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Jia-Sheng Li
Yong Tang
Zong-Tao Li, Member, IEEE
Jia-Xiao Chen
Xin-Rui Ding
Bin-Hai Yu

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Jia-Sheng Li, Yong Tang, Zong-Tao Li, Member, IEEE, Jia-Xiao Chen, Xin-Rui Ding, and Bin-Hai Yu

1Engineering Research Center of Green Manufacturing for Energy-Saving and New-Energy Technology, South China University of Technology, Guangdong 510640, China
2Foshan Nationstar Optoelectronics Company, Ltd., Foshan 528000, China

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Abstract: Phosphor-converted light-emitting diode (pcLED) is one of the most promising methods to generate white light. The interaction of the excitation light with the phosphor in pcLED is a complicated phenomenon, combining light absorption, light conversion, and scattering. An effective and efficient way to predict the optical behavior of pcLED is using the bidirectional scattering distribution function (BSDF). In this paper, the BSDF for pcLEDs with arbitrary phosphor concentration and thickness was established. The measured integrating energy and angular distributions of BSDF were successfully fitted by the interpolation method and the ABg model, respectively, realizing the calculation of BSDF values for arbitrary phosphor configurations. The proposed model was applied to a remote pcLED using an arbitrary phosphor configuration in Monte Carlo simulation. The output radiant power and the angular distributions of the blue and yellow light show good agreements between experiments and simulations; the errors of radiant power were all less than 4%, validating the effectiveness of the model. Consequently, this study provides a simple and useful tool to precisely predict the optical performance of pcLEDs with arbitrary concentration and thickness, it is essential for white LED design and fabrication.

Index Terms: Precise optical modeling, phosphor-converted white light-emitting diodes, bidirectional scattering distribution function, photoluminescence.

1. Introduction

Light-emitting diodes (LEDs) are regarded as the next-generation light source and have been broadly used in recent years attributed to their outstanding advantages such as high luminous efficiency, environmental friendliness, compact size, and long lifetime [1], [2]. There are mainly two methods to create white LED: one is to combine monochromatic LED chips with different colors (usually red, green, and blue), the other is to combine monochromatic LED chip with phosphor, namely, phosphor-converted LEDs (pcLEDs) [3]. The most promising configuration of pcLEDs is
the one using blue III-Nitride LED chip as pump excitation source and yellow YAG phosphor as luminary due to the high luminous efficiency of both [4], [5], [6]. The phosphor configuration will critically impact the optical performance of pcLEDs such in light output efficiency (LE), angular color uniformity (ACU), and color rendering index (CRI), etc. Thus, researchers have studied the effects of phosphor on pcLEDs numerically, in terms of the particles size [7], the concentration [8], the extinction coefficient [9], and especially the geometric morphology [10]–[12]. Most of these studies employ Monte Carlo method [13] to simulate light propagation inside the phosphor layer, where the optical properties of the phosphor are solved by the Mie scattering model [14]. However, the Mie model requires lots of microscopic parameters to be set during the simulation, some of which are rather difficult to acquire precisely. Moreover, the Mie model is an exact mathematical solution of the scattering property for the homogeneous spherical particle only, while there exists non-spherical particles in the phosphor layer [11], [15], [16]. Consequently, the simulation results of both the output blue and yellow light and their angular distribution may not accord well with the experiment, it is difficulty to precisely predict the light quality of pcLEDs.

To obtain the precise optical modeling for pcLEDs, Zhu et al. used a double integrating sphere system to investigate the radiant power of the transmitted and reflected light of a YAG phosphor layer illuminated by a fiber-guided source [17]. Wu et al. also measured the radiant flux of the transmitted and reflected light of series phosphor layers with a double integrating sphere system, and then used the result to optimize the Mie parameters in optical simulation [18]. Kang et al. [19] developed a 1-D model to calculate the light absorption and conversion in the phosphor layer and Hu et al. improved this model by further considering the scattering effect [20], [21]. But researches above did not pay attention to the angular distribution of the output light. Hung et al. [22] and Chen et al. [23] introduced the measured bidirectional scattering distribution function (BSDF) to characterize the optical properties of the phosphor layer. Their simulation results show an excellent agreement with the experiments in the intensity distribution of both blue and yellow light. Most importantly, the BSDF model can accurate simulate the light distributions without considering the light scattering and absorption processes in the phosphor layers, it is not limited by the shape of phosphor particles and can significantly save the simulation times compared with the bulk scattering method based on Mie scattering. Similarly, Huang et al. proposed a planar lighting system based on BSDF data [24]. Acuña et al. applied BSDF to a pcLED module and obtained a precise result of the output light power, and then further optimized the mixing cavity of the module [25], [26]. However, arbitrary phosphor concentration and thickness that can significantly influence the light scattering and absorption [27]–[29] are not considered in previous BSDF model, causing it difficult to be utilized in pcLEDs requiring various phosphor configurations.

