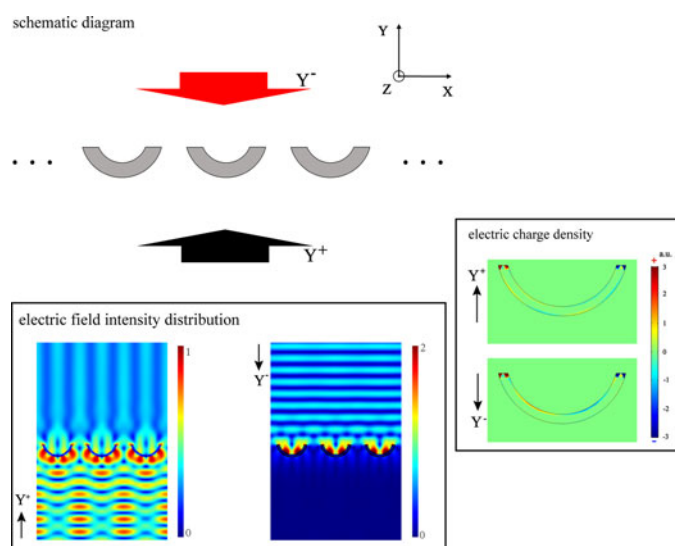


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# Unidirectional Optical Transmission in a Single-Layer Metallic Grating Consisting of Cambered Resonators

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**Abstract:** In order to realize unidirectional optical transmission (UOT), a single-layer metallic grating (SMG) consisting of cambered resonators (CRs) is proposed. Numerical simulation results show that the transmittance contrast ratio of the SMG can reach about 5000 with a high unidirectional transmittance of 39%. UOT results from diffraction enhancement or suppression in transmission direction. Diffraction is manipulated by the interference of electromagnetic radiation from local surface plasmons on CRs. Our single-layer structure has sub-100 nm thickness and can be used in ultracompact optical devices to manipulate light transmission.

**Index Terms:** Surface plasmons, unidirectional transmission, metallic grating.

## 1. Introduction

Unidirectional optical transmission (UOT) [1] means that light can transmit in one direction and is blocked in the reverse direction. The roles of UOT devices in photonic circuits are similar to that of electronic diodes in electronic circuits. UOT can be classified into two types. One runs counter to Lorentz reciprocity and the other follows reciprocity theory [2]. In the first type, the use of Magneto-optical [3], [4], liquid crystals [5], [6] or time-dependent [7] media inevitably comes up with an external either magnetic or electric field, which usually requires a precise and reliable control over the position, magnitude and direction of the field. Similarly, nonlinear media [8], [9] calls for high intensity. The second type is made of isotropic and linear materials and takes photonic crystals [10]–[12], gratings [13]–[15], and grating-photonic crystal [16], [17] as universal structures.

In the recent years, metallic micro/nano structures are popular and widely studied because surface plasmons (SPs) [18], i.e., oscillating electrons patterns, of metal allow for manipulating electromagnetic wave in nano-scale. Such as the insertion of metal slab in dielectric gratings [19], [20], asymmetric dual-gratings-structure [21], [22], metamaterials [23], [24], metallic slots [25] and slit [26], many novel structures are gradually emerging and used to realize UOT. Among them,

