A Quantum Particle Swarm Optimization Method with Fitness Selection Methodology for Electromagnetic Inverse Problems

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ABSTRACT The objective of the research is to extend the potential of standard Quantum Particle Swarm Optimization (QPSO) method for electromagnetic inverse problems. As, QPSO trapped into local optima while dealing with complex design problems. In order to address this type of issue, to avoid from trapping into local optima and tradeoff between the exploration and exploitation searches, a novel methodology is employed that includes the design of a new position updating formula, the introduction of a novel fitness selection methodology and the proposal of a dynamic parameter updating strategy. Nevertheless, the evaluated results as reported have revealed that the proposed MQPSO (Modified quantum inspired particle swarm optimization) method for global optimization and electromagnetic inverse problems can find better outcomes at initial stage of the iterating process as compared to other tested optimal methods.

INDEX TERMS Fitness selection, Particle swarm optimization, Quantum mechanics, Design optimization, Electromagnetic problem.

I. INTRODUCTION

The study of electromagnetic design problems surpasses more than decades. Generally, it indicates to the optimal design of electromagnetic devices that naturally rises in many practical engineering problems.

Recent tactic to solve the electromagnetic design problem is to split them into a number of direct problems and then to solve them by using a stochastic optimal technique. Thus, the numerical techniques and stochastic algorithms play primary role for the solution of electromagnetic design problems. Consequently, many efforts have been made to improve the general structure of the stochastic algorithms for solving these problems and many other real-world engineering optimization problems have been solved by using these stochastic techniques.

Recently, Tian proposed an improved ant lion optimization method and have successfully applied in hydraulic turbine governing system parameter identification. In the proposed method a chaotic mutation operation namely, logistic map is introduced for the elite to break out of the local optimum [1]. In [2], particle swarm optimization algorithm is applied for satisfying the vehicle power demand and to tradeoff between the energy consumption and battery health. To solve wireless sensor networks optimization problems in smart grid applications a new multi-objective optimization method based on sperm fertilization procedure has been applied [3]. A novel brain storm optimization algorithm with multi-information interactions was proposed for global optimization problems [4]. In [5], a deep feature optimization fusion method was introduced for extracting bearing degradation features. The state of power (SOP) estimation algorithm using genetic algorithm is proposed in [6] to deal with the long-time scale estimation for power management application. A new multi-objective quantum particle swarm optimization for electronic nose in wound infection was proposed [7]. A modified Quantum-inspired Particle Swarm Optimization (QPSO) algorithm for global optimizations of inverse problems was proposed in [8].
Moreover, in the manufacturing of an optimal design, it generally includes the optimal solution of inverse problem which consist of determining the global optimal solution of an objective function(s) under some given constraints. Since, the objective function is generally a multimodal one and because of the inefficiency of traditional deterministic and stochastic optimal algorithms in finding the global optimal solution of such a problem, the attentions of many researches are devoted to the development of new stochastic optimal methods. Consequently, the evolutionary algorithm (EA) has become the standard for solving global optimizations in different engineering disciplines because it can find global optimal solutions that are otherwise not obtainable using traditional optimal algorithms. Nevertheless, according to the no free lunch theorem there is no any universal optimizer that can solve all optimization problems. Thus, it is necessary to seek a new global optimizer for the study of inverse problems and there is a need to keep the diversity of evolutionary algorithm high in solving inverse problems.

Moreover, in engineering design optimization, most of the problems can be defined by nonlinear relations that often give rise to multiple local optima. In this regard, a standard benchmark problem to validate the robustness and performance of various optimization method is the TEAM problem 22 [9].

TEAM problem 22 is used to determine the optimal design of SMES (superconducting magnetic energy storage) device, to store a substantial amount of energy in the magnetic field by using a simple and reasonable coil arrangement that can be easily scaled up in size. The literature has several optimization techniques that have been applied to solve the TEAM workshop problem 22 [8,10,11,12].