In this study, the optical properties of YAG phosphor plates with series concentration and thickness are measured. The effect of concentration and thickness on the BSDF distributions were discussed. The measured results were used to establish the BSDF model considering phosphor with arbitrary concentration and thickness parameters within the research range. Finally, the model is applied to a cylindrical pcLED in simulation, verifying its accuracy and effectiveness.

2. Experiments

The commercial YAG:Ce3+ phosphor purchased from Intelmatix (YAG-04) with an average diameter of 13.7 μm is mixed with silicone. After stirring under a vacuum circumstance for 10 minutes, the silicone mixes with phosphor is cured at 150 °C for 3 hour to manufacture phosphor plates. The phosphor plates used in this paper has concentration ranging from 10% to 60% and thickness from 0.1 mm to 1.3 mm, as shown in Fig. 1.

Light scattering by phosphor in pcLED can be divided into three situations, i.e., blue light incident leading to blue light output (BIBO); blue light incident leading to yellow light output (BIYO); and yellow light incident leading to yellow light output (YIYO). It should be noticed that the BRDF data at incident angles around θi = −5° are approximately regarded as that at normal incident angles, which is due to the interference between the laser source and the detector head. The blue light is considered as the light emitted by the LED chip, and yellow light as the light converted by the
Fig. 1. Phosphor plates with concentration ranging from 10% to 60% and thickness from 0.1 mm to 1.3 mm.

Fig. 2. Measurement setup of the scattering characteristic.

phosphor. In the scattering measurement, we used lasers with the wavelengths of 450 nm and 550 nm as the blue and yellow light source respectively for simplification. These two wavelengths are selected because they are close to the peak emission wavelengths of the LED chip and the YAG: Ce phosphor. The scattering characteristics of the phosphor plates are measured with the equipment shown in Fig. 2. Its detection unit consists of a detector head connect with an optical fiber to a spectrometer. The center of the phosphor plate is irradiated by the blue laser beam and the yellow laser beam with different incident angles, respectively, and the radiant power of light escaping from this phosphor plate with different scattering angles $\theta_s$ are recorded. The output
power of lasers are 20 mW to avoid the thermal influence on phosphor plates during measurement [30]. Please note that the thermal effect on the refractive index [31], the emission spectra [32], and the scattering characteristics [33], [34] are not considered in this study.

3. Modeling Methods

3.1 BSDF Characterization of the Phosphor Plates

The scattering properties of the phosphor plates can be characterized with the BSDF, defined as:

$$\text{BSDF} (\theta_i, \phi_i, \theta_s, \phi_s) = \frac{dL_s(\theta_i, \phi_i, \theta_s, \phi_s)}{dE_i(\theta_i, \phi_i)} = \frac{dP_s(\theta_i, \phi_i, \theta_s, \phi_s)}{P_i(\theta_i, \phi_i) \cos \theta_s d\omega}$$  \hspace{1cm} (1)$$

where $\theta$, $\phi$ are the zenith and azimuth angle referring to the surface normal, $\omega$ the solid angle, $L$ the radiance, $E$ the irradiance, $P$ the radiant power. Subscripts $i$, $s$ mean incident light and scattered light respectively. BSDF can be further divided into forward scattering and backward scattering, which are called as bidirectional transmittance distribution function (BTDF) and bidirectional reflection distribution function (BRDF), respectively.

3.1.1 BSDF Characterization With the Light Source Incident at the Normal Direction: According to its definition, BSDF varies to the incident angle of the light source. However, since the spatial luminous intensity distribution of the blue LED chip is nearly Lambertian, which means the intensity is the strongest at normal direction, we chose $\theta_i = 0^\circ$ as the incident angle of the laser to acquire the most typical BSDF data. Please note that the interference is occurred between the light source and the detector when the laser is incident at $0^\circ$; we offset the incident angle to $-5^\circ$ for BRDF of BIBO and YIYO to obtain a more complete curve and take it as the result of normal incident.

