Magnetic Units

Memorandum on the M.K.S. System of Practical Units

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Abstract—The Committee for Electric and Magnetic Units of the International Electrotechnical Commission at its meeting in Paris, October 5th and 6th, 1933, voted unanimously in favour of the proposal to arrange the system of practical electrotechnical units into a complete absolute system (M.K.S. system), which is intended to remain in use simultaneously with the C.G.S. systems, in accordance with the suggestion advanced by the writer in 1901. The object of this Memorandum is to illustrate the proposal and to explain the principles on which the system is founded.

I. PRELIMINARY

We have now three groups of units in use: C.G.S. electrostatic, C.G.S. electromagnetic and the practical units.

The C.G.S. systems originated at a time when the whole theory of electric magnitudes and dimensions had not been perfectly developed. An "absolute" system was then regarded as a system where all units are derived from three fundamental units, those of length, mass and time. The first aim in building the C.G.S. system was to have a universal system for all purposes. Coulomb had discovered the laws $f = k(qq'/r^2)$ in electricity, $f = h(mm'/r^2)$ in magnetism. These laws were assumed to be corner stones for erecting the building of electric and magnetic units; in them, k and h appeared as unnecessary coefficients, to be abolished in an absolute system. But the two assumptions conflicted mutually, so the absolute system remained split into two, one so-called "electrostatic," another "electromagnetic," and both were necessarily dissymmetrical.*

Moreover, the derived units of the C.G.S. systems, even the mechanical ones, came out in very inconvenient sizes, so that it was necessary to adopt a third series of so-called "practical units," originally defined as multiples of the C.G.S. units.

The evil of the inconvenient size of the derived units has two origins: (1) the choice of the centimetre as fundamental unit of length, (2) the arbitrary value unity given to the coefficients k and h.

Maxwell's theory has shown how to regard things under a different light. Gradually, it became evident that the medium through which electric actions are transmitted has to be taken into account, and that both h and k represent physical properties of the free space or ether. It may be said that their reciprocals measure the "compliance" of the medium, its capability of storing energy under the action of a given field of force. Now the energy stored in the unit volume of the free space when it is subjected to a magnetic or an electric field of such intensity as we usually produce, is extremely small. Therefore, it was not wise to regard h and k as mere numeric coefficients, and still worse it was to attribute to them the value unity; it would have been better to recognise them as physical magnitudes and to give to them values very far removed from unity (see Section IV below).

Maxwell did not live long enough to develop the consequences of the fundamental ideas of his theory. He made a first attempt to complete the group of the practical units into an absolute system, by combining them with the quadrant ($= 10^7$ metres) and the mass of 10^{-11} grams; of course, that was not acceptable for practical use, as under it, many other units acquired an inconvenient size. The reason was that the "electromagnetic" scheme of foundation was adhered to, that is, the postulate of having $h = 1$ in Coulomb's formula for magnetic attraction was still preserved.

The full development of Maxwell's ideas was only achieved in O. Heaviside's electromagnetic theory; this theoretical scheme is admirable for its simplicity and clearness and is particularly adapted for the requirements of electrotechnics. But Heaviside did not lay stress on the advantages that would follow if we were to recognise a fourth fundamental dimension, and discard the so-called electromagnetic foundation of a system of units. His name is attached, however, to the history of units on account of the "rationalisation." He showed that in the formulæ of old electric theory a 4π was present in places where this coefficient ought not to be; by restoring this factor to its right place, a further simplicity and a perfect duality between electric and magnetic relations is secured. Since then, "rationalised" and "non-rationalised" formulæ have been employed by scientists and by electrical engineers, at random, but the correctness of the principle of the "rationalisation" has been generally recognised. In former times, it appeared, however, that in order to comply with the "rationalised" equations, it would have been necessary to change radically the whole of the units in use by multiplying or dividing most of them by $\sqrt{4\pi}$. Heaviside indeed advocated this change, but it was too radical for acceptance.

In the meantime, the theory of physical dimensions was beginning to be better understood, and the opinion was no longer held that everything in the physical world depended necessarily on three fundamental mechanical dimensions. Physicists recognised that entropy, temperature, loudness of a sound, light intensity, etc., brought into play some dimensions which were not dependent on [L], [M], [T]. Why ought not

*It is possible to have simultaneously $h = 1$, $k = 1$, and so to get a symmetrical structure of relations, provided a coefficient c (equal to the velocity of light) be introduced into the circuital equations of electromagnetism. The Gauss–Hertz system and the Lorentz–Einstein system are built on these lines. Systems of this kind may be of value in several investigations, pertaining to the highest branches of science, but nobody thinks of the possibility of adopting them for general use..

