.

The proposed virtual negative inductor stabilizing controller receives the source converter output current and generates the droop term for the output voltage reference value, which can be expressed as follows:

\[
v'_o = v_{\text{nom}}' - \left[ R_{\text{drop}} + (-L_{\text{drop}}) \cdot s / (\tau s + 1) \right] i_o
\]

(15)

where, \( v' \) is the Laplacian operator, \( v'_o \) is the output voltage reference, \( v_{\text{nom}}' \) is the nominal value of the bus voltage, and \( i_o \) is the output current.

Detailed control structure of the proposed stabilizing strategy has been shown in Fig. 6.

**B. SMALL-SIGNAL MODELING**

Both the stabilizer parameter designing and the system stability analysis require a detailed small-signal model of the dc microgrid system. Here followed will be the deriving process of the small-signal model for the studied dc microgrid system. Based on the average state-space model of the dc/dc converter derived in (4), the inner loop controlling functions obtained in (5), the CPL equivalent model expressed in (2), and the proposed virtual negative inductor stabilizing controller shown in (15), the small-signal model of the studied dc microgrid system can be expressed as follows:

![Diagram](Image)
\[ \Delta \dot{v}_o = \left( (1 - D) \Delta i_L - I_o \Delta d - \Delta i_v \right) / C_v \] (16)

\[ \Delta \dot{i}_L = \left[ \Delta v_o - R_v \Delta i_L - (1 - D) \Delta v_o + V_o \Delta d \right] / L_i \] (17)

\[ \Delta v_o = \left( \Delta v_o - R_v \Delta i_o - \Delta v_{\text{eq}} \right) / L_v \] (18)

\[ \Delta \dot{v}_{\text{eq}} = \Delta v^*_{\text{eq}} - R_{\text{eq}} \Delta i_{\text{eq}} + L_{\text{eq}} \Delta x_{\text{eq}} - \Delta v_{\text{eq}} \] (19)

\[ \Delta \dot{v}_{\text{v}} = k_v \left( \Delta v^*_v - R_{\text{v}} \Delta i_v + L_{\text{v}} \Delta x_{\text{v}} - \Delta v_v \right) \]

\[ + k_v \Delta x_{\text{v}} - \Delta i_v \] (20)

\[ \Delta \dot{x}_{\text{NF}} = \left[ \left( \Delta v^*_o - R_v \Delta i_o - \Delta v_{\text{NF}} \right) / L_v - \Delta x_{\text{NF}} \right] / \tau \] (21)

\[ \Delta \dot{i}_{\text{eq}} = \left( \Delta v_{\text{eq}} - R_{\text{eq}} \Delta i_{\text{eq}} - \Delta v_{\text{eq}} \right) / L_{\text{eq}} \] (22)

\[ \Delta \dot{v}_{\text{eq}} = \left( \Delta i_{\text{eq}} - \Delta i_{\text{CPL}} \right) / C_{\text{eq}} \] (23)

where, \( D, I_L \) and \( V_o \) represent the steady-state value of duty ratio, inductance current and output voltage, respectively. \( x_v, x_{\text{v}}, \) and \( x_{\text{NF}} \) are the inner states of the output voltage controller, the inductance current controller and the virtual negative inductor stabilizer, respectively.

The controlling functions and other algebraic equations of the dc microgrid system have also been stated here as follows:

\[ \Delta d = k_i \left[ k_P \left( \Delta v^*_o - R_v \Delta i_o + L_{\text{v}} \Delta x_{\text{v}} - \Delta v_v \right) \right] \]

\[ + k_i \Delta x_{\text{v}} - \Delta i_v + \Delta i_{\text{v}} \] (24)

\[ \Delta \dot{i}_o = -\left( P_{\text{CPL}} / V^2_{\text{eq}} \right) \cdot \Delta v_{\text{eq}} \] (25)

\[ \Delta v_{\text{eq}} = R_{\text{eq}} \left( \Delta i_{\text{eq}} - \Delta i_{\text{CPL}} \right) \] (26)

Substituting the algebraic equations (24), (25) and (26) into the differential equations (16)-(23), we can obtain the small-signal model of the studied dc microgrid system. For the clearance of description, the small-signal model of the studied dc microgrid system is expressed in the matrix form as follows:

\[ X_{\text{dc}} = A_{\text{dc}} X_{\text{dc}} \] (27)

where, \( X_{\text{dc}} \) represents for the whole system state,

\[ X_{\text{dc}} = [X_{\text{DG}}, X_{\text{CPL}}]^T \] (28)

where, \( X_{\text{DG}} \) represents the state of the dc/dc source converter with the proposed stabilizing controller

\[ X_{\text{DG}} = [\Delta v_o, \Delta i_L, \Delta i_o, \Delta x_{\text{NF}}, \Delta x_{\text{v}}, \Delta x_{\text{v}}]^T \] (29)

and \( X_{\text{CPL}} \) is the state of equivalent CPL.

