Tunable Magnetization Chains Induced With Annular Parabolic Mirrors

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Tunable Magnetization Chains Induced With Annular Parabolic Mirrors

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Abstract: Based on the inverse Faraday effect, tunable light-induced magnetization chains are investigated by focusing narrow annular azimuthally polarized vortex beams with a 4π focusing system composed of double annular parabolic mirrors. Using Richards–Wolf vector diffraction theory, approximate analytical expressions of all parameters of the magnetization chains are theoretically calculated, realizing that all parameters are tunable. The calculated results show that the parameters are independent of the amplitude distribution function of the incident beam and the apodization factor of the focusing system. Hence, different kinds of long (about 100λ) chains with sub-wavelength lateral size can be achieved by adjusting the central angular position, the angular width, and the phase difference of two counter-propagating incident beams. Compared with lenses, parabolic mirrors can generate multi-needle fields that lenses cannot.

Index Terms: Magneto-optical materials, polarization, parabolic mirror.

1. Introduction

In the past few years, the interaction between laser and magneto-optical materials has attracted intensive research interest, for it plays an important role in many areas, such as all-optical magnetic recording [1]–[3], particle manipulation [4], confocal and magnetic resonance microscopy [5], [6]. Longitudinal magnetization needles and chains with super-long longitudinal size and sub-wavelength lateral size are desirable to further promote these practical applications. Based on the inverse Faraday effect (IFE) in magneto-optic (MO) film, magnetization fields with sub-wavelength lateral size can be obtained by focusing beams [7]–[22]. If focused beams are azimuthally polarized vortex beams (APVBs), the obtained magnetization fields will be pure longitudinal [7]–[17]. In 2013, Jiang et al. first achieved a pure longitudinal magnetization field by tightly focusing an APVB with a high numerical aperture lens [7]. Later on, Wang et al. got a long (7.48λ) pure longitudinal magnetization needle with lateral super-resolution (0.38λ) by annular vortex binary optics [9]. Recently, Yan et al. used an azimuthally polarized beam (APB) modulated by multi-zone plate phase filter to achieve an ultra-long (107λ) pure longitudinal magnetization needle with super-resolution (0.37λ) [10]. Magnetization needles mentioned above are suitable for single particle trapping. In order to manipulate multiple particles simultaneously, some researchers are focusing on magnetization chains. Nie et al. achieved super-long (12λ) and sub-wavelength (0.416λ) longitudinal magnetization chains with single/dual channels by adjusting the radii of different rings of the single/cascaded vortex binary filters with a single lens [11]. However, a 4π tight focusing configuration is more usual.
to obtain magnetization chains [12], [13]. Gong et al. found a super-long (16λ) magnetization chain composed of 19 sub-wavelength (0.44λ) spherical spots by focusing an APVB with special amplitude modulation [12]. Later, Yan et al. attained an extra-long and three-dimensional super-resolution longitudinal magnetization chain with single/dual channels by selecting optimized parameters of a multi-Gaussian beam and topological charge of a spiral phase plate [13]. The methods above all require a complex modulation and have no analytical result. In other words, parameters in these papers can not be modified freely and further applications are limited. In 2017, Yan et al. reported an analytical result on magnetization fields [15]. But it requires that incident beams should be angular Gaussian beams and they did not give a method to change the parameter of incident beams, which means the obtained fields cannot be adjusted freely. In this paper, a 4π focusing system composed of double annular parabolic mirrors are used to obtain a magnetization chain by focusing narrow annular APVBs. The approximate expressions of all parameters of the magnetization chains are given, making them tunable by changing the angular thickness, the angular position and the phase difference of incident beams. A feasible method to adjust the parameters is proposed. Individual spots in a magnetization chain usually have both small lateral and longitudinal sizes [11], [12]. But in this paper, the individual spots’ longitudinal sizes have a large range from sub-wavelength to dozens of wavelengths by adjusting the parameters while keeping their lateral sizes being sub-wavelength.

