Joint Fair Resource Allocation of D2D Communication Underlaying Downlink Cellular System with Imperfect CSI

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Abstract—Device-to-device (D2D) communication as a feasible technique can improve the system efficiency without fixed infrastructures. In this paper, we focus on the fair resource allocation of D2D communication underlaying cellular system with channel uncertainty and propose a joint resource allocation policy about the resource block (RB) assignment and power allocation among the D2D and cellular users (CUs). We formulate a nominal optimization problem to improve the sum-rate of the D2D system and guarantee the quality of service (QoS) of the CUs. Moreover, taking the uncertainty of the channel state information (CSI) into consideration, we employ the robust optimization theory to define the uncertainty and construct a robust optimization problem with uncertain parameters in both objective function and constraints. To facilitate the solution, we segment the robust optimization problem into two subproblems. Furthermore, we utilize the chance constraint approach to cut down the cost of the robustness and achieve a tradeoff between the robustness and the optimality. An insight into the influence of the protection function to the achievable D2D sum-rate is obtained by sensitivity analysis. Additionally, the effect of the robustness on the D2D sum-rate with different channel uncertainty is presented in the simulation and a rate tradeoff between the D2D users and CUs is achieved by the proposed policy.

Index Terms—Resource allocation, D2D communication, convex optimization, robustness, channel uncertainty.

I. INTRODUCTION

Recently, as a key technology to achieve the vision of the fifth generation (5G) wireless communications, device-to-device (D2D) communication has gathered great attentions from both academic and industrial communities [1], [2]. D2D communication realizes the direct transmission between the adjacent users by reutilizing the cellular system resources so that it inherently improves the system spectrum efficiency and facilitates the development of 5G wireless networks. Besides the high spectrum efficiency, D2D communication also possesses several characteristics, such as better user traffic load and experience, shorter latency, less power consumption, more flexible physical layer design and so on. D2D communication underlaying cellular system is commonly composed of a base station (BS), multiple cellular users (CUs) and neighboring D2D user pairs, where each CU can occupy a specified resource block (RB) that may be reused by other D2D user pair. D2D system and CUs may reuse the same resources in non-orthogonal mode, which will cause interference to each other. Hence, resource allocation is essential for the networks to operate normally and efficiently [3], [4].

The issue of resource allocation in the underlaying D2D system has already attracted lots of interests from the scholars and researchers. The proposed resource allocation approaches in the underlaying D2D system are mainly differentiated in (a) distributed approaches, targeting at maximizing each user’s utility, e.g., achievable data rate or energy efficiency, and (b) centralized approaches, targeting at maximizing the system’s welfare. The related work can be found in [5]–[8]. In [5] and [6], the distributed approach has been employed to improve the energy efficiency and mitigate the interference of the D2D systems. [7] presented a new distributed approach for the provisional spectrum access in heterogeneous D2D networks. On the other hand, a mixed integer nonlinear programming problem was adopted to maximize the D2D system sum-rate and a high-performance suboptimal solution was proposed in [8]. Furthermore, both distributed and centralized D2D link scheduling algorithms in a cellular-aided in-band overlay D2D network have been investigated [9]. In this paper, our work will concentrate on the centralized fair resource allocation approach.

Common research on the joint RB and power allocation in the underlaying D2D communication system with non-orthogonal mode mainly focuses on the case that one D2D user pair reuses one RB of the CU [10]–[12]. Currently, the resource reuse mechanism has been widely studied in D2D communication because of its flexibility and high efficiency [13]–[15]. In [13], the authors investigated the downlink resource utilization among multiple D2D pairs and CUs with the purpose of enhancing the network utility of the D2D communication and guaranteeing the quality of service (QoS) of the CUs simultaneously. The optimal power allocation...
scheme was investigated in [14] for two D2D user pairs reusing the same sub-channel and an efficient sub-channel allocation scheme with a simple greedy algorithm was proposed. Besides the research based on the convex optimization theory above, the authors in [15] also studied the distributed resource allocation in D2D communication underlay cellular system based on Game theory.

Most of the aforementioned literatures are based on the perfect system information, such as channel state information (CSI). However, it is hard to have the CSI practically. To model and utilize the imperfect CSI, the robust optimization theory is introduced, which maps the nominal optimization problem to its robust counterpart without considering the channel uncertainty and characterizes every uncertain parameter as the sum of the estimated value and the additive error. There are two prevalent methods to deal with the effect of the channel uncertainty, the worst-case approach [16], [17] where the channel locates in the uncertain region of the estimated channel and the Bayesian method [16], [18] where the channel has a random quantity and the average condition guarantees the constraints. The Bayesian approach has been extensively utilized currently. However, it is more suitable for the worst-case approach when the constraints are satisfied in the error cases.

For D2D communication underlay cellular system, CUs commonly hold higher priority and their QoS should be guaranteed first [19]. Meanwhile, flexible resource reuse mechanism will lead to complex interference environment. Therefore, considering the requirements of the rate QoS of the CUs and the interference limited resource reuse mechanism, we aim to propose a joint fair resource allocation policy simultaneously optimizing the RB and power allocation of the system users to improve the D2D links' sum-rate. Furthermore, considering the uncertainty of the CSI, we formulate the robust optimization problem and adopt the appropriate worst-case optimization that does not require any statistical information to model the channel uncertainty and characterize the instantaneous CSI with errors. Because of the non-convexity of common nominal and robust optimization problems, we transform the non-convex problems into convex subproblems to get a more tractable form. With the application of Karush-Kuhn-Tucker (KKT) conditions and Lagrangian dual method [16], the closed-form solutions of the subproblems are obtained. Moreover, the optimal solution related to the original problem can be found by one dimensional search algorithm [20], [21] and verified through simulations.

In this paper, we extend the research about the resource allocation of the D2D communication underlay cellular system with channel uncertainty and take the tradeoff between the robustness and the system performance into consideration. The main contributions are described in the following,

1) Consideration of the two uncertain parameter sets: the definition of the uncertainty regions of the CSI is considered in both the objective function and the constraints. Under this condition, we map the nominal resource allocation problem to its robust counterpart.

2) Reduced computational complexity: the proposed robust resource allocation problem considering the two sets of the uncertain parameters has high complexity. Hence, the robust optimization problem is further segmented into two subproblems to simplify the robust formulation significantly. First, we consider the uncertain parameter in the objective function as a bounded random variable and derive a closed formulation. Second, we solve the robust optimization problem with the uncertain interference CSI (i.e., the uncertain parameters in the constraints) according to the protection function by one dimensional search algorithm and Lagrangian dual method.

3) Tradeoff between the robustness and system sum-rate performance: due to the consideration of the additive error to the maximum extent, the worst-case method is conservative and may incur the performance loss, such as the decrease of the D2D system sum-rates. Therefore, we also study the cost of the robust resource allocation. By deploying the chance constraints approach, the D2D links' sum-rate will be increased with higher violation probability of the constraints. Through the adjustment of the upper bound of the violation probability, the tradeoff between the robustness and the system rate performance is acquired compared with the worst-case approach.

The rest of the paper is organized as follows. The system model and the transmission rate analysis are shown in Section II. Section III formulates the nominal resource allocation problem and maps it to its robust counterpart. The robust resource allocation is illustrated and solved with different conditions in Section IV. We study the cost of the robustness, present a tradeoff mechanism for the robustness, and analyze the sensitivity in Section V. Section VI provides the numerical results and performance analysis. Finally, conclusions are summarized in Section VII.