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electric and magnetic magnitudes to be treated in the same way? The pretention of making them dependent on length, mass and time has led to such strange consequences of attributing to electrical resistance the dimensions of a velocity (in the electromagnetic system) or of the reciprocal of a velocity (in the electrostatic system). Accordingly, the principle of having a fourth fundamental dimension entering into the electric and magnetic magnitudes gradually became universally recognised. An independent dimension means also an independent and arbitrary unit; but this conclusion was not drawn at once.

II. ORIGIN OF THE M.K.S. SYSTEM

It occurred to the writer, as early as 1901, to remark that the solution of all difficulties accumulated up to then, could at once be obtained if the whole of the electrotechnical practical units were taken together with the metre as unit of length, the kilogram as unit of mass and the second as a unit of time, so that by adding one other arbitrary unit, a complete system of units of absolute character, was built up from four fundamentals. Indeed, the metre and the kilogram-mass are consistent with the joule and watt. By taking any one of the electrotechnical units (either the ampere or the volt or the coulomb, etc.) as arbitrary and fundamental, all other practical units in use necessarily follow. The resulting set of units is neither "electrostatic" nor "electromagnetic," it enjoys perfect symmetry towards electric and magnetic phenomena; it is in agreement with the electrophysical principle of duality; it permits the use of either "rationalised" or "non-rationalised" derived units and formulæ, without changing the concrete established units. It has, moreover, the singular advantage that it is entirely composed of units already in use (except perhaps one or two secondary ones); finally, all units, fundamental and derived, are of a convenient size.

The whole system constitutes thus a useful extension of the set of practical units, of such character that it is equally fit for electrotechnical, for scientific and for common use of everyday life. It may be employed as a universal absolute system of an independent origin. At the same time its recognition does not exclude the use of the C.G.S. systems or the Lorentz–Einstein system or any other system whenever it may be desired; on the contrary, the study of these other systems is facilitated, because their construction and the evaluation of their units may be more easily explained in terms of the practical set of units. It is not to be forgotten that the standards of measurements in all laboratories (even in laboratories of pure physics) are all rated in ohms, volts, farads, etc., so that all units of every system require to be referred to the standards of practical units.

The theory of dimensions becomes likewise very simple when it is explained in the new system. Besides [L], [M], [T], a fourth fundamental dimension is introduced; this may be any one of the electric or magnetic magnitudes, for instance, the quantity of electricity [Q]; then to write down the dimensions of all derived units becomes quite an easy matter, and does not require fractional exponents and complicated formulæ. The learning of the theory of electricity for the students of electrical engineering becomes very simple, and an enormous waste of time and mental effort is avoided.

On account of these reasons, when the writer published his system, or rather, his method of connecting together the existing units, he simply employed the title "Rational units of electromagnetism." Here, "rational" was not employed in Heaviside's meaning of "rationalised" but in a quite general signification; the writer wished to lay stress on the fact that the "electrostatic" and the "electromagnetic" ways of foundation having been simultaneously discarded, the dissymmetries and limitations and all other objectionable features of the preceding systems were removed. Afterwards, he wrote "M.K.S. system." Prof. Kennelly more completely said "M.K.S. Ω system." In this memorandum, the brief term "M.K.S. system" is used when speaking generally in order not to prejudice the ultimate decision as to what the fourth unit is to be. When Dr. George Campbell illustrated the proposals of the writer in 1924, he employed the names "definitive system" and "definitive units," thus conveying the idea that the system has gradually to supersede all other combinations of units. Also the denominations of "practical absolute" or "electrotechnical absolute" system have been employed.

Dr. Campbell has shown that by a happy circumstance all units of the system are of intermediate size between the smallest and largest magnitudes that present science has to measure; therefore, their size is the most convenient in every case; numbers between 10^{-12} and 10^{12} are sufficient to evaluate almost all magnitudes occurring in the physical science and in the description of the world. For these reasons, it may be said that the writer's system agrees equally well with the most rigorous deductions of pure science and with the requirements of practice. We are greatly indebted to Dr. Campbell for having illustrated the system so efficiently under wide points of view and with fresh arguments.