2. Configuration and Theory

Fig. 1 shows the geometry of a 4π focusing system made of two annular parabolic mirrors (PMs) illuminated by two collimated counter-propagating narrow annular APVBs. The incident beam is an APB. An annular APB can be obtained by two axicons and an diaphragm. The radius and the thickness of the annual APB are tunable by the axicons and the diaphragm, respectively. A spiral phase plate (SPP) is behind the diaphragm, which can turn an APB into an APVB. A SPP is a phase modulator that delays the phase of the incident beam from 0 to 2π, and its transmittance function is expressed as \( T(\phi) = \exp(i\phi) \) when its topological charge is 1. Then the beam will transform into two counter-propagating narrow annular APVBs by a beam splitter and three mirrors. A magneto-optic film with IFE is located at the confocal plane and a magnetization chain can be generated near the confocal spot. If the beam splitter, the mirrors and one of the PMs are removed, a magnetization needle will be obtained near the focus.

According to the Richards-Wolf vector diffraction theory [23], the electric field near the focus of an APVB can be expressed as [24]

\[
\vec{E}(r, \phi, z) = \begin{bmatrix} E_r \\ E_\phi \\ E_z \end{bmatrix} = A e^{i\phi} \begin{bmatrix} l_0 + l_2 \\ i(l_0 - l_2) \\ 0 \end{bmatrix},
\]

(1)

Fig. 1. Schematic diagram of a 4π tight focusing system. APB, collimated azimuthally polarized beams; A1,2, axicons; D, a diaphragm; SPP (cyan rectangle), a spiral phase plate; BS, a beam splitter; M1-3, mirrors; PM1,2, parabolic mirrors; MO (blue rectangle), a magneto-optic film. \( \alpha_0 \) and \( \Delta \alpha \) represent the angular position and the angular thickness of the incident APBs, respectively. They are tunable by the axicons and the diaphragm, respectively.
with
\[ l_n(r, z) = \int_{α_0 - Δα/2}^{α_0 + Δα/2} ll(α)q(α)J_n(kr \sin α) \times \sin α \exp(i kz \cos α) dα, \]
(2)

where \( A \) is a constant related to wavelength \( λ \), \( k = 2π/λ \) is wave number, \( α_0 \) represents the angular position of the central ray of the incident APBs and \( Δα \) is the angular thickness of the incident beams. The function \( q(α) \) is apodization factor, obtained from energy conservation. For a PM, \( q(α) = 2/(1 - \cos α) \) \([25], [26]\). The function \( l(α) \) is the amplitude distribution of the incident beam and \( J_n \) denotes the nth-order Bessel function of the first kind. Then, the electric field near the confocal spot can be expressed as \([12], [13]\)
\[ \vec{E}(r, φ, z) = \vec{E}_1(r, φ, z) + e^{iφ} \vec{E}_2(-r, φ, -z), \]
(3)

where \( \vec{E}_1 \) and \( \vec{E}_2 \) denote the electric fields focused by the left PM and right PM, respectively. The negative signs of \( r \) and \( z \) in \( \vec{E}_2 \) indicate the opposite directions of instantaneous polarization and propagation with respect to \( \vec{E}_1 \). \( ϕ \) is the phase difference between two counter-propagating APBs. According to IFE \([7]–[13]\), the induced magnetization filed near the confocal spot is expressed as
\[ M = iγ \vec{E} × \vec{E}^*, \]
(4)

where \( γ \), which will be omitted in the following, is the magneto-optic constant, \( \vec{E}^* \) is conjugate of the electric field \( \vec{E} \). Substituting Eq. (1)–(3) into Eq. (4), the magnetization field can be expressed as
\[ \vec{M}(r, z) = M_z(r, z) \hat{z} \]
\[ = 4|A|^2 \left[ |l_1|^2 - |l_2|^2 + Re[e^{-iφ} (l_0^2 - l_2^2)] \right] \hat{z}. \]
(5)

where \( \hat{z} \) is the unit vector along \( z \)-axis. Based on narrow annular azimuthally polarized illumination (\( Δα ≪ α_0 \)), it is possible to further analyze Eq. (5). Since \( Δα ≪ α_0 \), only the change of the phase term needs to be considered in Eq. (2). Hence, \( l_n \) can be approximately expressed as
\[ l_n(r, z) ≈ lqJ_n \int_{α_0 - Δα/2}^{α_0 + Δα/2} \sin α \exp(i kz \cos α) dα \]
\[ = \frac{2lqJ_n}{kz} \sin (kz sin α_0 \sin \frac{Δα}{2}) \exp \left( ikz \cos α_0 \cos \frac{Δα}{2} \right). \]
(6)

