Lasing With Resonant Feedback in Weakly Modulated Parity-Time Symmetric Lattices

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Abstract: We report on the lasing action in weakly modulated parity-time (PT) symmetric lattices. At the exceptional point in PT-symmetric lattices, the amplitude and phase of unidirectionally scattered waves are independently and flexibly determined by the PT modulations, thus, providing a distinctive route to construct a lasing mode in finite systems with weakly modulated structures. It is revealed that the resonant feedback mechanism plays a crucial role in building up the lasing action. Our finding would enrich the singularity dynamics in the weakly modulated PT systems.

Index Terms: Parity-time symmetry, lasing, resonant feedback.

1. Introduction

Lasing in weakly modulated media has long been considered impossible. However, Lethokhov et al. predicted the existence of laser-like emission from the weakly modulated random media through non-resonant intensity feedback [1], where some lucky photons with a very long flight time in the diffusive scattering media play a crucial role in drastically amplified spontaneous emission [2]. Later on, another type of laser-like emission from such media is proposed as lasing with resonant field feedback [3]. Since the phase of radiation field is coherent, these lasing modes are featured with narrow spectral lines on noisy fluorescence background. Those findings afterwards lead to the rising of random lasers, where the lasing modes basically follow the longest light paths populated in the system [4]–[10]. However, comparing with conventional lasers, random lasers are still limited by some drawbacks, such as a relatively low quality (Q) factor and rare chance to excite them [4].

In this paper, we study parity-time (PT) symmetric optical lattices in the weak-scattering regime. The topic of PT symmetry in optics has been rapidly developed during the past decade [11]–[20]. In this work, we present strict analytical calculations and numerical simulations which demonstrate an unusual type of lasing mode with a resonant feedback. The physics involves the nearly decoupled dependence of phase and amplitude of one-way scattered waves on PT modulations, when the open system is operating in the vicinity of an exceptional point (EP). Here the EP is defined by the degenerated point at which the eigenvalues of the scattering matrix are the same [11], [12]. Therefore, the reported lasing action can be well determined and controlled by the judiciously
designed PT modulations. In stark contrast to random lasers, the lasing modes in PT-symmetric structures are featured with extremely high Q factor, thanks to the theoretically infinite long trip and avalanche amplification of the triggered photons. Our finding provides an original and efficient way to accessing the high Q lasing action in weakly modulated PT-symmetric optical lattices, which may have potential applications on the aspect of single-mode low-threshold lasers.

An optical lattice is called PT-symmetric, when the complex index of refraction takes the form of $n(x) = n^*(-x)$ [11], [12]. Here we study a multilayered PT-symmetric structure in Fig. 1, where the periodic modulation of the complex refractive index $\delta n$ can be expressed into the summation of Fourier series

\[
\delta n = \frac{-4 \Delta n}{\pi} \sum_{m=1}^{\infty} \frac{i^{2m-1}}{2m-1} \exp \left( \frac{i^{2m-1} 2\pi (2m-1)}{L} \left(x + \frac{\Phi L}{2\pi}\right) \right),
\]

and $\Delta n$, $\Phi$ and $L$ are the amplitude, initial phase and period of PT modulation, respectively. In this study, the refractive index of the background medium $n = 3$, the period of PT modulation $L = 200 \text{ nm}$, and the wavelength of light $\lambda = 1.2 \mu \text{m}$. The amplitude of PT modulation $\Delta n < 0.003$, indicating that the PT-symmetric lattice is weakly modulated and realizable in the current technology [13]. The initial phase $\Phi$ is set in the range from 0 to $2\pi$. For example, $\Phi$ takes the value of 0 and $0.5\pi$ in Fig. 1(a) and (b), respectively.