III. THE M.K.S. SYSTEM AND ITS PRINCIPAL UNITS

As now explained, the system is intended to contain all usual electrotechnical units, together with the mechanical units in common use, connected into an absolute frame, capable of standing by itself, but not necessarily disconnected from the other systems in use.

The fact that the system is neither "electrostatic" nor "electromagnetic" means that the assumptions $k = 1$ and $h = 1$ in Coulomb's formulæ are both given up; so, no arbitrary unitary value is attributed to either of the constants of the free space. Instead of that, the principle of a fourth independent dimension being recognised, one of the existing electrotechnical units (no matter which of them) is considered as arbitrary and fundamental. This unit may be defined by material standards, just as the metre and the kilogram are. The questions about the effective choice of the fundamental electric unit and the definition of its standard will be discussed afterwards; the structure of the system is independent of that.

We are now going to enumerate the principal units of the proposed system, the mechanical ones, the electric and magnetic "rationalised" and "non-rationalised" ones. The names of the concrete units are put especially into evidence.*

(1) Mechanical Fundamental Magnitudes and Units

Length [L]: the *metre* as defined by the international platinum-iridium standard kept at Sèvres.

Mass [M]: the *kilogram* as defined by the international platinum standard kept at Sèvres.

*Distinction is to be made between concrete and specific magnitudes. The first ones are quantities such as mass, length, weight, resistance, conductance, magnetic flux, etc. Their units are generally represented by effective standards. Specific magnitudes generally arise as ratios between concrete quantities and length or area or volume. Thus, density, resistivity, conductivity, magnetic induction, gradient of potential, etc. It is preferable not to emphasise the size of specific units by giving special names to them. The need was never felt of giving a name to the unit of density. Specific magnitudes are better expressed in the form of ratios.

Time [T]: the *second* of mean solar time, defined as usual.

Remark. The metre and the kilogram are the original units of the metric system; the centimetre has not a convenient size for everyday use, and nobody employs it in common life for measuring large distances or for deriving multiples and submultiples.

Additional Note. It is a matter of choice to define and consider *angles* as pure numbers or as magnitudes having a dimension of their own. In the latter case, the unit will be the radian and the physical dimension will be indicated by [θ].

(2) Mechanical Derived Magnitudes and Units

Area [L^2]: the *square metre.*

Volume [L^3]: the *cubic metre.*

Velocity [LT^{-1}]: the *metre per second.*

Acceleration [LT^{-2}]: the *metre per second per second*, so that the absolute value of gravity in London is measured by 9.8.

Mass Density [ML^{-3}] defined as specific mass or mass per unit volume—the *kilogram per cubic metre*. The density of water is therefore 1000. This is in accordance with the use of all engineers and practical people in metric countries and is much more convenient than to have density of water equal to unity.

Force (mechanical or ponderomotive): dimensions [MLT^{-2}]. The unit will be the gravitational force on (or weight of) a kg-mass in a place* where the gravity would be 1 m/sec^2. Really this is the only important new unit of the proposed system. It equals about the weight of 102 grams at the surface of the earth. The name *vis* has been suggested by Professor Pistolesi for this unit. Awaiting that an international name be agreed upon we shall employ this word.

Pressure (mechanical) defined as ratio between force and area. Dimensions [$ML^{-1}T^{-2}$]. The unit is the *vis per square metre*, to be employed together with its multiples and submultiples.

Work or energy [ML^2T^{-2}]: the *joule*, which equals the product of a vis into a metre. The international joule now in use is a little less (about 3 part in 10,000) than the absolute or mechanical joule thus defined; it needs to be corrected.

Power [ML^2T^{-3}]: the *watt* or joule per second. The international watt needs the same correction as the joule in order to bring it into accordance with the mechanical watt.

Torque or *moment of a couple* [ML^2T^{-2}] or better [$ML^2T^{-2}\theta^{-1}$]: the *joule per radian*. In case the angles be regarded as pure numerals, and the dimensions written without θ, there is no distinction between it and the joule in the statement of dimensions.

(3) Electrical Units

The fact that one unit force (the vis) operating through one metre does an amount of work equal to one joule (= 10^7 ergs) makes a very convenient link with the established electrical units and enables them to be incorporated, as they stand, into an "absolute" system. We thus get a system which is absolute in its scientific basis and practical in the size of the units employed.