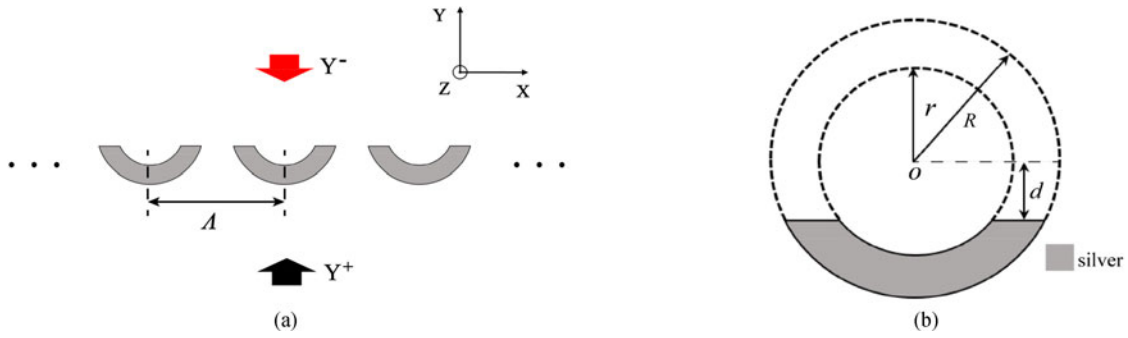


Fig. 1. Schematic of the (a) SMG and the geometry of a single (b) CR.

most structures are compounded of multiple layers and need special alignment of different layers. Single-layer structure is rare and the UOT performance of a single metal film [27] needs to be improved.

In this paper, a single-layer metallic grating (SMG), consisting of periodic cambered resonators (CRs), is proposed to realize UOT. The simulation results show that the transmittance contrast ratio between two contrary incident directions can reach about 5000, with a high unidirectional transmittance of 39%. It suggests that the SMG is a good component in UOT devices. In our work, it is demonstrated that the UOT results from enhanced or suppressed diffraction in transmission direction of the SMG. The difference in diffraction intensities is caused by the interference of electromagnetic radiation from local surface plasmons (LSPs) on CRs.

## 2. Structure and Parameters Definitions

Fig. 1(a) is the diagram of the proposed SMG, composed of periodic CRs. Fig. 1(b) illustrates the geometry of a CR. The CR is minor arc as a portion of a concentric ring. The inner and outer radii are denoted as  $R$  and  $r$ , respectively. The separation between the flat kerfs and the center of the ring is signed by  $d$ . The period, i.e., the grating constant of the SMG, is denoted by  $\Delta$ . The CRs are made of silver and supposed standing freely in the air. The frequency-dependent complex relative permittivity of silver is characterized by the Drude model [28]:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}. \quad (1)$$

Here, the plasma resonance frequency  $\omega_p$  is  $1.37 \times 10^{16}$  rad/s and the electrons characteristic collision frequency  $\gamma$  is  $3.21 \times 10^{13}$  Hz.  $\omega$  represents the angular frequency of the incident beam. The values of  $\omega_p$  and  $\gamma$  are chosen to fit with the experimental data of Johnson and Christy [29].

The incident light is transverse magnetic (TM) plan wave.  $Y^+$  ( $Y^-$ ) represents the incident light along positive (negative) direction of y-axis. The transmittance is

$$T = \frac{P_0}{P_i}, \quad (2)$$

where  $P_0$  is the output power and  $P_i$  is incident power.  $T^+$  and  $T^-$  represent the transmittance of a SMG for  $Y^+$  and  $Y^-$ , respectively. The transmittance contrast ratio  $\eta$  is used to describe the UOT performance and defined as:

$$\eta = \frac{T^+}{T^-}. \quad (3)$$

In our UOT research, we seek for the cases of  $\eta \gg 1$  or  $\eta \ll 1$ , while  $T^+$  or  $T^-$  has a relatively large value.

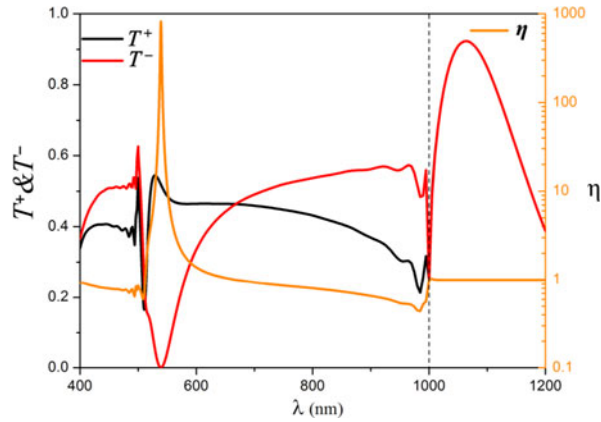


Fig. 2.  $T^+$ ,  $T^-$  and  $\eta$  of a SMG with  $r = 300$  nm,  $R = 350$  nm,  $d = 80$  nm and  $\Lambda = 1000$  nm.