In the weak modulation regime, light propagation can be well described by the coupled-mode theory [14]. As revealed in (1), the PT-symmetric lattice is actually a complex grating that provides a unidirectional modulation vector ($|\beta| = 2\pi/L$), pointing to the $+x$ direction. Therefore, the phase-matching condition is only satisfied in one direction but not the other. When the light is incident from one side, the total electric field can be expressed by the sum of incident and reflected fields

\[
E_z = E_{\text{in}}(x) \exp (ik_1 x) + E_{\text{re}}(x) \exp (ik_2 x),
\]

where $E_{\text{in}}$, $E_{\text{re}}$ are the electric field amplitudes of incident and reflected waves, and $k_1$, $k_2$ represent the wave vectors of incident and reflected waves, respectively. It should be pointed out that the electric field amplitudes of incident and reflected waves are varied along $x$ axis, given that the distribution of refractive index is inhomogeneous. We define a phase matching factor $\gamma = \beta + k_1 - k_2$. For the backward propagation with $k_1 = -2\pi n/\lambda$ and $k_2 = 2\pi n/\lambda$, the phase-matching condition is perfectly satisfied by $\gamma = 0$. In such case, the coupled-mode equation is reduced to [15]

\[
\begin{bmatrix}
\frac{dE_{\text{in}}}{dx} \\
\frac{dE_{\text{re}}}{dx}
\end{bmatrix} = ie^{i\Phi} \begin{bmatrix}
-\frac{\omega \Delta n}{2\pi c} \exp (i\beta x) & -\frac{\omega \Delta n}{2\pi c} \exp (2i\beta x) \\
\frac{\omega \Delta n}{2\pi c} & \frac{\omega \Delta n}{2\pi c} \exp (i\beta x)
\end{bmatrix} \begin{bmatrix}
E_{\text{in}} \\
E_{\text{re}}
\end{bmatrix},
\]

where $c$ is the speed of light in free space, $\omega$ the angular frequency, and $\kappa$ the coefficient related to the coupling strength. Here, the coupled-mode equation (3) is deduced by substituting (2) into the Maxwell’s equations, which quantitatively describes the energy conversion between the forward propagating and backward propagating wave fields. The conversion is actually caused by the reflection in the inhomogeneous dielectric medium. We note that (3), although it is defined for an infinitely long grating, can be applied to our finite-length structure on condition that the number of period $N$ is large. In (3), the fast oscillating terms $\exp (i\beta x)$ and $\exp (2i\beta x)$ are integrated to be zero,
Fig. 2. (a) Dependence of reflection phase on the parameters $\Phi$ and $\Delta n$ in PT modulations. The horizontal dashed lines correspond to $\Phi_1 \approx 0.1\pi$ and $\Phi_2 = 0.9\pi$. (b) Dependence of reflection amplitude on $\Phi$ and $\Delta n$ in PT modulations. The vertical dashed lines respectively correspond to $\Delta n_1 \approx 2.133 \times 10^{-3}$ and $\Delta n_2 = 2.930 \times 10^{-3}$.

Thus giving rise to

$$
\frac{dE_{\text{in}}}{dx} = 0, \quad \frac{dE_{\text{re}}}{dx} = i e^{\frac{\Phi_1 \Delta n}{2nc}} E_{\text{in}}. 
$$

However, for the forward propagation of light with $k_1 = 2\pi n/\lambda$ and $k_2 = -2\pi n/\lambda$, the phase matching factor $\gamma \neq 0$. We thus have $dE_{\text{in}}/dx = 0$ and $dE_{\text{re}}/dx = 0$. Previous works have shown that unidirectional phase matching, as a characteristic singularity effect, occurs only at the EPs in the PT-symmetric systems. As indicated by (4), it is interesting to find out that the amplitude and phase of backward reflection are related to $\Delta n$ and $\Phi$ of PT modulation respectively at the EPs, where the amplitude of reflection $E_{\text{re}} \propto \Delta n$ and the phase of reflection $\varphi_{\text{re}} = \Phi + 0.5\pi$. From (4), we find that $E_{\text{in}}$ remains unchanged during the propagation ($dE_{\text{in}}/dx = 0$). Therefore, the transmission through the PT-symmetric potential should be unitary ($E_{\text{in}}|_{x = L}/E_{\text{in}}|_{x = 0} = 1$). Such unitary transmission can also be interpreted by the fact that one-way reflectionless photons at the EPs will experience balanced loss and gain in the PT-symmetric potential.