\[ X_{\text{CPL}} = [\Delta i_{\text{eq}}, \Delta v_{\text{eq}}]^T \] (30)

C. PARAMETER DESIGNING

The PI controller parameters for the inductance current loop and output voltage loop have been designed above in section II as listed in Table II. Therefore, for the proposed stabilizing strategy, there are three parameters remaining to be designed, namely, the conventional droop coefficient, the virtual negative inductor, and the low-pass filter time constant.

For the conventional droop coefficient, there is always a compromise between the power sharing accuracy and the maximum allowable dc bus voltage deviation. A large droop coefficient may guarantee a better power sharing accuracy, but leaving with a large voltage deviation. For limiting the deviation of the dc bus voltage while enhancing the power sharing accuracy, the droop coefficient is usually determined as follows:

\[ R_{\text{droop}} = \frac{\Delta v_{\text{max}}}{i_{\text{max}}} \] (31)

where, \( \Delta v_{\text{max}} \) is the maximum allowable voltage deviation and \( i_{\text{max}} \) is the maximum output current of the source converter.

The purpose of the proposed virtual negative inductor stabilizing strategy is to ensure the stability of the dc microgrid system. Therefore, the stabilizer parameters should be designed to maximize the system stability margins. For achieving this goal, a root-locus-based parameter designing approach is proposed in this paper. Two cases are studied for assuring the optimal value of the virtual negative inductor and

**FIGURE 7.** (a) Root-locus diagram of the studied dc microgrid system with the virtual negative inductor varying from 0.2mH to 1.0mH; (b) Zoom.
FIGURE 8. (a) Root-locus diagram of the studied dc microgrid system with the low-pass filter time constant varying from 0.5ms to 1.5ms; (b) Zoom.

In the first case, the root-locus of the dc microgrid system is plotted by varying virtual negative inductance from 0.2mH to 1.0mH, as shown in Fig. 7. The constant power load is 5kW. The conventional droop coefficient is set to be 0.4. The time constant of the low-pass filter in the virtual negative inductor droop controller is arbitrarily preset to be 1.0ms here.

As we can see from Fig. 7, there is an inflection point \((L_{\text{droop}} = 0.46mH)\) in the system root-locus diagram, which is farthest point away from the right-half-plane. If the negative inductor goes larger or smaller than 0.46mH, the system root-locus will move towards right. As stated above, the virtual negative inductor should be designed to guarantee the maximum system stability margin. Therefore, the inflection point of system root-locus will be the optimal one for the virtual negative inductor, namely, \(L_{\text{droop}} = 0.46mH\).

Likewise, a similar parameter designing process is also taken for assuring the optimal value of the low-pass filter time constant. In this case, the virtual negative inductor is set to be 0.46mH, which is the optimal designing result obtained above. The time constant of the low-pass filter varies from 0.5ms to 1.5ms. The system root-locus is shown in Fig. 8.

As shown in Fig. 8, there is also an inflection point for the low-pass filter time constant in the system root-locus diagram. Coincidentally, the inflection point of the low-pass filter time constant is exactly 1.0ms, which is same with the preset value used for designing the virtual negative inductor. Therefore, the optimal value of the low-pass filter time constant is confirmed to be 1.0ms for the studied dc microgrid system.

However, it should be mentioned that the coincidence stated above does simplify the optimal designing process of the stabilizer parameters for the studied dc microgrid system. However, the same words cannot be said for all cases. If there is a difference between the preset value and the optimal value for the low-pass filter time constant, the above virtual negative inductor should be redesigned. The total parameter designing process stated above may be iterated twice or more times until the optimal pair of stabilizer parameters are assured. Due to the space limitation, this iterating designing process will not be specified here.

For the clearance of description, the final designing results of the proposed stabilizing controller parameters have been listed in Table II.