The proposed SMG is numerically simulated by COMSOL Multiphysics, which can accurately calculate the electromagnetic interaction between light and matter based on Finite Element Method [30].

### 3. Numerical Simulation Results and Analyses

Fig. 2 shows  $T^+$ ,  $T^-$  and  $\eta$  of a SMG with  $r = 300$  nm,  $R = 350$  nm,  $d = 80$  nm and  $\Lambda = 1000$  nm. A critical wavelength  $\lambda_c$  is marked by a vertical black dash line and it equals the grating constant  $\Lambda$ . The transmission spectrum can be separated into two regions according to  $\lambda_0$ . When  $\lambda > \lambda_c$ ,  $T^+ = T^-$  and thus  $\eta = 1$ . By contrast, when  $\lambda \leq \lambda_c$ ,  $T^+ \neq T^-$  and  $\eta$  changes non-monotonously with the decrease of  $\lambda$ .  $\eta$  reaches the minimum 0.4406 at 985 nm due to  $T^+ = 0.21$  and  $T^- = 0.48$ , and reaches its maximum 819.7 at 539 nm while  $T^+ = 0.52$  and  $T^- = 6.3 \times 10^{-4}$ . So, the SMG realizes perfect UOT for the case of  $\eta \gg 1$ . In the following discussion,  $\lambda$  is fixed between 500 nm to 650 nm.

The diffraction of a grating is produced by the interference between grating cells and follows

$$m\lambda_0 = \Lambda(n_o \sin \theta_m - n_i \sin \theta_i), \quad (4)$$

where  $m = 0, \pm 1, \pm 2, \dots$ ,  $\lambda_0$  is free-space wavelength.  $n_o$  and  $n_i$  are the refractive indexes at output- and input-side of light, respectively.  $\theta_i$  is incidence angle and  $\theta_m$  is angle of  $m$ -order diffraction. For our simulation, Eq. (4) can be simplified as

$$m\lambda = \Lambda \sin \theta_m, \quad (5)$$

because of  $n_o = n_i = 1$ ,  $\lambda = \lambda_0$  and  $\theta_i = 0$ . Eq. (5) shows  $\theta_m$  is dependent on the period of grating cells. However, the efficiency of  $m$ -order diffraction is dependent on concrete size and shape of the resonator in a grating cell.

Actually, it is the non-reciprocal diffraction that leads to the UOT of the SMG. Transmittance, in Eq. (2), is calculated by integrating the Poynting vector on an observation line in our simulation and the energies of forward-propagating diffraction waves are included in. The diffraction intensities in transmittance are further calculated using diffraction ports. Fig. 3 shows the diffraction intensities of a SMG with the same geometry parameters in Fig. 2. When  $\lambda$  is between 500 nm and 650 nm, 0, +1 and -1 order diffractions exist. 0-order transmission is represented by  $T_0^+$  for  $Y^+$  and  $T_0^-$  for  $Y^-$ . +1 and -1 order diffraction intensities are equal because of the vertically incidence and are represented by a common symbol  $T_1^+$  for  $Y^+$  ( $T_1^-$  for  $Y^-$ ). In Fig. 3,  $T_0^+ = T_0^-$  at any  $\lambda$ , but  $T_1^+ \neq T_1^-$  because of non-reciprocal diffraction. The transmittance calculated by integral is the sum of forward diffractions. Therefore, the expansion of Eq. (3) is  $\eta = (T_0^+ + 2T_1^+)/ (T_0^- + 2T_1^-)$ . The necessary condition for  $\eta \gg 1$  is  $T_1^+ \gg T_1^-$  while  $T_0^+ = T_0^- \approx 0$ . In Fig. 3, when  $\lambda$  is 539 nm,  $T_0^+ = T_0^- \approx 0$  implying the incident light is totally reflected backward and absorbed by CRs.