II. SYSTEM MODEL

In this paper, the scenario of the D2D communication underlaying downlink cellular system is considered. Specially, we take a hybrid single-cell network composed of $M$ D2D pairs and $N$ orthogonal downlink CUs into consideration, which is
shown in Fig. 1. It is assumed that each CU is assigned a specified RB and the information is conveyed from the BS to the nth CU with the nth RB, n ∈ A and A = \{1, 2, ..., N\}. At the same time, the D2D user pairs possessing no specified RBs will transmit the information signal through the reuse of the RBs of the CUs. BS is responsible for assigning a D2D link the cellular resources and adjusts the transmit power for both the CUs and the D2D user pairs. Every D2D user pair can reuse multiple RBs and each RB must be assigned to at most one D2D user pair. We denote the RB assignment indicator as α_{m,n} ∈ \{0, 1\}, m ∈ Ψ and Ψ = \{1, 2, ..., M\}. We have α_{m,n} = 1 if the RB of the nth CU is assigned to the nth D2D link. Otherwise, α_{m,n} = 0. Hence, we have Σ_{m=1}^M Σ_{n=1}^N α_{m,n} ≤ 1.

The interference case of the nth D2D link reusing the RB of the CU n is shown as the circular area in Fig. 1. We assume that all the channels follow Nakagami distribution as [22] in this paper. The channel complex gain of the nth RB link can be expressed as h_{n}(t) = G_{t} \exp(-\beta \phi_{t}), where G_{t} is the channel gain with Nakagami distribution and \phi_{t} is the phase-shift with uniform distribution within [0, 2\pi]. The probability density function (pdf) of \phi_{t} is related to the Gamma distribution Γ(\cdot) and can be given by f_{\phi_{t}}(x) = \frac{\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\alpha x}, x ≥ 0, where \alpha is the fading parameter with positive integer. Furthermore, we define h_{m,n} as the noise-normalized channel gain of the nth D2D link on the RB of the nth RU. Similarly, h_{n,m}, s_{n,m}, and g_{m,n} represent the normalized channel gains from the BS to the nth RU, from the BS to the nth D2D receiver on the RB of the nth RU, and from the nth D2D transmitter to the nth RU, respectively. Moreover, the transmit power of the nth D2D link on the associated RB of the nth RU is denoted as P_{m,n} ≥ 0 and the transmit power from the BS to the nth RU is denoted as P_{n} ≥ 0. So the data rate of the nth D2D link is written by

\[ r_{n}^{D} = \sum_{n=1}^{N} \alpha_{m,n} \log_{2} \left( 1 + \frac{P_{m,n}h_{n,m}}{\sigma_{D}^{2} + s_{n,m}P_{n}} \right) \]  

(1)

where \sigma_{D}^{2} and \sigma_{C}^{2} are the noise power of the D2D receivers and CUs, respectively. The bit rate from the BS to the CU n can be denoted as

\[ r_{n}^{C} = \log_{2} \left( 1 + \frac{P_{n}h_{n}}{\sigma_{C}^{2} + \sum_{m=1}^{M} \alpha_{m,n}g_{m,n}P_{m,n}} \right). \]  

(2)

Generally, we regard D2D communication as a complement to the conventional cellular system. Therefore, the CUs commonly keep higher priority and their rate QoS should be guaranteed first, i.e., \( r_{n}^{C} \geq R_{n}^{C} \), where \( R_{n}^{C} \) is the minimal information transmission rate from the BS to the nth RU. Considering the monotonicity of the logarithmic operation, we set \( \beta_{n} = 2^{R_{n}^{C}} - 1 \) and have

\[ \sum_{m=1}^{M} \alpha_{m,n}g_{m,n}P_{m,n} + \sigma_{C}^{2} \leq \frac{h_{n}}{\beta_{n}} P_{n}, n \in \Lambda. \]  

(3)

We will further study the optimal joint RB and power allocation policy to improve the rate performance of the D2D system and ensure the QoS of the CUs. With the establishment of a nominal resource allocation problem, we analyze its characteristics and solve its robust counterpart.

III. PROBLEM FORMULATION OF RESOURCE ALLOCATION

A. Problem Formulation of Nominal Resource Allocation

In this subsection, we first present the problem formulation of the nominal resource allocation of the D2D communication underlaying cellular system. The objective is to achieve the maximum overall bit rate of the D2D users and guarantee the CUs’ rate QoS. The optimization vectors are defined as \( \alpha = \{\alpha_{m,n}, m \in \Psi, n \in A\} \), \( P_{1} = \{P_{m,n}, m \in \Psi, n \in A\} \), and \( P_{2} = \{P_{n}, n \in A\} \). Firstly, we can formulate the nominal resource allocation problem as

\[ \begin{align*}
\text{(P1)} & \max_{\{\alpha, P_{1}, P_{2}\}} \sum_{m=1}^{M} \sum_{n=1}^{N} \alpha_{m,n} \log_{2} \left( 1 + \frac{P_{m,n}h_{n,m}}{\sigma_{D}^{2} + s_{n,m}P_{n}} \right) \\
\text{s.t.} & (3) \sum_{m=1}^{M} \alpha_{m,n} \leq 1, n \in \Lambda \quad (4b) \\
& \alpha_{m,n} \in \{0, 1\}, m \in \Psi, n \in \Lambda \quad (4c) \\
& P_{1} \succeq 0, P_{2} \succeq 0. \quad (4d)
\end{align*} \]

The constraint (3) ensures the CUs’ rate QoS, the constraint (4b) satisfies the requirement of the RB assignment, (4c) sets the indicator of the RB assignment to be 0 or 1, and (4d) makes the non-negative transmit power.

The optimization problem (P1) is a mixed-integer non-linear problem with non-convex property and is computationally intractable. In order to achieve a practical joint optimization, we relax the constraint that one RB can be only assigned to one D2D link with a time-sharing factor \( \alpha_{m,n} \) [23] which denotes the portion of the time that the nth D2D link takes the nth RB. Hence, we can temporarily relax \( \alpha_{m,n} \) as \( \alpha_{m,n} \in [0, 1] \) and substitute \( P_{1} \) with a new auxiliary vector \( \chi = \{\chi_{m,n} = P_{m,n}\alpha_{m,n}, m \in \Psi, n \in \Lambda\} \). Apparently, \( \chi_{m,n} \) is the actual power assigned to the nth D2D link on the nth RB and \( P_{m,n} \) is the power when the RB n is only occupied by the D2D link m. In addition, we set \( t \geq \sigma_{D}^{2} + s_{n,m}P_{n} > 0, m \in \Omega_{n}, n \in \Lambda \), where \( \Omega_{n} = \{m|\alpha_{m,n} = 1, m \in \Psi\} \) denotes the set of D2D user pairs reusing the RB of the nth RU. Hence, the problem (P1) can be relaxed to the maximization problem (P2) below