Please note that two cases are discussed in this section as examples. One is that phosphor plates have the thickness of 0.3 mm and the concentration ranging from 10% to 60%, as shown in Fig. 3; the other is that phosphor plates with concentration of 20% and thickness ranging from 0.1 mm to 1.3 mm, as shown in Fig. 4. As for the former configuration, the BTDF of BIBO, BIYO, and YIYO at normal incident angle are shown in Fig. 3(a)–(c), respectively, while the BRDF of
BIBO, BIYO, and YIYO at normal incident angle are shown in Fig. 3(d)–(f), respectively. As for the later configuration, the BTDF of BIBO, BIYO, and YIYO at normal incident angle are shown in Fig. 4(a)–(c), respectively, while the BRDF of BIBO, BIYO, and YIYO at normal incident angle are shown in Fig. 4(d)–(f), respectively. As can be seen in Fig. 3(a) and Fig. 4(a), the BTDF curve of BIBO exhibits a distinct peak within the range of ±10° around the specular transmitted direction (i.e., $\theta_s = 0^\circ$), and values at other angles are relatively small. As the concentration and the thickness increase, the peak value of the curve decreases gradually to 0. This is because that the probability of blue light encountering the phosphor particles increases, leading to much more absorption events.

As can be seen in Figs. 3(b) and 4(b), the distribution angle range of BTDF curve of BIYO is distinctly broader compared with the blue light. Besides, the BTDF curve rises first and then declines with the increase of the thickness of the phosphor plates. This is because when the amount of phosphor reaches a certain level, the transmitted blue light is completely absorbed and the energy of the converted yellow light reaches the peak. After that, if the amount of the phosphor continues to increase, it will be more difficult for the yellow light to transmit forward, so the BTDF curve will decline. Moreover, it is worth noting that when the thickness (concentration) of the phosphor plate is small, the curve is relatively flat and even concave around the normal direction. As the thickness (concentration) increases, the curve becomes more and more steep, and eventually the shape of the curve approximates cosine distribution. This is because when the phosphor plate is relatively thin, it has less absorption to the blue light. When scattered at the angle of $0^\circ$, the absorption of the blue light is the least due to the shortest travel distance. As the scattering angle increases, the distance of the blue light traveling within the phosphor layer becomes longer, so the absorbed proportion of blue light is higher. Although the scattered blue light can be regarded as Lambertian distribution, the converted yellow light at larger angle could be more comparing with the normal direction after considering the absorbed proportion, which leads to the concave shape of the scattering distribution curve, as shown in Fig. 5(a). When the phosphor plate is thick enough, the blue light scattered at $0^\circ$ can also be absorbed adequately, resulting in the maximum converted yellow light. Thus, the scattering distribution curve approximates cosine distribution eventually, as shown in Fig. 5(b). Similar phenomena could be observed in low phosphor concentrations to high phosphor concentrations.
As evident from Figs. 3(c) and 4(c), the BTDF curve of YIYO exists a sharp peak around the specular transmission direction (i.e., $\theta_s = 0^\circ$) in the curve while the phosphor film is relatively thin. As the thickness increases, the peak gradually weaken and disappear, and the curve becomes smooth, uniform and eventually approximates cosine distribution. The reason to this phenomenon is that when the phosphor is less, the scattering to light is not complete, and part of the incident yellow light transmits through the phosphor layer directly without scattering. Thus the light at specular transmission direction has a high energy density, which represents as the peak in the curve. When the amount of phosphor increases, the scattering enhances and thus flattens the curve.

As shown in Figs. 3(d) and 4(d), the BRDF curves of BIBO of different plates are almost coincident with each other, which show a peak shape around the specular reflection with a peak value at $\theta_s = 5^\circ$. The data outside the peak shape are almost 0. We considered that the BRDF in the specular reflection direction was caused by the reflection of the surface of the phosphor plates. When the blue light is irradiated onto the surface of the phosphor plates, a part of blue light transmits into the phosphor layer, and then is absorbed, converted and scattered by the phosphor particles; the other part of the blue light is directly reflected back along the specular direction. The scattered blue light will suffer from secondary absorption and scattering when encountering the phosphor particles. Thus the energy of the scattered blue light outside the specular direction is quite limited to 0. However, the amount of reflection light is related to the incident angle and the refractive indexes of the two propagation media according to Fresnel reflection. For phosphor plates with different parameters, their refractive indexes are almost the same. Besides, the incidence angles of the laser source are all $-5^\circ$. As a result, the energy of the reflected blue light is basically the equal for different phosphor plates, which represents the overlap of the scattering distribution curves.