IV. STABILITY ANALYSIS

In this section, the stability of the proposed virtual negative inductor stabilizing strategy is analyzed. An explicit Nyquist stability condition for the studied dc microgrid system is first specified in detail, with the system minor loop gain carefully derived. Stability of the dc microgrid system are then analyzed in three cases, which includes the variation of CPL power, the variation of droop coefficient, and the variation of CPL-side capacitance. As a comparison, the same stability analyses have also been taken for the conventional droop controlled system.

A. EXPLICIT NYQUIST STABILITY CONDITION

The equivalent circuit of the studied dc microgrid system considering the inner loop dynamics can be depicted as shown in Fig. 9, where, \(Z_o\) is the output impedance of the dc/dc converter, \(Z_s\) is the line impedance, and \(Y_2\) is the input admittance of the CPL, which can be expressed as follows.
According to the Thevenin’s theorem, the voltage \( V \) and the current \( I \) in the equivalent circuit can be expressed as follows:

\[
I = \frac{V - V_1}{Z_L + Y_L} = \frac{V}{1 + Z_S Y_L}
\]

\[
V = V_1 + I_z (Z_L + Y_L)
\]

where, \( Z_S \) is the source impedance, and \( Y_L \) is the load admittance, which can be expressed as

\[
Z_S = Z_o + Z_e
\]

\[
Y_L = 1 / R_{dc} + 1 / (Z_e + Y_e)
\]

The source system and load system are usually designed to be stable alone. Therefore, as shown in (34), the whole system stability will be related to the right-half-plane zeros (RHZ) of the denominator \( 1 + Z_S Y_L \), which are also the right-half-plane poles (RHP) of total system. The system will be stable if there is no RHZ in \( 1 + Z_S Y_L \). Here, a minor loop gain is usually defined as follows

\[
T_M = Z_S Y_L
\]

According to the classical control theory, the number of RHZ in denominator can be estimated by the Nyquist trajectory of the system minor loop gain \( T_M \), which can be expressed as follows:

\[
\text{RHZ} (1 + Z_S Y_L) = N(1 + Z_S Y_L) + \text{RHP}(1 + Z_S Y_L)
\]

\[
= N(T_M)
\]

where, \( \text{RHZ} (1 + Z_S Y_L) \) is the number of right-half-plane zeros of \( 1 + Z_S Y_L \), \( \text{RHP}(1 + Z_S Y_L) \) is the number of right-half-plane poles of \( 1 + Z_S Y_L \), and \( N(T_M) \) is the number of times that Nyquist trajectory of \( T_M \) encircles point (-1,0) in clockwise direction.

It should be mentioned that because the source and load are stable when operating alone, \( \text{RHP}(1 + Z_S Y_L) \) is zero in (38).

As stated above, the following explicit sufficient and necessary Nyquist stability condition for the studied dc microgrid system can be constructed:

1. If the Nyquist trajectory of the minor loop gain \( T_M \) does not encircle the critical point (-1,0), the studied dc microgrid system will be stable;
2. Otherwise, the studied dc microgrid system will be unstable.

The following will be the detailed deriving process of the minor loop gain for the studied dc microgrid system.

**B. MINOR LOOP GAIN**

As shown in (34) and (35), the output impedance model of the dc/dc converter is required for obtaining the minor loop gain of the studied dc microgrid system. According to (16)-(26), the small-signal model diagram of the dc/dc converter can be depicted as shown in Fig. 10, where,

\[
A_v(s) = \frac{\Delta v_o(s)}{\Delta v_i(s)} = \frac{1 - D}{L_w C_o s^2 + R_w C_o s + (1 - D)^2}
\]

\[
Z_{out}(s) = -\frac{\Delta v_o(s)}{\Delta i_o(s)} = \frac{L_w s + R_w}{L_w C_o s^2 + R_w C_o s + (1 - D)^2}
\]

\[
G_{vd}(s) = \frac{\Delta v_d(s)}{\Delta d(s)} = \frac{L_w I_s - R_w I_L + V_i}{L_w C_o s^2 + R_w C_o s + (1 - D)^2}
\]

\[
Y_{io}(s) = \frac{\Delta i_o(s)}{\Delta v_i(s)} = \frac{C_i s}{L_w C_o s^2 + R_w C_o s + (1 - D)^2}
\]

\[
A_{io}(s) = -\frac{\Delta i_o(s)}{\Delta i_i(s)} = \frac{L_w C_i s^2 + R_w C_i s + (1 - D)^2}{(1 - D)}
\]

\[
G_{id}(s) = \frac{\Delta i_d(s)}{\Delta d(s)} = \frac{C_i V_i + I_s (1 - D)}{L_w C_o s^2 + R_w C_o s + (1 - D)^2}
\]

\[
Z_{droop}(s) = R_{droop} + (-L_{droop} \cdot s) \frac{s}{\tau s + 1}
\]

![FIGURE 10. Small-signal model diagram of the dc/dc converter with the proposed stabilizing strategy](image)
Therefore, as stated above, the stability problem of the studied dc microgrid system is then transformed into a question of whether the Nyquist trajectory of the minor loop gain encircles the point (-1, 0). For analyzing the system stability, the Nyquist diagram of the minor loop gain $T_m$ needs to be plotted.