Finite element simulations were then conducted to validate the relation of backward reflection with PT modulation. In simulations, the number of period of PT modulation $N = 300$, and the total length of PT modulation is $60 \mu m$. For normally incident plane waves, the electric field component has a unitary amplitude and an initial phase of zero. The pseudo-color map plotted in Fig. 2(a) presents the relation between the reflection phase $\varphi_{\text{re}}$ and the parameters $\Phi$, $\Delta n$ of PT modulation, while Fig. 2(b) displays the reflection amplitude response $E_{\text{re}}$ to those two parameters. The simulation results unequivocally show that $\varphi_{\text{re}}$ and $E_{\text{re}}$ can cover the ranges of $(0.5\pi, 2.5\pi)$ and $(\sim 0.8, \sim 1.2)$, when $\Phi$ and $\Delta n$ are tuned in $(0, 2\pi)$ and $(0.002, 0.003)$, respectively. The ergodicity in phase-amplitude space enables us to design two PT-symmetric lattices 1 and 2, where the phases and amplitudes of reflection simultaneously satisfy the relations

$$
\varphi_{\text{re}1} + \varphi_{\text{re}2} = 2m\pi, \quad E_{\text{re}1}E_{\text{re}2} = 1. 
$$

To be specific, we first set the initial phases of PT modulation in lattices 1 and 2 to be $\Phi_1 \approx 0.1\pi$ and $\Phi_2 = 0.9\pi$, as marked by the horizontal dashed lines in Fig. 2(a). The numerical simulation suggests that $\varphi_{\text{re}1} + \varphi_{\text{re}2} = 2\pi$, in a good agreement with (4). In the next step, we set the amplitudes of PT modulation in lattices 1 and 2 to be $\Delta n_1 \approx 2.133 \times 10^{-3}$ and $\Delta n_2 = 2.930 \times 10^{-3}$, as marked by the vertical dashed lines in Fig. 2(b). Without altering the preset phases of reflection, the reflection amplitudes obey the relation $E_{\text{re}1}E_{\text{re}2} = 1$. In Fig. 2(b), we put the cross points of horizontal and vertical dashed lines into two categories, viz., the black triangle/square and the white triangle/square. It is worthy to be mentioned that (5) still holds for the PT-symmetric lattices with parameters $\Phi_3 \approx 0.1\pi$, $\Delta n_3 = 2.930 \times 10^{-3}$ (the black triangle) and $\Phi_4 = 0.9\pi$, $\Delta n_4 \approx 2.133 \times 10^{-3}$.
Fig. 3. (a) Schematic of a lasing mode formed by the resonant feedback at the interface of PT-symmetric lattices 1 and 2. The reflections $r_{R(L)}$ and $r_{1(2)}$ are marked by the arrows. (b) Absolute left-side reflection coefficient $|r_1|$ of the composite lattice 1 + 2 vs. $\Phi_1$ and $\Delta n_1$ in lattice 1. (c) Absolute right-side reflection coefficient $|r_2|$ of the composite lattice 1 + 2 vs. $\Phi_1$ and $\Delta n_1$ in lattice 1. (d) Absolute transmission coefficient $|t|$ of the composite lattice 1 + 2 vs. $\Phi_1$ and $\Delta n_1$ in lattice 1. Here, the parameters $\Phi_2$ and $\Delta n_2$ in lattice 2 are set to be 0.9$\pi$ and 2.930 $\times$ 10$^{-3}$, respectively.