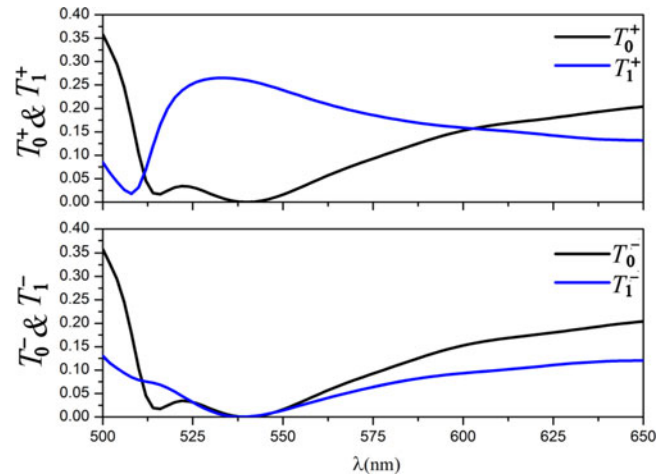


Fig. 3. The intensities of 0 and 1 order diffraction for  $Y^+$  and  $Y^-$ . Here,  $r = 300$  nm,  $R = 350$  nm,  $d = 80$  nm and  $\Delta = 1000$  nm.

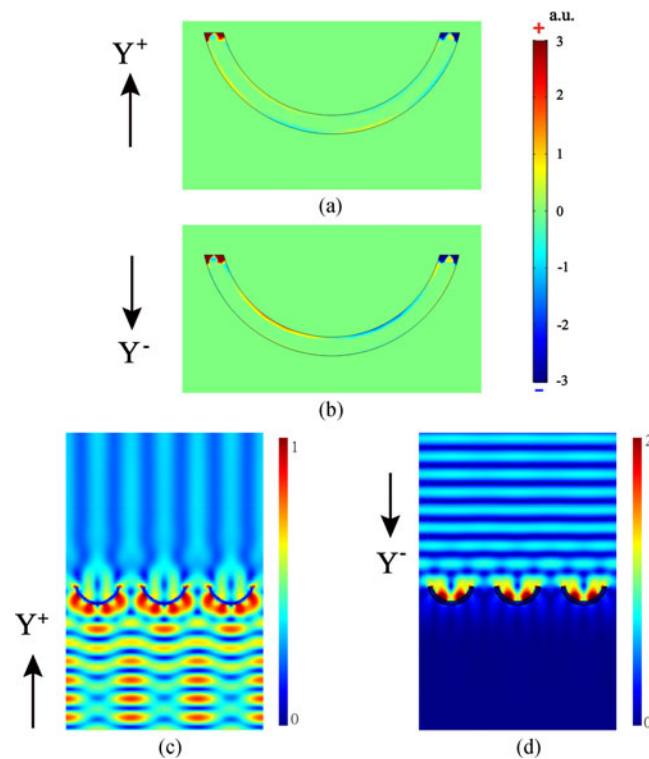


Fig. 4. The charge distributions on a CR and electric field intensity distributions around the SMG. (a) and (c) are for  $Y^+$ . (b) and (d) are for  $Y^-$ . Here,  $\lambda$  is 539 nm,  $r = 300$  nm,  $R = 350$  nm and  $\Delta = 1000$  nm, and  $d = 80$  nm.

Meanwhile,  $T_1^+ = 0.26$  and  $T_1^- = 2.7 \times 10^{-4}$ . It is corresponding to  $T^+ = 0.52$  and  $T^- = 6.3 \times 10^{-4}$  at  $\lambda = 539$  nm in Fig. 2.

