Ultra-Low Level Light Detection Based on the Poisson Statistics Algorithm and a Double Time Windows Technique with Silicon Photomultiplier

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Abstract—In this article, we present a method to detect ultra-low level light ranging from several mean photoelectrons (MPEs) down to 10^{-5} MPEs. It is based on the Poisson statistics algorithm and a double time windows technique with a silicon photomultiplier (SiPM) and a field programmable gate array (FPGA). It measures the mean incident photoelectron number of pulsed light with zero peak statistics of the Poisson distribution to reduce the influence of the correlation noises and the dark count rate (DCR) fluctuation. The linear measurement ranges from ~10^4 MPEs to ~10 MPEs (i.e., ~97 dB) were demonstrated with double 35-ns time windows at room temperature. Its upper detection limit is determined by the availability for the zero peak counts of the Poisson distribution, and the lower detection limit is mainly determined by the DCR of the SiPM. By narrowing the time window to ~550 ps or decreasing the operating temperature to ~30 °C, the detection limit can be further decreased to ~10^8 MPEs. This method, with capability to record the arrival time of incident photons, demonstrated an instrument response function (IRF) of ~214.9 ps (FWHM), showing its compatibility to the time correlated photon counting (TCPC) technique and the time of flight (TOF) measurement.

Index Terms—Low-level light detection, Poisson statistics, Silicon photomultiplier (SiPM), Field programmable gate array (FPGA), Time correlated photon counting (TCPC)

I. INTRODUCTION

Photodetectors are widely applied in scientific research and various industrial fields. Avalanche photodiodes (APD) and p-i-n, which are popular in photodetection, are not sensitive enough to detect low-level light down to few photons due to the thermal noises of the detector and the readout electronics. Photomultiplier tube (PMT) can detect low-level light down to a single photon operated in photon counting mode. However, PMT needs high-bias voltage and possesses poor photon number resolution; it is also bulky, fragile, sensitive to magnetic fields, and difficult to integrate with readout electronics [1]. One alternative is single photon avalanche photodiode (SPAD), which is a solid-state single photon counting detector [2]. However, since SPAD has no photon number discriminating capability, photons pile up may degrade the accuracy of photon counting results; moreover, the area of SPAD is small (e.g., diameter is ≤ 200 μm [3]), leading to limited applications. In past two decades, a new type of detector, called silicon photomultiplier (SiPM) has been intensively investigated due to its high gain, high photon detection efficiency (PDE), perfect photon number resolving capability, low operating voltage, compactness and convenience for integration [4]. It has been replacing PMT in high-energy physics [5], nuclear medicine [6], fluorescence detection [7]; it also may replace SPAD to realize time correlated photon counting (TCPC) technique with higher photon counting efficiency than conventional time correlated single photon counting (TCSPC) technique [8]. However, there are some shortages of SiPMs. One drawback of SiPM is large noises, including dark noises (dark counts) and correlated noises (afterpulsing and crosstalk) in comparison with PMT [1]. It may degrade the sensitivity and accuracy of low-level light detection. The Poisson statistics algorithm was thus adopted to reduce the influence of the correlated noises to the accurate measurement of PDE [9]. Another drawback of SiPM is its sensitivity to the operating temperature, which may increase the dark count rate (DCR) and cause the variation of the breakdown voltage and the gain [10]. Temperature stabilization measures or a temperature compensation bias circuit may alleviate this effect; however, these extra hardware increases the cost and the area of the readout electronics [11].

In contrast to conventional photon counting technique that counts the photoelectron pulses directly [12], or integrates the charges of signal pulses outputted from a SiPM [13], or measures the mean photoelectron number more than one mean photoelectrons (MPEs) via Poisson statistics algorithm [9], we propose a method to detect ultra-low level light by measuring the mean photoelectron number ranging from several MPEs down to 10^{-5} MPEs in this study. This method, based on the Poisson statistics algorithm with a double time windows technique, is easy to implement with a cheap field programmable gate array (FPGA) operating in the low voltage differential signaling (LVDS) receiver mode for algorithm implementation, measurement control and data processing. The advantages of this method include the reduction of the influence of the correlation noises and the DCR fluctuation, thus high signal-to-noise ratio (SNR) can be achieved even for...
ultra-low level light detection by using SiPMs. It is compatible
to TCPC technique and can be applied in time correlated or
time of flight (TOF) measurements. It also features compact
structure, simple readout electronics and low cost. By
narrowing the time window and/or decreasing the operating
temperature, the dynamic range (DR) of the measurement can
be significantly extended in the low-level-light region. It may
find applications in compact and cost-effective fluorescence
spectroscopy [14], time-correlated Raman spectroscopy [15],
and point-of-care testing (POCT) [16].