\[ \begin{align*}
\text{(P2)} & \max_{\{\chi, P_{2}\}} \sum_{m=1}^{M} \sum_{n=1}^{N} \alpha_{m,n} \log_{2} \left( 1 + \frac{\chi_{m,n}h_{n,m}}{t \cdot \alpha_{m,n}} \right) \\
\text{s.t.} & \sum_{m=1}^{M} \alpha_{m,n} \leq 1, n \in \Lambda \quad (5b) \\
& \sum_{m=1}^{M} \chi_{m,n}g_{m,n} + \sigma_{C}^{2} \leq \frac{h_{n}}{\beta_{n}} P_{n}, n \in \Lambda \quad (5c) \\
& \sigma_{D}^{2} + s_{n,m}P_{n} \leq t, m \in \Omega_{n}, n \in \Lambda \quad (5d) \\
& \alpha_{m,n} \in [0, 1], m \in \Psi, n \in \Lambda \quad (5e) \\
& \chi \succeq 0, P_{2} \succeq 0. \quad (5f)
\end{align*} \]
With large enough number of the RB, the duality gap of the optimization problem that satisfies the time sharing condition can be ignored [23]. Then, the solution of the problem (P2) tends to be asymptotically optimal. If $t$ is fixed, we can set $r_m = \alpha_{m,n} \log_2 \left(1 + \frac{\chi_{m,n}h_{m,n}}{t_{\alpha_{m,n}}} \right)$ and find that $r_m$ is concave in $P_{m,n}$. Hence, (5a) is concave in $P_{m,n}$ following the composition rule in [16]. Since $r_m$ is a function of variable $\alpha_{m,n}$, we can temporarily relax the value of $\alpha_{m,n}$ to be within the interval $[0, 1]$ and replace $P_{m,n}$ with $\chi_{m,n} = \alpha_{m,n} P_{m,n}$. The constraints in (P2) are also convex in $\{\alpha_{m,n}, \chi_{m,n}\}$ and $r_m(\chi_{m,n})$ is concave considering that $r_m(\alpha_{m,n}, \chi_{m,n})$ is the perspective function of $r_m(\chi_{m,n})$ [16]. The objective function is concave and its constraints are affine. So the optimization problem (P2) becomes convex and we can get a unique optimal solution. The convex optimization problem with fixed $t$ can be processed by Lagrangian dual decomposition method and the solution of (P2) which is guaranteed to be optimal for its analytical properties can be obtained through one dimension search algorithm.

B. Problem Formulation of Robust Resource Allocation

To construct the problem illustrated in Section III-A, we need to know the perfect CSI at the BS, i.e., the exact values of $h_{m,n}, \chi_{m,n},$ and $s_{n,m},$ $m \in \Psi, n \in \Lambda$. Nevertheless, all the parameters are subject to channel uncertainty in practice. In this subsection, we employ the worst-case robust optimization method and consider the channel uncertainty to investigate the robust resource allocation problem [24], [25]. With the worst-case robust optimization method, we denote every uncertain parameter as the sum of the estimate and an additive error which falls within an uncertainty region. Then, the three parameters are represented as

$$h_{m,n} = \hat{h}_{m,n} + h_{m,n}, h_{m,n} \in \mathcal{R}_{h_{m,n}}$$

$$g_{m,n} = \hat{g}_{m,n} + g_{m,n} \in \mathcal{R}_{g_{m,n}}$$

$$s_{n,m} = \hat{s}_{n,m} + s_{n,m}, s_{n,m} \in \mathcal{R}_{s_{n,m}}$$

respectively, where $\hat{g}_{m,n} = [g_{1,m,n} g_{2,m,n} \cdots g_{M,m,n}], m \in \Psi, n \in \Lambda,$ denotes the vector of the interference channel gains between the D2D transmitters and the CUs on the nth RB. $\mathcal{R}_{g_{m,n}}, \mathcal{R}_{s_{n,m}},$ and $\mathcal{R}_{h_{m,n}}$ are the corresponding uncertainty sets of the uncertain parameters $g_{m,n}, s_{n,m},$ and $h_{m,n}$, respectively. Moreover, $(\cdot)$ and $(\cdot)$ respectively denote the estimated value and the bounded error of $(\cdot)$.

To achieve the robust solution, the uncertainty sets can be regarded as the distances or differences between the real values and the estimates and calculated by the general norm operation [26]. In this case, the uncertainty sets are given by

$$\mathcal{R}_{h_{m,n}} = \{h_{m,n} \parallel U_{h_{m,n}}(h_{m,n} - \hat{h}_{m,n}) \parallel \leq \psi_1 \}$$

$$\mathcal{R}_{g_{m,n}} = \{g_{m,n} \parallel W_{g_{m,n}}(g_{m,n} - \hat{g}_{m,n}) \parallel \leq \psi_2 \}$$

$$\mathcal{R}_{s_{n,m}} = \{s_{n,m} \parallel V_{s_{n,m}}(s_{n,m} - \hat{s}_{n,m}) \parallel \leq \psi_3 \}$$

respectively, where $\parallel \cdot \parallel$ denotes the general norm, $\psi_1, \psi_2, \psi_3$ respectively represent the upper bounds of the uncertainty regions, and $W_{g_{m,n}}$ is the invertible $M \times M$ weight matrix of $g_{m,n},$ $U_{h_{m,n}}$ and $V_{s_{n,m}}$ denote the weight variables of $\hat{h}_{m,n}$ and $\hat{s}_{n,m},$ respectively. Based on the condition that the channel uncertainty and channel gain within $g_{m,n}$ are i.i.d random variables [27], $W_{g_{m,n}}$ becomes a diagonal matrix.

According to the worst-case robust optimization method [24], we formulate the robust optimization problem as

$$(P3) \quad \max_{\{\alpha_n, \mathbf{P}_2\}} \sum_{m=1}^{M} \sum_{n=N}^{N} \min_{\alpha_{m,n}} \alpha_{m,n} \log_2 \left(1 + \frac{\chi_{m,n}h_{m,n}}{t_{\alpha_{m,n}}} \right)$$

s. t.  \hspace{0.5cm} (5b) - (5f)

$$(12a)$$

$$g_{m,n} \in \mathcal{R}_{g_{m,n}}, n \in \Lambda \quad (12b)$$

$$s_{n,m} \in \mathcal{R}_{s_{n,m}}, m \in \Psi, n \in \Lambda \quad (12c)$$

The problem (P3) is a robust counterpart of (P2) with perfect CSI, i.e., the estimated values can be viewed to be the practical values and the additive errors are neglected. From the problem (P3), two sets of the uncertain parameters exist in the robust optimization problem, such as the objective function $(h_{m,n})$ and the constraints $(g_{m,n}, s_{n,m})$. Hence, there exists much computational complexity to deal with the robust problem (P3). We can utilize the protection function instead of the uncertainty set to present the closed-form constraints. In order to further reduce the complexity, we divide the problem (P3) into two subproblems. Firstly, we consider $h_{m,n}$ as a bounded random variable and derive a closed formulation for $h_{m,n}$. Second, we solve the robust optimization problem with uncertain interference CSI based on the notion of protection function with given $h_{m,n}$.