Figs. 3(e) and 4(e) show that the BRDF curves of BIYO are quite similar to the forward distribution curves of BIYO discussed above. The symmetric center is at the normal direction rather than the specular reflection direction (i.e., $\theta_s = 5^\circ$). This is because the yellow light scattering is mainly caused by the emitting of phosphor, which is similar to diffuse scattering. Thus, the scattering distribution is similar to Lambertian distribution eventually. Moreover, the BRDF curve rises with the increase of the thickness (concentration) of the phosphor plate because on one hand, the conversion and scattering of yellow light increase, and on the other, it is more difficult for the yellow light to transmit forward but easier backward.

The BRDF curve of YIYO depicted in Figs. 3(f) and 4(f) can be divided into two parts, one is the significant peak around the specular reflection direction (i.e., $\theta_s = 5^\circ$), and the other is the diffuse scattering except the peak value. The specular reflection part of the curve is identical with the curve of BIBO above, which is caused by the reflection of the surface of the phosphor plate. Thus, the curves of different plates are almost coincident with each other as well. The diffuse scattering part is caused by the scattering of the phosphor particles inside the plates. The curve rises with increase of the thickness (concentration) of phosphor, because backward scattering becomes stronger with the increase of phosphor. For example, the plate with the concentration of 20% and the thickness of 0.3 mm, the diffuse reflection part of the BRDF curve is successfully fitted by the cosine curve, as shown in Fig. 6. The result shows a great agreement between the
measured and fitted curves, so we regard the diffuse scattering part of the curve in BRDF of YIYO as cosine distribution. In addition, the BRDF values are closed to 0 near $\theta_s = 5^\circ$, this is due to the interference between the laser source and the detector.

3.1.2 BSDF Characterization With the Light Source Incident at Different Angles: After performing the BSDF measurements with the light source at a normal incident angle, we selected the phosphor plate with a thickness of 0.3 mm and a concentration of 20% to investigate the influence of the incident angle on the BSDF. The incident angles $-15^\circ$, $-30^\circ$, $-45^\circ$, $-60^\circ$ were studied for these measurements.

The BTDF and BRDF of BIBO at different incident angles is shown in Fig. 7(a) and (b), respectively. As the deviation of the incident angle from the normal direction increases, the BTDF curve will decline. This is caused by two reasons: First, an increase in the incident angle leads to an increase in the reflectance and a decrease in the transmittance according to Fresnel’s law. Next, the travel path of light inside the phosphor layer lengthens at larger incident angle, which enhances the chance of light being absorbed, and then lower the transmitted energy of the blue light. The shapes of BRDF curves with different incident angles are similar to each other, which all have a peak value in the specular reflection direction. As the incident angle increase, the curve rises and the reflected

![Fig. 6. Cosine fitting for the diffuse scattering part of BRDF of BIYO.](image)

![Fig. 7. (a) BTDF (b) BRDF of BIBO with different incident angles. The phosphor plate has a concentration of 20% and thickness of 0.3 mm.](image)
energy enhances, consistent with Fresnel law. According to our study above, we consider that the BRDF of BIBO is the same for different phosphor plates.

The BTDF and BRDF of BIYO at different incident angles is shown in Fig. 8(a) and (b), respectively. The curve shapes of the BSDF at different incident angles are almost the same, which are all symmetric about the normal axis. The curve declines as the incident angle increases, this is because that the transmitted blue light decreases, leading to a reduction in the converted yellow light.

The BTDF and BRDF of YIYO at different incident angles is shown in Fig. 9(a) and (b), respectively. The BTDF of YIYO is similar to the BTDF of the BIBO above. The specular reflection part of the BRDF of YIYO is identical to the BRDF of the BIBO, and curve of the diffuse scattering part almost remains the same at different incident angles.