C. NYQUIST DIAGRAM BASED STABILITY ANALYSIS

A sufficient and necessary explicit Nyquist stability condition for the studied dc microgrid system has been established above. With this explicit condition, the stability of the studied dc microgrid system can be analyzed with the help of the Nyquist diagram. Three conditions are considered here, which include the variation in CPL power, the variation in the droop coefficient, and the variation in the CPL-side capacitance. As a comparison, the same conditions are also analyzed for the conventional droop controlled system.

1) STABILITY ANALYSIS WITH A VARYING CPL POWER

In the first case, a varying CPL power condition is studied for the comparative stability analysis of the proposed stabilizing strategy and the conventional droop control method, as shown in Fig. 11. The droop coefficients for two comparative systems are set to be exactly same, namely, 0.4. The virtual negative inductor coefficient for the proposed stabilizing strategy is 0.46mH, and the low-pass filter time constant is 1.0ms, which are the optimal results obtained from the root-locus based parameter designing approach. Three CPL power conditions are considered, which are 0.8kW, 1.8kW, and 2.8kW, respectively. Other parameters have been listed in Table. I.

As shown in Fig. 11(a), for the CPL power of 0.8kW, the Nyquist trajectory does not encircle the critical point (-1, 0), which means the conventional droop controlled system is stable with 0.8kW CPL. However, for the 1.8kW CPL, the Nyquist trajectory crosses the point (-1, 0). For this condition, any system parameter perturbation or external disturbance may make the Nyquist trajectory encircle the point (-1, 0), causing the system unstable. Therefore, the 1.8kW is the critical stable CPL power for the conventional droop controlled system. For the CPL of 2.8kW, the Nyquist trajectory of the system minor loop gain encircles the point (-1, 0), which means that there is a RHP in the studied dc microgrid system. Therefore, the conventional droop controlled system is unstable with the 2.8kW CPL.

Fig. 11(b) shows the Nyquist trajectory of the system minor loop gain under the same condition with the proposed virtual negative inductor stabilizing strategy. As we can see, all of the Nyquist trajectories in Fig. 11(b) do not encircle the point (-1, 0), which means that the system is stable with the CPL power of 0.8kW, 1.8kW and 2.8kW. Moreover, as shown in Fig. 11(b), the gain margins of the studied dc microgrid system are almost unchanged for three different CPL power conditions, which further confirms the robustness of the proposed stabilizing strategy to the CPL power variation.

From the comparative stability analysis results depicted in Fig. 11, we can see that the proposed virtual negative inductor stabilizing strategy shows a better robustness to the CPL power variations than the conventional droop method.

2) STABILITY ANALYSIS WITH A VARYING DROOP COEFFICIENT

In the conventional droop-controlled dc power system, there is always a tradeoff between the power sharing accuracy and the system stability when designing the droop coefficient. A large droop coefficient may guarantee a better power sharing accuracy, but at the cost of a degraded system stability. Moreover, some adaptive droop control methods try to balance the state-of-charge (SoC) of energy storage devices by adjusting the droop coefficient online. The droop coefficient may be varying during the system operation. Therefore, the impact of the droop coefficient variation on the system stability should be analyzed.

In this case, a varying droop coefficient condition is considered for the stability analysis of the studied dc microgrid
system. The CPL power is set to be 1kW in this case. Three droop coefficients are considered, which are 0.4, 0.6 and 0.8, respectively. The optimal parameter designing results of the virtual negative inductor and the low-pass filter time constant obtained from the root-locus parameter designing approach are utilized here. As a comparison, the same conditions are also considered for the conventional droop controlled system. The Nyquist trajectories of the system minor loop gain with three different droop coefficients have been depicted as shown in Fig. 12.