IV. ROBUST RESOURCE ALLOCATION

A. Robust Optimization Problem with Given $h_{m,n}$

Aiming to solve the maximin robust optimization problem in (P3), we need to solve the inner minimization

$$\min_{\alpha_{m,n}} \alpha_{m,n} \log_2 \left(1 + \frac{\chi_{m,n}h_{m,n}}{t_{\alpha_{m,n}}} \right)$$

first. Following the same argument as in [28], $h_{m,n}, m \in \Psi, n \in \Lambda$ is assumed as i.i.d. random variable with the pdf denoted as $f(h_{m,n})$. In this situation, we transform the uncertainty set into the form of $h_{m,n} \in \left[\frac{\psi_1}{\chi_{m,n}}, \frac{\psi_2}{\chi_{m,n}}\right]$. Now, the inner minimization is converted into the following optimization problem

$$\min_{u_{m,n}} \alpha_{m,n} u_{m,n}$$

s. t. \hspace{0.5cm} $\Pr \left\{ \log_2 \left(1 + \frac{\chi_{m,n}h_{m,n}}{t_{\alpha_{m,n}}} \right) \geq u_{m,n} \right\} \geq \eta_{m,n},$ $m \in \Psi, n \in \Lambda$ \hspace{0.5cm} (13a)

$$h_{m,n} \in \left[\frac{-\psi_1}{U_{h_{m,n}}}, \frac{\psi_2}{U_{h_{m,n}}}\right], m \in \Psi, n \in \Lambda$$ \hspace{0.5cm} (13b)

$$\eta_{m,n} \in [0, 1], m \in \Psi, n \in \Lambda$$ \hspace{0.5cm} (13c)

where $u_{m,n}$ is an auxiliary variable denoting the upper bound of achievable bit rate for any D2D user pair $m$ on the nth RB with probability $\eta_{m,n} \in [0, 1]$.

Moreover, (13b) is rewritten as

$$\Pr \left\{ h_{m,n} \leq \frac{(2m_{m,n} - 1) \cdot t_{\alpha_{m,n}}}{\chi_{m,n}} \right\} \geq \eta_{m,n}.$$ \hspace{0.5cm} (14)
We can further use $F_{h,m,n}$ to represent the distribution function of $h_{m,n}$ and get
\[
\left(\frac{2\psi_{m,n}}{\bar{\chi}_{m,n}} - 1\right) \cdot \frac{t \cdot \alpha_{m,n}}{\chi_{m,n}} \geq F_{h,m}^{-1}(\eta_{m,n}),
\]
where $F_{h,m,n}^{-1}(\cdot)$ is an inverse function of $F_{h,m,n}(\cdot)$ and we have
\[
u_{m,n} \geq \log_{2} \left(1 + F_{h,m,n}^{-1}(\eta_{m,n}) \frac{2}{\alpha_{m,n}}\right).
\]
Considering that $f_{h}(h_{m,n})$ is a uniform distribution over the interval $\left[-\psi_{m,n}/\bar{\chi}_{m,n}, \psi_{m,n}/\bar{\chi}_{m,n}\right]$ as in [29], [30], we can write
\[
F_{h,m,n}^{-1}(x) = 2 \cdot \frac{\psi_{m,n}}{U_{h,m,n}} - \frac{1}{U_{h,m,n}}.
\]
Accordingly, the solution of the inner minimization in regard to $h_{m,n}$ is
\[
h_{m,n} = 2 \cdot \frac{\psi_{m,n}}{U_{h,m,n}} \eta_{m,n} + \bar{h}_{m,n} - \frac{1}{U_{h,m,n}}, m \in \Psi, n \in \Lambda.
\]
Based on the concept of the protection function [25], [26], the protection function against the channel uncertainty on $h_{m,n}$ can be represented as
\[
\Delta h_{m,n} = \left(2 \cdot \frac{\psi_{m,n}}{U_{h,m,n}} \eta_{m,n} - \frac{1}{U_{h,m,n}}\right) \cdot \frac{t \cdot \alpha_{m,n}}{\chi_{m,n}}.
\]
It is a function of $\psi_{m,n}$ and $\eta_{m,n}$. $\psi_{m,n}$ and $\eta_{m,n}$ affect the sum-rate of the D2D system, which means that a tradeoff between the performance and the robustness can be achieved. Furthermore, substituting $h_{m,n}$ in (17) into the solution of the Problem (P1), we can get the solution of the robust optimization problem.

With $h_{m,n}$ obtained by (17), the robust optimization problem is reconstructed as
\[
\begin{align}
(P4) \quad \max_{\{\alpha, \chi, P\}} & \sum_{m=1}^{M} \sum_{n=1}^{N} \alpha_{m,n} \log_{2} \left(1 + \frac{\alpha_{m,n} h_{m,n}}{t \cdot \alpha_{m,n}}\right) \\
\text{s. t.} & \quad (5b) - (5f) \\
& \quad (12b), (12c)
\end{align}
\]
Its solution is robust against uncertainties if and only if the optimal solution satisfies the constraints in (5c) and (5d) for any realization of $g_{s} \in \mathcal{R}_{g_{s}}$ and $s_{m,n} \in \mathcal{R}_{s_{m,n}}, m \in \Psi, n \in \Lambda$. The uncertainty constraints in (5c) and (5d) are satisfied as in [31] if and only if
\[
\begin{align}
\max_{g_{s} \in \mathcal{R}_{g_{s}}} & \sum_{m=1}^{M} \chi_{m,n}(g_{m,n} - g_{m,n}) + \sigma_{D}^{2} \leq \frac{h_{n}}{\beta_{n}} P_{n}, n \in \Lambda & \quad (20a) \\
\max_{s_{m,n} \in \mathcal{R}_{s_{m,n}}} & \sigma_{D}^{2} + s_{m,n} P_{n} \leq t, m \in \Omega_{n}, n \in \Lambda & \quad (20b)
\end{align}
\]
Equation (20) can be transformed to
\[
\sum_{m=1}^{M} \chi_{m,n}(g_{m,n} - g_{m,n}) + \Delta g_{n} \leq \frac{h_{n}}{\beta_{n}} P_{n} - \sigma_{D}^{2}, n \in \Lambda
\]
\[
P_{n} s_{m,n} + \Delta s_{m,n} \leq t - \sigma_{D}^{2}, m \in \Omega_{n}, n \in \Lambda
\]
where
\[
\Delta g_{n} = \max_{g_{s} \in \mathcal{R}_{g_{s}}} \sum_{m=1}^{M} \chi_{m,n}(g_{m,n} - g_{m,n})
\]
\[
\Delta s_{m,n} = \max_{s_{m,n} \in \mathcal{R}_{s_{m,n}}} P_{n} s_{m,n} - \bar{s}_{m,n}
\]
are the protection functions of (5c) and (5d), respectively, which will be impacted by the shape and size of the uncertainty sets.

Let the general norms as (10) and (11) represent the protection functions of the uncertainty sets and substitute them into (P4), the optimization problem is further written as
\[
\begin{align}
(P5) \quad \max_{\{\alpha, \chi, P\}} & \sum_{m=1}^{M} \sum_{n=1}^{N} \alpha_{m,n} \log_{2} \left(1 + \frac{\alpha_{m,n} h_{m,n}}{t \cdot \alpha_{m,n}}\right) \\
\text{s. t.} & \quad (5b), (5e), (5f) \\
& \quad \sum_{m=1}^{M} \chi_{m,n}(g_{m,n} - g_{m,n}) + \psi_{2} \|W_{g_{n}}^{-1} \chi_{n}^{T}\| \leq \frac{h_{n}}{\beta_{n}} P_{n} - \sigma_{D}^{2}, \\
& \quad \psi_{2} \max_{v_{n}, \|v_{n}\| \leq 1} \chi_{n} W_{g_{n}}^{-1} v_{n} = \psi_{2} \|W_{g_{n}}^{-1} \chi_{n}^{T}\|.
\end{align}
\]
Besides, the protection function of the uncertainty set in (11) is obtained similarly. Because the dual norm is convex, the problem (P4) maintains the convexity.