3.2 BSDF Modeling the Phosphor Plates With Arbitrary Concentration and Thickness

3.2.1 BSDF Modeling With the Light Source Incident at the Normal Direction: The BTDF (BRDF) energy ratio refers to the ratio of the integrated energy of BTDF (BRDF) to the total incident energy. The BTDF energy ratio and BRDF energy ratio are defined by hemisphere transmittivity $T$ and
hemisphere reflectivity $R$, respectively:

\[
\begin{align*}
T &= \int_{2\pi} f_r(\theta_i, \phi_i, \theta_r, \phi_r) \cos \theta_r d\omega_r \\
R &= \int_{2\pi} f_t(\theta_i, \phi_i, \theta_t, \phi_t) \cos \theta_t d\omega_t
\end{align*}
\]

(2)

where $f_r$, $f_t$ are the value of BRDF and BTDF, and subscripts $r$, $t$ represent reflection and transmission respectively. In this section, we build a calculation model for the energy ratio ($T$ and $R$) and take the normal-incident condition as an example. This model is based on the interpolation method and the concentration $x$ and thickness $y$ of the phosphor plate are used as independent variables. Through the fitting curve equations with determined interpolation coefficients, the energy ratio of phosphor plate with arbitrary parameters can be acquired.

Fig. 10(a) shows the BTDF energy ratio of BIBO $T_{BIBO}$ obtained by integrating according to equations (2). The curved surface equation of the BTDF energy ratio of BIBO is then fitted. The fitting surface is shown in Fig. 10(b), and its equation is (3) with a determination coefficient $R^2$ of 0.9983.

\[
T_{BIBO}(x, y) = 0.10240 - \frac{0.05056}{x} - \frac{0.07604}{y} + \frac{0.00391}{x^2} + \frac{0.00621}{y^2} + \frac{0.03146}{xy} - \frac{5.04593 \times 10^{-5}}{x^3} - \frac{1.04807 \times 10^{-4}}{y^3} - \frac{0.11255}{xy^2} - \frac{0.00118}{x^2y}
\]

(3)

The BRDF curves of BIBO for phosphor plates with different concentration and thickness parameters are coincident as shown in Figs. 3(d) and 4(d). Therefore, the BRDF curves of BIBO are the same, and there is no need to analyze their energy ratios for different phosphor configurations.

Fig. 10(a) shows the BTDF energy ratio of BIYO $T_{BIYO}$ obtained by integrating according to equations (2). The curved surface equation of the BTDF energy ratio of BIYO is then fitted. The fitting surface is shown in Fig. 10(b), and its equation is (3) with a determination coefficient $R^2$ of 0.9983.

\[
T_{BIYO}(x, y) = 0.14565 + 1.43543x - 2.71810x^2 + 2.04896x^3 + 0.09981 \ln y + 1 + 0.74259x - 2.07370x^2 + 1.78447x^3 + 0.03899 \ln y
\]

(4)

The BRDF curves of BIYO for phosphor plates with different concentration and thickness parameters are coincident as shown in Figs. 3(d) and 4(d). Therefore, the BRDF curves of BIYO are the same, and there is no need to analyze their energy ratios for different phosphor configurations.

The BTDF energy ratio of BIYO $T_{BIYO}$ is shown in Fig. 12. As can be seen, the curves of different concentrations are inconsistent with each other, which is inconvenient for surface fitting. Therefore, we add the energy of BRDF and BTDF together to investigate the total energy ratio of the BSDF of BIYO $S_{BIYO}$. The result shown in Fig. 13(a) reveals a good similarity between the curves of different concentrations. The fitting curved surface of the energy ratio of the BSDF of BIYO is shown in

---

Fig. 10. (a) BTDF energy ratio of BIBO and (b) its fitting curved surface for arbitrary concentration and thickness.
Fig. 11. (a) BRDF energy ratio of BIYO and (b) its fitting curved surface for arbitrary concentration and thickness.

Fig. 12. BTDF energy ratio of BIYO.

Fig. 13. (a) BSDF energy ratio of BIYO and (b) its fitting curved surface for arbitrary concentration and thickness.
Fig. 14. (a) Peak value and (b) its fitted curved surface of the diffuse scattering part of BRDF for arbitrary concentration and thickness.

From the discussion above in Figs. 3(f) and 4(f), we know that the BRDF of the YIYO can be divided into specular reflection part and diffuse scattering part. Its specular reflection part is identical to that of the BRDF of BIBO, which is constant and determined by the Fresnel reflection. However, the diffuse scattering part is various at different phosphor configurations, exhibits a cosine distribution as shown in Fig. 6, i.e., \( f_r = a \cos \theta_s \) where \( a \) is the peak value of the curve. Consequently, the diffuse scattering part of BRDF of YIYO is fitted with cosine distribution curve to obtain the values of \( a \) for different phosphor plates, as shown in Fig. 14(a). The distribution of the peak values \( a \) is fitted into a curved surface as shown in Fig. 14(b). The fitting equation is as (6) with a determination coefficient \( R^2 \) of 0.9988.