As we can see from Fig. 12(a), the increasing droop coefficient has a significant adverse impact on the system stability. For the condition of the droop coefficient equal to 0.4, the Nyquist trajectory does not encircle the point (-1, 0), which means the conventional droop controlled system is stable with the droop coefficient of 0.4. However, for the droop coefficient of 0.6 and 0.8, both two Nyquist trajectories encircle the point (-1, 0), which means that the conventional droop controlled system is unstable with the droop coefficient larger than 0.6.

Fig. 12(b) shows the same stability analysis results of the studied dc microgrid system with the proposed stabilizing strategy. As we can see, all of three Nyquist trajectories do not encircle the point (-1, 0), which means that the proposed stabilizing strategy controlled system is stable with the variation of the droop coefficient from 0.4 to 0.8.

Therefore, as we can see from the comparative stability analysis results shown in Fig. 12, the proposed stabilizing strategy controlled system shows a better robustness to the variation of the droop coefficient than the conventional droop controlled system.

3) STABILITY ANALYSIS WITH A VARYING CPL-SIDE CAPACITANCE

Usually, the studied dc microgrid system is preferred to have a plug-and-play feature, where the load side dc bus capacitance may be varying by adding in or cutting off several

![Nyquist Diagram](image1)

**FIGURE 12.** Comparative Nyquist stability analysis result of the studied dc microgrid system with the droop coefficient variations. (a) Conventional droop controlled system. (b) Proposed virtual negative inductor stabilizing strategy controlled system.

![Nyquist Diagram](image2)

**FIGURE 13.** Comparative Nyquist stability analysis result of the studied dc microgrid system with the CPL-side capacitance variations. (a) Conventional droop controlled system. (b) Proposed virtual negative inductor stabilizing strategy controlled system.
active loads. Therefore, in this condition, a stability analysis for the studied dc microgrid system with a varying CPL-side capacitance will be preferred.

In this case, the CPL power is set to be 2.9kW. Three CPL-side capacitances are considered, which are 2200 μF, 1100 μF, and 470 μF, respectively. The Nyquist trajectories of the system minor loop gain have been plotted as shown in Fig. 13. As a comparison, the stability of the conventional droop controlled system is also studied here under the same condition.

As shown in Fig. 13(a), the variation in the CPL-side capacitance has a large impact on the system stability. Different from the conventional idea that a large dc bus capacitance may enhance the system stability, we can find out that the conventional droop controlled system is stable with the CPL-side capacitance of 470 μF, but unstable with the CPL-side capacitance of 2200 μF, as shown in Fig. 13(a).

The same conditions are also studied for the proposed stabilizing strategy controlled dc microgrid system. As shown in Fig. 13(b), all of the Nyquist trajectories of the system minor loop gain do not encircle the critical point of (-1, 0), which means that the proposed stabilizing strategy controlled system remains stable with the variation of the CPL-side capacitance from 2200 μF to 470 μF. Therefore, we can find out that the proposed stabilizing strategy controlled dc microgrid system has a better robustness to the variation of the CPL-side capacitance than that of the conventional droop controlled system.

For the clarity of description, the stability analysis results stated above and corresponding parameter variations have been summarized, as listed in Table. III.

### TABLE III

<table>
<thead>
<tr>
<th>Case</th>
<th>System Parameters</th>
<th>Conventional</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$P_{\text{CPL}}$</td>
<td>0.8kW</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>$R_{\text{drop}} = 0.4, C_{\text{eq}} = 2200\mu F$</td>
<td>1.8kW</td>
<td>Unstable</td>
</tr>
<tr>
<td></td>
<td>$L_{\text{drop}} = 0.46mH, \tau = 1.0ms$</td>
<td>2.8kW</td>
<td>Unstable</td>
</tr>
<tr>
<td>2</td>
<td>$P_{\text{CPL}} = 1.0kW, C_{\text{eq}} = 2200\mu F$</td>
<td>0.4</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>$P_{\text{CPL}} = 0.46mH, \tau = 1.0ms$</td>
<td>0.6</td>
<td>Unstable</td>
</tr>
<tr>
<td>3</td>
<td>$C_{\text{eq}}$</td>
<td>2200μF</td>
<td>Unstable</td>
</tr>
<tr>
<td></td>
<td>$P_{\text{CPL}} = 2.9kW, R_{\text{drop}} = 0.4$</td>
<td>1100μF</td>
<td>Unstable</td>
</tr>
<tr>
<td></td>
<td>$L_{\text{drop}} = 0.46mH, \tau = 1.0ms$</td>
<td>470μF</td>
<td>Stable</td>
</tr>
</tbody>
</table>

V. SIMULATION VALIDATIONS

Numerical simulations are conducted for validating the effectiveness of the proposed virtual negative inductor based stabilizing strategy. The models of the studied dc microgrid depicted in (2)-(5) are implemented on the AppSIM real-time simulator, as shown in Fig. 14, which enables a real-time simulation for the switching details of power electronic converters.