The linear norm simplifying the robust optimization problem can be defined as $\|y\|_{\alpha} = \left(\sum_{i} |y_{i}|^{\alpha}\right)^{\frac{1}{\alpha}}$, where $\alpha (\alpha \geq 1)$ is the order and $|\cdot|$ means the absolute operation. Since the dual norm of a linear norm with order $\alpha$ is still a linear norm with order $\beta = 1 + \frac{1}{\alpha-1}$, the dual norm in the protection function (24) can be formulated as
\[
\|W_{g_{n}}^{-1} \chi_{n}^{T}\|^{*} = \left(\sum_{m=1}^{M} \left(W_{g_{n}}^{-1} (m,:) \cdot \chi_{n}^{T}\right)^{\beta}\right)^{\frac{1}{\beta}}
\]
the robust resource allocation with one-dimensional search of $t$.

$$\text{(P6)} \quad \max_{\{\alpha, \chi, P_2\}} \sum_{m=1}^{M} \sum_{n=1}^{N} \alpha_{m,n} \log_2 \left(1 + \frac{\chi_{m,n} h_{m,n}}{t \cdot \alpha_{m,n}} \right)$$

s. t. $$(5b), (5e), (5f)$$

$$\sum_{m=1}^{M} \chi_{m,n} \bar{g}_{m,n} + \psi_2 \sum_{m=1}^{M} w_{m,m,n} \chi_{m,n} \leq h_n P_n - \sigma_2^2, \quad n \in \Lambda \quad (26b)$$

$$P_n s_{n,m} + \psi_3 v_{s,n,m} P_n \leq t - \sigma_2^2 D, \quad m \in \Omega_n, \quad n \in \Lambda. \quad (26c)$$

B. Joint Optimization of Power Allocation and RB Assignment with Fixed $t$

We employ Lagrangian dual decomposition method to investigate the nature of the power allocation and define the Lagrangian function as

$$\mathcal{L}(\Upsilon, \Theta) = \sum_{m=1}^{M} \sum_{n=1}^{N} \alpha_{m,n} \log_2 \left(1 + \frac{\chi_{m,n} h_{m,n}}{t \cdot \alpha_{m,n}} \right) - \sum_{n=1}^{N} \beta_n \sum_{m=1}^{M} \chi_{m,n} \bar{g}_{m,n} + \psi_2 \sum_{m=1}^{M} w_{m,m,n} \chi_{m,n} - \frac{h_n P_n + \sigma_2^2 C}{\beta_n}$$

$$- \sum_{n=1}^{N} \sum_{m=1}^{M} \gamma_{m,n} \left(P_n s_{n,m} + \psi_3 v_{s,n,m} P_n - t + \sigma_2^2 D \right), \quad n \in \Omega_n.$$

(27)

We denote the optimization vector as $\Upsilon = \{\alpha, \chi, P_2\}$ and the vector of the dual variables as $\Theta = \{\phi_n, \rho_n, \gamma_{m,n}, \alpha_{m,n}, n \in \Lambda, m \in \Omega_n\}$, where $\phi_n, \rho_n$, and $\gamma_{m,n}$ are the Lagrange multipliers for the constraints $(5b), (26b)$, and $(26c)$, respectively. The boundary constraints $(5e)$ and $(5f)$ can be absorbed in the KKT conditions as shown in $(28)$, where $\alpha_{m,n}, \chi_{m,n},$ and $P_n^*$ are the optimal solutions of the robust optimal problem with fixed $t$ and $\phi_n, \rho_n,$ and $\gamma_{m,n}$ are the optimal values of the dual variables.

$$\frac{\partial L(\Upsilon, \Theta)}{\partial \chi_{m,n}} = 0, \quad m \in \Psi, \quad n \in \Lambda$$

$$\frac{\partial L(\Upsilon, \Theta)}{\partial P_n^*} = 0, \quad \alpha_{m,n}^* > 0, \quad n \in \Lambda$$

$$\frac{\partial L(\Upsilon, \Theta)}{\partial \alpha_{m,n}} = 0$$

$$n \in \Psi, \quad m \in \psi, \quad n \in \Lambda\quad (28a)$$

$$n \in \Lambda\quad (28b)$$

$$n \in \Lambda\quad (28c)$$

With the assumption that the D2D link $m$ reutilizes the RB of the CU $n$, the optimal power allocation of the D2D link $m$ on the RB of the $n$th CU with KKT conditions is given by

$$P_n^* = \left\{ F_{m,n} - \frac{\sigma_2^2 C}{(g_{m,n} + \psi_2 w_{m,m,n})} \right\}^+$$

where $F_{m,n} = \frac{\beta_n (s_{n,m} + \psi_3 v_{s,n,m})(g_{m,n} + \psi_2 w_{m,m,n})}{\epsilon_n h_n}$ and $\{\epsilon\}^+ = \max\{0, \epsilon\}$. The optimal power allocation from the BS to the CU $n$ is obtained by

$$P_n = \left\{ \frac{t - \sigma_2^2 D}{s_{n,m} + \psi_3 v_{s,n,m}} \right\}^+$

(30)

For a given power allocation scheme, the RB assignment is determined as [33]

$$\alpha_{m,n}^* = \begin{cases} 1, & m = \arg\max_m H_{m,n} \\ 0, & \text{otherwise} \end{cases}\quad (31)$$

with $H_{m,n} = \log_2 \left(1 + \frac{P_{m,n}^* h_{m,n}}{t}\right) - \frac{P_{m,n}^* h_{m,n}}{t \cdot m(n^m + n^m_m + t)}$. We can assign the RB of the CU $n$ to the D2D link having the largest $H_{m,n}$.

After obtaining the optimal solution of the robust resource allocation problem with fixed $t$, i.e., $P_{m,n}^*, \alpha_{m,n}^*$, and $\alpha_{m,n}^*$, we can utilize one dimensional search method to get the solution of $(19)$ which is guaranteed to be optimal by its analytical properties.

C. Solution of Joint Optimization

If the RB assignment is given, $\Omega_n, n \in \Lambda$ becomes certain. Then, we can rewrite the objective function in $(19)$ and substitute $(29)$ into it. Hence, the D2D links’ sum-rate can be calculated as

$$R(t) = \sum_{n=1}^{N} \sum_{m \in \Omega_n} \log_2 \left( A_{m,n} - \frac{B_{m,n}}{t} \right)$$

(32)

where $A_{m,n} = 1 + \frac{\beta_n (s_{n,m} + \psi_3 v_{s,n,m})(g_{m,n} + \psi_2 w_{m,m,n})}{\psi_2}$ and $B_{m,n} = \frac{\beta_n (s_{n,m} + \psi_3 v_{s,n,m})(g_{m,n} + \psi_2 w_{m,m,n})}{\psi_2} + \sigma_2^2 n$, $m \in \Omega_n, n \in \Lambda$. We observe that the sum-rate of the D2D links is a function of $t$. If $t > 0$, $R(t)$ is a quasi-concave function which increases monotonically and tends to be stable with the increase of $t$. Hence, with one dimensional search method, we can set the search starting-point $t$ and increase it successively to get the maximum sum-rate at the corresponding optimal $t^*$. We summarize the joint RB and power allocation algorithm in Algorithm 1.