\[
a(x, y) = 0.14625 + 0.01403 \ln x - \frac{0.01402}{y} - \frac{0.00400 \ln^2 x}{y^2} + \frac{0.00203 \ln y}{y} + 0.00685 \ln x - 0.00054 \ln^3 x - \frac{0.00011 \ln x}{y^3} - \frac{0.00022 \ln y}{y^2} + \frac{0.00128 \ln^2 x}{y} \quad (6)
\]

Fig. 13(b), the fitting equation is as (5) with a determination coefficient \( R^2 \) of 0.9964. Once we calculate the energy ratios of BRDF and BSDF of BIYO using (4) and (5), the energy ratio of BTDF can be obtained by \( S_{\text{BIYO}} - R_{\text{BIYO}} \).

\[
S_{\text{BIYO}}(x, y) = \frac{0.55380 + 0.24464 \ln x + 0.19919 \ln y + 0.03066 \ln^2 x + 0.01878 \ln^2 y + 0.04306 \ln x \ln y}{1 + 0.47111 \ln x + 0.39317 \ln y + 0.07446 \ln^2 x + 0.04872 \ln^2 y + 0.11073 \ln x \ln y} \quad (5)
\]

\[
T_{\text{YIYO}}(x, y) = -0.26256 + \frac{0.29729}{x} - 0.24421 \ln y - \frac{0.04306}{x^2} - 0.01966 \ln^2 y + \frac{0.01086 \ln y}{x} + \frac{0.00214}{x^3} - 2.5534 \times 10^{-4} \left( \ln^3 y + \frac{\ln^2 y}{x} + \ln y \right) \quad (7)
\]

Consequently, the integrated BTDF and BRDF energy for different phosphor concentration and thickness can be achieved once the total incident energy is given. This means that the total energy of forward emission blue light, backward emission blue light, forward emission yellow light, backward emission yellow light are determined after inputting the total incident energy of blue light.
3.2.2 BSDF Modeling With Light Source Incident at Different Angles: The BRDF of BIBO for different phosphor plate are the same, just as shown in Fig. 7(b). Besides, the specular reflection part for the BRDF of YIYO is identical with the BRDF of BIBO, and the diffuse scattering part does not change with the incident angle. Thus, there is no need to model the BSDF of these two cases (BRDF of BIBO, BRDF of YIYO) for different incident angles.

The symmetry of the BSDF curve decreases gradually with increasing incident angles, as shown in Figs. 7 and 9. Therefore, we use the shift-invariant scattering model (or Harvey-Shack model) to characterize the variation in BSDF at different incident angles. For many optical surfaces, the curve of BSDF will not change in shape but will only shift in distribution position, when the scattering direction vectors are used instead of the scattering angle as variables for BSDF [35]. If we denote the projection of the unit vector in the scattering direction on the sample surface as $\hat{\beta}$, and the projection of the unit vector in the specular direction as $\hat{\beta}_0$, then:

$$f_s = \frac{A}{B + |\hat{\beta} - \hat{\beta}_0|} A, B, g \geq 0$$  \hspace{1cm} (8)

where $A$, $B$, and $g$ are parameters that can be used to fit the formula of the measured BSDF data. From the form of the formula, we can see that the parameter $A$ is equivalent to a proportional coefficient, which only determines the height or energy of the BSDF curve but not changing the shape of the BSDF curve. Thus, as long as we fit the $ABg$ parameters using the BSDF data of the normal incident direction, the parameters of other incident angles can be obtained by calculating $A$ value according to the multiple relations between the energy of BSDF, while remaining the values of $B$ and $g$ unchanged. This relation can be simply defined by the ratio of $T_{BIBO}$ at arbitrary $\theta_i$ to $T_{BIBO}$ at $\theta_i$ of 0°, which refers to $n$. As a result, the fitted $A$ at arbitrary $\theta_i$ can be achieved by the product of the $n$ and the fitted $A$ at $\theta_i$ of 0°. We also use the phosphor plate with a concentration of 20% and thickness of 0.3 mm as an example. The fitted parameters of BTDF curve of BIBO at normal incident direction in Fig. 7(a) are $A = 0.02147$, $B = 0.04095$, $g = 1.2223$. Accordingly, Table 1 shows their fitted $A$ values of different incident angles. The fitted and the measured BTDF curves achieved by a phosphor plate with a concentration of 20% and thickness of 0.3 mm are shown in Fig. 16, revealing a great agreement.