When we have one coulomb of electricity at a pressure of one volt, we have a store of energy equal to one joule. Whether the energy has been obtained by electrical work (a quantity of Q coulombs being displaced through a difference of potential of V volts) or has been obtained by doing mechanical work it possesses the same dimensions. Therefore we may write [QV] = [ML^2T^{-2}]. If we adopt the coulomb as a fundamental unit of dimensions [Q], then we may write [V] = [$ML^2T^{-2}Q^{-1}$]. Or if we adopt the volt as a fundamental unit, then we may give the dimensions of [Q] = [$ML^2T^{-2}V^{-1}$]. We may also adopt the ampere as fundamental, so that [A] = [QT^{-1}] is amperage (electric current); then [Q] = [AT], and power in watts is VA, so that we have [VA] = [ML^2T^{-3}] and [A] = [$ML^2T^{-3}V^{-1}$]· In fact, any one of the electrotechnical units may be taken as fundamental, and all others become derived units. For theoretical purposes, perhaps the best way is to explain everything in terms of the coulomb. Practically, as the standards of resistance are the easiest to preserve and to compare, the ohm would be preferable. However, in order to leave the choice of the fourth unit free, we have here denoted the system simply as the M.K.S. system, fully realising that the complete designation should be in the general form "M.K.S.E. system," where E is the fourth unit.

For the sake of writing down the dimensions easily, a very simple way is to introduce [V], [A] and even [R] simultaneously into the dimensional formulæ, on the understanding that if [Q] is the fundamental, then [A] is a short symbol for [QT^{-1}] and similarly [V] is short for [$ML^2T^{-2}Q^{-1}$], and [R] means [VA^{-1}] = [$ML^2T^{-1}Q^{-2}$]. Here, we shall write the dimensions in terms of [V] and [A].

It will be seen that we avoid all difficult questions as to what are the dimensions of the space constants ε and μ by simply adopting [Q] or [A] or [V] or [R] = [VA^{-1}] as fundamental dimension. It is generally recognised that (with our present knowledge) four fundamentals are necessary, and it does not matter whether we put the fourth fundamental into the permittivity of a dielectric or take the coulomb or the ohm or the volt itself as fundamental except that the latter course leads to the simpler results.

We can now set out the electrical units and their dimensions and first we state them as rationalised units because in all probability it will be the rationalised units that will be preferred. Afterwards, we can consider what changes of values are necessary to put the units in the "unrationalised" form.

Rationalised Electrical Units in the M.K.S. system

Quantity of electricity [Q] = [AT]: the *coulomb*, which may be standardised and defined by the silver voltameter or by comparison with the C.G.S. electromagnetic unit, or in any other way.

Electric current [A] = [QT^{-1}], defined as the ratio between quantity of electricity and time; the *ampere* equal to the coulomb per second.

Density of electric current [AL^{-2}], or ratio between current and area: the *ampere per square metre*, together with its decimal multiples and submultiples.

Electromotive force or *voltage* or *difference of potential* [V], which may be defined as a ratio between work and quantity of electricity. The unit is the *volt*, equal to joule/coulomb or watt/ampere.

Electromotive impulsion [VT] defined as the time integral of E.M.F., or product of E.M.F. and time: the *volt–second*.

Electric force of an electrostatic field or *gradient of electric potential* (sometimes called also *intensity of electric field of force*): dimensions [VL^{-1}]: the unit is the *volt per metre*, with its decimal multiples and submultiples.

Electric displacement or *electric induction* or *polarisation* vector of an electric field (in the "rationalised" formulæ there is no distinc-

* About 20,000 kilometres from the centre of the earth.

tion between these three); dimensions [ATL^{-2}]; defined by the ratio between quantity of electricity and area; the unit is one *coulomb per square metre*, with its multiples and submultiples.

Electric resistance [VA^{-1}] defined as the ratio between voltage and current; the unit is the *ohm*.

Electric conductance [AV^{-1}]: the unit is the *ohm*$^{-1}$.

Electric capacity or *permittance* of a condenser, defined as the ratio between quantity of electricity and voltage: dimensions [ATV^{-1}]; the unit is the *farad*.

Electric permittivity or *specific capacity* or *electric constant* of a dielectric medium defined as the capacity of a cube of unitary size, or as the ratio between electric displacement and electric force: dimensions [$ATV^{-1}L^{-1}$]: the *farad per metre*.