The SMG periodically modulates the amplitude and phase of the incident light and splits the incident wave into diffraction waves traveling forward. LSP resonances on CRs lead to field amplification in the near-field zone and further influence the interference between adjacent CRs which

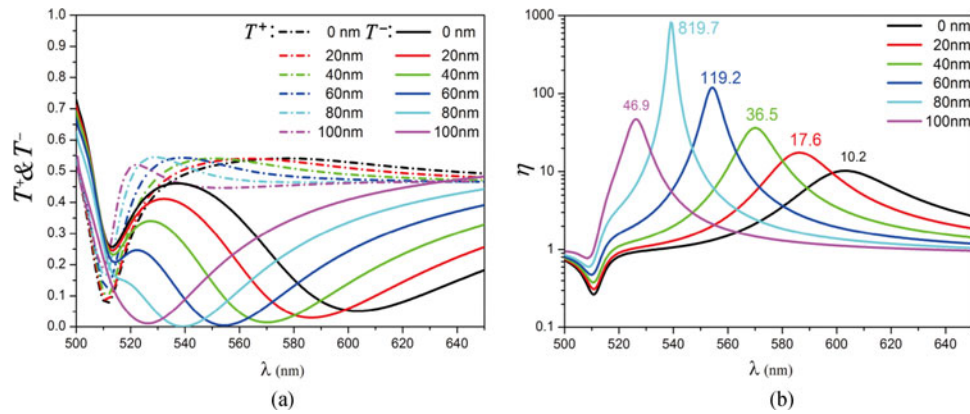


Fig. 5. The influence of  $d$  on (a)  $T^+$ ,  $T^-$  and (b)  $\eta$ . Here,  $r = 300$  nm,  $R = 350$  nm,  $\Lambda = 1000$  nm and  $d = 0$  nm, 20 nm, 40 nm, 60 nm, 80 nm and 100 nm, respectively.

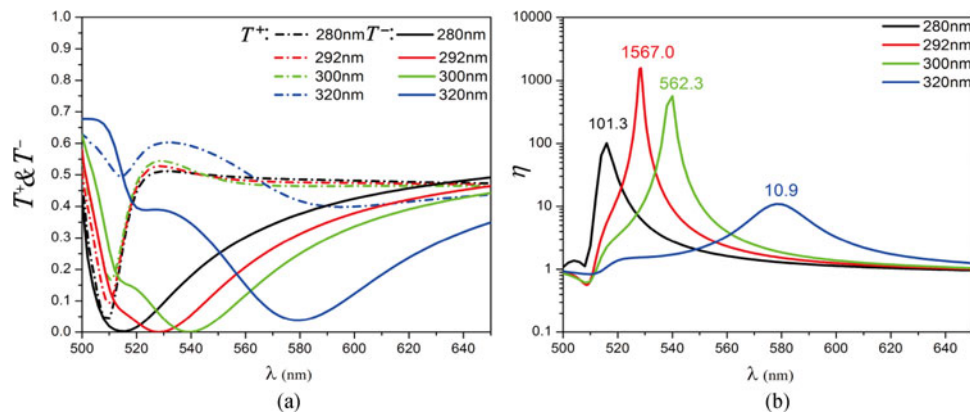


Fig. 6. The influence of  $r$  on (a)  $T^+$ ,  $T^-$  and (b)  $\eta$ . Here,  $R = 350$  nm,  $\Lambda = 1000$  nm  $d = 80$  nm and  $r = 280$  nm, 292 nm, 300 nm and 320 nm, separately.

plays an important role for light transmission. Fig. 4(a) and (b) show the charge distributions on a CR at  $\lambda = 539$  nm for  $Y^+$  and  $Y^-$ , respectively. The corresponding electric field intensity distributions are shown in Fig. 4(c) and (d). The LSPs are produced on CRs through the interaction between CRs and incident light. Charge distributions show that the excitations efficiency of LSP for the same surface of a CR are different for  $Y^+$  and  $Y^-$ . The electric field distributions indicate that the interference of electromagnetic radiation from LSPs on the CRs is important for light transmission. For  $Y^+$ , the inner arc and two flat kerfs of CRs act as the exit surface. LSPs at the exit surface are obvious in Fig. 4(a) and the electromagnetic radiation from LSPs on the exit surface leads to much light is diffracted as  $\pm 1$  order which explain for the periodic intensity distributions along forward direction in Fig. 4(c). For  $Y^-$ , the outer arc of the CR acts as the exit surface and LSPs existing on the outer arc are weak seen from Fig. 4(b). Contrarily, LSPs on the entrance surface (the inner arc and two flat kerfs) are strong, leading to high reflectance and low transmittance. So, it is the peculiar geometrical shape and the radiation of LSPs of CRs that modulates the diffraction intensity and result in UOT of the SMG. The interference of electromagnetic radiation from LSPs that making 0 order transmission suppressed and  $\pm 1$  order diffractions enhanced or suppressed.