However, Particle Swarm Optimization (PSO) technique is an addition to the evolutionary algorithms. It was originated by Kennedy and Eberhart [13], based on the social behavior of birds flocking and fish schooling in their hunt for foods. Thus, the PSO is similar to the evolutionary algorithms in that it works with population. In PSO, the population is called a swarm and each individual is known as a particle. The PSO algorithm is very easy in concepts and implementation. It has been applied successfully to solve various engineering inverse problems. However, the PSO method encounter a premature convergence when solving a complex optimization problem, this is due to the improperly balance between the local and global searches. To solve such difficulties the quantum version of particle swarm optimization (QPSO) was proposed [14].

In QPSO, the behavior of particles follows the principles of quantum mechanics instead of Newtonian mechanics imposed in PSO. Thus, instead of Newtonian random walk some type of quantum motion is incorporated into the evolution process of QPSOs to bring a good balance between local and global searches. The QPSO is a broadly used global convergence algorithm for engineering electromagnetic problems. However, there are still many disputes in QPSO that should be solved.

In this regard, a novel fitness selection methodology is introduced into QPSO based on dynamic control parameter for the optimization of SMES design problems to tradeoff between exploration and exploitation searches as reported in this work.

II. QPSO METHOD

The trajectory analysis [16] reveals that the PSO convergence speed can be guaranteed if each individual converges to its local attractor

\[
\mathbf{p}_{\text{best}} = (p_{\text{best},1}, p_{\text{best},2}, \ldots, p_{\text{best},d}),
\]

of which the coordinates are

\[
p_{\text{best},i}(k) = (c_i \times p_{\text{best},i}(k) + c_z \times \mathbf{p}_i(k)) / (c_i + c_z) \tag{1}
\]

or

\[
p_{\text{best},i}(k) = \phi \times p_{\text{best},i}(k) + (1 - \phi) \times \mathbf{p}_i(k) \tag{2}
\]

where \(\phi = c_{z,i} / (c_{z,i} + c_{z,z})\). It has been shown that the local attractor is a stochastic particle \(i\) and lies in a hyper rectangle with \(p_{\text{best},i}\) and \(p_i\) being the two ends of its diagonal. In [14] (Sun et al., 2004), a parameter \(L(k_i)\) is defined as

\[
L(k_i) = 2.\beta \left| p_{\text{best},i}(k_i) - x_i(k_i) \right| \tag{3}
\]

where \(\beta\) is known as the contraction expansion (CE) parameter, which is used to control the convergence behavior of the algorithm and is represented by,

\[
\beta = 0.5 + (1.0 - 0.5)(k_{\text{max}} - k_i) / k_{\text{max}} \tag{4}
\]

where \(k_i\) is the current iteration and \(k_{\text{max}}\) is the maximum iteration.

To evaluate \(L(k_i)\), the Mainstream thought or mean best position is defined as the center of personal best position of the swarm. i.e.

\[
m(k_i) = (m_1(k_i), m_2(k_i), \ldots, m_d(k_i)) = \left( \frac{1}{M_{\text{sc}}} \sum_{i=1}^{N} p_{\text{best},1}(k_i), \frac{1}{M_{\text{sc}}} \sum_{i=1}^{N} p_{\text{best},2}(k_i), \ldots, \frac{1}{M_{\text{sc}}} \sum_{i=1}^{N} p_{\text{best},d}(k_i) \right) \tag{5}
\]

where \(M_{\text{sc}}\) is the population size, \(k_i\) is the current iteration and \(N\) is the dimension of problem. Thus, parameter \(L\) will become,

\[
L(k_i) = 2.\beta \left| m(k_i) - x_i(k_i) \right| \tag{6}
\]

Hence, the particle’s position will be updated according to the following equation,

\[
x_i(k_i + 1) = p_{\text{best},i}(k_i) \pm \beta \left| m(k_i) - x_i(k_i) \right| \cdot \ln(1/u_i). \tag{7}
\]