![Configuration of the AppSIM Real Time Simulator](image1)

**FIGURE 14.** Configuration of the AppSIM Real Time Simulator.

A. CASE I: ACTIVATION AND DEACTIVATION OF THE PROPOSED STABILIZING STRATEGY

In the first case, transient simulations are conducted for validating the effectiveness of the proposed stabilizing strategy, as shown in Fig. 15. Before $t = 1.5s$, the proposed virtual negative inductor stabilizing strategy is not activated. Therefore, the apparent instability issue can be found in the simulation results of the dc microgrid system with 1.8kW CPL. At $t = 1.5s$, the proposed virtual negative inductor stabilizer is activated, and the studied dc microgrid system is restored to the new stable state within 50ms. At $t = 3.5s$, the proposed virtual negative inductor stabilizing strategy is deactivated. Due to the destabilizing effect of the CPL, the system becomes unstable again. Therefore, the effectiveness of the proposed

![Numerical simulation results of the studied dc microgrid system with the proposed stabilizing strategy deactivated and activated](image2)

**FIGURE 15.** Numerical simulation results of the studied dc microgrid system with the proposed stabilizing strategy deactivated and activated.
virtual negative inductor stabilizing strategy can be validated from the numerical simulation results stated above.

**B. CASE II: CHANGING OF OPERATING POINTS**

Both the proposed optimal parameter designing approach and the Nyquist stability analysis of the proposed virtual negative inductor stabilizing strategy are based on the small-signal model of the studied dc microgrid system. In order to validate the effectiveness of the proposed stabilizing strategy to the dc system with nonlinear constant power loads, it is necessary to verify the stabilizer performance with different operating points. Continuous step load change is tested here for the proposed stabilizing strategy controlled dc microgrid system, as shown in Fig. 16. The CPL varies following the sequence of 1.8kW, 2.8kW, 3.8kW, 4.8kW and 5.8kW. As shown in Fig. 16, stable operation of the studied dc microgrid system is guaranteed during the whole process of the step load change by using the proposed virtual negative inductor stabilizing strategy. Therefore, the transient stability of the proposed stabilizing strategy is validated.

**C. CASE III: TESTING OF PARAMETER VARIATION**

For testing the performance of the proposed virtual negative inductor stabilizing strategy with system parameter variations, the droop coefficient and CPL-side capacitance are changed in this numerical simulation case. In particular, the droop coefficient varies following the sequence of 0.4, 0.6, and 0.8, as shown in Fig. 17. It can be seen that the proposed stabilizing strategy controlled dc microgrid system remains stable with the variation of the droop coefficient.

The CPL-side capacitance varies following the sequence of 470 µF, 1100 µF, and 2200 µF, as shown in Fig. 18. It can be seen from the simulation results that the stable operation of the studied dc microgrid system is also ensured by the proposed virtual negative inductor stabilizing strategy with different CPL-side capacitances. Therefore, the effectiveness of the proposed virtual negative inductor stabilizing strategy with system parameter variation is validated.

**VI. CONCLUSIONS**

A novel virtual negative inductor stabilizing strategy for the dc microgrid with constant power loads is proposed in this paper. The proposed stabilizing strategy constructs a virtual negative inductor on the source side converter by modifying the traditional droop control method. The constructed virtual negative inductor counteracts the line inductance, hence enhancing the system damping effect from the system point of view. A low-pass filter is built in series with the proposed virtual negative inductor for eliminating the high frequency oscillations brought by the pure differentiator. Small-signal model of the studied dc microgrid system is derived, and a root-locus based parameter designing approach is proposed. Optimal stabilizer parameters are obtained with the help of the system root-locus diagram, which maximize the system stability margins. A sufficient and necessary Nyquist stability
condition for the studied dc microgrid system is established. The system minor loop gain of the system is carefully derived from the diagram of the system small-signal model. System stability with a changing CPL power, droop coefficient variations and the CPL equivalent capacitance perturbations are studied with the help of Nyquist diagram. From the analysis results, we find out that the proposed stabilizing strategy shows a better robustness to the parameter variations over the conventional droop method. Finally, detailed numerical simulations are conducted for validating the effectiveness of the proposed virtual negative inductor stabilizing strategy.

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