Fig. 8 shows that the distributions of BSDF of BIYO are almost the same at different incident angles, which can be directly used without ABg fitting. Instead, the BSDF curves at arbitrary $\theta_i$ can be achieved by scaling the curve of the normal incident according to the multiple relations of the BSDF energy ratios. The energy ratios and their multiple relations of BTDF and BRDF are shown in Tables 2 and 3, respectively.
TABLE 1
Energy Ratios, Multiple Relations, and Fitted A Values of BTDF of BIBO at Different Incident Angles. The Phosphor Plate has a Concentration of 20% and Thickness of 0.3 mm.

<table>
<thead>
<tr>
<th>Incident angle $\theta_i$</th>
<th>0°</th>
<th>-15°</th>
<th>-30°</th>
<th>-45°</th>
<th>-60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy ratios $T_{BIBO}$</td>
<td>0.1135</td>
<td>0.1074</td>
<td>0.0901</td>
<td>0.0663</td>
<td>0.0486</td>
</tr>
<tr>
<td>Multiple relations $n$</td>
<td>1.0</td>
<td>0.9462</td>
<td>0.7930</td>
<td>0.5836</td>
<td>0.4279</td>
</tr>
<tr>
<td>Fitted $A$ values</td>
<td>0.02147</td>
<td>0.02031</td>
<td>0.01703</td>
<td>0.01253</td>
<td>0.00919</td>
</tr>
</tbody>
</table>

![BTDF curves of BIBO](image)

Fig. 16. ABg fitting result of BTDF of BIBO at different incident angles. The phosphor plate has a concentration of 20% and thickness of 0.3 mm.

TABLE 2
Energy Ratios and Their Multiple Relations of BTDF of BIYO at Different Incident Angles. The Phosphor Plate has a Concentration of 20% and Thickness of 0.3 mm.

<table>
<thead>
<tr>
<th>Incident angle $\theta_i$</th>
<th>0°</th>
<th>-15°</th>
<th>-30°</th>
<th>-45°</th>
<th>-60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy ratios $T_{BIYO}$</td>
<td>0.2235</td>
<td>0.2227</td>
<td>0.2182</td>
<td>0.2088</td>
<td>0.1884</td>
</tr>
<tr>
<td>Multiple relations $n$</td>
<td>1.0</td>
<td>0.9964</td>
<td>0.9763</td>
<td>0.9342</td>
<td>0.8430</td>
</tr>
</tbody>
</table>

TABLE 3
Energy Ratios and Their Multiple Relations of BRDF of BIYO at Different Incident Angles. The Phosphor Plate has a Concentration of 20% and Thickness of 0.3 mm.

<table>
<thead>
<tr>
<th>Incident angle $\theta_i$</th>
<th>-5°</th>
<th>-15°</th>
<th>-30°</th>
<th>-45°</th>
<th>-60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy ratios $R_{BIYO}$</td>
<td>0.2147</td>
<td>0.2134</td>
<td>0.2093</td>
<td>0.2017</td>
<td>0.1826</td>
</tr>
<tr>
<td>Multiple relations $n$</td>
<td>1.0</td>
<td>0.9939</td>
<td>0.9748</td>
<td>0.9395</td>
<td>0.8505</td>
</tr>
</tbody>
</table>

Similar to BTDF curves of BIBO, the BTDF curves of YIYO at normal incident angle can also be fitted with ABg model and the fitted result is $A = 0.09537$, $B = 0.12016$, $g = 1.2865$. Table 4 shows the multiple relations of the BSDF energy ratios and the fitted $A$ values at other incident angles. The fitted and the measured BTDF curves are shown in Fig. 17.

Consequently, the BSDF curves at arbitrary incident angles for phosphor plates with various configurations can be achieved according to the above ABg fitting method.
TABLE 4
Energy Ratios, Multiple Relations, and Fitted $A$ Values of BTDF of YIYO at Different Incident Angles.
The Phosphor Plate has a Concentration of 20% and Thickness of 0.3 mm

<table>
<thead>
<tr>
<th>Incident angle $\theta_i$</th>
<th>0°</th>
<th>-15°</th>
<th>-30°</th>
<th>-45°</th>
<th>-60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy ratios $T_{YIYO}$</td>
<td>0.6618</td>
<td>0.6503</td>
<td>0.6269</td>
<td>0.5868</td>
<td>0.5307</td>
</tr>
<tr>
<td>Multiple relations $n$</td>
<td>1.0</td>
<td>0.9827</td>
<td>0.9473</td>
<td>0.8468</td>
<td>0.7620</td>
</tr>
<tr>
<td>Fitted $A$ values</td>
<td>0.09537</td>
<td>0.09372</td>
<td>0.09344</td>
<td>0.08076</td>
<td>0.07267</td>
</tr>
</tbody>
</table>

Fig. 17. ABg fitting result of BTDF of YIYO at different incident angles. The phosphor plate has a concentration of 20% and thickness of 0.3 mm.