II. THEORY AND MECHANISM

A. Poisson statistics algorithm with double time windows

The photoelectron number of SiPM in low-level light
detection follows a Poisson distribution, and the mean
photoelectron number ($\mu$) is calculated by zero peak statistics
which is almost not affected by the correlation noises [17]

$$P(k) = \frac{N_k}{N_{total}} = e^{-\mu} \frac{\mu^k}{k!}$$

(1)

$$\mu = -\ln P(0) = -\ln \frac{N_0}{N_{total}}$$

(2)

Where $P(k)$ is the probability to detect $k$ photoelectrons, given
by a Poisson distribution; $N_0$ is the number of the zero
photoelectron events; $N_k$ and $N_{total}$ present the number of the $k$
photoelectron events and the total number of the statistic events,
respectively.

The zero peak statistics technique, based on Eq. (2), is not
care for the exact photoelectron number of each incident light
pulse, it just needs to distinguish baseline (zero photoelectron
pulse) from the light signal in SiPM can be reduced. Only those zero
photoelectron events that located within a pre-set time span
correlated to the light time window were recorded as the
number of the zero photoelectron events $N^0_{dc+ph}$ in dark periods.
Similarly, the number of the zero photoelectron events $N^0_{ph}$ in
dark periods and the $N_{total}$ (derived from the trigger signal of the
pulsed light source) can be obtained. Finally, the mean incident
photoelectron number ($\mu_{net}$) of the light signal is given by Eq.
(3) with the dark counting effects subtracted synchronously.

$$\mu_{net} = \mu_{dc+ph} - \mu_{dc} = \ln \frac{N^0_{dc+ph}}{N^0_{dc}}$$

(3)

Where $\mu_{net}$, $\mu_{dc+ph}$ and $\mu_{dc}$ are the light, total and dark mean
photoelectron number, $N^0_{dc+ph}$ and $N^0_{dc}$ are the number of zero
photoelectron events for dark and light periods, respectively.

C. Dynamic range

Figures of merit for the performance of a photodetector
include the SNR, Noise Equivalent Power (NEP) and DR [2].
The DCR is the major noise of SiPM as an ultra-low level light
detector, which may cause false counts. In this work, the TCPC
technique was adopted and the DCR needed to be reconsidered
as Eq. (4).

$$\text{Light signal}$$

$$\text{Trigger signal}$$

$$\text{Time window}$$

$$\text{SiPM analog pulse}$$

$$\text{SiPM digital pulse}$$

Fig. 1. Schematic and timing diagram of the Poisson statistics algorithm with
double time windows technique.
Substituting Eq. (4) into Eq. (7), we have

$$DC_{\Delta t} = \Delta t \cdot N_{total} \cdot DCR_{SiPM}$$  

(4)

Where $DC_{SiPM}$ is the dark count rate of SiPM, $DC_{\Delta t}$ is the dark counts of the SiPM within the specific time window $\Delta t$.

Thus, SNR is given by

$$SNR = \frac{S}{\sqrt{S + 2 \cdot N}} = \frac{\mu_{ph}}{\sqrt{\mu_{ph} + 2 \cdot DC_{\Delta t}}}$$  

(5)

Where $S$ is the intensity of pulsed light signals and $N$ is the intensity of dark noises within the specific time window $\Delta t$.

NEP is defined as the minimum signal intensity required to attain a unity SNR within 1-s integration time, it is given by Eq. (6) for SPAD [18].

$$NEP = \frac{2 \cdot DC_{\Delta t}}{PDE \cdot f}$$  

(6)

We translate NEP into NE$\mu$, which is defined as the minimum detectable photoelectron number per pulsed light signal required to attain a unity SNR within 1-s integration time for SiPM.

$$NE\mu = \frac{PDE \cdot NEP}{hv \cdot f} = \frac{2 \cdot DC_{\Delta t}}{f}$$  

(7)

Substituting Eq. (4) into Eq. (7), we have

$$NE\mu = \frac{\sqrt{2 \cdot \Delta t \cdot DCR_{SiPM}}}{f}$$  

(8)

Where $f$ is the frequency of the narrow pulsed light signal, which is equivalent to the $N_{total}$ within 1-s integration time.

The minimum detectable photoelectron number $\mu_{min}$ is determined by the NE$\mu$. The maximum detectable incident photoelectron number $\mu_{max}$ is determined by the availability for the zero peak counts of the Poisson statistics algorithm. $N_0^c$ and $N_0^{c+ph}$ can be only taken positive integers in actual measurement.