III. PROPOSED MQPSO METHOD

A. SELECTION OF FITTEST PARTICLE

To further intensify the QPSO performance in terms of both solution quality and convergence behavior, many efforts have been done and different variants of QPSO have been developed. However, most of these optimization techniques are problem oriented. Thus, there is a need to research and develop a new optimizer for the optimization of SMES design problems. In this regard, a novel QPSO-FSM
optimizer is proposed in this work for the optimization of SMES design problems. In the proposed method a novel fitness selection methodology is used to choose the fittest particle among the population. As, in other selection techniques the fitness function allocates a fitness to the promising solution. This fitness level is used to associate a probability of selection with each particle as given by,

\[ P_{new,i} = \frac{f(P_{best,i})}{M_{sz}} \]  

(8)

where \( N \) is the number of particles in the population, \( M_{sz} \) is the swarm size, \( f \) is the fitness function, \( P_{best} \) is the personal best position of a particle and \( P_{new,i} \) is the probability of selection of a particle in the population.

Then, a new particle will be generated in the search domain by using the following methodology,

\[ F_{best,i}(k_j) = x_i(k_j) + P_{new,i}(k_j) \times E_i \]  

(9)

where \( x_i(k_j) \) is the current particle, \( E_i \) is the random number with exponential probability distribution and \( F_{best,i} \) is the new best particle in the current population.

The new best particle generated will further take part in the evolution process and is incorporated into the QPSO position updating equation, defined as,

\[ x_i(k_j + 1) = P_{ad}(k_j) \pm \beta \left| m(k_j) \times F_{best,i}(k_j) - x_i(k_j) \right| \ln(1/u_i) \]  

(10)

where \( u_i \) is a uniform random number.

Table 1

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Standard Benchmark Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1(x) = \sum_{i=1}^{n} x_i^2 )</td>
<td>( x \in [-100, 100]^n )</td>
</tr>
<tr>
<td>( f_2(x) = \sum_{i=1}^{n} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2) )</td>
<td>( x \in [-100, 100]^n )</td>
</tr>
<tr>
<td>( f_3(x) = \sum_{i=1}^{n} i x_i^4 )</td>
<td>( x \in [-100, 100]^n )</td>
</tr>
<tr>
<td>( f_4(x) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos(\frac{x_i}{\sqrt{i}}) + 1 )</td>
<td>( x \in [-100, 100]^n )</td>
</tr>
<tr>
<td>( f_5(x) = \sum_{i=1}^{n}</td>
<td>x_i</td>
</tr>
<tr>
<td>( f_6(x) = \sum_{i=1}^{n} [x_i^2 - 10 \cos(2\pi x_i) + 10] )</td>
<td>( x \in [-5.12, 5.12]^n )</td>
</tr>
<tr>
<td>( f_7(x) = -a \exp \left( -b \left[ \sum_{i=1}^{n} x_i^2 \right] - \exp \left( \frac{1}{n} \sum_{i=1}^{n} \cos(c \cdot x_i) \right) + a + \exp(1) \right) )</td>
<td>( x \in [-32, 32]^n )</td>
</tr>
<tr>
<td>( f_8(x) = \sum_{i=1}^{n} z_i^2 + f_{bias}, z = x - a, )</td>
<td>( x \in [-100, 100]^n )</td>
</tr>
</tbody>
</table>

The incorporation of the new best particle \( F_{best,i} \) into the position updating equation of QPSO is because at the early stage of evolution process, the diversity of the population is high but later on it reduces rapidly. The reason for reducing the diversity is that initially the gap between mean best position \( m(k_j) \) and current particle position \( x(k_j) \) is large. However, at the later phase of optimization this distance is reduced quickly and the algorithm trapped into local optima. Thus, the aforementioned strategy will refresh the mean best position \( m(k_j) \) of particle to enlarge the gap between the mean best position \( m(k_j) \) and current particle position \( x(k_j) \) and in this way will avoid the algorithm to trap to local minima.