V. ANALYSIS OF THE ALGORITHM

A. Cost of Robust Algorithm with Uncertain Interference CSI

For the robust resource allocation, an important issue is the consequent decrease in the D2D links’ sum-rate, given by $d_\Delta = \| R^* - R^*_\Delta \|_2$, where $R^*$ and $R^*_\Delta$ are the optimal achievable D2D links’ sum-rates acquired by solving the nominal and the robust resource allocation problem with uncertain interference CSI, respectively.
Algorithm 1: Joint RB assignment and power allocation algorithm

1: Initialization: Set the search starting-point \( t_1 > 0 \), the search step size \( \tau \), and the allowable error \( \epsilon \).
2: Repeat:
3: Set \( t_2 = t_1 + \tau \).
4: Calculate \( P^*_{m,n} \), \( P^\ast_{m,n} \), and \( \alpha^*_{m,n} \) by (29), (30), and (31) with \( t = t_1 \) and \( t = t_2 \), respectively.
5: Calculate the maximum sum-rates, \( R(t_1) \) and \( R(t_2) \), of the D2D users with \( t = t_1 \) and \( t = t_2 \), respectively.
6: Set \( t_1 = t_2 \).
7: Until: the accuracy requirement \( R(t_2) - R(t_1) \leq \epsilon \) is met.
8: Output: the optimal power \( P^*_{m,n} \) and \( P^\ast_{m,n} \), the RB assignment indicator \( \alpha^*_{m,n} \) with \( t = t_2 \), and the maximum sum-rate \( R(t_2) \) of the D2D links.

Note that \( \rho^*_n \) and \( \gamma^*_n,m \) are the optimal values of the Lagrange multipliers for the constraints (5c) and (5d), respectively. For all the values of the protection functions \( \Delta g_n \) and \( \Delta s_{n,m} \), the decrease in the achievable sum-rate with robust resource allocation can be approximately expressed as

\[
d_\Delta \approx \sum_{n=1}^{N} \rho^*_n \Delta g_n + \sum_{n=1}^{N} \sum_{m \in \Omega_n} \gamma^*_{n,m} \Delta s_{n,m}.
\]  

(33)

It shows that \( d_\Delta \) can be controlled by adjusting the size of \( \Delta g_n \) and \( \Delta s_{n,m} \). So \( d_\Delta \) relies on the size of the uncertainty set.

Proof: Note that (P5) is the perturbed version of (P2) with protection functions in (5c) and (5d). By perturbing the constraints of (P5), we employ the local sensitivity analysis to establish (33) (Section 5.6 in [16], Chapter IV in [34]). Let the elements of the disturbance vectors \( a, b \) contain \( \Delta g_n \) and \( \Delta s_{n,m} \) and the cost disturbance function \( R^*(a, b) \) is expressed as

\[
R^*(a, b) = \inf \left\{ \max_{\alpha, P^m} \sum_{m=1}^{M} \sum_{n=1}^{N} \alpha_{m,n} \log_2 \left( 1 + \frac{\chi_{m,n} P^m_n}{1 + \alpha_{m,n}} \right) \right\}
\]

\[
\sum_{m=1}^{M} \chi_{m,n} g_{m,n} + \Delta g_n \leq \frac{h_n}{\beta_n} P^m_n - \sigma_C^2, P^m_n + \Delta s_{n,m} \leq t - \sigma_d^2 \right\}.
\]  

(34)

We can differentiate \( R^*(a, b) \) in regard to the perturbation vectors \( a \) and \( b \) with small \( \Delta g_n \) and \( \Delta s_{n,m} \) (Chapter IV in [34]). With Taylor series, (34) is further written as

\[
R^*(a, b) = R^*(0, 0) + \sum_{n=1}^{N} a_n \frac{\partial R^*(0, b)}{\partial a_n} + \sum_{n=1}^{N} \sum_{m \in \Omega_n} b_{n,m} \frac{\partial R^*(a, 0)}{\partial b_{n,m}} + o(\epsilon)
\]  

(35)

where \( R^*(0, 0) \) denotes the optimal value of (P2) and \( o(\epsilon) \) is the truncation error of the Taylor series expansion. \( R^*(a, b) \) and \( R^*(0, 0) \) are actually equal to \( R^*_a \) and \( R^* \), respectively. Because of the convexity of \( (P5) \), \( R^*(a, b) \) can be acquired from the Lagrange dual function of \( (P5) \). Using the sensitivity analysis [34], we get

\[
\frac{\partial R^*(0, b)}{\partial a_n} = -\rho^*_n \quad \text{and} \quad \frac{\partial R^*(a, 0)}{\partial b_{n,m}} = -\gamma^*_{n,m}.
\]

From (35), we obtain

\[
R_A^* - R^* \approx - \sum_{n=1}^{N} \rho^*_n \Delta g_n - \sum_{n=1}^{N} \sum_{m \in \Omega_n} \gamma^*_{n,m} \Delta s_{n,m}.
\]  

(36)

Since the Lagrange multipliers \( \rho^*_n \) and \( \gamma^*_{n,m} \) are both non-negative, the achievable D2D links’ sum-rate will be cut down compared with the case where the perfect CSI is available.

B. Tradeoff Between Robustness and Achievable Sum-rate

The robust worst-case resource allocation tackling the channel uncertainties is conservative and may lead to inefficient resource utilization, which makes it unfeasible in many cases. Therefore, we desire to make a tradeoff between the robustness and the overall sum-rate of the D2D users. Through the modification of the worst-case approach, we can choose the uncertainty set in such manner that the probability of violating the constraints (5c) and (5d) should be held below the predesigned values and the D2D sum-rate maintains close to the optimal nominal case. Hence, the constraints (5c) and (5d) in (P2) can be modified as

\[
P \left( \sum_{m=1}^{M} \chi_{m,n} g_{m,n} + \sigma_C^2 \leq \frac{h_n}{\beta_n} P^m_n \right) \leq \Theta_n, n \in \Lambda
\]  

(37a)

\[
P \left( \sigma_d^2 + s_{n,m} P_n^m \geq t \right) \leq \Theta_{n,m}, m \in \Omega_n, n \in \Lambda
\]  

(37b)

respectively, where \( \Theta_n \) and \( \Theta_{n,m} \) are given probabilities of the violation of the constraints (5c) and (5d), respectively. By reducing \( \Theta_n \) and \( \Theta_{n,m} \), the system turns to be more robust against channel uncertainty. While if we increase them, the D2D sum-rate is improved. Hence, with the change of \( \Theta_n \) and \( \Theta_{n,m} \), the tradeoff between the robustness and the optimality is achieved. Here, we use the chance constrained approach [25] to deal with this tradeoff. Furthermore, if the constraints are affine functions, it is shown that we can replace (5c) and (5d) by convex functions with less calculations as their safe approximations [25]. Applying this approach, we obtain

\[
\sum_{m=1}^{M} \chi_{m,n} g_{m,n} = \sum_{m=1}^{M} \chi_{m,n} g_{m,n} + \sum_{m=1}^{M} \zeta_{m,n,1} \chi_{m,n} g_{m,n}, n \in \Lambda
\]  