Inductance of an electric circuit or coefficient or self-induction defined as the ratio between an electromotive impulsion and a current: dimensions [$VA^{-1}T$]; the unit is the *henry*.

(4) Rationalised Magnetic Units of the M.K.S. System

Magnetic units are derived from electric ones. When "rationalised" equations are employed, perfect symmetry is secured between the two sets. As fundamental equations connecting electricity and magnetism, it is best to accept the two circuital laws, viz.:

(a) The induced E.M.F. in an electric circuit equals the rate of decrease of the magnetic flux interlinked with it.

(b) The induced M.M.F. in a magnetic circuit equals the total current (or sum of currents) interlinked with it.

(In old theoretical schemes, this second law was deduced from Laplace's law or from Neumann's equations and was written with a 4π in the second member. Accordingly, "rationalised" or "non-rationalised" units of magnetism may be deduced. In the modern schemes, both laws are accepted as resulting directly from experiment).

The following dimensions and units are deduced:

Magnetic flux: the magnetic flux interlinked with a closed line may be defined as the electromotive impulsion (time-integral of E.M.F.), which is induced in a material electrically conducting circuit following that line when it is moved and deformed in such a way that its area is reduced to nil, or, more generally, it is that quantity the rate of decrease of which with respect to time equals the induced E.M.F. Therefore, it is a product of a voltage into a time, with dimensions [VT]. When the electric current interlinks several times with the magnetic flux, the latter is measured by this product divided by a number of turns of the electric circuit. The unit accordingly is the *volt–second*, or more completely the *volt–second per turn*. The name proposed for it by the International Committee is *weber*.

Quantity of magnetism or strength of magnetic pole. In the "rationalised" system this quantity is considered identical with magnetic flux, because a magnetic pole is measured by the flux radiated from it.

Magnetic induction, [B], is the magnetic flux per unit area. Being the ratio between a flux and an area, its dimensions are [$VL^{-2}T$]. The resulting unit is the *weber per square metre*, with its decimal multiples and submultiples. It corresponds to 10^4 gausses of the C.G.S. system.

Magnetomotive force. In rationalised formulæ, the M.M.F. along a magnetic circuit is measured by the *total* electric current interlinked with the magnetic circuit. Where a wire carrying a current passes in many turns around the same magnetic circuit, the M.M.F. is the product of the single current into the turns. The dimension is [A]. The unit is the *amp-turn*.

Magnetic force of a field [H]: called also the intensity of magnetic field of force, or gradient of magnetic potential, is the M.M.F. per unit length of path. As it is a ratio of M.M.F. to length, its dimensions are [AL^{-1}]. The "rationalised" unit is the *amp-turn per metre*, with its decimal multiples and submultiples. Two of these multiples, amp-turn per centimetre and amp-turn per millimetre are extensively used.

Permeance of a magnetic circuit as defined by the ratio between flux and M.M.F. Dimensions [$VA^{-1}T$]. Where there is only one turn of the electric circuit, the permeance may be measured by the inductance of that turn. Where there are a number of turns, the permeance is the inductance in henries divided by the square of the number of turns. The "rationalised" unit is the *weber/amp–turn* or *henry per turn*^{2}.

Reluctance of a magnetic circuit, as defined by the ratio between the M.M.F. and the flux; or reciprocal of permeance. Dimensions [$V^{-1}AT^{-1}$]. The "rationalised" unit is equal to *amp–turn/weber*.

Permeability or (better) magnetic inductivity, or magnetic "constant" of a medium, defined as the permeance of a cube of unit edge, or the ratio of flux density to magnetic force; this is a physical quantity, to be distinguished from the relative permeability which is a pure number. The "rationalised" unit is the *henry per metre*.

(5) Electric and Magnetic "Non-Rationalised" Units

It is possible under the M.K.S. system to preserve the "irrational" features found in the C.G.S. system if it is so desired. We would then have among electric units the vector "electric induction" different from the vector "displacement" and equal to 4π times the latter. The dielectric constant of a medium similarly would split into two, of which one is the capacitivity as defined, the other is the ratio between electric induction and electric force, that is 4π times the capacitivity; its reciprocal is analogous to Coulomb's coefficient *k*.

Among magnetic units, the M.M.F. would be measured by a unit 4π times smaller than the amp-turn. The units of magnetic force, of reluctance and of reluctivity are accordingly varied. The reciprocal of the reluctivity becomes analogous to Coulomb's coefficient *h*. The units of permeance and permeability become 4π times larger, so that the magnetic constant of a medium splits into two. Similarly, the unit for the intensity of a pole becomes 4π webers, the vector of magnetic polarisation becomes different from the magnetic induction, differing by a factor 4π.