B. PARAMETER UPDATING STRATEGY

Moreover, the contraction expansion coefficient \( \beta \) is the only control parameter for the QPSO and is used to tune the algorithm. The \( \beta \) play an imperative role to control the convergence behavior of the QPSO algorithm. Therefore, different researchers have proposed different strategies to adjust the \( \beta \) parameter [15,16]. The most common value of \( \beta \) is that initially set it to 1 and then reduced linearly to 0.5. Also, \( \beta \) play a vital role to keep balance between the local and global searches of the algorithm.

However, improper adjustment of \( \beta \) would make the local and global searches disturb, as a consequence the algorithm will trapped into local minima. Thus, to address this type of issue, a proper adjustment of \( \beta \) parameter is important, for this purpose in this work, a new dynamic control parameter is proposed based on the new fittest particle \( F_{best,i} \). The proposed strategy will keep a good balance between the exploration and exploitation searches and will avoid the algorithm to stuck into local minima. The proposed dynamic control parameter is defined as,

\[ \beta(k_j) = 0.2 + \frac{0.9}{1 + \exp(F_{best,i} + 0.6)} \]  

(11)

It should be noted that the functions supposed in this work is strictly positive in a minimization problem. The relationship between \( \beta \) parameter and \( F_{best,i} \) particle is

\[ x \in [x_1, x_2, ..., x_n] \]
shown in figure 1. The explanation of this scheme is straightforward: if the gap between the mean best position \(m(k)\) and current particle \(x(k)\) is large i.e. if the particle is far away from mean best \(m(k)\), then one expects a small \(\beta\) to help it come back; while if the particle is very near to mean best \(m(k)\), then the distance between mean best \(m(k)\) and current particle \(x(k)\) will be small even negative and one prefers a large \(\beta\) to force it to bounce away and bring a good balance between the local and global searches.

**IV. EXPERIMENTAL RESULTS**

**A. STANDARD BENCHMARK FUNCTIONS**

In order to demonstrate the performance of the proposed method, some well-known benchmark functions as tabulated in Table 1, are solved using the proposed algorithm. The problems are divided into three categories: unimodal, multimodal, and shifted. The functions \(f_1, f_2,\) and \(f_3\) are unimodal. The functions \(f_4\) and \(f_5\) are continuous, convex and unimodal. The function \(f_3\) is a mono-modal function for small dimension but can also be treated as a multimodal for high dimensional problems. Its global minimum lies in a narrow parabolic valley. However, even still this valley is easy to find but the convergence to the minimum is difficult.

**Table 2**

<table>
<thead>
<tr>
<th>Test function</th>
<th>QPSO</th>
<th>GPQPSO</th>
<th>LIQPSO</th>
<th>MQPSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1) Min(best)</td>
<td>1.4168×10^{-10}</td>
<td>4.9300×10^{-10}</td>
<td>7.5216×10^{-10}</td>
<td>0</td>
</tr>
<tr>
<td>Mean</td>
<td>1.1087×10^{-9}</td>
<td>1.9849×10^{-9}</td>
<td>1.4891×10^{-9}</td>
<td>1.4822×10^{-9}</td>
</tr>
<tr>
<td>SD</td>
<td>2.9477×10^{-10}</td>
<td>6.8768×10^{-10}</td>
<td>1.3858×10^{-9}</td>
<td>0</td>
</tr>
<tr>
<td>Worst</td>
<td>1.124×10^{-9}</td>
<td>2.6708×10^{-9}</td>
<td>6.8234×10^{-9}</td>
<td>1.9763×10^{-9}</td>
</tr>
<tr>
<td>(f_2) Min(best)</td>
<td>22.162</td>
<td>26.484</td>
<td>28.2316</td>
<td>26.762</td>
</tr>
<tr>
<td>Mean</td>
<td>35.114</td>
<td>27.041</td>
<td>28.975</td>
<td>27.281</td>
</tr>
<tr>
<td>(f_3) Min(best)</td>
<td>27.401</td>
<td>2.2833×10^{-1}</td>
<td>0.1783</td>
<td>4.9805×10^{-2}</td>
</tr>
</tbody>
</table>