(38a)

\[
P_n s_{n,m} = P_n s_{n,m} + \zeta_{m,n,2} s_{n,m}, m \in \Omega_n, n \in \Lambda
\]  

(38b)

respectively, where \( \zeta_{m,n,1} = \frac{g_{m,n} - g_{m,n}}{g_{m,n}} \) and \( \zeta_{m,n,2} = \frac{s_{n,m} - s_{n,m}}{s_{n,m}} \) vary within the range \([-1, +1]\). Under the assumption of uncorrelated fading channels, \( \zeta_{m,n,1} \) and \( \zeta_{m,n,2} \) are independent with each other and follow the specific probability distribution \( Q_{m,n,1} \) and \( Q_{m,n,2} \), respectively. Hence, the constraints (5c) and (5d) could be substituted with Bernstein.
TABLE I: Values of $\eta_{Q_j}^+$ and $\tau_{Q_j}$ for Typical Probability Distribution $Q_j$

<table>
<thead>
<tr>
<th>$Q_j$</th>
<th>$\eta_{Q_j}^+$</th>
<th>$\tau_{Q_j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sup{Q_j} \in [-1, 1]$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\sup{Q_j}$ is unimodal and $\sup{Q_j} \in [-1, 1]$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{\sqrt{2}}$</td>
</tr>
<tr>
<td>$\sup{Q_j}$ is unimodal and symmetric</td>
<td>0</td>
<td>$\frac{1}{\sqrt{2}}$</td>
</tr>
</tbody>
</table>

approximations of the chance constraints [25] as

\[
\sum_{m=1}^{M} \chi_{m,n} \hat{g}_{m,n} + \Delta g_n = \hat{h}_n P_n - \sigma^2_n C, n \in \Lambda \tag{39a}
\]

\[
P_n \hat{s}_{n,m} + \Delta s_{n,m} \leq t - \sigma^2 D, m \in \Omega_n, n \in \Lambda \tag{39b}
\]

respectively, where the protection functions $\Delta g_n$ and $\Delta s_{n,m}$ are given respectively by

\[
\Delta g_n = \sum_{m=1}^{M} \eta_{Q_m,n,1}^+ \chi_{m,n} \hat{g}_{m,n} \\
+ \sqrt{2 \ln \frac{1}{\theta_n}} \left( \sum_{m=1}^{M} \tau_{Q_m,n,1}^2 (\chi_{m,n} \hat{g}_{m,n})^2 \right)^{\frac{1}{2}}, n \in \Lambda \tag{40a}
\]

\[
\Delta s_{n,m} = \eta_{Q_m,n,2}^+ P_n \hat{s}_{n,m} \\
+ \sqrt{2 \ln \frac{1}{\theta_n}} \left( \tau_{Q_m,n,2}^2 (P_n \hat{s}_{n,m})^2 \right)^{\frac{1}{2}}, m \in \Omega_n, n \in \Lambda \tag{40b}
\]

It is seen from (40a) and (40b) that the protection functions relation to $\Theta_n$ and $\Theta_{n,m}$. The variables $\eta_{Q_j}^+$ and $\tau_{Q_j}$ with $0 \leq \eta_{Q_j}^+ \leq 1$ and $\tau_{Q_j} \geq 0$ are utilized for the safe approximation of the chance constraints and rely on $Q_j$. Given a specific probability distribution $Q_j$, the corresponding values of $\eta_{Q_j}^+$ and $\tau_{Q_j}$ are given in Table I [25].

Note that the constraints in (39a) and (39b) transform the resource allocation problem to a low complexity conic quadratic programming problem [35]. With $\| y \|_2 \leq \| y \|$, we can obtain the optimal resource allocation about the RB and power in a distributed way which is similar to that in Algorithm 1.

The optimal power allocation for the D2D link $m$ on the RB of the CU $n$ is obtained with the KKT conditions as

\[
P_{m,n}^* = \left\{ \frac{h_n (t - \sigma^2 D)}{\beta_n C_{m,n} D_{n,m} - \sigma^2 C} \right\}^+ \tag{41}
\]

where

\[
C_{m,n} = \tilde{g}_{m,n} + \eta_{Q_{m,n,1}}^+ \hat{g}_{m,n} + \sqrt{2 \ln \frac{1}{\theta_n}} \tau_{Q_{m,n,1}} \hat{g}_{m,n} \tag{42}
\]

\[
D_{n,m} = \tilde{g}_{m,n} + \eta_{Q_{m,n,2}}^+ \hat{g}_{m,n} + \sqrt{2 \ln \frac{1}{\theta_n}} \tau_{Q_{m,n,2}} \hat{g}_{m,n} \tag{43}
\]

and the optimal power assignment from the BS to the CU $n$ is $P_n^* = \left\{ \frac{-\sigma^2 D}{\beta_n \sigma^2} \right\}^+$. Similarly, for a given power allocation, the RB of the $m$th CU of the D2D link $m$ can be obtained by (31).

C. Sensitivity Analysis

It is shown in Section V-B that the protection functions depend on $\Theta_n$ and $\Theta_{n,m}$. We now analyze the sensitivity of $d_{\Delta}$ and its relationship with the tradeoff parameters. For the protection functions (40a) and (40b), $d_{\Delta}$ is expressed as

\[
d_{\Delta} = \sum_{n=1}^{N} \rho_n^* \Delta g_n + \sum_{n=1}^{N} \sum_{m=1}^{M} \gamma_{n,m}^* \Delta s_{n,m} \tag{44}
\]

Differentiating (44) with respect to the tradeoff parameters $\Theta_n$ and $\Theta_{n,m},$ respectively, we can get the sensitivity of $d_{\Delta}$ as

\[
S_{\Theta_n}(d_{\Delta}) = \frac{\partial d_{\Delta}}{\partial \Theta_n} = -\frac{\rho_n^* \left( \sum_{m=1}^{M} \tau_{Q_m,n,1}^2 (\chi_{m,n} \hat{g}_{m,n})^2 \right)^{\frac{1}{2}}}{\Theta_n \sqrt{2 \ln \frac{1}{\theta_n}}} \tag{45a}
\]

\[
S_{\Theta_{n,m}}(d_{\Delta}) = \frac{\partial d_{\Delta}}{\partial \Theta_{n,m}} = -\frac{\gamma_{n,m}^* \left( \tau_{Q_m,n,2}^2 (P_n \hat{s}_{n,m})^2 \right)^{\frac{1}{2}}}{\Theta_{n,m} \sqrt{2 \ln \frac{1}{\theta_{n,m}}}} \tag{45b}
\]

Furthermore, (45a) and (45b) can be summarized as

\[
S_{\Theta_j}(d_{\Delta}) = \frac{\partial d_{\Delta}}{\partial \Theta_j} = \frac{D_j}{\Theta_j \sqrt{2 \ln \frac{1}{\theta_j}}} \tag{46}
\]

where $D_j$ is an expression which is irrelevant to $\Theta_j$. Hence the sensitivity of $d_{\Delta}$ versus $\Theta_j$ with normalized $D_j$ is shown in Fig. 2.