CRs surface topography influences the LSPs resonances and modulates the interference of electromagnetic radiation from LSPs. The difference between upper and lower surface topography of CRs makes the electromagnetic radiation interference different, leading to UOT. The influence of geometric parameter  $d$  on UOT is investigated. We keep  $r = 300$  nm,  $R = 350$  nm and  $\Lambda = 1000$  nm.  $T^+$ ,  $T^-$ , and  $\eta$  are displayed in Fig. 5 for  $d = 0$  nm, 20 nm, 40 nm, 60 nm, 80 nm and



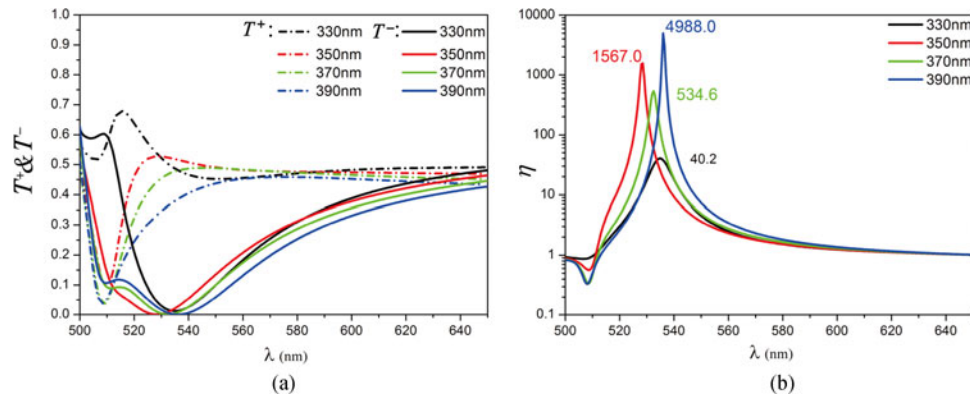


Fig. 7. The influence of  $R$  on (a)  $T^+$ ,  $T^-$  and (b)  $\eta$ . Here,  $r = 292$  nm,  $\Lambda = 1000$  nm,  $d = 80$  nm and  $R = 330$  nm,  $350$  nm,  $370$  nm and  $390$  nm, separately.

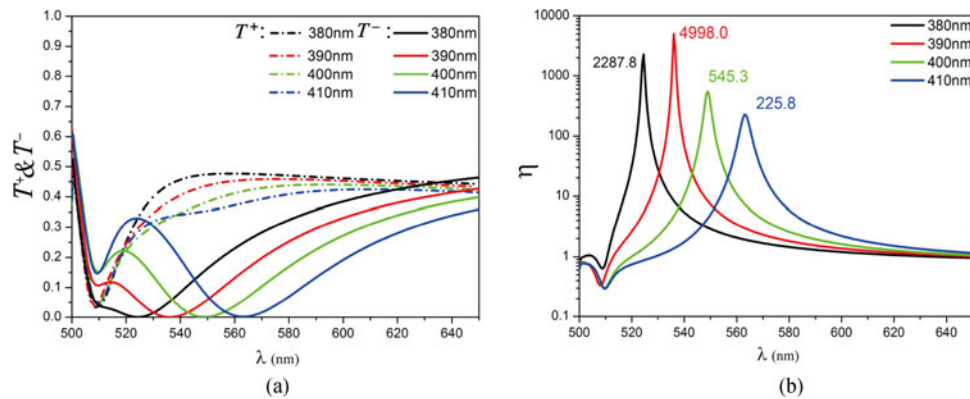


Fig. 8. When  $R-r = 98$  nm, the influence of  $R$  on (a)  $T^+$ ,  $T^-$  and (b)  $\eta$ . Here,  $\Lambda = 1000$  nm,  $d = 80$  nm and  $R = 380$  nm,  $390$  nm,  $400$  nm and  $410$  nm, respectively.