In the limit case, the minimum positive integer of $N_0^{c+ph}$ is 1 and $N_0^c$ is near to $N_{total}$ in the dark condition. So the $\mu_{max}$ is given by

$$\mu_{max} = \ln \frac{N_0^c}{N_0^{c+ph}} \approx \ln N_{total} = \ln f$$  

(9)

The DR is defined as the ratio of $\mu_{max}$ over $\mu_{min}$ [2], and is given by the Eq. (10) according to the Poisson statistics algorithm.

$$DR = 20 \log_{10} \frac{\mu_{max}}{\mu_{min}} = 20 \log_{10} \frac{\sqrt{f} \cdot \ln f}{\sqrt{2 \cdot \Delta t \cdot DCR_{SiPM}}}$$  

(10)

III. EXPERIMENTAL

The design schematic of the detector module, based on the Poisson statistics algorithm and a double time windows technique with a FPGA operating in the LVDS receiver mode, is shown in Fig. 2. The signal pulses, outputted from the SiPM and amplified by a low-noise transimpedance amplifier (NDL AMP-40-1) [21], were inputted to the LVDS comparator in an Altera EP4CE6E22C8 FPGA. The LVDS comparator has a positive and a negative input, its output is determined by the relative voltage of the two inputs [20]. A pulse-width modulating step-up DC-DC converter was employed to generate the high voltage power supply (HV). The threshold voltages of the comparator and the voltages of the HV for the SiPM bias were adjusted and set by a digital-to-analog converter (DAC) with two channels of analog outputs. The DAC was controlled by the FPGA logics and outputted the appropriate reference voltages as needed. A synchronous signal provided by the pulsed light source was fed to an input port of the FPGA as a trigger for configuring the double time windows. The trigger signal from the light source and the light response signal from the SiPM were inputted to two different TDCs implemented in the FPGA. As for the TDCs, the bin width of coarse time counter was 5 ns by using a 200 MHz clock, and the bin width of fine time counter based on carry delay chain of FPGA was estimated to be ~83 ps (i.e., time measurement accuracy by the TDC of FPGA) [20]. And the time-to-digital quantization logics calculated and outputted the binary codes with the time information of the signals to the events identification and Poisson statistics logics for data processing. For a valid zero peak statistics for dark or light event, the relative arrival time of a response pulse of SiPM was within the pre-set time span of the dark time window or the light time window. When the data acquisition logics acquired enough statistics events, the zero peak statistics data were packed and send to PC via a USB cable.

In this study, a complete detector module, including SiPM detector element, signal amplification element, programmable high voltage power element and reconfigurable threshold voltage element, and data acquisition and control element based on the Poisson statistics algorithm with double time windows technique, was designed and fabricated by NDL (Novel Device Laboratory, Beijing). The SiPM under test was NDL 11-1010C with peak photon detection efficiency 31% at 420 nm and effective area of 1 mm × 1 mm [22]. The breakdown voltage of the SiPM was approximately 27.5 V and the maximal over-voltage was approximately 8 V at room temperature.

The DR of the detector module was characterized with a calibrated photodiode (Hamamatsu S2381) used to measure and calibrate the photocurrent. An integrating sphere with three input/output ports was employed to adapt the calibrated photodiode, the detector module and a LED (420 nm) light source respectively. The photocurrent, as a reference for the light intensity of the LED, was measured by a Keithley 2635B SMU (Source-Measure Unit). Considering the measuring accuracy of the SMU and the complete DR measurement of the SiPM, an optical attenuator was put in front of the detector module. A pulse generator was employed to drive the LED. Adjusting the light intensity and measuring the photocurrent
and the mean incident photoelectron number, the dependence of the mean photoelectron number on the photocurrent can be obtained.

IV. RESULTS AND DISCUSSIONS

The counting rates versus different threshold levels and overvoltages at room temperature without illumination are shown in Fig. 3 (a). It was obtained by the detector module with the reference voltages of LVDS comparator calibrated. The amplitude of electronic noise is evaluated to be less than 10 mV from the curves. The DCR is about 500 KHz/mm² and the optical crosstalk probability is approximately 4% at over-voltage 5 V and room temperature.

Fig.3 (b) shows the instrument response function (IRF) of the detector module when the SiPM was biased at over-voltage 5 V, illuminated by a 420-nm LED driven by a pulse generator with 30-ns electrical pulse width and 1-MHz frequency; the pulsed light intensity was adjusted to ~0.05 MPEs. The IRF, determined by the full width at half maximum (FWHM) of the Gauss fit curve, was ~13.7 ns and the σ (FWHM ≈ 2.355 σ) was 5.8 ns. As we know, if the width of the time window is no less than 6σ, every pulsed light signal can almost be recorded by the module. Therefore, the double time windows were both set as 35 ns (i.e., seven coarse time bins). As f, Δt and DCR_{SiPM} were 1 MHz, 35 ns and 500 KHz in this study, the NE_{F}, μ_{max} and DR were calculated to be ~1.9 × 10^4 MPEs, ~13.8 MPEs and ~97 dB by Eq.(8)-(10), respectively.