As schematically shown in Fig. 3(a), it is possible to build up a very strong localization of light at the interface of the PT-symmetric lattices 1 and 2, according to (5), by setting $\phi_{R(L)} = \phi_{1(2)}$ and $r_{R(L)} = E_{1(2)}$. In this case, the reflection components of PT lattices will lead to a resonant feedback, thus forming self-sustaining interface states with the Q factor up to infinity. Although similar in mathematics, it should be emphasized that the interface state in the composite lattice 1 + 2 is in stark contrast to cavity modes, viz., standing waves in a lossless cavity with unitary reflection mirrors. Note that the designed PT-symmetric lattice is weakly modulated ($\Delta n < 0.003$). To give a demonstration, we first consider a lossless dielectric multilayered medium with the modulation strength $\Delta n = 0.003$ and the number of period $N = 300$. The calculated reflection of light is less than 30% near the Bragg condition ($\sim$1.2 $\mu$m), indicating that the lossless multilayered structure is almost transparent. Then we study the scattering property of the composite lattice 1 + 2 to show that unlike lossless cavity modes that are ideally confined without energy dissipation or radiation, the resonant interface state is itself a lasing mode. Suppose a normally incident plane wave of unitary amplitude is launched from the left-side of the composite lattice 1 + 2. By employing (5), we can qualitatively deduce that the reflection coefficient $|r_1| = m|r_L|$ and the transmission coefficient $|t| = m$ by summing up all the reflected and transmitted parts. Since $m$ stays infinite at resonant feedback, we will obtain $|r_1| \rightarrow \infty$ and $|t| \rightarrow \infty$. In a similar way, $|r_2| \rightarrow \infty$ can be derived at the resonant feedback, where the plane wave is incident from the right-side. In Fig. 3(b)–(d), we employ the transfer matrix method [21]–[23] to rigorously map out the relations of the absolute left-side reflection coefficient $|r_1|$, absolute right-side reflection coefficient $|r_2|$, and absolute transmission coefficient $|t|$ with respect to the material parameters $\Phi_1$ and $\Delta n_1$ in lattice 1, respectively, where $\Phi_2$ and $\Delta n_2$ in lattice 2 are set to be 0.9$\pi$ and 2.930 $\times$ 10$^{-3}$. The results clearly show that there exists a remarkably sharp peak (or a lasing mode) at $\Phi_1 \approx 0.1\pi$ and $\Delta n_1 \approx 2.133 \times 10^{-3}$. In the
in numerical simulation, for the lattices 1 and 2, the periodic modulations of the complex refractive indices $\delta n_1$ and $\delta n_2$ can be expressed into

$$
\delta n_1 \approx -\frac{8.532 \times 10^{-3}}{\pi} \sum_{m=1}^{\infty} \frac{j^{2m-1}}{2m-1} \exp \left[ j^{2m-1} \frac{\pi (2m-1)}{100} (x + 10) \right],
$$

$$
\delta n_2 \approx -\frac{1.172 \times 10^{-2}}{\pi} \sum_{m=1}^{\infty} \frac{j^{2m-1}}{2m-1} \exp \left[ j^{2m-1} \frac{\pi (2m-1)}{100} (x + 90) \right],
$$

where the variable $x$ takes the unit of nm.

In this paper, the lasing stems from a well localization of light, where even one photon will trigger avalanche amplification during its infinite long flight at the resonant feedback, eventually much lowering the lasing threshold in comparison with traditional DFB lasers [24]–[27]. Also the lasing only occurs at the EP, while the relations in (5) must be simultaneously satisfied. Therefore, the proposed PT laser is a purposely constructed lasing action with an intrinsic single-mode property. For the composite lattice 1 + 2, the singular property of scattering also can be revealed from the sum of absolute eigenvalues $|\lambda_1| + |\lambda_2|$ of the scattering matrix (S-matrix) versus $\Phi_1$ and $\Delta n_1$ in lattice 1 [see Fig. 4(a)], where eigenvalues of the S-matrix are determined by [28]

$$
\lambda_{1,2} = t \pm i \sqrt{t_1 t_2}.
$$

The bright speckle, marked by the arrow in Fig. 4(a), corresponds to a singularity point of the S-matrix at the same position $\Phi_1 \approx 0.1 \pi$ and $\Delta n_1 \approx 2.133 \times 10^{-3}$ in Fig. 3(b)–(d). To study the lasing action in more details, we numerically calculated the normalized intensity field distribution of the predicted lasing mode in Fig. 4(b). The result shows that the lasing mode is indeed localized at the interface of the PT-symmetric lattices 1 and 2, while lasing in both directions.

In conclusion, we have demonstrated a distinct lasing action in weakly modulated PT-symmetric optical lattices. Our results show that the amplitude and phase of one-way scattered waves can be engineered in a decoupled way by a generalized PT potential at the EPs, which offers great material design flexibility and additional freedoms for novel lasing devices. In our work, the PT potentials may not necessarily be the same in the composite structure and are not limited to sinusoidal modulations. In practice, the weakly modulated multi-layered PT-symmetric potential as well as the decoupled manipulation of the reflection amplitude and phase at the EPs will greatly favor the experimental realization of a well-determined lasing. For example, to achieve resonant feedback or lasing action, we can first construct two PT-symmetric lattices with the preset reflection phases satisfying (5) and modulation intensity tunable by another factor. Then we can further engineer the modulation intensity to ensure the reflection amplitudes also obey the relation of (5), without
affecting the preset reflection phases. The current work extends the traditional laser paradigm into a new avenue for the development of single-mode low-threshold lasers.

References