It is thus seen that the whole of the system proposed by the writer may be brought into agreement with "rationalised" and with "non-rationalised" formulæ, but the advantage of the utmost simplicity lies with the first.

IV. CONSTANTS OF THE FREE SPACE

Let us write ε and μ for the capacitivity and the permeability of a medium and ε_0, μ_0 for those of free space. The resulting values of the latter are:

$$\varepsilon_0 = 8.88542 \times 10^{-12} \text{ farad/m}$$

$$\mu_0 = 1.25607 \times 10^{-6} \text{ henry/m}$$

These very small values are not an abnormality of the system. They are an expression of the fact already alluded to, viz.: when a volume of free space of human dimensions is acted upon by an electric or a magnetic gradient of potential of ordinary intensity, the stores of energy per unit volume are exceedingly small, much smaller in the first case than in the second.

Free space, considered from the human standpoint, has a very small compliance for magnetic actions and a still smaller one for electrostatic actions. These facts are almost concealed in the C.G.S. systems, just in the same way as the very high speed of light c does not come into evidence in those systems where c is taken as the unit of velocity.

In the C.G.S. electrostatic system, we had

$$\varepsilon_0 = (4\pi)^{-1} \qquad \mu_0 = 1.398 \times 10^{-20}$$

and in the C.G.S. electromagnetic system it was

$$\varepsilon_0 = 0.885 \times 10^{-22} \qquad \mu_0 = 4\pi$$

so that even in these systems one at least of these constants is largely different from unity, and in both the dissymmetry arises.

The equation

$$c = \sqrt{\frac{1}{\varepsilon_0 \mu_0}} = \text{velocity of light} = 2.99792 \times 10^{10}\ \text{cm/sec}$$
$$= 2.99792 \times 10^{8}\ \text{m/sec}$$

is valid in the three systems equally.

V. PROPOSALS

It is not proposed to discard the existing systems of units, C.G.S. electrostatic, C.G.S. electromagnetic and other systems in scientific use. Each one will be employed according to the requirements of the subject and the preference of the user.

The suggestion is to extend the group of practical electrotechnical units already in use in order to have them fitted into a coherent M.K.S. system as described, capable of standing as an absolute system by itself.

This will result in a great simplification of all practical calculations and of the learning of electrical theory in the schools. A great deal of waste of time and intellectual fatigue will be saved.

Future practice will show which units are the most convenient for every particular purpose and the law of the "survival of the fittest" will receive application.

APPENDIX I. CHOICE OF THE FOURTH FUNDAMENTAL UNIT FOR THE M.K.S. SYSTEM

Two or three points of view have to be separately considered and it is not necessary that the answer be the same in the two cases.

Theoretical Point of View

For the purpose of exposing the theory of electric magnitudes, units and dimensions, it is almost immaterial which among the existing electrotechnical units be assumed as fundamental and arbitrary, because in conjunction with the watt, which is defined mechanically, all other units necessarily follow. The most obvious choice appears to be the coulomb, but for exhibiting the dimensional formulæ, it is best to employ simultaneously two fundamentals, connected by a relation, for instance, as shown in the preceding text, the ampere and the volt, connected by amp. $\times$ volt $=$ watt.

Metrological Points of View

Many very important questions are here involved, concerning the ultimate fixation of the values of the units and the determination of the material standards.

These questions were very hastily discussed at the meeting in Paris and the prevailing opinion was that the electrotechnical units ought to be defined by comparison with the electromagnetic C.G.S. units in such a way as to be multiples or submultiples of them, according to integral powers of 10, that is, conforming to their original definitions.

The writer does not agree with this resolution and he suggests that all the sides of the question be carefully considered.