Fig. 18. (a) The remote phosphor LED lamp and its (b) schematic diagram and (c) blue source.

4. Model Validation
We apply the proposed BSDF model to a remote phosphor LED lamp as shown in Fig. 18. The light source is composed of 9 blue LEDs mounted on an aluminum core print circuit board (ACPCB), with a working current at 350 mA. We model the LED light in the commercial software of TracePro, setting its inner surfaces as mirror material with a reflectivity of 0.98. The reflective index of the silicone lens is set to 1.54, and the reflectivity of the aluminum board is set to 0.90.

The phosphor plate with concentration of 20% and thickness of 0.25 mm is selected and then applied to the LED lamp. Please note that this is an arbitrary phosphor configuration that has not been used in BSDF modeling. The BSDF curves of this phosphor plate is obtained by the BSDF model established in this paper, as shown in Fig. 19.

Then we set the calculating BSDF data as the surface scattering property of the phosphor plate in TracePro. The total radiant flux of the blue source is set as 4.041 W with a Lambertian
Fig. 19. Calculating BSDF of Phosphor plate with concentration of 20% and thickness of 0.25 mm (a) BIBO (b) BIYO (c) YIYO.

TABLE 5
Radiant Power of Blue and Yellow Light of the Remote Phosphor LED Lamp With Concentration of 20% and Thickness of 0.25 mm

<table>
<thead>
<tr>
<th>Radiant powers (W)</th>
<th>Blue</th>
<th>Yellow</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured values</td>
<td>0.655</td>
<td>1.428</td>
<td>2.093</td>
</tr>
<tr>
<td>Simulated values</td>
<td>0.648</td>
<td>1.375</td>
<td>2.023</td>
</tr>
<tr>
<td>Errors</td>
<td>1.07%</td>
<td>3.71%</td>
<td>3.34%</td>
</tr>
</tbody>
</table>

Fig. 20. Spatial angular distributions of the blue (B) and yellow (Y) light for remote phosphor LED lamp with concentration of 20% and thickness of 0.25 mm.

intensity distribution, and the number of rays is set to 1 million to ensure an adequate simulation accuracy. Furthermore, the wavelengths of the blue and yellow light are set as 450 nm and 550 nm respectively. The simulated radiant powers are shown in Table 5, comparing with the measured values. As can be seen, the errors between the simulated and measured data are within 4%, indicating a high accuracy of our simulation method. Moreover, to further verify our model, the spatial angular distributions of the blue and yellow light are demonstrated both numerically and experimentally as shown in Fig. 20. The simulation result shows a quite good agreement with the experimental one. Consequently, this method can effectively predict the optical power and angular
light distributions for pcLEDs with arbitrary phosphor concentration and thickness, which is essential for white LED design and fabrication.

5. Conclusion

In this paper, a BSDF calculation model for phosphor plate with arbitrary parameters of thickness and concentration is established. The BSDFs of the YAG: Ce phosphor plates with various concentration and thickness were measured under different scattering conditions. We find that the BSDF integral energy of different phosphor thickness and concentration can be fitted using interpolation method. However, the BRDF integral energy of blue light at BIBO keeps constant for different phosphor thickness and concentration, which can be calculated by the Fresnel reflection law. Moreover, the BSDF of BIYO is almost the same for different incident angles, and the BRDF of BIBO and YIYO for different incident angles can be obtained by the Fresnel reflection law. Therefore, only the BTDF of BIBO and YIYO varies from different incident angles, which is successfully calculated using ABg model. According to the proposed BSDF model, a Monte Carlo ray-tracing simulation is performed to achieve the optical performance of the LED lamp using a phosphor plate with arbitrary parameters. Compared with experimental results, the errors of the simulated radiant powers were all less than 4%, and the spatial distributions can be predicted with high accuracy in simulation. Consequently, this study provides a useful tool to precisely predict the optical power and light distributions for pcLEDs with arbitrary concentration and thickness, which is essential for white LED design and fabrication.

References