where \( l = ll(α_0) \), \( q = q(α_0) \), \( J_n = J_n(κr \sin α_0) \). Eq. (6) is continuous in the limit \( z = 0 \). Then, the magnetization can be approximately expressed as
\[ M_z(r, z) ≈ M_0(r, z) \cos^2 \left( kz \cos α_0 \cos \frac{Δα}{2} \right) \]
(7)

with
\[ M_0 = \frac{32|A|^2 \rho^2 q^2 (J_0^2 - J_2^2)}{(kz)^2} \sin^2 \left( kz \sin α_0 \sin \frac{Δα}{2} \right). \]
(8)

\( M_0 \), in Eq. (7) and (8), is proportional to the magnetization needle field generated by one PM. Considering that \( Δα ≪ α_0 \), \( sin α_0 \sin \frac{Δα}{2} < \cos α_0 \cos \frac{Δα}{2} \) will be obtained as long as \( α_0 \) is not too close to \( \frac{π}{2} \). Thus, The magnetization chain field \( M_z \) can be regarded as a magnetization needle modulating a carrier term \( \cos^2(kz \cos α_0 \cos \frac{Δα}{2}) \). The magnetization needle \( M_0 \), the envelope of the magnetization chain, can be regarded a diffraction term by one PM. Meanwhile, the interference of two counter-propagating APVBs cause the carrier term.
3. Discussion

To check the approximation by Eq. (6), an axial profile at $r = 0$ and a lateral profile at $z = 0$ of magnetization chains obtained by different methods are shown in Fig. 2. The solid red line is generated from Eq. (1)~(4) by numerical integration; by contrast, the dashed green line is produced by Eq. (7) and (8). As is shown in Fig. 2, two methods lead to consistent results for the magnetization chain, which verifies the reliability of the approximation. It is noted that parameters ($\alpha_0 = 50^\circ \approx 0.8727$ rad, $\Delta \alpha = 0.12$ rad, $\varphi = \pi/4$, and $l(\alpha) = 1$) in Fig. 2 are not good for approximation, since a good approximation by Eq. (6) requires $\Delta \alpha \ll \alpha_0$. Therefore, the approximation will be more valid if a larger $\alpha_0$ and a smaller $\Delta \alpha$ are adopted. In addition, $\varphi = \pi/4$ is a general value, which means that it can be replaced by any other value. If $\varphi$ changes, the position of the maximum in Fig. 2(a) will change simultaneously. For example, the maximum will be at $z = 0$ if $\varphi = 0$.

According to Eq. (7) and Eq. (8), a magnetization chain is a magnetization needle modulating a carrier. Hence, it is reasonable to characterize the length of a chain with the longitudinal full width at half maximum (LFWHM) of a needle. With the help of Eq. (8), an analytical expression for the LFWHM can be found. It is obtained by twice the longitudinal coordinate $z$ that is solution of the equation $M_0(r, z) = \frac{1}{2}M_0(r, 0)$. Thus, the length of the chain can be expressed as:

$$L_{FWHM} \approx \frac{0.8743 \lambda}{\sin \alpha_0 \Delta \alpha}. \quad (9)$$

Hence, the length of the magnetization chain is inversely proportional to the angular width $\Delta \alpha$ and the sine value of the angle $\alpha_0$. As shown in Fig. 3, the length of the magnetization chain mainly changes with $\Delta \alpha$, especially when $\alpha_0 > 75^\circ$. Thus, the length of the magnetization chain is tunable by altering $\Delta \alpha$. In addition, $\alpha_0$ also needs to be considered when $\alpha_0 < 75^\circ$.

Similarly, the transverse full width at half maximum (TFWHM) of the magnetization chain can be calculated by solving the equation $M_0(r, z) = \frac{1}{2}M_0(0, z)$. The result is:

$$TFWHM \approx \frac{0.3498 \lambda}{\sin \alpha_0}. \quad (10)$$

Eq. (10) shows the TFWHM of the magnetization is only dependent on wavelength $\lambda$ and the focusing angle $\alpha_0$. Its minimum is $0.3498\lambda$ at $\alpha_0 = 90^\circ$. It increase by 3.5% at $\alpha_0 = 75^\circ$ and 41% at $\alpha_0 = 45^\circ$. It is indicated that a small lateral size of the magnetization requires a large $\alpha_0$ as shown in Fig. 4, which reflects an advantage of PMs compared with lens [26]. An interesting result is that
Fig. 3. LFWHM of a needle as a function of the angular position ($\Delta \alpha = 0.01$ rad) (a) and the angular thickness (b) of an annular APB. These lines are produced by Eq. (9).