Drawbacks of this resolution are: 1st, the existing international units and standards ought to be changed because the ohm ought to be altered by 1 in 2000 in order to comply with the C.G.S. definition; this requires in the whole world an enormous amount of expense and complication, the evil consequences of which cannot sufficiently be estimated now; 2nd, the standards for measurements would be dependent on the so-called "absolute measurements," the precision of which is much less than the precision obtainable in the comparison of material standards and a never ceasing fluctuation of values would necessarily follow; 3rd, there is no scientific reason for adhering now to the original theoretical definition of the practical units; since we have recognised the principle of the fourth fundamental dimension, it is scientifically right to assume the corresponding unit as arbitrary; 4th, the original definition based on C.G.S. units makes the units dependent on the physical properties of free space; now modern physics shows, on theoretical and even on experimental grounds, that the physical properties of free space may be subject to change as functions of space and time; therefore, in order not to have a variable standard and in order to be able to express these variations, it is right that the standards be not founded on the so-called "absolute measurements" which are nothing else than measurements of the properties of free space; 5th, the evil of the 4π and irrationality of units would be perpetuated, because the permeability of free space would receive the exact value $4\pi \times 10^{-7}$ instead of being the result of a measurement.

The question is the same as that of the standardisation of the metre and the kilogram. Originally, they had a theoretical definition, but nobody now would agree to change the standard of length in order to agree with the measurements of the earth meridian, or to change the kilogram in order to have it exactly equal to the mass of a cubic decimetre of water. But to change now the ohm, farad, henry, etc., would result in a much greater trouble than to change the metre or the kilogram, because the electrical standards in every laboratory involve a much greater amount of expense and labour than those of length and mass.

Therefore, the writer suggests that both for scientific and practical reasons, the original way of making the practical units dependent on the C.G.S. electromagnetic units ought to be discarded and the fourth fundamental unit ought to be fixed in order to comply with the existing international units. Probably, the ohm is the best fundamental unit, because the comparisons of resistance can be made with an accuracy as high as 1 in 1,000,000, which is not obtainable in the measurement of electrical magnitudes of any other kind, and if mercury standards alone or metallic standards alone are open to some objections, the practical ways of keeping the values of the standards fixed may be investigated by some special committee (just as was done for the standard of length).

To keep for the ohm the same value as now in use means to keep fixed a whole group of units, among which are the farad and the henry and all the units of such magnitudes as allow of more accurate comparison and measurement. There are two other groups of units, beginning from the ampere and the volt, which need to be retouched in order to agree with the watt defined mechanically; but the change required is very small and within the limits of the accuracy of laboratory measurements; the magnitudes corresponding to these two groups do not allow of accuracies exceeding 1 in 5,000 generally, and are not represented in the laboratories by material standards, so that in retouching slightly their units, there is not the same inconvenience as there would be in altering the values of the units of the first group.

These proposals are in agreement with the plan for the standardisation of all electric units which has been elaborated by Dr. G. Campbell, and the reasons developed in his pamphlet deserve to be carefully considered.

Concluding, the opinion of the writer is that the questions connected with the choice and the standardisation of the fourth fundamental unit be deferred to a further discussion.

APPENDIX II. EXAMPLE OF A PRACTICAL CALCULATION WITH OLD AND NEW METHOD

Calculation of the Capacity of the Earth (Earth Assumed as a Spherical Conductor)

(A) With Old Method

1) Radius of Earth

$$R = 637\,122\,000 \text{ cm}$$

2) Electric constant of free space

$$(\varepsilon_0) = 1 \text{ (in C.G.S. electrostatic units)}$$

3) Capacity of Earth in electrostatic C.G.S. units

$$(C) = \frac{R}{(\varepsilon_0)} = 6.37122 \times 10^8$$

4) Coefficient for transformation from electrostatic to electromagnetic value

$$c^2 = 8.98752 \times 10^{20}$$

5) Capacity of Earth in electromagnetic C.G.S. units

$$[C] = \frac{(C)}{c^2} = \frac{6.37122}{8.98752} \times 10^{-12}$$

6) Conventional ratio between C.G.S. electromagnetic and practical value

$$\alpha = 10^9$$

7) Capacity of Earth in practical units, conventional value

$$\overline{C} = \alpha[K] = 0.000\,708\,85$$

8) Coefficient of correction for evaluation in international units

$$\beta = 1.0005$$

9) Capacity of Earth in practical international units

$$C = \beta\overline{C} = 0.000\,709\,20 \text{ farads}$$

(B) With New Method

1) Radius of Earth

$$R = 6\,371\,220 \text{ metres}$$

2) Electrostatic constant of free space (in practical units)

$$\varepsilon_0 = 8.8581 \times 10^{-12} \text{ farads/metre}$$

3) Capacity of Earth (in practical units)

$$C = 4\pi\,\varepsilon_0 = 0.000\,709\,20 \text{ farads}$$