**Figure 1.** Relationship between \(F_{\text{best}}(i)\) and contraction expansion (\(\beta\)) parameter

<table>
<thead>
<tr>
<th>Test function</th>
<th>QPSO</th>
<th>GPQPSO</th>
<th>LIQPSO</th>
<th>MQPSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1) Min(best)</td>
<td>3.1195×10^{-10}</td>
<td>1.8771×10^{-10}</td>
<td>5.6320×10^{-10}</td>
<td>0</td>
</tr>
<tr>
<td>SD</td>
<td>1.5831×10^{-10}</td>
<td>1.8337×10^{-10}</td>
<td>9.7253×10^{-10}</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 3**

Performance comparison of different optimal algorithms on multimodal functions for 30-dimension problems

<table>
<thead>
<tr>
<th>Test function</th>
<th>QPSO</th>
<th>GPQPSO</th>
<th>LIQPSO</th>
<th>MQPSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1) Min(best)</td>
<td>7.1620×10^{-10}</td>
<td>9.8316×10^{-10}</td>
<td>2.1059×10^{-10}</td>
<td>3.9204×10^{-10}</td>
</tr>
<tr>
<td>SD</td>
<td>1.2294×10^{-10}</td>
<td>3.5494×10^{-10}</td>
<td>4.9801×10^{-10}</td>
<td>2.8666×10^{-10}</td>
</tr>
<tr>
<td>Worst</td>
<td>3.6843×10^{-10}</td>
<td>1.3747×10^{-9}</td>
<td>5.6307×10^{-10}</td>
<td>1.1102×10^{-9}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test function</th>
<th>QPSO</th>
<th>GPQPSO</th>
<th>LIQPSO</th>
<th>MQPSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1) Min(best)</td>
<td>7.2547×10^{-10}</td>
<td>5.3490×10^{-10}</td>
<td>1.6208×10^{-10}</td>
<td>0</td>
</tr>
<tr>
<td>SD</td>
<td>3.7836×10^{-10}</td>
<td>1.9400×10^{-9}</td>
<td>5.1728×10^{-10}</td>
<td>0</td>
</tr>
<tr>
<td>Worst</td>
<td>1.4691×10^{-9}</td>
<td>7.5944×10^{-10}</td>
<td>2.7430×10^{-10}</td>
<td>7.4604×10^{-10}</td>
</tr>
</tbody>
</table>

**Table 4**

Performance comparison of different optimal algorithms on shifted functions for 30-dimension problems