The linear response of the detector module based on the Poisson statistics algorithm and double time windows technique is given in Fig. 4, in which the horizontal axis represents the relative intensity of the light source. Fig.4 shows the linear DR from 2.4 × 10^4 MPEs to 2.2 MPEs, which agrees with the theoretical result (i.e., ~1.9 × 10^4 MPEs and ~13.8 MPEs). In order to compare our method with conventional photon counting technique for ultra-low level light detection, the 420-nm LED was driven by the pulse generator with 5-ms electrical pulse width and 100-Hz frequency. The double time windows were both set as 5 ms. The incident photon number was directly obtained within 1-s integration time by counting the number of response pulses of the SiPM and subtracting the dark counts. In this condition, a conventional photon counting technique is simulated with the same detector module. Figure 4 shows that the linear response curve of the “pseudo” photon counting technique is poorer than that obtained by the Poisson statistics algorithm and double time windows technique. This can be attributed to the error counts of the delayed correlation noises at the low-level light; the curve gradually appears saturation at the high-level light because of the pile-up effect and without distinguishing the multi-photoelectron pulses. These features bring the measurement errors and limit the DR for the direct photon counting technique.

According to the Eq. (10), the DR is determined by f, DCR_{SiPM} and Δt. Higher frequency of the narrow pulsed light leads to larger μ_{max} and smaller μ_{min}; however, increase f is not very effective to enhance μ_{max} due to the logarithmic relationship between μ_{max} and f. In addition, f cannot be increased very high to decrease μ_{min} since the correlation noises of light signals is possibly coupled to the next time window of Poisson statistics algorithm, leading to error counts. The more efficient ways to extend the DR of the detector module are to decrease μ_{min} by operating the device at lower temperature to decrease the DCR_{SiPM} or/and to set the smaller time window with high precision TDCs. Fig. 5(a) shows DCR at different threshold levels and over-voltages as cooling the detector module down to -30 °C. The breakdown voltage of the SiPM was approximately 26.5 V and the maximal over-voltage was approximately 7 V at -30 °C. At this temperature, the NE_{F} and DR were derived to be ~3.2 × 10^3 MPEs and ~113 dB when f, Δt and DCR_{SiPM} were 1 MHz, 35 ns and 15 KHz. As for narrowing the time windows to extend the DR, a pulsed light source with wavelength of 420 nm and pulse width of 100 ps (FWHM) was used; the device was biased at 8 V over-voltages and operated at room temperature; the light intensity was adjusted to ~0.05 MPEs. As shown in Fig. 5 (b), the IRF of the detector module, measured with leading edge discrimination, is ~214.9 ps (FWHM); or equivalently, σ is 91.3 ps. As the double time windows Δt was both set as ~550 ps (i.e., seven fine time bins), f and DCR_{SiPM} were 1 MHz and 800 KHz at room temperature.

![Fig. 3. (a) The count rates versus different threshold levels and overvoltages at room temperature without illumination. (b) The instrument response function (IRF) of the detector module under a LED illumination driven by a pulse generator with 30-ns electric pulse width.](image)

![Fig. 5. (a) The count rates versus different threshold levels and overvoltages at -30 °C without illumination. (b) The instrument response function (IRF) of the detector module under a pulsed light illumination with pulsed width of 100 ps (FWHM).](image)
temperature, the $NE_{\mu}$ and $DR$ were obtained to be $3.0 \times 10^{-5}$ MPEs and $-113$ dB, respectively.

V. CONCLUSION

The method based on the Poisson statistics algorithm and a double time windows technique may reduce the effects of the correlated noises and the DCR fluctuation of the SiPMs, thus the sensitivity and the DR can be greatly improved in ultra-low level light detection. The related electronics can be effectively simplified by employing a FPGA operating in the LVDS receiver mode and applying zero peak statistics technique. The linear measurement ranges from $10^4$ MPEs to $10$ MPEs (i.e., $97$ dB) was demonstrated with double 35-ns time windows at room temperature. By narrowing the time window to $550$ ps or decreasing the operating temperature of SiPM to $-30$ °C, the detection limit can be further decreased to $10^{-5}$ MPEs. This method can also record the arrival time of incident photons and demonstrated an IRF of $214.9$ ps (FWHM), indicating its compatibility to the TCPC technique and the TOF measurement.

REFERENCES