The aspect ratio of the magnetization is independent of $\alpha_0$, but only dependent of $\Delta \alpha$, which may have special applications in magnetic storage and atomic trapping [10].

The diffraction term, $M_0$, describes the envelope of a magnetization chain, while the interference/carryer term characterizes every magnetization spot of the chain (Fig. 2(a)). It is easily calculated that $\text{Period} = \frac{\lambda}{2 \cos \alpha_0 \cos \frac{\Delta \alpha}{2}} \approx \frac{\lambda}{2 \cos \alpha_0}$ from Eq. (7). The expression presents that the period is tunable by changing $\alpha_0$. Moreover, the number of the magnetization spot can be estimated as:

$$N \approx \frac{1.7486}{\Delta \alpha \tan \alpha_0},$$

(11)

where $N$ is the number of the magnetization spots. The error of the estimation does not exceed 1.

To solve $\cos^2(kz \cos \alpha_0 \cos \frac{\Delta \alpha}{2} - \frac{\phi}{2}) = \frac{1}{2}$, the LFWHM of a magnetization spot is approximately equal to $\frac{\lambda}{4 \cos \alpha_0}$, precisely half of the period. Then, the aspect ratio (AR) of every magnetization spot of the chain can be expressed as:

$$AR \approx 0.7147 \tan \alpha_0.$$  

(12)

Eq. (12) provides that AR will increase as $\alpha_0$ increases. When $\alpha_0$ approaches 90°, the magnetization chain will vanish and become a magnetization needle. One interesting finding is that the product of AR and the number of the magnetization spots, $N$, is only dependent of $\Delta \alpha$. Once $\Delta \alpha$ is
By applying different $\alpha_0$, various magnetization chains can be obtained. If a large $\alpha_0$ is employed, multi-needles appear as shown in Fig. 5(a1) and (a2). Fig. 5(a1) shows a double-needle with $\varphi = \pi$ and Fig. 5(a2) shows a triple-needle with $\varphi = 0$. If the subsidiary maximum in Fig. 5(a1) is acceptable, a four-needle can be achieved. These magnetization fields can enhance the efficiency on the occasion where magnetization needles are needed. The ARs in the figures are both about 41. If PMs are replaced by aplanatic lenses, the ARs will be too small. This is because $\alpha_0 = 89^\circ$ means NA = 0.99985 in the air while NA = 0.95 is a big number for a lens. As shown in Fig. 5(b1) and (b2), when NA of lenses is 0.95, $AR \approx 2.2$, which is too little for practical applications [10]. If NA decreases, the result will be worse. The phase difference $\varphi$ decides where the principal maximum is.

When $\varphi = 0$, the principal maximum is at $z = 0$. As $\varphi (\neq \pi)$ grows, the principal maximum gradually deviates from the origin. In the extreme condition of $\varphi = \pi$, two principal maximums appears as shown in Fig. 5(a1).

If $AR = 1$, i.e., $\alpha_0 = 54.45^\circ$, a spherical magnetization chain, of which the diameter of every magnetization spot is 0.43$\lambda$, is obtained as shown in Fig. 6. The figure displays 18 spherical spots in which the maximum magnetization of the rightmost/leftmost spot is 0.9885. If the uniformity tolerance can be adjusted to 0.9, satisfactory spots will increase to 50. Further, the number of the spots will be $N \approx 125$ by Eq. (11) if half maximum is acceptable.
4. Conclusions

In summary, we propose the generation of a tunable ultra-long (about 100λ) magnetization chain by focusing annular APVBs based on a 4π system. All parameters of the chain, including the length and period of the chain, the AR and the number of the magnetization spots and the position of the maximum, are tunable by changing the angular position ω₀, the angular width Δω and the phase difference ψ. Besides, our conclusion is not only for parabolic mirrors but also for any aplanatic systems such as anaplastic lenses, since the apodization function q(ω) is not important.

Li Hang and Kaixi Huang contributed equally to this work.

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