<table>
<thead>
<tr>
<th>Test function</th>
<th>QPSO</th>
<th>GPQPSO</th>
<th>LIQPSO</th>
<th>MQPSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1) Min(best)</td>
<td>7.0628×10^{-2}</td>
<td>7.6773×10^{-1}</td>
<td>6.2016×10^{-1}</td>
<td>2.4248×10^{-1}</td>
</tr>
<tr>
<td>Mean</td>
<td>4.0758×10^{-1}</td>
<td>1.8311</td>
<td>1.3167</td>
<td>2.5674×10^{-1}</td>
</tr>
<tr>
<td>SD</td>
<td>1.2936×10^{-2}</td>
<td>8.0815×10^{-2}</td>
<td>6.2054×10^{-1}</td>
<td>7.1819×10^{-1}</td>
</tr>
<tr>
<td>Worst</td>
<td>5.0783×10^{-2}</td>
<td>3.6957</td>
<td>4.8930</td>
<td>2.6865×10^{-1}</td>
</tr>
<tr>
<td>(f_1) Min(best)</td>
<td>2.2204×10^{-10}</td>
<td>4.1652</td>
<td>6.4129</td>
<td>2.5121×10^{-9}</td>
</tr>
<tr>
<td>Mean</td>
<td>9.0494×10^{-10}</td>
<td>6.3451</td>
<td>1.0813</td>
<td>2.6092×10^{-10}</td>
</tr>
<tr>
<td>SD</td>
<td>2.8392×10^{-10}</td>
<td>1.2379</td>
<td>9.2064</td>
<td>6.0575×10^{-10}</td>
</tr>
<tr>
<td>Worst</td>
<td>1.0849×10^{-12}</td>
<td>8.5300</td>
<td>11.0267</td>
<td>2.7270×10^{-10}</td>
</tr>
<tr>
<td>(f_1) Min(best)</td>
<td>1.1327×10^{-10}</td>
<td>1.1336×10^{-10}</td>
<td>5.8627×10^{-10}</td>
<td>1.1339×10^{-10}</td>
</tr>
<tr>
<td>Mean</td>
<td>1.1364×10^{-10}</td>
<td>1.1439×10^{-10}</td>
<td>8.4139×10^{-10}</td>
<td>1.1341×10^{-10}</td>
</tr>
</tbody>
</table>
The functions $f_4$, $f_5$, $f_6$ and $f_7$ are multimodal. The function $f_4$ has many widespread local minima. The $f_5$ function is a complex multimodal function that has many local minima that are located far away from the global optimum point. It will be very difficult to find the global optima if some particles fall into one of the local minimum point. The function $f_6$ also has a large number of local optima. Function $f_7$ is a complex multimodal function described by almost flat outer region and a large hole at the center. It has also many widespread local optima.

The functions $f_5$, $f_6$ $f_{10}$ and $f_{11}$ are shifted version. These functions have taken from [17]. The details are given in table 1.

<table>
<thead>
<tr>
<th>Function</th>
<th>SD</th>
<th>Worst</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>2.9100×10⁻¹</td>
<td>1.1609×10⁻¹</td>
</tr>
<tr>
<td>$f_4$</td>
<td>6.0278×10⁻¹</td>
<td>9.4494×10⁻²</td>
</tr>
<tr>
<td>$f_5$</td>
<td>9.9374×10⁻¹</td>
<td>1.1357×10⁻¹</td>
</tr>
<tr>
<td>$f_6$</td>
<td>6.2016×10⁻¹</td>
<td>4.6167×10⁻¹</td>
</tr>
<tr>
<td>$f_7$</td>
<td>1.0526×10⁻²</td>
<td>6.0615×10⁻²</td>
</tr>
<tr>
<td>$f_8$</td>
<td>5.8577×10⁻³</td>
<td>8.3437×10⁻⁴</td>
</tr>
<tr>
<td>$f_9$</td>
<td>3.0582×10⁻²</td>
<td>9.9374×10⁻¹</td>
</tr>
<tr>
<td>$f_{10}$</td>
<td>8.9634×10⁻¹</td>
<td>1.4622×10⁻¹</td>
</tr>
<tr>
<td>$f_{11}$</td>
<td>1.4622×10⁻¹</td>
<td>1.4622×10⁻¹</td>
</tr>
</tbody>
</table>

B. PERFORMANCE COMPARISON OF THE PROPOSED METHOD

The proposed method is then compared with standard QPSO [14], (Gaussian Quantum Behaved Particle Swarm Optimization approaches for constrained engineering design problems) GQPSO [18] and (An Improved Quantum behaved Particle Swarm Optimization Algorithm based on Linear Interpolation) LIQPSO [19]. In this case study the population size is 40 with corresponding dimension of 30. The number of iterations is set to 2000. We have 30 trial runs for each instance and the minimum, worst, mean and standard deviations (SD) are recorded in tables 2 to 4. Moreover, Figures 2–9 demonstrate the convergence trajectories of different optimal algorithms (30 runs) in logarithmic scale of best objective function for the aforementioned standard benchmark problems.