Note that if $\Theta_j < 0.2$, $S_{\Theta_j}(d_{\Delta})$ is quite sensitive to $\Theta_j$. While for larger $\Theta_j$, the sensitivity of $d_{\Delta}$ is relatively independent of $\Theta_j$. From (46), we observe that the increase in $\Theta_j$ leads the proportional decreases in $d_{\Delta}$, which will increase the sum-rate of the D2D links. Therefore, small $\Theta_j$ makes the system more robust against channel uncertainty, while large $\Theta_j$ increases the D2D links’ sum-rate. Therefore, a tradeoff between the optimal performance and the robustness can be achieved by adjusting $\Theta_j$ within the range of $(0, 0.2)$. 

Fig. 2: Sensitivity of $d_{\Delta}$ versus $\Theta_j$ with normalized $D_j$
VI. SIMULATION RESULTS

In this section, we provide the numerical results to evaluate the system performance of the studied fair robust resource allocation policy and take the Nakagami channel with mean 0.5, variance 1.0 dBm, and $m = 1$ into consideration. With one dimensional search method, the allowable error $\epsilon$ is set as 0.5 bits/s/Hz, and the search starting-point $t_1$ and the search step size $\tau$ are both set as 2.0 dBm. Note that if $\psi_1 = \psi_2 = \psi_3 = 0$, the proposed robust optimization problem will turn to be the nominal resource allocation problem without uncertainty.

Fig. 3 shows the error performance of the D2D link channel gain $h_{m,n}$ versus the upper bound $\psi_1$ of the uncertainty region and the safe approximation probability $\eta_{m,n}$ of the chance constraints. It is seen from Fig. 3, when the weight variable $U_{h_{m,n}}$ is set to be 1, the error value of $h_{m,n}$ is affected by $\psi_1$ and $\eta_{m,n}$. For $0 \leq \eta_{m,n} < 0.5$, the error value of $h_{m,n}$ is less than zero and decreases with the increase of $\eta_{m,n}$ and the search starting-point dimensional search method, the allowable error $\epsilon$ is set as 0.5 bits/s/Hz, and the search starting-point $t_1$ and the search step size $\tau$ are both set as 2.0 dBm. Note that if $\psi_1 = \psi_2 = \psi_3 = 0$, the proposed robust optimization problem will turn to be the nominal resource allocation problem without uncertainty.

The D2D links’ sum-rate versus the upper bound $\psi_1$ is shown in Fig. 4. It provides the insight into the effect of the channel uncertainty on $h_{m,n}$. When $\eta_{m,n}$ is set as 0.5, the D2D links’ sum-rate is stable in value without considering the channel uncertainty (perfect channel case). For $0 \leq \eta_{m,n} < 0.5$, the D2D links’ sum-rate falls with increasing $\psi_1$ and it also drops compared to the case without channel uncertainty. For $0.5 \leq \eta_{m,n} < 1$, the actual value of $h_{m,n}$ is greater than the estimated value and the error value increases with $\psi_1$.

The D2D links’ sum-rate versus the upper bound $\psi_1$ is shown in Fig. 4. It provides the insight into the effect of the channel uncertainty on $h_{m,n}$. When $\eta_{m,n}$ is set as 0.5, the D2D links’ sum-rate is stable in value without considering the channel uncertainty (perfect channel case). For $0 \leq \eta_{m,n} < 0.5$, the D2D links’ sum-rate falls with increasing $\psi_1$ and it also drops compared to the case without channel uncertainty. For $0.5 \leq \eta_{m,n} < 1$, the actual value of $h_{m,n}$ is greater than the estimated value and the error value increases with $\psi_1$.

Furthermore, we investigate the influence of the uncertainty of the CSI of the interference channels on the system performance. For the D2D links, we have $\psi_1 = 0$. The impact of the channel uncertainty about $g_{m,n}$ and $s_{n,m}$ on the sum-rate of the D2D links is shown in Fig. 5 and Fig. 6, respectively. In the simulation, the values of the non-zero diagonal weight matrix $W_g$, reflecting the uncertainty region of $g_{m,n}$, are set to be the same and denoted as $w$. Fig. 5 shows that the increase of $w$, the uncertainty region of $g_{m,n}$, reduces the D2D links’ sum-rate obviously and smaller weight value $w$ will lead to higher sum-rate. With the worst-case approach, the performance of the interference channel relates to the uncertainty region denoted as $\psi_2$ and $\psi_3$. In another words, more uncertainty exits with larger uncertainty region. It is observed in Fig. 6 that increasing $\psi_2$ and $\psi_3$ monotonically reduces the D2D links’ sum-rate for the robust resource allocation, compared with the nominal resource allocation with $\psi_2 = \psi_3 = 0$. Additionally, Fig. 6 also shows that the D2D links’ sum-rate decreases obviously as the rate limitation $R_n^C$ increases. Hence, the proposed algorithm also achieves a transmission rate tradeoff between the CUs and the D2D users. It is found from Fig. 5 and Fig. 6 that increasing the uncertainty in $g_{m,n}$ and $s_{n,m}$ monotonically reduces the network utility as what we can expect from (33).
Fig. 6: Sum-rate of the D2D links versus the minimum transmission rate of the CUs with $M = N = 15, \psi_1 = 0, w = 1, v = 1$.

Fig. 7: Sum-rate of the D2D links with chance constrained approach versus $\Theta$ with $M = N = 15, R_n^C = 1\text{bit/s/Hz}$, $\psi_1 = 0$.

Fig. 8: Sum-rate of the D2D links versus the number of D2D links with $M = 30$.

To validate the superiority of the proposed joint resource allocation policy, we compare our policy with the other two typical schemes studied in [36] and [37], respectively. The policy in [36] consists of two steps, RB allocation and power allocation. The policy in [37] takes a greedy RB allocation and we further enhance it with the employment of the power allocation approach in [36]. We denote the combined scheme as “Greedy RA + PA of [37]”. Fig. 8 shows the sum-rate versus the number of the D2D links with $M = 30$. When the D2D link number is 18, the proposed policy improves the D2D links’ sum-rate to 38.6% and 26.5%, compared with the policy in [36] and the combined scheme in [37], respectively. The gaps of the D2D system sum-rate decrease with increasing the number of the D2D links. Furthermore, we can observe that all the schemes show upward trend with the increase of the D2D link number and the proposed policy outperforms the other two policies especially with small D2D link number.

VII. CONCLUSIONS

We have proposed a joint fair resource allocation policy with channel uncertainty in the D2D communication underlying cellular system to enhance the sum-rate of the D2D links and guarantee the QoS of the CUs. With the robust optimization problem formulation, we utilize one dimensional search algorithm to balance the robustness cost defined as the decrease of the D2D system sum-rate and provide a tradeoff mechanism to allocate the resources efficiently. Moreover, we obtain the insight of the effects of the protection function on the achievable D2D system sum-rate and also the tradeoff between the optimality and robustness. Extensive simulation results also prove that the proposed policy achieves a tradeoff in transmission rate between the CUs and the D2D users and the influence of the robustness on the D2D system sum-rate with different uncertainty sets has been discussed.

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