100 nm. Fig. 5(a) shows  $d$  has a significant influence on  $T^-$ . A dip in  $T^-$  curves has obvious blue shift along with the increase of  $d$ . The blue shift of the dip in  $T^-$  results to blue shift of the peak in  $\eta$ . In Fig. 5(b), the maximum of  $\eta$  increases non-monotonically along with the increase of  $d$  and is about 820 at  $\lambda = 539$  nm when  $d$  is 80 nm.

The influence of  $r$  on UOT of a SMG with  $R = 350$  nm,  $\Lambda = 1000$  nm and  $d = 80$  nm is shown in Fig. 6.  $T^-$  changes greatly with  $r$ . In Fig. 6(a), the dip in  $T^-$  curves has obvious red shift along with the increase of  $r$ . By comparing Figs. 6(a) with 5(a), the influences of increasing  $r$  and decreasing  $d$  on  $T^-$  are similar. The red shift of the dip in  $T^-$  curves, shown in Fig. 6(a), results to red shift of the peak of  $\eta$ , shown in Fig. 6(b). The maximum of  $\eta$  reaches 1567.0 at  $\lambda = 529$  nm when  $r$  is 292 nm while  $T^-$  reaches the minimum  $3.4 \times 10^{-14}$ .

Fig. 7 shows the influence of  $R$  on the performance of a SMG with  $r = 292$  nm,  $\Lambda = 1000$  nm and  $d = 80$  nm. In Fig. 7(a), the wavelength of the dip in  $T^-$  curves has a small change around 530 nm and  $T^+$  has an obvious change along with the increase of  $R$ . The influences of  $R$  on the dip position in  $T^-$  curves is much less than the influence of  $d$  and  $r$ .  $T^+$  depends more on  $R$  and  $d$  than  $r$ . The dip position in  $T^-$  curves determines the peak position of  $\eta$ . In Fig. 7(b), the maximum of  $\eta$  reaches about 5000 due to  $T^- = 0.77 \times 10^{-4}$  and  $T^+ = 0.39$ , at  $\lambda = 536$  nm when  $R$  is 390 nm and  $R-r = 98$  nm.

When  $R-r = 98$  nm, the influence of  $R$  on the UOT of a SMG with  $\Lambda = 1000$  nm and  $d = 80$  nm is shown in Fig. 8. Fig. 8(a) shows that wavelength of the dip in  $T^-$  curves red shifts and the maximum

of  $T^+$  decreases along with the increase of  $R$ . In Fig. 8(b), the peak position of  $\eta$  red shifts with  $R$  the increases. However, the maximum of  $\eta$  doesn't gets larger than 5000. Seen from Figs. 5–8, the size of inner arc and the wavelength of the dip in  $T^-$  curves have positive correlation, e.g., LSPs resonance on inner arc of CRs count for UOT. This confirms Fig. 4(b). Our investigation shows that the UOT of a SMG results from non-reciprocal diffraction and non-reciprocal diffraction efficiency is influenced by the structure size of a CR. In essence, the structure size influences LSPs resonance on CRs.

## 4. Conclusions

In this paper, a single-layer metallic grating consisting of cambered resonators is proposed to realize unidirectional optical transmission. It is the interference of electromagnetic radiation from local surface plasmons on the cambered resonators that makes diffraction enhanced or suppressed, leading to unidirectional optical transmission. In our research, the transmittance contrast ratio of this single-layer metallic grating can reach about 5000 with a high unidirectional transmittance of 39%. The characteristic of single layer and the good performance of unidirectional optical transmission indicate that the single-layer metallic grating is a potential candidate for unidirectional optical transmission devices.

## Acknowledgment

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