C. RESULTS AND DISCUSSION

1). Unimodal Problems: From the results, it can be observed for unimodal problems, that the proposed MQPSO has better results on $f_1$ and $f_2$ than the original QPSO and LIQPSO. However, on $f_2$, MQPSO and GQPSO have achieved similar best results and shows better global searching capability than other optimal method. The LIQPSO has better outcomes than original QPSO and GQPSO on $f_1$ and $f_3$. Similarly, GQPSO has perform significantly better than other optimal methods on $f_2$.

2). Multimodal Problems: The proposed MQPSO obtained better performance on all the four multimodal functions as compared to other optimal algorithms. MQPSO surpasses all other methods on functions $f_4$, $f_5$, $f_6$ and $f_7$, and also significantly improves the performance on functions $f_8$, $f_9$ and $f_{10}$. Thus, the MQPSO achieved better results on most of the tested problems where other well-designed stochastic methods miss the global optimum point. The $f_3$ function is an example, where all other methods stuck into local optima and the MQPSO successfully avoids falling to trapped into local optima which is far away from the global optimum point.

3). Shifted Problems: On the four shifted problems MQPSO and the original QPSO performs better. The original QPSO significantly improved its performance especially on $f_5$ and $f_6$ as compared to other tested optimal methods. The proposed MQPSO beats all the tested algorithms on $f_{10}$ and $f_{11}$. Nevertheless, the GQPSO and LIQPSO completely fails and could not generate good results on the shifted problems and stuck into local optimum point.

4). Statistical analysis and Discussion: Comparing the results and convergence plots among these four QPSO algorithms. In this context, the proposed MQPSO found an appropriate mean behavior in approximately initial generations on most of the tested problems during the search process while all other optimal methods stuck into local minima. Thus, the convergence plots also demonstrate that the convergence speed of the proposed method is very fast and the proposed MQPSO has better global searching capability on many tested functions. LIQPSO converges faster than GQPSO and original QPSO. However, the original QPSO and GQPSO yield to a balanced performance between the local and global versions. Thus, among the four algorithms, MQPSO has perform significantly better on unimodal, multimodal and some shifted version. However, its performance affected by the shifting problems, it still performs the best on two shifted problems. LIQPSO also yields comparatively better than GQPSO and original QPSO on unimodal and multimodal problems, but original QPSO is significantly improved on the shifted version problems. Also, the GQPSO and LIQPSO failed on the shifted problems.
Fig 3. Convergence plots. Comparison of different optimal algorithms on $f_2$.

Fig 4. Convergence plots. Comparison of different optimal algorithms on $f_4$.

Fig 5. Convergence plots. Comparison of different optimal algorithms on $f_5$.

Fig 6. Convergence plots. Comparison of different optimal algorithms on $f_6$.

Fig 7. Convergence plots. Comparison of different optimal algorithms on $f_7$.

Fig 8. Convergence plots. Comparison of different optimal algorithms on $f_9$. 


V. NUMERICAL APPLICATION

To validate the high applicability and competency of the proposed MQPSO method for electromagnetic inverse problems. It is used to solve a well-known benchmark TEAM workshop Problem 22 as stated in [8,9,12,13,14].

The TEAM workshop problem 22 is a SMES (superconducting magnetic energy storage system) design optimization as shown in Fig 10. The system consists of two concentric coils carrying current in the opposite directions. The inner main solenoid and the outer shielding solenoid that is used to minimize the stray field. The optimal design of SMES is to achieve a desired stored energy with negligible stray field. Therefore, the design should fulfill:

1. The energy stored in the device should be 180 MJ,
2. The magnetic field produced inside the solenoids must not violate certain physical condition to ensure the superconductivity,
3. The mean stray field at 22 measurement points along line A and line B at distance of 10 m should be as small as possible.

To assure the superconductivity of the superconductors, the constraint equation between the current density of the two solenoids and magnetic flux density should fulfill:

\[ J_i \leq (6.4(B_{\text{max}})) + 54(A/\text{mm}^2) \quad (i = 1, 2) \]  

where \( J_i \) and \( B_{\text{max}} \) are the current density and maximal magnetic flux density in the \( i \)th coil.

In the three-parameter optimization problem of SMES design, the inner solenoid is fixed at \( r_1 = 2m, h_1 = 2/2 = 0.8m, d_1 = 0.27m \). The dimensions of the outer solenoid are optimized following the constraints as: \( 2.6m < r_2 < 3.4m \), \( 0.204m < h_2 < 1.1m \), \( 0.1m < d_2 < 0.4m \). Furthermore, the current densities for the two coils are set to be 22.5 A/mm\(^2\) in opposite directions. Also, for the convenience of numerical implementation, equation (12) can be simplified to \( |B_{\text{max}}| \leq 4.92T \).

The numerical results and statistical analysis demonstrate the superiority of the proposed MQPSO method on other well-designed stochastic approaches. The convergence trajectories also illustrate that the convergence speed of the proposed MQPSO is very fast and the proposed method converges quickly at the initial stage and is capable to jump out the trap and further explore the design space.

\[
\min f = \frac{E_{\text{ref}}}{E_{\text{norm}}} + \left| \text{Energy} - E_{\text{ref}} \right| \quad \text{subject to} \quad |B_{\text{max}}| \leq 4.92T
\]  

where \( E_{\text{ref}} = 180MJ \), \( B_{\text{norm}} = 3 \times 10^{-3}T \), Energy is the energy stored in SMES device, \( B_{\text{max}} \) is the maximum magnetic flux density, \( B_{\text{norm}}^{\text{stray}} \) is evaluated at 22 equidistance points along line A and line B as shown in Fig 10, defined as,

\[
B_{\text{norm}}^{\text{stray}} = \frac{\sum_{i=1}^{22} B_{\text{norm}}^{\text{stray}_i}}{22}
\]  

In the numerical implementation, the performance parameters as required in (13) and (14), are determined using 2-D finite element method.

For performance comparison, this case study is solved using the proposed MQPSO, original QPSO [14], GQPSO [18] and LIQPSO [19]. The optimal results of different stochastic approaches for 10 random runs are recorded in table 5. In this case study, the swarm size is 15. The stopping criteria for each algorithm is 2000 evaluations.

**Table 5. Performance comparison of different optimal methods on Team problem 22**

<table>
<thead>
<tr>
<th>Optimizer</th>
<th>( r_2 )</th>
<th>( h_2/2 )</th>
<th>( d_2 )</th>
<th>Cost Function</th>
<th>Function calls</th>
</tr>
</thead>
<tbody>
<tr>
<td>GQPSO</td>
<td>3.1723</td>
<td>0.2319</td>
<td>0.3892</td>
<td>0.1222</td>
<td>2000</td>
</tr>
<tr>
<td>QPSO</td>
<td>3.0786</td>
<td>0.2414</td>
<td>0.3795</td>
<td>0.1077</td>
<td>2000</td>
</tr>
<tr>
<td>LIQPSO</td>
<td>3.0214</td>
<td>0.2732</td>
<td>0.3419</td>
<td>0.0959</td>
<td>2000</td>
</tr>
<tr>
<td>MQPSO</td>
<td>3.1396</td>
<td>0.3160</td>
<td>0.2871</td>
<td>0.0716</td>
<td>2000</td>
</tr>
</tbody>
</table>

**Figure 10. SMES configuration.**
VI. CONCLUSION

In this work, a new approach of fitness selection methodology with dynamic control parameter is proposed to intensify the performance of QPSO algorithm. The new method has been validated by two case studies. The experimental outcomes on the case studies demonstrates the merit and high applicability of the proposed MQPSO method. Moreover, for future work it should be investigated to find other optimal methods for the study of electromagnetic inverse problems.

REFERENCES

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